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Single-integral expressions for electron cyclotron damping, relativistically and with an arbitrary distribution function

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The relativistic expressions for the dielectric-tensor elements can be expressed as a double integral over both parallel and perpendicular momenta. The anti-Hermitian (damping) parts can be integrated over a delta function for either one of these two momenta, leaving a single integral over the other momentum variable. We review these expressions for either integration option, and compare them with statements in the literature. We give the limits on the remaining integral explicitly, for all positive or negative values of the parallel refractive index, whose magnitude can be less or greater than one. This single-integral form can be applied to electron cyclotron harmonic damping for any distorted electron momentum distribution function. As an example, we recover the results for a Maxwellian distribution. We also provide approximate results when the distribution function can be taken as a Maxwellian only in the perpendicular-momentum direction. A computer program will be written for a distribution function subjected to distortion by both electron cyclotron and lower-hybrid waves.

1. Introduction

Tokamak machines for fusion energy will require electromagnetic (EM) power to heat and drive current in the plasma. Such EM sources include electron cyclotron (EC) and lower-hybrid (LH) waves. These two EM sources are used for example in the experimental Tokamak de Varennes (TdeV) machine (Côté et al. 1998). Lower-hybrid current-drive (LHCD) systems distort the electron distribution function to yield the characteristic plateau region in the tail of the distribution. Upon combining the LHCD and EC systems, the distribution function is further distorted in both the parallel- and perpendicular-momentum directions. Figure 1 shows an example for TdeV of computed contours in momentum space of a highly distorted non-Maxwellian electron distribution, subjected to both LH and EC high-power waves ($\omega \approx 2\omega_c$), including trapping effects (Shoucri et al. 1997).

For the EC wave, we apply a ray-tracing code (Le Clair et al. 1997), which uses a relativistic formulation for both the real and imaginary parts of the dielectric tensor elements, corresponding to ray tracing and absorption. The dielectric-tensor elements are calculated for the mildly relativistic case using a Maxwellian distribution in momentum space involving the Shkarofsky (1966,

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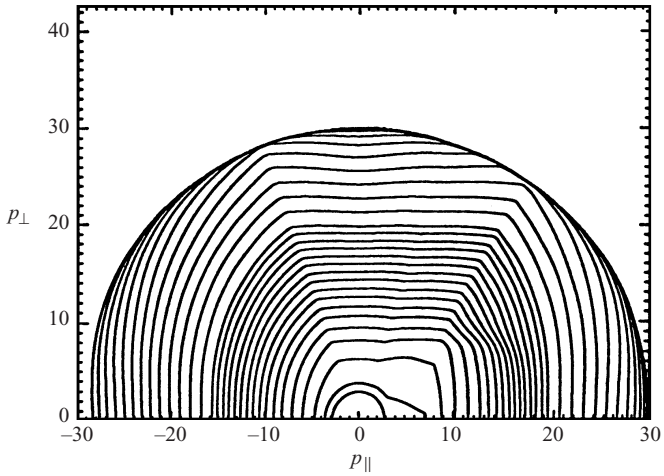


Figure 1. Contour plot of the distribution function for combined electron cyclotron and lower-hybrid power inputs, as derived from a Fokker–Planck calculation in Shoucri et al. (1997).

1986) functions. As can be seen from Fig. 1, the use of a Maxwellian distribution may be inaccurate, especially for the calculation of damping, which can be affected by the form of the distribution function outside the bulk.

Here we review the expressions for the anti-Hermitian parts of the dielectric-tensor elements, which provide the damping of an EC wave whose frequency is close to an nth harmonic of the cyclotron resonance frequency for arbitrary distribution functions.

2. General expressions

The relativistically correct expressions for the dielectric-tensor elements ε_{ij} are given for example in Befeki (1966):

$$\boldsymbol{\epsilon} = \mathbf{I} + 2\pi \frac{\omega_p^2}{\omega^2} \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{0}^{\infty} dz_\parallel \, dz_\perp \, z_\perp^2 \, \mathbf{V} U \Big/ \left(1 - \frac{n_\parallel z_\parallel}{\gamma} - \frac{n\omega_c}{\gamma\omega}\right), \tag{1}$$

where

$$z_{\parallel,\perp} \equiv \frac{p_{\parallel,\perp}}{mc}, \qquad n_{\parallel,\perp} \equiv \frac{k_{\parallel,\perp} c}{\omega}, \qquad \gamma^2 = 1 + z_\parallel^2 + z_\perp^2,$$

$$\begin{aligned} V_{11} &= V_1 V_1, & V_{12} &= -V_1 V_2 & V_{13} &= a V_1 V_3, \\ V_{21} &= V_2 V_1, & V_{22} &= -V_2 V_2, & V_{23} &= a V_2 V_3, \\ V_{31} &= V_3 V_1, & V_{32} &= -V_3 V_2, & V_{33} &= a V_3 V_3 \end{aligned}$$

$$V_1 = \frac{n J_n(x)}{x}, \qquad V_2 = \frac{i d J_n(x)}{dx}, \qquad V_3 = \frac{z_\parallel}{z_\perp} J_n(x), \tag{2}$$

$$a = \frac{z_\perp W}{z_\parallel U}, \tag{3a}$$

$$x = \frac{n_\perp z_\perp \omega}{\omega_c}, \tag{3b}$$

$$U \equiv \gamma \frac{\partial f}{\partial z_\perp} + n_\parallel \left(z_\perp \frac{\partial f}{\partial z_\parallel} - z_\parallel \frac{\partial f}{\partial z_\perp} \right), \tag{4a}$$

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$$W \equiv \gamma \frac{\partial f}{\partial z_\parallel} - \frac{n\omega_c}{\omega z_\perp}\left(z_\perp \frac{\partial f}{\partial z_\parallel} - z_\parallel \frac{\partial f}{\partial z_\perp}\right). \tag{4b}$$

Here $f = f(z_\parallel, z_\perp)$ is the electron distribution, and is normalized so that $f = f_{\text{actual}}(mc)^3$ and $2\pi \int f d^2z = 1$.

The damping arises from the resonance in (1), namely at

$$\gamma^{\text{res}} = n_\parallel z_\parallel + \frac{n\omega_c}{\omega}. \tag{5}$$

Then in (3a), $a = 1$ and

$$U^{\text{res}} = \left(\frac{z_\perp}{z_\parallel} W\right)^{\text{res}} = \left(\frac{n\omega_c}{\omega}\frac{\partial f}{\partial z_\perp} + n_\parallel z_\perp \frac{\partial f}{\partial z_\parallel}\right)^{\text{res}}. \tag{6}$$

We have the option of first integrating over either the $z_\perp$ or the $z_\parallel$ direction. The two options are done respectively in Secs 3 and 4.

3. Integration first over the perpendicular direction

We are interested in damping close to an nth cyclotron harmonic $\omega \approx n\omega_c$, with $n \geqslant 1$. All the expressions below for the anti-Hermitian dielectric-tensor elements ε^a_{ij} are to be understood to be summed over all such positive n values of concern, although we focus our attention on a specific n.

3.1. Case of $1 > n_\parallel > 0$ *or* $-1 < n_\parallel \leqslant 0$

Solving (5) at the resonance singularity for $z_\perp$ yields

$$z_\perp^{\text{res}}(z_\parallel) = [(1 - n_\parallel^2)(z_\parallel - z_\parallel^-)(z_\parallel^+ - z_\parallel)]^{1/2}, \tag{7}$$

where

$$z_\parallel^\pm \equiv \frac{n_\parallel}{1 - n_\parallel^2}\frac{n\omega_c}{\omega} \pm \frac{(n_\parallel^2 - 1 + n^2\omega_c^2/\omega^2)^{1/2}}{|1 - n_\parallel^2|}. \tag{8}$$

We have $z_\parallel^- < z_\parallel < z_\parallel^+$ and $n_\parallel^2 - 1 + n^2\omega_c^2/\omega^2 \geqslant 0$. Then the anti-Hermitian dielectric-tensor elements become equal to

$$\boldsymbol{\epsilon}^a = -i2\pi^2 \frac{\omega_p^2}{\omega^2} \int_{z_\parallel^-}^{z_\parallel^+} dz_\parallel \, z_\perp^{\text{res}} [\mathbf{V}^{\text{res}} U^{\text{res}}]_{z_\perp = z_\perp^{\text{res}}}. \tag{9}$$

Here U^{res} is given in (6), and the elements of $\mathbf{V}^{\text{res}}$ are obtained from (2) and (3a) with $a = 1$, both evaluated at the value of $z_\perp = z_\perp^{\text{res}}$ shown in (7).

3.2. Case of $n_\parallel > 1$ *or* $n_\parallel < -1$

The resonance singularity in (5) now occurs at

$$z_\perp^{\text{res}}(z_\parallel) = [(n_\parallel^2 - 1)(z_\parallel - z_\parallel^-)(z_\parallel - z_\parallel^+)]^{1/2}, \tag{10}$$

where

$$z_\parallel^\pm \equiv -\frac{n_\parallel}{n_\parallel^2 - 1}\frac{n\omega_c}{\omega} \pm \frac{(n_\parallel^2 - 1 + n^2\omega_c^2/\omega^2)^{1/2}}{|n_\parallel^2 - 1|}. \tag{11}$$

Note that (10) is the same as (7) and that (11) is the same as (8). We have $n_\parallel^2 - 1 + n^2\omega_c^2/\omega^2 \geqslant 0$. Now the anti-Hermitian dielectric-tensor elements, for $n_\parallel > 1$ when $z_\parallel^- < z_\parallel^+ \leqslant z_\parallel$, are given by

$$\boldsymbol{\epsilon}^a = -i2\pi^2 \frac{\omega_p^2}{\omega^2} \int_{z_\parallel^+}^{\infty} dz_\parallel \, z_\perp^{\text{res}} [\mathbf{V}^{\text{res}} U^{\text{res}}]_{z_\perp = z_\perp^{\text{res}}}, \tag{12a}$$

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whereas, for $n_\parallel < -1$ when $z_\parallel < z_\parallel^- < z_\parallel^+$, we have instead

$$\boldsymbol{\epsilon}^a = -i2\pi^2 \frac{\omega_p^2}{\omega^2} \int_{-\infty}^{z_\parallel^-} dz_\parallel \, z_\perp^{\rm res} [\mathbf{V}^{\rm res} U^{\rm res}]_{z_\perp = z_\perp^{\rm res}}. \tag{12b}$$

4. Integration first over the parallel direction

4.1. Case of $1 > n_\parallel > 0$ *or* $-1 < n_\parallel \leqslant 0$

Solving (5) at the resonance singularity for $z_\parallel$ yields the following two values, say $z_a^{\rm res}$ and $z_b^{\rm res}$:

$$z_{a,b}^{\rm res}(z_\perp) \equiv \frac{n_\parallel}{|1-n_\parallel^2|} \frac{n\omega_c}{\omega} \pm \frac{[(1-n_\parallel^2)\,(z_{\perp\max}^2 - z_\perp^2)]^{1/2}}{|1-n_\parallel^2|}, \tag{13}$$

where

$$z_{\perp\max}^2 = \frac{(n\omega_c/\omega)^2}{1-n_\parallel^2} - 1, \tag{14}$$

with $z_\perp \leqslant z_{\perp\max}$ and $z_{\perp\max} > 0$ or $n_\parallel^2 - 1 + n^2\omega_c^2/\omega^2 \geqslant 0$. Then the anti-Hermitian dielectric-tensor elements are given by

$$\boldsymbol{\epsilon}^a = -i2\pi^2 \frac{\omega_p^2}{\omega^2} \int_0^{z_{\perp\max}} \frac{dz_\perp \, z_\perp^2}{[(1-n_\parallel^2)\,(z_{\perp\max}^2 - z_\perp^2)]^{1/2}} \{[\mathbf{V}^{\rm res} U^{\rm res}]_{z_\parallel = z_a^{\rm res}} + [\mathbf{V}^{\rm res} U^{\rm res}]_{z_\parallel = z_b^{\rm res}}\}. \tag{15}$$

Here $U^{\rm res}$ is given by (6), and the matrix elements of $\mathbf{V}^{\rm res}$ are obtained from (2) and (3a) with $a = 1$. Both are evaluated, and their product is summed over the two values of $z_\parallel$ shown in (13).

4.2. Case of $n_\parallel > 1$ *or* $n_\parallel < -1$

The resonance singularity in (5) now occurs at

$$z_{a,b}^{\rm res}(z_\perp) \equiv -\frac{n_\parallel}{n_\parallel^2 - 1} \frac{n\omega_c}{\omega} \pm \frac{[(n_\parallel^2 - 1)\,(z_k^2 + z_\perp^2)]^{1/2}}{n_\parallel^2 - 1}, \tag{16}$$

where

$$z_k^2 = \frac{(n\omega_c/\omega)^2}{n_\parallel^2 - 1} + 1. \tag{17}$$

Then the anti-Hermitian dielectric-tensor elements for $n_\parallel > 1$ are

$$\boldsymbol{\epsilon}^a = -i2\pi^2 \frac{\omega_p^2}{\omega^2} \int_0^\infty \frac{dz_\perp \, z_\perp^2}{[(n_\parallel^2 - 1)\,(z_k^2 + z_\perp^2)]^{1/2}} [\mathbf{V}^{\rm res} U^{\rm res}]_{z_\parallel = z_a^{\rm res}}, \tag{18a}$$

whereas for $n_\parallel < -1$ they are

$$\boldsymbol{\epsilon}^a = -i2\pi^2 \frac{\omega_p^2}{\omega^2} \int_0^\infty \frac{dz_\perp \, z_\perp^2}{[(n_\parallel^2 - 1)\,(z_k^2 + z_\perp^2)]^{1/2}} [\mathbf{V}^{\rm res} U^{\rm res}]_{z_\parallel = z_b^{\rm res}}, \tag{18b}$$

5. Expressions in the literature

Bornatici *et al.* (1990) give (9) in their equation (B 1). However instead of (12a) or (12b), they state that for $n_\parallel^2 > 1$, the $z_\parallel$ integration extends over the two intervals $(-\infty, z_\parallel^-)$ and $(z_\parallel^+, \infty)$, rather than over one of these, depending on whether $n_\parallel$ is negative or positive respectively.

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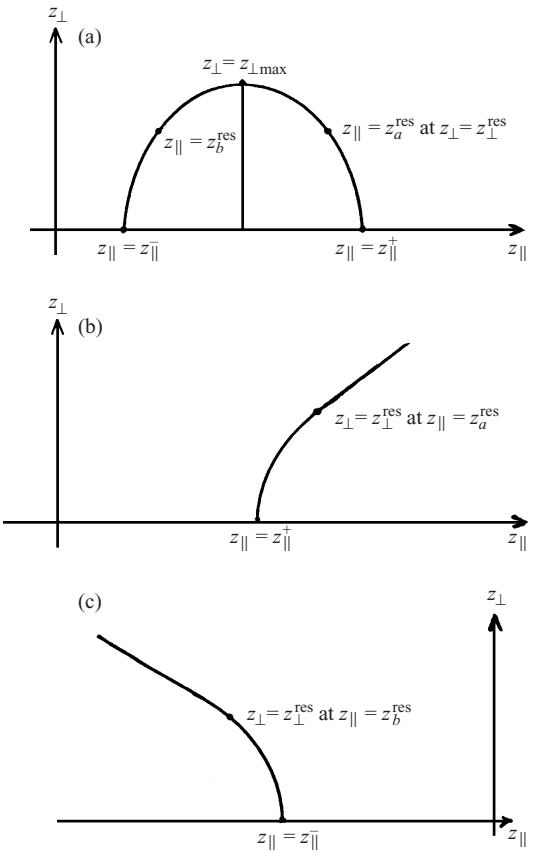


Figure 2. (a) Zeros of (5) form an ellipse in the $(z_\perp, z_\parallel)$ plane when $1 > n_\parallel > 0$ or $-1 < n_\parallel < 0$. (b) Zeros of (5) form a hyperbola in the $(z_\perp, z_\parallel)$ plane, starting from $z_\parallel^+$, when $n_\parallel > 1$. (c) Zeros of (5) form a hyperbola in the $(z_\perp, z_\parallel)$ plane, starting from $z_\parallel^-$, when $n_\parallel < -1$, so that the sign of $z_\parallel$ is the same as that of $n_\parallel$ as $|z_\parallel|$ goes to infinity.

Granata and Fidone (1991) also give (9) in their equation (3). Our expression in (15) appears after their equation (3), where they state that $z_\parallel$ is evaluated at $z_{a,b}^{\rm res}$. This agrees with our (15) if they mean that one should sum over these two $z_\parallel$ values.

Stix (1992) poses (12a) and (12b) or (18a) and (18b) as a problem, namely that only one of the two roots is to be used, in contrast to (9) or (15), where both roots are used. The different situations are illustrated here in Figs 2(a, b and c).

6. Example: a relativistic Maxwellian

The following relativistic Maxwellian distribution is taken for example, where K is a modified Hankel function, $A \equiv \mu/[4\pi K_2(\mu)]$, $\mu \equiv (c/v_t)^2$, $v_t^2 = T/m$, and γ is given below (1):

$$f(z_\perp, z_\parallel) = \frac{\mu}{4\pi K_2(\mu)} \exp(-\mu\gamma) \equiv A \exp(-\mu\gamma). \tag{19}$$

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We expand the Bessel functions in (2) by

$$J_p(x)\,J_q(x)=\sum_{k=0}^{\infty}\frac{(-1)^k x^{p+q+2k}(p+q+2k)!}{2^{p+q+2k}k!(p+q+k)!(p+k)!(q+k)!}. \tag{20a}$$

Note that Granata and Fidone (1991) expand differently. We look at the ε_{11}^a element. (See the Appendix for the results for all the elements, which can be similarly derived.) We perform both types of integration – first for the case when $1>|n_\parallel|$, and later for $|n_\parallel|>1$. At the resonance, (5) shows the value of $\gamma^{\rm res}$. Then

$$U^{\rm res}=-\mu z_\perp^{\rm res}A\exp\left[-\mu\left(n_\parallel z_\parallel+\frac{n\omega_c}{\omega}\right)\right], \tag{20b}$$

$$V_1\,V_1=\sum_{k=0}^{\infty}B(z_\perp^{2k+2n-2})^{\rm res}, \tag{20c}$$

$$B=(-1)^k\left(\frac{k_\perp c}{\omega_c}\right)^{2k+2n-2}\frac{n^2(2k+2n)!}{2^{2k+2n}k![(k+n)!]^2\,(k+2n)!}. \tag{20d}$$

First, for $n_\parallel^2<1$, (9) yields (see Gradshteyn and Ryzhik 1980, equation 3.383.2), subject to $n_\parallel^2-1+n^2\omega_c^2/\omega^2\geqslant 0$,

$$\begin{aligned}\varepsilon_{11}^a&=i2\pi^2A\left(\frac{\omega_p}{\omega}\right)^2\mu\,e^{-\mu n\omega_c/\omega}\sum_{k=0}^{\infty}B(1-n_\parallel^2)^{k+n}\int_{z_\parallel^-}^{z_\parallel^+}dz_\parallel(z_\parallel-z_\parallel^-)^{k+n}(z_\parallel^+-z_\parallel)^{k+n}e^{-\mu n_\parallel z_\parallel}\\
&=i\frac{\pi\mu n^2\omega_p^2}{\omega^2(1-n_\parallel^2)^{1/2}}\frac{(\pi/2\mu)^{1/2}}{e^\mu K_2(\mu)}\exp\left[\frac{\mu(1-n_\parallel^2-n\omega_c/\omega)}{1-n_\parallel^2}\right]\sum_{k=0}^{\infty}\frac{(-1)^k(2k+2n)!}{2^{k+n}\,k!(k+n)!(k+2n)!}\\
&\quad\times\left(\frac{k_\perp^2c^2}{\mu\omega_c^2}\right)^{k+n-1}\left(\frac{n_\parallel^2-1+n^2\omega_c^2/\omega^2}{n_\parallel^2}\right)^{(2k+2n-1)/4}I_{k+n+1/2}\left(\frac{\mu|n_\parallel|}{1-n_\parallel^2}\left(n_\parallel^2-1+\frac{n^2\omega_c^2}{\omega^2}\right)^{1/2}\right),\end{aligned} \tag{21}$$

where I is a modified Bessel function. When $n_\parallel^2\ll 1, n\omega_c/\omega\approx 1, \mu\gg 1$ and $k=0$ (that is, $(k_\perp v_t/\omega_c)^2\ll 1$), this reduces to equations (4a) and (18b) in Shkarofsky (1966).

The same result as in (21) can be also derived from (15). Here we have to integrate

$$\begin{aligned}&\varepsilon_{11}^a\Big/\left\{i2\pi^2A\left(\frac{\omega_p}{\omega}\right)^2\mu(1-n_\parallel^2)^{-1/2}\exp\left[-\frac{\mu n\omega_c}{\omega(1-n_\parallel^2)}\right]\right\}\\
&\quad=\sum_{k=0}^{\infty}B\int_0^{z_{\perp\max}}dz_\perp\frac{z_\perp^{2k+2n+1}}{(z_{\perp\max}^2-z_\perp^2)^{1/2}}\left\{\exp\left[-\mu n_\parallel\left(\frac{z_{\perp\max}^2-z_\perp^2}{1-n_\parallel^2}\right)^{1/2}\right]\right.\\
&\qquad\left.+\exp\left[\mu n_\parallel\left(\frac{z_{\perp\max}^2-z_\perp^2}{1-n_\parallel^2}\right)^{1/2}\right]\right\}\\
&\quad=\sum_{k=0}^{\infty}B\int_{-1}^{1}dt\,(1-t^2)^{k+n}z_{\perp\max}^{2k+2n+1}\exp\left[-\frac{t|n_\parallel|\mu z_{\perp\max}}{(1-n_\parallel^2)^{1/2}}\right],\end{aligned} \tag{22}$$

which again leads (see Gradshteyn and Ryzhik 1980, equation 3.387.1) to (21).

Next for $n_\parallel^2>1$, (12a) with limits on the integral from $z_\parallel^+$ to ∞, or (12b) with

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limits from $-\infty$ to $z_\parallel^-$, yields (see Gradshteyn and Ryzhik 1980, equation 3.383.3)

$$\varepsilon_{11}^a = i2\pi^2 A\left(\frac{\omega_p}{\omega}\right)^2 \mu\, e^{-\mu n\omega_c/\omega} \sum_{k=0}^{\infty} B(n_\parallel^2-1)^{k+n}\int dz_\parallel (z_\parallel - z_\parallel^-)^{k+n}(z_\parallel - z_\parallel^+)^{k+n} e^{-\mu n_\parallel z_\parallel}$$

$$= i\frac{\mu n^2\omega_p^2}{\omega^2(n_\parallel^2-1)^{1/2}}\frac{(\pi/2\mu)^{1/2}}{e^\mu K_2(\mu)}\exp\left[\frac{\mu(n_\parallel^2-1+n\omega_c/\omega)}{n_\parallel^2-1}\right]\sum_{k=0}^{\infty}\frac{(-1)^k(2k+2n)!}{2^{k+n}\,k!(k+n)!(k+2n)!}$$

$$\times\left(\frac{k_\perp^2 c^2}{\mu\omega_c^2}\right)^{k+n-1}\left(\frac{n_\parallel^2-1+n^2\omega_c^2/\omega^2}{n_\parallel^2}\right)^{(2k+2n+1)/4} K_{k+n+1/2}\left(\frac{\mu|n_\parallel|}{n_\parallel^2-1}\left(n_\parallel^2-1+\frac{n^2\omega_c^2}{\omega^2}\right)^{1/2}\right). \tag{23}$$

The same result as in (23) can be also derived from (18a) or (18b). Here we have to integrate

$$\varepsilon_{11}^a \Big/ \left\{ i2\pi^2 A\left(\frac{\omega_p}{\omega}\right)^2 \mu(n_\parallel^2-1)^{-1/2}\exp\left[\frac{\mu n\omega_c}{\omega(n_\parallel^2-1)}\right]\right\}$$

$$= \sum_{k=0}^{\infty} B\int_0^\infty dz_\perp \frac{z_\perp^{2k+2n+1}}{(z_\perp^2+z_k^2)^{1/2}}\exp\left[-\mu|n_\parallel|\left(\frac{z_\perp^2+z_k^2}{n_\parallel^2-1}\right)^{1/2}\right]$$

$$= \sum_{k=0}^{\infty} B z_k^{2k+2n+1}\int_1^\infty dt\,(t^2-1)^{k+n}\exp\left[-\frac{t|n_\parallel|\mu z_k}{(n_\parallel^2-1)^{1/2}}\right], \tag{24}$$

which again leads to (23) (see Gradshteyn and Ryzhik 1980, equation 3.387.3).

In the limit when the arguments of the K function in (23) are large and $(k_\perp v_t/\omega_c)^2 \ll 1$, we obtain

$$\varepsilon_{11}^a = i\frac{\mu n^2\omega_p^2}{2^n n!\omega^2}\left(\frac{\pi}{2\mu}\right)^{1/2}\left(\frac{k_\perp^2 v_t^2}{\omega_c^2}\right)^{n-1}\frac{1}{|n_\parallel|^{n+1}}\left(n_\parallel^2-1+\frac{n^2\omega_c^2}{\omega^2}\right)^{n/2}$$

$$\times\exp\left\{-\frac{\mu}{n_\parallel^2-1}\left[1-n_\parallel^2-\frac{n\omega_c}{\omega}+n_\parallel\left(n_\parallel^2-1+\frac{n^2\omega_c^2}{\omega^2}\right)^{1/2}\right]\right\}. \tag{25}$$

This agrees with equations (4a) and (24) in Shkarofsky (1966). If, furthermore, $n_\parallel^2 \gg 1$ and $n_\parallel^2 \gg |(n\omega_c/\omega)^2-1|$, (25) reduces to the well-known nonrelativistic result

$$\varepsilon_{11}^a = i\frac{n^2\omega_p^2}{2^n n!\omega|k_\parallel|v_t}\left(\frac{\pi}{2}\right)^{1/2}\left(\frac{k_\perp^2 v_t^2}{\omega_c^2}\right)^{n-1}\exp\left[-\left(\frac{\omega-n\omega_c}{\sqrt{2}|k_\parallel|v_t}\right)^2\right]. \tag{26}$$

7. Other relativistic distributions

For other distributions, it is more convenient to use different forms instead of (9) and (12). These are obtained by integrating by parts the parallel derivative in $U^{\rm res}$. Since $f = f(z_\perp, z_\parallel)$, we obtain

$$\int dz_\parallel\, z_\perp^{\rm res}\mathbf{V}^{\rm res}\left(n_\parallel z_\perp\frac{\partial f}{\partial z_\parallel}\right)^{\rm res} = -\int n_\parallel\, dz_\parallel\left[f\frac{\partial}{dz_\parallel}(z_\perp^2\,\mathbf{V}) + z_\perp^2\,\mathbf{V}\frac{\partial z_\perp}{\partial z_\parallel}\frac{\partial f}{\partial z_\perp}\right]^{\rm res}. \tag{27}$$

Equation (9) or (12) becomes, with the same upper and lower limits,

$$\boldsymbol{\epsilon}^a = i2\pi^2\frac{\omega_p^2}{\omega^2}\int dz_\parallel\left\{f n_\parallel\frac{\partial}{\partial z_\parallel}(\mathbf{V}z_\perp^2) + (n_\parallel^2-1)\,\gamma\mathbf{V}z_\perp\frac{\partial f}{\partial z_\perp}\right\}^{\rm res}_{z_\perp = z_\perp^{\rm res}}, \tag{28}$$

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where, within V_{ij}, the argument x of the Bessel functions, is related to $z_\perp$ by (3b), and the derivative of either with respect to $z_\parallel$ can be obtained from (7) or (10). Thus

$$z_\perp \frac{\partial z_\perp}{\partial z_\parallel} = -z_\parallel(1-n_\parallel^2)+\frac{n_\parallel n\omega_c}{\omega}, \tag{29a}$$

$$\frac{\partial}{\partial z_\parallel}\left(z_\perp \frac{\partial z_\perp}{\partial z_\parallel}\right) = -(1-n_\parallel^2). \tag{29b}$$

We can likewise integrate by parts the perpendicular derivative in $U^{\rm res}$, resulting in

$$\int dz_\parallel\, z_\perp^{\rm res}\, \mathbf{V}^{\rm res}\left(\frac{\partial f}{\partial z_\perp}\right)^{\rm res} = -\int dz_\parallel \left[f\frac{\partial}{\partial z_\parallel}\left(\frac{z_\perp \mathbf{V}}{\partial z_\perp/\partial z_\parallel}\right)+\left(\frac{z_\perp \mathbf{V}}{\partial z_\perp/\partial z_\parallel}\right)\frac{\partial f}{\partial z_\parallel}\right]^{\rm res}. \tag{30}$$

Inserting (30) into (9) or (12) gives

$$\boldsymbol{\epsilon}^a = i2\pi^2\frac{\omega_p^2}{\omega^2}\int dz_\parallel \left\{f\frac{n\omega_c}{\omega}\frac{\partial}{dz_\parallel}\left(\frac{z_\perp \mathbf{V}}{\partial z_\perp/\partial z_\parallel}\right)+\left(\frac{(1-n_\parallel^2)\,\gamma z_\perp \mathbf{V}}{\partial z_\perp/\partial z_\parallel}\right)\frac{\partial f}{\partial z_\parallel}\right\}_{z_\perp=z_\perp^{\rm res}}^{\rm res}. \tag{31}$$

Note that

$$\left\{\frac{\partial}{\partial z_\parallel}\left(\frac{z_\perp \mathbf{V}}{\partial z_\perp/\partial z_\parallel}\right)\right\}^{\rm res} = \left\{\frac{(1-n_\parallel^2)\,z_\perp^2\,\mathbf{V}}{(z_\perp\,\partial z_\perp/\partial z_\parallel)^2}+\frac{(\partial/\partial z_\parallel)\,(z_\perp^2\,\mathbf{V})}{z_\perp\,\partial z_\perp/\partial z_\parallel}\right\}^{\rm res}. \tag{32}$$

Also, referring to (2),

$$\frac{\partial}{\partial z_\parallel}(z_\perp^2\,V_{i,j}) = \frac{\partial z_\perp}{\partial z_\parallel}\left[\frac{\partial}{\partial z_\perp}(z_\perp^2\,V_{i,j})\right]+\frac{z_\perp^2\,V_{i,j}}{z_\parallel}(\delta_{i,3}+\delta_{j,3}). \tag{33}$$

Both expressions for $\boldsymbol{\epsilon}^a$, namely (28) and (31), are given in Granata and Fidone (1991). In the following, we shall be using the form in (28).

8. A relativistic bi-Maxwellian

We now give the equivalent integral to that in (21) for the case when the distribution function is a bi-Maxwellian, although we do not have a closed-form expression for the integral in terms of known functions. We define different temperatures $1/\mu_\perp = T_\perp/mc^2$ and $1/\mu_\parallel = T_\parallel/mc^2$ respectively in the perpendicular and parallel directions. The following form for the distribution function is a possible relativistic counterpart to the nonrelativistic bi-Maxwellian. Other counterparts have been suggested, but the form applied here actually reduces to the nonrelativistic form when $T_\perp = T_\parallel$. This form was previously applied by Yoon (1989). Let

$$f = A\exp[-\mu_\perp(1+z_\perp^2+z_\parallel^2)^{1/2}+\Delta\mu(1+z_\parallel^2)^{1/2}], \tag{34}$$

with $\Delta\mu = \mu_\perp - \mu_\parallel$. The constant A is obtained from $\int f d^3z = 1$, giving

$$A = \frac{\mu_\perp}{4\pi}\left[\frac{1}{K_2(\mu_\parallel)-\Delta\mu\,K_1(\mu_\parallel)/\mu_\parallel\mu_\perp}\right]. \tag{35}$$

We now insert this distribution function into (28) and illustrate the resulting integral for the 11 element. For $n_\parallel^2 < 1$, (28) yields, with A given by (35) and B by (20d),

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$$\varepsilon_{11}^a = i2\pi^2 A\left(\frac{\omega_p}{\omega}\right)^2 e^{-\mu_\perp n\omega_c/\omega}\sum_{k=0}^{\infty} B(1-n_\parallel^2)^{k+n}\int_{z_\parallel^-}^{z_\parallel^+} dz_\parallel \exp[-\mu_\perp n_\parallel z_\parallel + \Delta\mu(1+z_\parallel^2)^{1/2}]$$
$$\times (z_\parallel - z_\parallel^-)^{k+n-1}(z_\parallel^+ - z_\parallel)^{k+n-1}[(k+n)\, n_\parallel (z_\parallel^- + z_\parallel^+ - 2z_\parallel)$$
$$+\mu_\perp(1-n_\parallel^2)\,(z_\parallel - z_\parallel^-)\,(z_\parallel^+ - z_\parallel)]. \tag{36}$$

Also using (28), we find for (12a) or (12b) with its appropriate limits, when $n_\parallel^2 > 1$,

$$\varepsilon_{11}^a = i2\pi^2 A\left(\frac{\omega_p}{\omega}\right)^2 e^{-\mu_\perp n\omega_c/\omega}\sum_{k=0}^{\infty} B(n_\parallel^2-1)^{k+n}\int dz_\parallel \exp[-\mu_\perp n_\parallel z_\parallel + \Delta\mu(1+z_\parallel^2)^{1/2}]$$
$$\times (z_\parallel - z_\parallel^-)^{k+n-1}(z_\parallel - z_\parallel^+)^{k+n-1}[(k+n)\, n_\parallel (2z_\parallel - z_\parallel^- - z_\parallel^+)$$
$$-\mu_\perp(n_\parallel^2-1)\,(z_\parallel - z_\parallel^-)\,(z_\parallel - z_\parallel^+)]. \tag{37}$$

As mentioned above, a closed form of these integrals has not yet been found.

9. A relativistic Maxwellian in the perpendicular direction

We now consider the case when the distribution function has an arbitrary form, $g(z_\parallel)$, in the $z_\parallel$ parallel direction but is a Maxwellian in the perpendicular direction. We adopt a distribution function of the following form, with $1/\mu_\perp = T_\perp/mc^2$:

$$f = Ag(z_\parallel)\exp[-\mu_\perp(1+z_\perp^2+z_\parallel^2)^{1/2} + \mu_\perp(1+z_\parallel)^{1/2}]. \tag{38}$$

The constant A is obtained from $\int f d^3z = 1$, giving

$$A = \mu_\perp^2 \Big/ \left\{2\pi\int_{-\infty}^{\infty} dz_\parallel [1+\mu_\perp(1+z_\parallel^2)^{1/2}]\, g(z_\parallel)\right\}. \tag{39}$$

For $n_\parallel^2 < 1$, (28) yields, with (39) for A and (20d) as before for B,

$$\varepsilon_{11}^a = i2\pi^2 A\left(\frac{\omega_p}{\omega}\right)^2 e^{-\mu_\perp n\omega_c/\omega}\sum_{k=0}^{\infty} B(1-n_\parallel^2)^{k+n}\int_{z_\parallel^-}^{z_\parallel^+} dz_\parallel\, g(z_\parallel)$$
$$\times \exp[-\mu_\perp n_\parallel z_\parallel + \mu_\perp(1+z_\parallel^2)^{1/2}]$$
$$\times (z_\parallel - z_\parallel^-)^{k+n-1}(z_\parallel^+ - z_\parallel)^{k+n-1}[(k+n)\, n_\parallel (z_\parallel^- + z_\parallel^+ - 2z_\parallel)$$
$$+\mu_\perp(1-n_\parallel^2)\,(z_\parallel - z_\parallel^-)\,(z_\parallel^+ - z_\parallel)]. \tag{40}$$

Also, we have for (12a) or (12b) with its appropriate limit, when $n_\parallel^2 > 1$,

$$\varepsilon_{11}^a = i2\pi^2 A\left(\frac{\omega_p}{\omega}\right)^2 e^{-\mu_\perp n\omega_c/\omega}\sum_{k=0}^{\infty} B(n_\parallel^2-1)^{k+n}\int dz_\parallel\, g(z_\parallel)$$
$$\times \exp[-\mu_\perp n_\parallel z_\parallel + \mu_\perp(1+z_\parallel^2)^{1/2}]$$
$$\times (z_\parallel - z_\parallel^-)^{k+n-1}(z_\parallel - z_\parallel^+)^{k+n-1}[(k+n)\, n_\parallel (2z_\parallel - z_\parallel^- - z_\parallel^+)$$
$$-\mu_\perp(n_\parallel^2-1)\,(z_\parallel - z_\parallel^-)\,(z_\parallel - z_\parallel^+)]. \tag{41}$$

10. A relativistic Maxwellian in the perpendicular direction and having a plateau in the parallel direction

In lower-hybrid current-drive situations, a plateau occurs over an intermediate range in $g(z_\parallel)$, as indicated in Fig. 3, even including electron-cyclotron heating with $n_\parallel^2 < 1$. The distribution is elevated in this region, and this important

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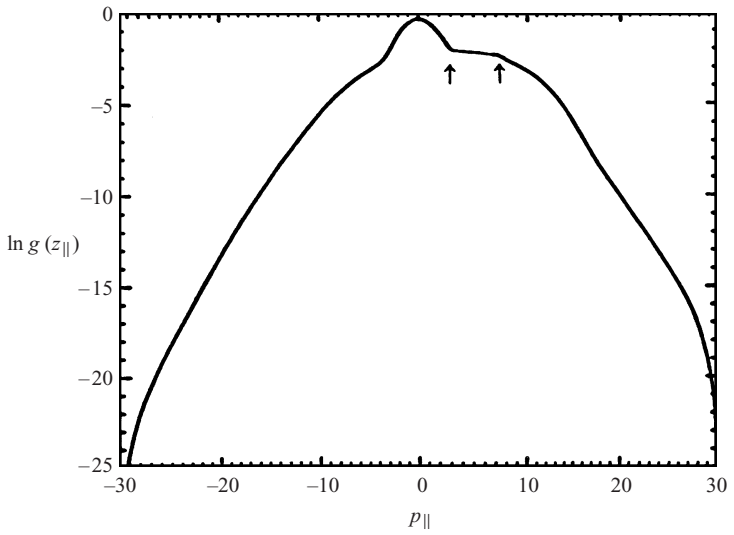


Figure 3. The one-dimensional parallel distribution function corresponding to Fig. 1. The arrows indicate the extent of the plateau region.

contribution to the damping is now considered. (The contribution of the bulk can be treated as in Sec. 6, i.e. as a full Maxwellian.) The limits in (40) are now $z_{\min} = \max(z_{\parallel}^{-}, z_1)$ and $z_{\max} = \min(z_{\parallel}^{+}, z_2)$, where the plateau extends from z_1 to z_2. Values of $z_{1,2} = \gamma_{1,2} v_{1,2}/c$ are based on given velocity $v_{1,2}$ limits of the plateau. These curves intersect the resonance curve $\gamma_{1,2} = n_{\parallel} z_{1,2} + n\omega_c/\omega$ at

$$z_{1,2} = \frac{n\omega_c}{\omega} \Big/ \left(\frac{c}{v_{1,2}} - n_{\parallel} \right), \tag{42}$$

provided that at the intersection point $z_{\perp}^2 > 0$, or

$$\left(\frac{n\omega_c}{\omega} \right)^2 \left[\left(\frac{c}{v_{1,2}} \right)^2 - 1 \right] > \left(\frac{c}{v_{1,2}} - n_{\parallel} \right)^2. \tag{43}$$

The extent of the resonance curve in Fig. 2(a) is decreased by one or both intersection points. (If $z_{\min} > z_{\max}$, the plateau does not contribute to the damping.) Let us assign to the argument of $g(z_{\parallel})$, which is more or less a flat function in the plateau region, the value at the middle of the plateau region, namely $z_{\text{av}} = \frac{1}{2}(z_{\min} + z_{\max})$. In this case, $g(z_{\text{av}})$ can now be taken outside the integral, yielding

$$\begin{aligned} \varepsilon_{11}^{a} = i2\pi^2 A \left(\frac{\omega_p}{\omega} \right)^2 e^{-\mu_{\perp} n\omega_c/\omega} g(z_{\text{av}}) \sum_{k=0}^{\infty} B(1 - n_{\parallel}^2)^{k+n} \\ \times \int_{z_{\min}}^{z_{\max}} dz_{\parallel} \exp[-\mu_{\perp} n_{\parallel} z_{\parallel} + \mu_{\perp}(1 + z_{\parallel}^2)^{1/2}] \\ \times (z_{\parallel} - z_{\parallel}^{-})^{k+n-1} (z_{\parallel}^{+} - z_{\parallel})^{k+n-1} [(k+n)\, n_{\parallel} (z_{\parallel}^{-} + z_{\parallel}^{+} - 2z_{\parallel}) \\ + \mu_{\perp}(1 - n_{\parallel}^2)\, (z_{\parallel} - z_{\parallel}^{-})\, (z_{\parallel}^{+} - z_{\parallel})]. \end{aligned} \tag{44}$$

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Then the damping is proportional to $g(z_{\rm av})$. Further approximations can possibly be made in (44), as well as in (36), (37), (40) and (41). However, at some stage, because of the limits on the integral, a computer evaluation is needed. We have therefore gone back to (9) and are writing a computer program based on the full two-dimensional form of the distribution function, an example of which is shown in Fig. 1.

11. Conclusions

The anti-Hermitian parts of the relativistic dielectric tensor, which provide the damping close to a cyclotron harmonic frequency, have been written as a single integral over either parallel or perpendicular momentum. The limits on the integral have been shown for either case, and depend also on the sign and magnitude of the parallel refractive index. Comparisons have been made with expressions in the literature. For the case of a relativistic Maxwellian distribution, the integrals can be evaluated exactly and analytical expressions obtained. These reduce to previous results in certain limits, including the nonrelativistic limit. For a relativistic bi-Maxwellian distribution function, we can only write the result as an integral. This integral form can then be generalized to a distribution that is Maxwellian only in the perpendicular direction, with an arbitrary form in the parallel direction. A computer program is being written to evaluate in general the single-integral expressions for complex distribution functions, such as those that result in the presence of lower-hybrid and electron cyclotron high-power waves.

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Appendix. ε^a_{ij} for a relativistic Maxwellian

The following relativistic Maxwellian distribution is adopted where I and K are respectively modified Bessel and Hankel functions, $v_t^2 = T/m$ and $\mu \equiv (c/v_t)^2$. We require that $n_\parallel^2 - 1 + n^2\omega_c^2/\omega^2 \geqslant 0$. First, relations are given for the case when $1 > |n_\parallel|$. We define

$$Q_{nk} \equiv a_{nk} \frac{\pi\mu\omega_p^2}{\omega^2(1-n_\parallel^2)^{1/2}} \frac{(\pi/2\mu)^{1/2}}{e^\mu K_2(\mu)} \left(\frac{k_\perp^2 v_t^2}{\omega_c^2}\right)^{k+n-1} \left(\frac{n_\parallel^2 - 1 + n^2\omega_c^2/\omega^2}{n_\parallel^2}\right)^{(2k+2n+1)/4} \times \exp\left[\frac{\mu(1-n_\parallel^2 - n\omega_c/\omega)}{1-n_\parallel^2}\right], \tag{A 1}$$

$$a_{nk} \equiv \frac{(-1)^k(2k+2n)!}{2^{k+n}\,k!(k+n)!(k+2n)!} = \frac{(-1)^k(n+k-\frac{1}{2})!}{2^n\,k!(n+k/2)!(n+\frac{1}{2}k-\frac{1}{2})!}, \tag{A 2}$$

$$Z_n \equiv \mu|n_\parallel| \frac{(n_\parallel^2 - 1 + n^2\omega_c^2/\omega^2)^{1/2}}{1-n_\parallel^2}. \tag{A 3}$$

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Then the various elements, including that in (21), are

$$\varepsilon^a_{11} = i\sum_{k=0}^{\infty} Q_{nk}\, n^2 I_{k+n+1/2}(Z_n), \tag{A 4}$$

$$\varepsilon^a_{12} = -\varepsilon^a_{21} = \sum_{k=0}^{\infty} Q_{nk}\, n(k+n) I_{k+n+1/2}(Z_n), \tag{A 5}$$

$$\varepsilon^a_{22} = i\sum_{k=0}^{\infty} Q_{nk}\left[n^2 + \frac{2k(n+k-1)\,(2n+k)}{2n+2k-1}\right] I_{k+n+1/2}(Z_n), \tag{A 6}$$

$$\varepsilon^a_{13} = \varepsilon^a_{31} = i\sum_{k=0}^{\infty} Q_{nk}\, n\frac{k_\perp c}{\omega_c}\frac{n_\parallel}{1-n_\parallel^2}\left[\frac{n\omega_c}{\omega} I_{k+n+1/2}(Z_n) - \left(\frac{n_\parallel^2 - 1 + n^2\omega_c^2/\omega^2}{n_\parallel^2}\right)^{1/2} I_{k+n+3/2}(Z_n)\right]. \tag{A 7}$$

$$\varepsilon^a_{32} = -\varepsilon^a_{23} = \sum_{k=0}^{\infty} Q_{nk}(k+n)\frac{k_\perp c}{\omega_c}\frac{n_\parallel}{1-n_\parallel^2}\left[\frac{n\omega_c}{\omega} I_{k+n+1/2}(Z_n) - \left(\frac{n_\parallel^2 - 1 + n^2\omega_c^2/\omega^2}{n_\parallel^2}\right)^{1/2} I_{k+n+3/2}(Z_n)\right], \tag{A 8}$$

$$\varepsilon^a_{33} = i\sum_{k=0}^{\infty} Q_{nk}\left(\frac{k_\perp c}{\omega_c}\right)^2 \frac{1}{(1-n_\parallel^2)^2}\left\{n_\parallel^2\left(\frac{n\omega_c}{\omega}\right)^2 I_{k+n+1/2}(Z_n) + \left[n_\parallel^2 - 1 + \left(\frac{n\omega_c}{\omega}\right)^2\right] I_{k+n+5/2}(Z_n) + \left[n_\parallel^2 - 1 + \left(\frac{n\omega_c}{\omega}\right)^2\right]^{1/2}\left[-\frac{2|n_\parallel| n\omega_c}{\omega} + \frac{1-n_\parallel^2}{\mu|n_\parallel|}\right] I_{k+n+3/2}(Z_n)\right\} \tag{A 9a}$$

or

$$\varepsilon^a_{33} = i\sum_{k=0}^{\infty} Q_{nk}\left(\frac{k_\perp c}{\omega_c}\right)^2 \frac{1}{(1-n_\parallel^2)^2}\left\{\left[n_\parallel^2 - 1 + (n_\parallel^2+1)\left(\frac{n\omega_c}{\omega}\right)^2\right] I_{k+n+1/2}(Z_n) - 2\left[n_\parallel^2 - 1 + \left(\frac{n\omega_c}{\omega}\right)^2\right]^{1/2}\left[\frac{|n_\parallel| n\omega_c}{\omega} + \frac{(k+n+1)\,(1-n_\parallel^2)}{\mu|n_\parallel|}\right] I_{k+n+3/2}(Z_n)\right\}. \tag{A 9b}$$

Next, for $n_\parallel^2 > 1$, we change the definitions of Q_{nk} and Z_n to the following:

$$Q_{nk} \equiv a_{nk}\frac{\mu\omega_p^2}{\omega^2(n_\parallel^2-1)^{1/2}}\frac{(\pi/2\mu)^{1/2}}{e^\mu K_2(\mu)}\left(\frac{k_\perp^2 v_t^2}{\omega_c^2}\right)^{k+n-1}\left(\frac{n_\parallel^2 - 1 + n^2\omega_c^2/\omega^2}{n_\parallel^2}\right)^{(2k+2n+1)/4} \times \exp\left[\frac{\mu(n_\parallel^2 - 1 + n\omega_c/\omega)}{n_\parallel^2 - 1}\right], \tag{A 10}$$

$$Z_n \equiv \frac{\mu|n_\parallel|(n_\parallel^2 - 1 + n^2\omega_c^2/\omega^2)^{1/2}}{n_\parallel^2 - 1}. \tag{A 11}$$

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Then the various elements, including that in (23), are

$$\varepsilon^a_{11} = i\sum_{k=0}^{\infty} Q_{nk}\, n^2 K_{k+n+1/2}(Z_n), \tag{A 12}$$

$$\varepsilon^a_{12} = -\varepsilon^a_{21} = \sum_{k=0}^{\infty} Q_{nk}\, n(k+n) K_{k+n+1/2}(Z_n), \tag{A 13}$$

$$\varepsilon^a_{22} = i\sum_{k=0}^{\infty} Q_{nk}\left[n^2 + \frac{2k(n+k-1)\,(2n+k)}{2n+2k-1}\right] K_{k+n+1/2}(Z_n), \tag{A 14}$$

$$\varepsilon^a_{13} = \varepsilon^a_{31} = -i\sum_{k=0}^{\infty} Q_{nk}\, n \frac{k_\perp c}{\omega_c}\frac{n_\parallel}{n_\parallel^2-1}\left[\frac{n\omega_c}{\omega} K_{k+n+1/2}(Z_n) - \left(\frac{n_\parallel^2 - 1 + n^2\omega_c^2/\omega^2}{n_\parallel^2}\right)^{1/2} K_{k+n+3/2}(Z_n)\right], \tag{A 15}$$

$$\varepsilon^a_{32} = -\varepsilon^a_{23} = -\sum_{k=0}^{\infty} Q_{nk}(k+n)\frac{k_\perp c}{\omega_c}\frac{n_\parallel}{n_\parallel^2-1}\left[\frac{n\omega_c}{\omega} K_{k+n+1/2}(Z_n) - \left(\frac{n_\parallel^2 - 1 + n^2\omega_c^2/\omega^2}{n_\parallel^2}\right)^{1/2} K_{k+n+3/2}(K_n)\right], \tag{A 16}$$

$$\varepsilon^a_{33} = i\sum_{k=0}^{\infty} Q_{nk}\left(\frac{k_\perp c}{\omega_c}\right)^2 \frac{1}{(n_\parallel^2-1)^2}\left\{n_\parallel^2\left(\frac{n\omega_c}{\omega}\right)^2 K_{k+n+1/2}(Z_n) + \left[n_\parallel^2 - 1 + \left(\frac{n\omega_c}{\omega}\right)^2\right] K_{k+n+5/2}(Z_n) - \left[n_\parallel^2 - 1 + \left(\frac{n\omega_c}{\omega}\right)^2\right]^{1/2}\left[\frac{2|n_\parallel| n\omega_c}{\omega} + \frac{n_\parallel^2-1}{\mu|n_\parallel|}\right] K_{k+n+3/2}(Z_n)\right\} \tag{A 17a}$$

or

$$\varepsilon^a_{33} = i\sum_{k=0}^{\infty} Q_{nk}\left(\frac{k_\perp c}{\omega_c}\right)^2 \frac{1}{(n_\parallel^2-1)^2}\left\{\left[n_\parallel^2 - 1 + (n_\parallel^2+1)\left(\frac{n\omega_c}{\omega}\right)^2\right] K_{k+n+1/2}(Z_n) - 2\left[n_\parallel^2 - 1 + \left(\frac{n\omega_c}{\omega}\right)^2\right]^{1/2}\left[\frac{|n_\parallel| n\omega_c}{\omega} - \frac{(k+n+1)\,(n_\parallel^2-1)}{\mu|n_\parallel|}\right] K_{k+n+3/2}(Z_n)\right\}. \tag{A 17b}$$

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