efforts done above, ridiculous owing to the fact that simply pressing the 'exp' key and then 1, one can get value of e to larger number of significant figures.

Acknowledgement

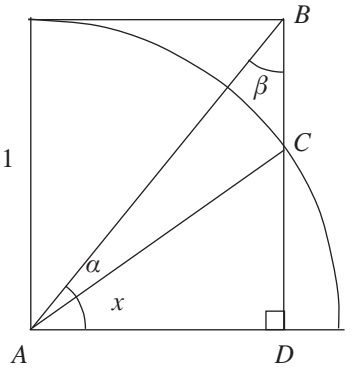
We are highly grateful to anonymous referee for his/her extremely useful suggestions to improve the clarity of the content.

10.1017/mag.2025.10087 © The Authors, 2025
Published by Cambridge University Press
on behalf of The Mathematical Association

K. B. SUBRAMANIAM
'Ganit Ashram'
135, Fine Avenue Phase 1
Nayapura, Kolar Road
Bhopal, INDIA 462042
e-mail: *kbsubramaniam.1950@gmail.com*
AMARNATH MURTHY
503, Sharan Residency
C.G Road
Chand Kheda
Ahmedabad, INDIA 382424
e-mail: *amarnath.1960@gmail.com*

109.22 A constant function

This Note was inspired by my attempt in a Calculus I course at the University of Zanjan to find a "proof without words simplification" of the composed quantity $\tan^{-1}(\cos x)$. After some effort with students, we generated the following diagram, including the quantity under question as the angle β.



https://doi.org/10.1017/mag.2025.10088 Published online by Cambridge University Press

Although, the above diagram gives no simplification of $\tan^{-1}(\cos x)$, but based on it we could to obtain a "proof without words" of the rather complicated identity

$$x + \tan^{-1}(\cos x) + \sin^{-1}\left(\frac{(1 - \sin x)\cos x}{\sqrt{1 + \cos^2 x}}\right) = \frac{\pi}{2} \qquad \left(0 < x < \frac{\pi}{2}\right).$$

For, we compute the area of the triangle ΔABC in two different ways. First, from the most common method, we get

$$2 \times \text{area}(\Delta ABC) = |BC||AD| = (1 - \sin x)\cos x.$$

Then, based the fact that the area of a given triangle is equal to half of product of two arbitrary sides into the sinus of the angle between them, we deduce that

$$2 \times \text{area}(\Delta ABC) = |AB||AC| \sin\alpha = \sqrt{1 + \cos^2 x}\,\sin\alpha.$$

Therefore, $\sqrt{1+\cos^2 x}\sin\alpha = (1-\sin x)\cos x$. Also, we have $\tan\beta = \cos x$. Since $x + \alpha + \beta = \frac{\pi}{2}$, we obtain the above identity.

10.1017/mag.2025.10088 © The Authors, 2025
Published by Cambridge University Press
on behalf of The Mathematical Association

MEHDI HASSANI
Department of Mathematics,
University of Zanjan,
University Blvd., 45371-38791,
Zanjan, Iran
e-mail: *mehdi.hassani@znu.ac.ir*

109.23 The derivative of the integral, redux

Let $f : [a, b] \to \mathbb{R}$ be a continuous function. According to the first fundamental theorem of calculus,

$$\frac{d}{dx}\left(\int_a^x f(t)\,dt\right)\Big|_{x=x_0} = f(x_0) \text{ for all } x_0 \in (a, b). \tag{1}$$

According to the second fundamental theorem of calculus, (1) still holds, assuming that f is Riemann integrable (only) and admits an antiderivative on $[a, b]$. However, there are many discontinuous functions that are Riemann integrable but lack any antiderivatives at all; and many functions that have antiderivatives but are not Riemann integrable, cf. [1, §3 and §4].

The following result shows that one can still compute the left-hand side in (1), even if f does not satisfy the hypotheses of the first and second fundamental theorem of calculus.

Proposition: If $f : [a, b] \to \mathbb{R}$ is Riemann integrable on $[a, b]$ and has a limit at a point $x_0 \in (a, b)$, say $L(x_0)$, then

$$\frac{d}{dx}\left(\int_a^x f(t)\,dt\right)\Big|_{x=x_0} = L(x_0). \tag{2}$$

https://doi.org/10.1017/mag.2025.10088 Published online by Cambridge University Press