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Whistler wave emission by a modulated electron beam injected into a plasma column surrounded by a uniform medium

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Abstract. The efficiency of whistler wave radiation by a density modulated and thin electron beam of finite length injected parallel with respect to the constant ambient magnetic field into a cylindrical plasma column surrounded by a uniform isotropic medium and aligned along the magnetic field is studied. A rigorous analytical derivation of the wave fields excited by the beam in the plasma column is presented. The time-averaged power radiated by the beam at the modulation frequency is determined. In particular, numerical calculations performed for physical conditions relevant to laboratory magnetized discharge plasmas show that, in the presence of a plasma column, the power lost by the modulated beam can be efficiently enhanced owing to resonant Cherenkov excitation of guided whistler modes at the beam modulation frequency.

1. Introduction

During the past four decades, a large number of papers have been published regarding the linear and nonlinear theory of whistler wave excitation by pulsed and density modulated electron beams injected in unbounded homogeneous magnetized plasmas (e.g., [1–16] and references therein). Recently [17–20], the authors studied the efficiency of whistler radiation by such beams injected along the background magnetic field into density ducts artificially created in laboratory or space plasmas [21–28]. In particular, they showed that the time-averaged power lost by the beam in the presence of a duct of enhanced plasma density can increase noticeably owing to resonant Cherenkov excitation of guided whistler modes at the beam modulation frequency [17, 18].

This paper studies the coherent spontaneous whistler emission by a modulated thin electron beam injected into a magnetic-field-aligned cylindrical plasma column surrounded by any uniform isotropic medium (dielectric, free space, metal, etc.). As is well known, particle beams can be considered in some circumstances as extended current-carrying antennas which might replace usual emitters [29–32]. Indeed, by modulating the beam current density at some appropriate frequency, one can generate a multi-front propagating structure of charged particles which is capable of radiating coherently low-frequency electromagnetic waves. In magnetized

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plasmas, wave emissions by such processes are particularly efficient when the beam modulation frequency lies in the whistler range. Moreover, an axially magnetized plasma column surrounded by a dielectric or by free space is able to guide waves propagating in this frequency range. As already shown for the case of modulated beam emission into artificially created density ducts [17, 18], the authors expect that a noticeable increase of whistler radiation power should be observed at Cherenkov resonance conditions when the beam is injected into a plasma column surrounded by a dielectric medium, compared to the case when it is injected into a homogeneous unbounded background magnetoplasma. In this paper we aim to demonstrate this fact and to determine the beam radiation power as a function of the medium dielectric permittivity, the main characteristics of the plasma column (radius, plasma density and electron collision frequency) and of the beam (radius and energy).

The interest into such investigations has been mainly motivated by recent laboratory experiments [33–41] aimed at studying wave emission by electron beams modulated in the whistler frequency range and injected into magnetized plasma columns. We should also note that, in typical laboratory experiments involving whistler excitation by beams or antennas, the discharge plasma volume is usually separated from the walls of the set-up chamber by a layer of rarefied plasma (see also [21,22]). Moreover, the investigation of beam emission into a magnetized plasma waveguide embedded in a dielectric medium may be oriented towards the possibility of performing wave diagnostics in discharge plasmas with the help of dielectric waveguides [42].

In this work, we restrict ourselves to the description of the linear stage of the wave excitation, assuming that the beam is given and ignoring its evolution during its interaction with the excited waves. To elucidate the main aspects of the problem analytically, we study the case of a beam injected into an axisymmetrical cylindrical magnetized plasma column with a step-shaped radial density profile and surrounded by a uniform medium. A rigorous analytical expression is obtained for the beam-excited fields which allows us to calculate the average radiated power. The main attention is focused on the estimates of the power radiated by the beam when its modulation frequency lies in the whistler range.

2. Wave fields excited by the beam

Let us consider an infinitely long cylindrical magnetized plasma column of radius a immersed in a uniform isotropic medium of permittivity $\widetilde{\varepsilon}$ (dielectric, metal, free space or plasma). The column axis is parallel to the constant external magnetic field $\mathbf{B}_0 = B_0\mathbf{u}_z$; $(\mathbf{u}_\rho, \mathbf{u}_\phi, \mathbf{u}_z)$ and (ρ, ϕ, z) are the cylindrical unit vectors and the corresponding coordinates, respectively. The plasma inside the column ($\rho < a$), whose density n_p is supposed to be constant, is described by the dielectric tensor $\hat{\varepsilon}$,

$$\hat{\varepsilon} = \epsilon_0 \begin{pmatrix} \varepsilon & -ig & 0 \\ ig & \varepsilon & 0 \\ 0 & 0 & \eta \end{pmatrix}, \tag{1}$$

where ϵ_0 is the permittivity of free space. For the whistler frequency range, i.e. $\omega_\mathrm{lh} \ll |\omega - i\nu_\mathrm{e}| \lesssim \omega_\mathrm{c} \ll \omega_\mathrm{p}$, the tensor elements ε, g and η in (1) can be written

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as [43]

$$\varepsilon = 1 + \frac{\omega_{\rm p}^2(\omega - i\nu_{\rm e})}{\omega(\omega_{\rm c}^2 - (\omega - i\nu_{\rm e})^2)}, \quad g = -\frac{\omega_{\rm p}^2\omega_{\rm c}}{\omega(\omega_{\rm c}^2 - (\omega - i\nu_{\rm e})^2)}, \quad \eta = 1 - \frac{\omega_{\rm p}^2}{(\omega - i\nu_{\rm e})\omega}, \tag{2}$$

where $\omega_{\rm p}$, $\omega_{\rm c}$ and $\omega_{\rm lh}$ are the electron plasma, the electron cyclotron and the lower hybrid frequencies, respectively; ω and $\nu_{\rm e}$ are the wave and the plasma collision frequencies ($\nu_{\rm e} \ll \omega$). The ions' contributions have been neglected in (2) owing to the range of wave frequencies considered. Note that $\hat{\varepsilon}$ in (1) corresponds to a monochromatic wave with a time dependence of $e^{i\omega t}$.

We consider a density modulated and thin electron beam of finite length L injected parallel to $\mathbf{B}_0$, forming a cylindrical current tube. As the electron motion in the plane perpendicular to $\mathbf{B}_0$ is not significant, the internal structure of the beam can be neglected (see also [17,18]). So, only Cherenkov resonant excitation is considered here and physical effects related to cyclotron resonance harmonics are not taken into account. The evolution of the beam during its interaction with the waves is ignored (linear approximation). The beam current density can be written as

$$\mathbf{j} = j(\rho, z, t)\mathbf{u}_z = j_0(\rho)H(z)H(t - z/v_{\rm b})[1 + \sin(\omega_0(t - z/v_{\rm b}))]e^{-\beta z}\mathbf{u}_z, \tag{3}$$

$$j_0(\rho) = I_0[1 - H(\rho - b)]/\pi b^2, \tag{4}$$

where $I_0 = -en_{\rm b}v_{\rm b}\pi b^2$ is the total beam current, b is the radius of the cylindrical beam ($b < a$), ω_0 and $v_{\rm b}$ are the modulation frequency and the modulus of the beam velocity, respectively, $n_{\rm b}$ is the beam density, which is constant for $\rho < b$ and sufficiently small ($n_{\rm b} \ll n_{\rm p}$) so that its influence on the plasma dispersion properties can be neglected, H is the Heaviside function, e is the magnitude of the electron charge; and β is the inverse coherence length of the beam [1,2]. The parameter β is assumed to be infinitesimally small in the forthcoming calculations and, therefore, it will be set equal to zero in the resultant formulas. Nevertheless, the factor $\exp(-\beta z)$ should however be kept in the calculations for proper integration in the complex plane.

Let us now calculate the wave fields radiated by the beam. Since the plasma is a dispersive medium, the relation between the beam current and the excited wave fields has an integral form obtainable using inverse Fourier–Laplace transforms. Bearing in mind that the electric and magnetic fields $\mathbf{E}$ and $\mathbf{B}$ excited by the current $\mathbf{j}$ in (3) are independent of the azimuthal angle ϕ, we define the two-dimensional Fourier–Laplace transform of the function $f(\rho, z, t)$ as

$$f(\rho, k_z, s) = \int_{-\infty}^{+\infty} dz\, e^{ik_z z} \int_0^{\infty} f(\rho, z, t)e^{-st}\, dt, \tag{5}$$

where $s = i\omega$ is a complex variable and k_z is the parallel wave number. When calculating the Fourier–Laplace transforms of the fields, we use the fact that $\mathbf{E}(\rho, z, t)$ and $\mathbf{B}(\rho, z, t)$ are identically equal to zero for $t \leqslant 0$.

Using the Maxwell equations, the radial and the parallel components of the Fourier–Laplace transforms $\mathbf{E}(\rho, k_z, s)$ and $\mathbf{B}(\rho, k_z, s)$ can be readily expressed as functions of the azimuthal components [44] $E_\phi(\rho, k_z, s)$ and $B_\phi(\rho, k_z, s)$, which verify the second-order coupled differential equations inside the column ($\rho < a$),

$$\frac{\partial}{\partial\rho}\left[\frac{1}{\rho}\frac{\partial(\rho E_\phi)}{\partial\rho}\right] - \frac{s^2}{\varepsilon c^2}\left\{\left[\varepsilon\left(\varepsilon + \frac{k_z^2c^2}{s^2}\right) - g^2\right]E_\phi - \frac{gk_zc^2}{s}B_\phi\right\} = 0, \tag{6}$$

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$$\frac{\partial}{\partial\rho}\left[\frac{1}{\eta\rho}\frac{\partial(\rho B_\phi)}{\partial\rho}\right]-\frac{s^2}{\varepsilon c^2}\left[\frac{gk_z}{s}E_\phi+\left(\varepsilon+\frac{k_z^2c^2}{s^2}\right)B_\phi\right]=\frac{Z_0}{c}\frac{\partial}{\partial\rho}\left(\frac{j}{\eta}\right), \tag{7}$$

as well as those in the outer region $\rho > a$,

$$\frac{\partial}{\partial\rho}\left(\frac{1}{\rho}\frac{\partial(\rho B_\phi)}{\partial\rho}\right)-\frac{s^2\widetilde{q}^2}{c^2}B_\phi=0,\quad \frac{\partial}{\partial\rho}\left(\frac{1}{\rho}\frac{\partial(\rho E_\phi)}{\partial\rho}\right)-\frac{s^2\widetilde{q}^2}{c^2}E_\phi=0. \tag{8}$$

The Fourier–Laplace transform of the current j in (7) is given by

$$j=j(\rho,k_z,s)=\frac{j_0(\rho)v_\mathrm{b}}{s-i(k_z+i\beta)v_\mathrm{b}}\left(\frac{1}{s}+\frac{\omega_0}{s^2+\omega_0^2}\right). \tag{9}$$

$c=(\epsilon_0\mu_0)^{-1/2}$ is the velocity of light in free space, $Z_0=(\mu_0/\epsilon_0)^{1/2}$ and μ_0 are, respectively, the characteristic impedance and permeability of free space and $\widetilde{q}=(\widetilde{\varepsilon}+c^2k_z^2/s^2)^{1/2}$ is the normalized perpendicular wave number in the outer region $\rho > a$. The dependence of the tensor elements ε, g and η on s is obtained by replacing ω by $-is$ in (2). Note that (6) and (7) are applicable for arbitrary axisymmetrical distributions of plasma density inside the column. However, we consider in this paper the simplest case of a uniform plasma column, for which it is obvious that the solutions of (6)–(8) may be obtained analytically for each region $\rho < b$, $b < \rho < a$ and $\rho > a$. It is required that the solutions are regular at $\rho = 0$ and satisfy the radiation condition at infinity, $\rho \to \infty$. The boundary conditions at $\rho = b$ and $\rho = a$ can be written for the parallel and the azimuthal electric and magnetic field components $E_{\phi,z}$ and $B_{\phi,z}$ as follows:

$$\begin{aligned}&E_{\phi,z}(b-0,k_z,s)=E_{\phi,z}(b+0,k_z,s),\quad B_{\phi,z}(b-0,k_z,s)=B_{\phi,z}(b+0,k_z,s),\\ &E_{\phi,z}(a-0,k_z,s)=E_{\phi,z}(a+0,k_z,s),\quad B_{\phi,z}(a-0,k_z,s)=B_{\phi,z}(a+0,k_z,s).\end{aligned} \tag{10}$$

If the right-hand side of (7) vanishes (no current density), the solutions of (6) and (7) can be represented up to a factor independent of ρ in terms of cylindrical functions,

$$E_\phi=i\mathscr{Z}_1(k_{\perp1,2}\rho),\quad B_\phi=-(n_{1,2}/c)\mathscr{Z}_1(k_{\perp1,2}\rho), \tag{11}$$

where

$$n_{1,2}=-\frac{ic}{s}\frac{\varepsilon}{k_zg}\left[k_{\perp1,2}^2+k_z^2-\frac{s^2}{c^2}(g^2/\varepsilon-\varepsilon)\right]. \tag{12}$$

$\mathscr{Z}_1$ stands for a Bessel function of the first order if $\rho < b$ (respectively, for Bessel and Neumann functions of the first order, if $b < \rho < a$); $k_{\perp1}$ and $k_{\perp2}$ are the two perpendicular wave vectors in the uniform magnetoplasma [44], verifying $k_{\perp1,2}^2=-(s/c)^2q_{1,2}^2$, where

$$q_k=\frac{1}{\sqrt{2}}\left\{\varepsilon-\frac{g^2}{\varepsilon}+\eta-\left(\frac{\eta}{\varepsilon}+1\right)p^2-(-1)^k\left(\frac{\eta}{\varepsilon}-1\right)[(p^2-P_\mathrm{b}^2)(p^2-P_\mathrm{c}^2)]^{1/2}\right\}^{1/2}, \tag{13}$$

$$P_\mathrm{b,c}=\left\{\varepsilon-\frac{g^2(\eta+\varepsilon)}{(\eta-\varepsilon)^2}+\frac{2\chi_\mathrm{b,c}}{(\eta-\varepsilon)^2}[\varepsilon\eta g^2(g^2-(\eta-\varepsilon)^2)]^{1/2}\right\}^{1/2}, \tag{14}$$

with $p=ick_z/s$ and $\chi_\mathrm{b}=-\chi_\mathrm{c}=-1$. For definiteness, the square root in (14) is chosen to have a positive real part.

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Applying the boundary conditions (10) and performing a considerable amount of algebraic manipulations, one obtains the expressions for the azimuthal components E_ϕ and B_ϕ in the form

$$\begin{bmatrix} E_\phi \\ B_\phi \end{bmatrix} = \mathscr{E} \sum_{k=1}^{2} \begin{bmatrix} i \\ -n_k/c \end{bmatrix} \left[\mathscr{B}_k J_1(Q_k \rho/a) + \frac{(-1)^k}{n_2 - n_1} G_k^{(1)}(\rho) H(\rho - b) \right], \quad \rho < a, \tag{15}$$

$$\begin{bmatrix} E_\phi \\ B_\phi \end{bmatrix} = \mathscr{E} \begin{bmatrix} -i\mathscr{C}_1 \\ \tilde{\varepsilon}\mathscr{C}_2/c \end{bmatrix} K_1(S\rho/a), \quad \rho > a, \tag{16}$$

with

$$\mathscr{E} = \frac{bZ_0 e n_\mathrm{b} v_\mathrm{b}^2}{s - i(k_z + i\beta)v_\mathrm{b}} \left(\frac{1}{s} + \frac{\omega_0}{s^2 + \omega_0^2} \right), \tag{17}$$

$$G_k^{(\ell)}(\rho) = (\pi/2) Q_k^{1-\ell} [J_\ell(Q_k \rho/a) Y_1(Q_k b/a) - J_1(Q_k b/a) Y_\ell(Q_k \rho/a)], \quad \ell = 0, 1, \tag{18}$$

where $Q_k^2 = -(sa/c)^2 q_k^2$ and $S^2 = (sa/c)^2 \tilde{q}^2$ (note that the radiation condition at infinity requires that the quantities S are chosen such that $\mathrm{Re}(S) > 0$); J_n and K_n are the nth-order Bessel function and modified Bessel function of the second kind, respectively, and Y_n is the nth-order Neumann function. The excitation coefficients $\mathscr{B}_{1,2}$ and $\mathscr{C}_{1,2}$ are given by

$$\mathscr{B}_{1,2} = \frac{B_{1,2}^{(\Delta)}(k_z, s)}{\Delta(k_z, s)}, \quad \mathscr{C}_{1,2} = \frac{C_{1,2}^{(\Delta)}(k_z, s)}{\Delta(k_z, s)}, \tag{19}$$

with

$$\begin{aligned} B_{1,2}^{(\Delta)}(k_z, s) = {} & \frac{1}{(n_2 - n_1) Q_{1,2} J(Q_{1,2})} \\ & \times \{K(S)[K(S)\tilde{\varepsilon}(n_{1,2} - n_{2,1}) - J(Q_{2,1})(\eta n_{2,1} - \tilde{\varepsilon} n_{1,2})] G_{1,2}^{(0)}(a) \\ & + [J(Q_{2,1})\eta(n_{1,2} - n_{2,1}) + K(S)(\eta n_{1,2} - \tilde{\varepsilon} n_{2,1})] \\ & \times G_{1,2}^{(1)}(a) - K(S) n_{2,1}(\eta - \tilde{\varepsilon}) \mathscr{J}(Q_{2,1})\}, \end{aligned} \tag{20}$$

$$C_1^{(\Delta)}(k_z, s) = -\frac{1}{SK_0(S)} (n_1 [J(Q_2) + K(S)] \mathscr{J}(Q_1) - n_2 [J(Q_1) + K(S)] \mathscr{J}(Q_2)), \tag{21}$$

$$C_2^{(\Delta)}(k_z, s) = -\frac{1}{SK_0(S)} ([\eta J(Q_2) + \tilde{\varepsilon} K(S)] \mathscr{J}(Q_1) - [\eta J(Q_1) + \tilde{\varepsilon} K(S)] \mathscr{J}(Q_2)), \tag{22}$$

$$\begin{aligned} \Delta(k_z, s) = {} & K(S)[J(Q_1)(n_1\eta - \tilde{\varepsilon} n_2) - J(Q_2)(n_2\eta - \tilde{\varepsilon} n_1)] \\ & + (n_1 - n_2)(\eta J(Q_1) J(Q_2) + \tilde{\varepsilon} K^2(S)), \end{aligned} \tag{23}$$

where

$$J(Q_k) = \frac{J_1(Q_k)}{Q_k J_0(Q_k)}, \quad \mathscr{J}(Q_k) = \frac{J_1(Q_k b/a)}{Q_k J_0(Q_k)}, \quad K(S) = \frac{K_1(S)}{SK_0(S)}. \tag{24}$$

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For the case of a metal boundary ($E_\phi = E_z = 0$ for $\rho > a$), (20) and (23) reduce to the simple form

$$B_{1,2}^{(\Delta)}(k_z, s) = \frac{n_{1,2} J(Q_{2,1}) G_{1,2}^{(0)}(a) - n_{2,1} G_{1,2}^{(1)}(a) + n_{2,1} \mathscr{J}(Q_{2,1})}{(n_2 - n_1) Q_{2,1} J_0(Q_{2,1})}, \tag{25}$$

$$\Delta(k_z, s) = J(Q_2) n_1 - n_2 J(Q_1). \tag{26}$$

Note that such a kind of boundary condition was also used in early theories devoted to whistler excitation in plasma columns [45].

Finally, combining (15), (16) and (19), one obtains a rigorous solution to the problem of wave field excitation by a modulated beam into a magnetoplasma column surrounded by a uniform medium. Applying the inverse Fourier–Laplace transform to $\mathbf{E}$ (E_ρ, E_ϕ, E_z) and $\mathbf{B}$ (B_ρ, B_ϕ, B_z), one gets

$$\begin{bmatrix} \mathbf{E}(\rho, z, t) \\ \mathbf{B}(\rho, z, t) \end{bmatrix} = \int_{-\infty}^{+\infty} \frac{dk_z}{2\pi} e^{-ik_z z} \int_{\sigma_0 - i\infty}^{\sigma_0 + i\infty} \begin{bmatrix} \mathbf{E} \\ \mathbf{B} \end{bmatrix} e^{st} \frac{ds}{2\pi i}, \tag{27}$$

where $\sigma_0 > 0$ is a real constant. We do not dwell here on the calculation of the integrals in (27), as the aim of the paper is to estimate the time-averaged power radiated by the beam.

3. Average power radiated at the beam modulation frequency

The power lost by the beam can be expressed as

$$P = -\int_V j(\rho, z, t) E_z(\rho, z, t) \rho \, d\rho \, d\phi \, dz, \tag{28}$$

where the integration is performed on the beam volume V. Using the inverse Fourier–Laplace transform of E_z and expressing $E_z(\rho, z, t)$ as a function of the azimuthal component of the magnetic field $B_\phi(\rho, z, t)$, (28) can be written in the form

$$P = -\int_0^b \rho \, d\rho \int_0^L dz \, j(\rho, z, t) \int_{-\infty}^{+\infty} dk_z \, e^{-ik_z z} \times \frac{1}{2\pi i} \int_{\sigma_0 - i\infty}^{\sigma_0 + i\infty} \frac{c}{s\eta} \left[Z_0 j(\rho, k_z, s) - \frac{c}{\rho} \frac{\partial}{\partial \rho} (\rho B_\phi(\rho, k_z, s)) \right] e^{st} \, ds, \tag{29}$$

where $L = v_\mathrm{b} t$ is the instantaneous beam length. Performing the integration with respect to ρ using (3), (15) and (19), one obtains

$$P = Z_0 (e n_\mathrm{b} v_\mathrm{b})^2 c b^2 \int_0^L dz \, e^{-\beta z} \sin[\omega_0 (t - z/v_\mathrm{b})] \int_{-\infty}^{+\infty} dk_z \, e^{-ik_z z} \times \frac{1}{2\pi i} \int_{\sigma_0 - i\infty}^{\sigma_0 + i\infty} \frac{1}{s^2 \eta} \frac{v_\mathrm{b}}{s - i(k_z + i\beta) v_\mathrm{b}} \left(1 + \frac{s\omega_0}{s^2 + \omega_0^2} \right) \frac{\mathscr{H}_\phi(b, k_z, s)}{\Delta(k_z, s)} e^{st} \, ds, \tag{30}$$

where we define the complex function $\mathscr{H}_\phi$ as

$$\mathscr{H}_\phi(b, k_z, s) = -\sum_{k=1}^{2} n_k B_k^{(\Delta)}(k_z, s) J_1(Q_k b/a). \tag{31}$$

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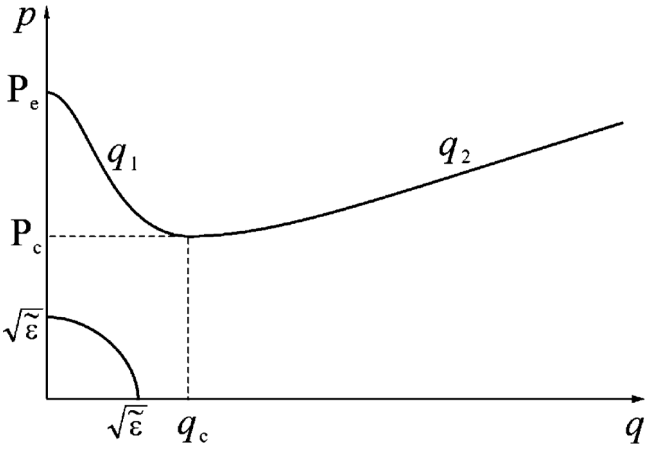


Figure 1. Refractive index surfaces for whistler waves propagating in a collisionless magnetoplasma, in the frequency range $\omega_{\rm lh} \ll \omega < \omega_{\rm c}/2 \ll \omega_{\rm p}$ (upper curve), and (ii) for electromagnetic waves propagating in a uniform medium of permittivity $\tilde{\varepsilon}$ (lower curve).

As we are interested in the time-averaged power lost by the beam at the modulation frequency ω_0, the dc component of the current in (30) has been discarded.

We now perform the integration over s neglecting the contributions of the possible implicit roots of the equations $\Delta(k_z, s) = 0$ and $s^2\eta(s) = 0$, of the pole $s = (ik_z - \beta)v_{\rm b}$ and of the branch points of the integrand. All of them do not contribute to the forced regime which is discussed here. Using the Cauchy's theorem of residues, we get the time-averaged power lost at the modulation frequency ω_0,

$$\langle P\rangle = Z_0(en_{\rm b}v_{\rm b})^2\frac{cb^2}{2i}\left\langle\int_0^L dz\, e^{-\beta z}\sin[\omega_0(t - z/v_{\rm b})]\right.$$

$$\left.\times\int_{-\infty}^{+\infty}[\mathscr{I}(k_z,\omega_0) - \mathscr{I}(k_z,-\omega_0)]\, e^{-ik_z z}\, dk_z\right\rangle, \tag{32}$$

$$\mathscr{I}(k_z,\omega) = \frac{\mathscr{H}_\phi(b,k_z,\omega)}{\eta(\omega)\omega\Delta(k_z,\omega)}\frac{e^{i\omega t}}{k_z + i\beta - \omega/v_{\rm b}}, \tag{33}$$

where $\langle\cdot\rangle$ represents time averaging over the period $2\pi/\omega_0$. The properties of the functions $\mathscr{H}_\phi$ and Δ in (33) depend on the band in which the modulation frequency ω_0 lies. Here we assume that ω_0 belongs to the whistler band. It is worth recalling that within this range and for $\rho < a$, the only propagating electromagnetic mode is the so-called extraordinary (whistler) mode, while the ordinary mode is evanescent. Figure 1 shows the corresponding refractive index surfaces in the case of a collisionless magnetoplasma and of an isotropic dielectric medium, for the frequency interval $\omega_{\rm lh} \ll \omega < \omega_{\rm c}/2 \ll \omega_{\rm p}$. The lower curve corresponds to the electromagnetic waves propagating in the dielectric medium of permittivity $\tilde{\varepsilon}$. The whistler mode refractive index surface (upper curve of Figure 1) is described by $q = q_1(p)$ when $0 \leqslant q \leqslant q_{\rm c}$ and by $q = q_2(p)$ when $q_{\rm c} < q$; $p = ck_z/\omega$ (respectively, q) is the normalized parallel (respectively, perpendicular) wave number and the point $q_{\rm c} = q_{1,2}(P_{\rm c})$ (14) corresponds to the so-called conical refraction whistler waves. As is well known, the branch $q_1(p)$ (respectively, $q_2(p)$) describes helicon (respectively, quasi-electrostatic) waves for $P_{\rm c} < p < P_{\rm e}$ (respectively, $p > P_{\rm c}$); $P_{\rm e} = \sqrt{\varepsilon - g}$ is the normalized parallel

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wave number of the helicon wave propagating along $\mathbf{B}_0$ in a collisionless homogeneous magnetoplasma. Note that, in the whistler range, one has $P_\mathrm{b} \simeq 0$ and $P_\mathrm{b} \ll P_\mathrm{c}$.

We should take into account that, in the whistler band, a plasma column described by the dielectric tensor $\hat{\varepsilon}$ in (1) can support slightly attenuated guided propagating eigenmodes, whose parallel wave numbers $k_{z,\nu} = k'_{z,\nu} - ik''_{z,\nu}$ are roots of the equation $\Delta(k_z, \omega) = 0$, with $\nu = 1, 2, \ldots$. Since $\Delta(k_z, \omega)$ stands in the denominator of (33), the integrand of (32) possesses poles at the points where $\Delta(k_z, \omega)$ vanishes. It is obvious that a collisionless magnetized plasma column is capable of supporting transversely localized eigenmodes with real parallel wave numbers p if $p > \sqrt{\bar{\varepsilon}}$ (in this case the quantity S is purely real). For $\sqrt{\bar{\varepsilon}} < p < P_\mathrm{c}$, the column can support a single eigenmode of surface type ($q_1 = q_2^*$) whose field inside the column is localized near the column boundary. As a result, this mode is weakly excited by a thin electron beam injected along the axis of the column and one can neglect its contribution to the time-averaged power $\langle P \rangle$ even if the Cherenkov resonance condition holds. Note that the presence of collisional losses in the plasma column only slightly affects the field structure of this mode. Here we are mostly interested in the modes verifying $\mathrm{Re}(P_\mathrm{c}) < ck'_{z,\nu}/\omega < \mathrm{Re}(P_\mathrm{e})$, or even $ck'_{z,\nu}/\omega \gtrsim \mathrm{Re}(P_\mathrm{e})$. The eigenmodes are of volume type if $\mathrm{Re}(P_\mathrm{c}) < ck'_{z,\nu}/\omega < \mathrm{Re}(P_\mathrm{e})$; in this interval, both perpendicular wave numbers q_1 and q_2 are real for a loss-free plasma column. If $ck'_{z,\nu}/\omega \gtrsim \mathrm{Re}(P_\mathrm{e})$, q_1 is imaginary but q_2 is real. Note that the dispersion properties and the field structures of whistler modes guided by a cylindrical magnetoplasma column surrounded by free space are discussed in [46].

Now we carry out the integration over k_z in (32) passing to the limit $\beta \to 0^+$ and applying the Plemelj' formula for analytic continuation [47]. Upon time averaging and integrating the resultant expression with respect to z using the relation

$$\frac{\mathscr{H}_\phi(b, -k_z^*, -\omega_0)}{\eta(-\omega_0)\Delta(-k_z^*, -\omega_0)} = \frac{\mathscr{H}_\phi^*(b, k_z, \omega_0)}{\eta^*(\omega_0)\Delta^*(k_z, \omega_0)}, \tag{34}$$

we obtain a general expression for the average power $\langle P \rangle$ lost by the beam at the modulation frequency ω_0,

$$\begin{aligned}\langle P \rangle = \frac{P_0 L c}{2\pi\omega_0 b^2} \Bigg\{ & \mathrm{Im}\left(\frac{\mathscr{H}_\phi(b, \omega_0/v_\mathrm{b}, \omega_0)}{\eta\Delta(\omega_0/v_\mathrm{b}, \omega_0)} \right) \\ & + \int_{-\infty}^{+\infty} \mathrm{Im}\left(\frac{\mathscr{H}_\phi(b, \kappa_z + \omega_0/v_\mathrm{b}, \omega_0)}{\eta\Delta(\kappa_z + \omega_0/v_\mathrm{b}, \omega_0)} \right) \frac{\sin^2(\kappa_z L/2)}{\pi\kappa_z^2 L/2}\, d\kappa_z \\ & + \mathrm{v.p.} \int_{-\infty}^{+\infty} \mathrm{Re}\left(\frac{\mathscr{H}_\phi(b, \kappa_z + \omega_0/v_\mathrm{b}, \omega_0)}{\eta\Delta(\kappa_z + \omega_0/v_\mathrm{b}, \omega_0)} \right) \frac{\sin(\kappa_z L)}{\pi\kappa_z L} \frac{d\kappa_z}{\kappa_z} \Bigg\}, \end{aligned} \tag{35}$$

where $P_0 = Z_0 I_0^2$; v.p. represents the principal value of the integral. Passing to the limit $L \to \infty$ in (35) gives the expression for the power radiated per unit length in the column by an infinitely long beam,

$$\lim_{L\to\infty} \frac{\langle P \rangle}{L} = \frac{cP_0}{\pi\omega_0 b^2} \mathrm{Im}\left(\frac{\mathscr{H}_\phi(b, \omega_0/v_\mathrm{b}, \omega_0)}{\eta\Delta(\omega_0/v_\mathrm{b}, \omega_0)} \right). \tag{36}$$

Equation (36) represents exactly the power radiated per unit length at the modulation frequency ω_0 whereas (35), which refers to a beam of increasing but finite length, comprises both the total power radiated and the power stored per unit time. As L increases, the latter becomes almost independent of L, while the former increases proportionally with L. As a result, the quantity $\langle P \rangle / L$, in the limit of

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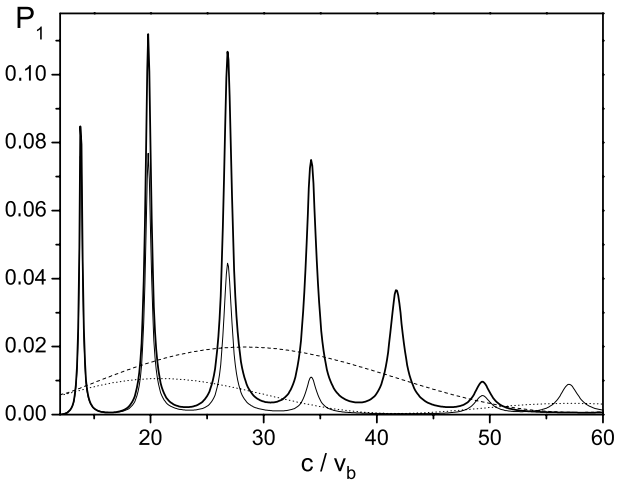


Figure 2. Variation of the normalized average power radiated per unit length, $P_1 = \langle P\rangle/LP_0$ (in arbitrary units), as a function of the normalized parameter c/v_b, for different values of the beam radius b, when the beam is injected (i) into a magnetoplasma column of radius a ($b = 0.005$ m, thick solid line; $b = 0.007$ m, thin solid line), and (ii) into an unbounded magnetoplasma ($b = 0.005$ m, dashed line; $b = 0.007$ m dotted line. The main physical parameters are: $\widetilde{\varepsilon} = 1$, $a = 0.03$ m, $\omega_\mathrm{c}/2\pi = 7 \times 10^8$ Hz, $\omega_\mathrm{p}/2\pi = 4 \times 10^9$ Hz, $\omega_0/2\pi = 2.5 \times 10^8$ Hz, $\nu_\mathrm{e}/2\pi = 4 \times 10^6$ Hz.

$L \to \infty$, goes over into the radiated power per unit length, as expected. It should be noted that, in the range close to the resonant points $k'_{z,\nu} = \omega_0/v_\mathrm{b}$, the power $\langle P\rangle/L$ approaches the result obtained in the limit $L \to \infty$ if the beam length becomes much greater than the characteristic mode attenuation length $k''^{-1}_{z,\nu}$. However, if the value ω_0/v_b is not close enough to the resonant points, $\langle P\rangle/L$ becomes independent of L if L significantly exceeds the wavelength of the whistler mode traveling along $\mathbf{B}_0$, which is usually smaller than the characteristic damping length of the guided modes (see, for more details, [18, 19]).

4. Discussion

Let us present some relevant numerical calculations of the time-averaged power radiated by beams for physical conditions typical of discharge plasma laboratory experiments involving whistler wave propagation in rather dense or weakly magnetized plasmas [33–39]. The calculations have been performed for the case when the beam length L is not too small so that the power $\langle P\rangle/L$ can be estimated by (36). We are mostly interested in the radiation by beams modulated at frequencies ω_0 lying in the range $\omega_\mathrm{lh} \ll \omega_0 < \omega_\mathrm{c}/2 \ll \omega_\mathrm{p}$, where a magnetic-field-aligned plasma column is capable of guiding several slightly attenuated whistler wave modes. In particular, the contribution of such modes to the radiated power is significant when Cherenkov resonance conditions $\omega_0/v_\mathrm{b} \simeq k'_{z,\nu}$ are realized between the beam and the wave modes. Figures 2–5 show the typical behavior of the normalized average power radiated by the beam per unit length, $P_1 = \langle P\rangle/LP_0$, as a function of c/v_b, for different values of the characteristic parameters: the beam and the plasma column radii, the surrounding medium permittivity and the electron collision frequency inside the plasma column. Note that the range of variation $20 < c/v_\mathrm{b} < 70$ corresponds to total beam energies of 50–700 eV.

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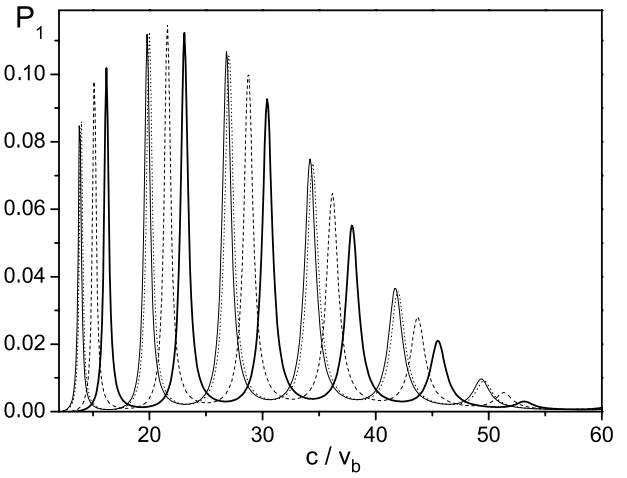


Figure 3. Variation of the normalized average power radiated per unit length, $P_1 = \langle P \rangle / LP_0$ (in arbitrary units), as a function of c/v_b, for different values of the dielectric permittivity of the isotropic surrounding medium: $\widetilde{\varepsilon} = 1$ (thin solid line), $\widetilde{\varepsilon} = 10$ (dotted line) and $\widetilde{\varepsilon} = 100$ (dashed line); the case of a metallic boundary is represented by the thick solid line. The main physical parameters are: $b = 0.005$ m, $a = 0.03$ m, $\omega_\mathrm{c}/2\pi = 7 \times 10^8$ Hz, $\omega_\mathrm{p}/2\pi = 4 \times 10^9$ Hz, $\omega_0/2\pi = 2.5 \times 10^8$ Hz, $\nu_\mathrm{e}/2\pi = 4 \times 10^6$ Hz.

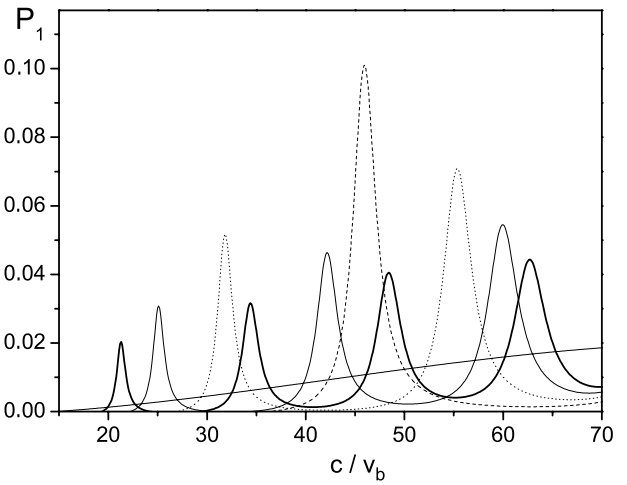


Figure 4. Variation of the normalized average power radiated per unit length, $P_1 = \langle P \rangle / LP_0$ (in arbitrary units), as a function of c/v_b, for different values of the plasma column radius a: $a = 0.02$ m (dashed line), $a = 0.03$ m (dotted line), $a = 0.04$ m (thin solid line), $a = 0.05$ m (thick solid line), $a \to \infty$ (thin solid line with smooth variation). The main physical parameters are: $\widetilde{\varepsilon} = 1, b = 0.005$ m, $\omega_\mathrm{c}/2\pi = 2.2 \times 10^8$ Hz, $\omega_\mathrm{p}/2\pi = 1.5 \times 10^9$ Hz, $\omega_0/2\pi = 7 \times 10^7$ Hz, $\nu_\mathrm{e}/2\pi = 2 \times 10^6$ Hz.

Figure 2 shows the variation of P_1 as a function of c/v_b for two different values of the beam radius b. The solid (respectively dashed and dotted) lines correspond to the case when the beam is injected into a magnetoplasma column surrounded by a uniform medium (respectively into an unbounded uniform magnetoplasma). When the plasma is unbounded, the radiated power varies smoothly as a function of c/v_b.

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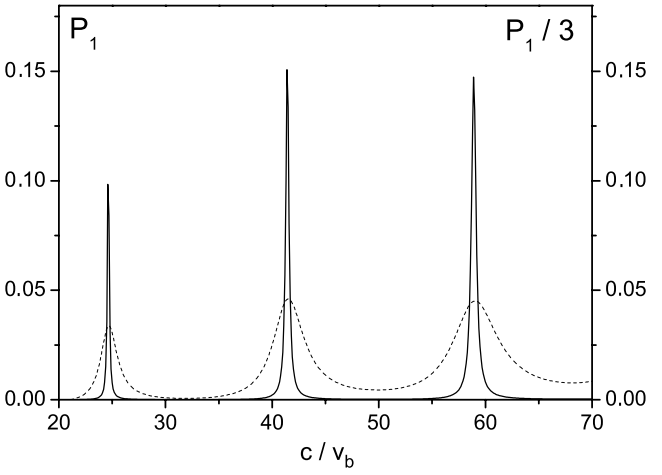


Figure 5. Variation of the normalized power radiated per unit length, $P_1 = \langle P \rangle / LP_0$ (in arbitrary units), as a function of c/v_b, for two different values of the plasma collision frequency: $\nu_\mathrm{e}/2\pi = 4 \times 10^5$ Hz (solid lines) and $\nu_\mathrm{e}/2\pi = 4 \times 10^6$ Hz (dashed lines). Note that the left (respectively right) vertical axis represents P_1 (respectively $P_1/3$) and corresponds to the frequency $\nu_\mathrm{e}/2\pi = 4 \times 10^6$ Hz (respectively $\nu_\mathrm{e}/2\pi = 4 \times 10^5$ Hz). The main physical parameters are: $\widetilde{\varepsilon} = 1$, $b = 0.005$ m, $a = 0.03$ m, $\omega_\mathrm{c}/2\pi = 3 \times 10^8$ Hz, $\omega_\mathrm{p}/2\pi = 2 \times 10^9$ Hz, $\omega_0/2\pi = 10^8$ Hz.

However, the presence of a boundary at $\rho = a$ strongly affects the dependence of P_1 on the beam velocity v_b: at the points $c/v_\mathrm{b} = ck'_{z,\nu}/\omega_0$, the power reveals resonance peaks which are related to the Cherenkov excitation of guided modes propagating with the parallel wave numbers $k_{z,\nu} = k'_{z,\nu} - ik''_{z,\nu}$. The slightly attenuated modes guided by the plasma column belong, for our set of parameters, to the interval $\mathrm{Re}(P_\mathrm{e}) \lesssim ck'_{z,\nu}/\omega_0$ (see also Fig. 1). As expected, the excitation coefficients of the waves (19) depend significantly on the beam radius (at $c/v_\mathrm{b} \simeq 42$, for example, only the peak corresponding to one value of b can be observed), what is not the case for the location of the resonant peaks.

As shown by Fig. 3, the influence of the dielectric permittivity $\widetilde{\varepsilon}$ of the surrounding medium on the power radiated at the guided modes is rather weak if $|\widetilde{\varepsilon}| \lesssim (ck'_{z,\nu}/\omega_0)^2$. The limiting case of infinite $\widetilde{\varepsilon}$, which corresponds to metallic boundaries, is represented by the thick solid line. Then, in laboratory experiments, the nature of the surrounding medium should not influence significantly the efficiency of the beam emission of guided modes. Moreover, as checked by our calculations, no noticeable difference between the power radiated exists when $|\widetilde{\varepsilon}|$ varies roughly from 1 to 10 (see the almost indistinguishable thin solid and dotted lines in Fig. 3).

The number of resonance peaks depends mainly on the radius a of the plasma column, as shown by Fig. 4 which presents the variation of P_1 as a function of c/v_b for different values of a, the rest of the parameters being fixed. One observes that the number of guided modes contributing to the radiated power increases with a. When the magnetoplasma is unbounded, the variation of P_1 is smooth (see the thin solid and almost straight line). The level of the power radiated by the beam at the resonant peaks is enhanced when the plasma is confined in a column surrounded by a uniform medium, compared to the case when it is unbounded. For some values

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of the beam velocity, this enhancement can reach one order of magnitude (see, for example, the dashed line).

Fig. 5 shows the variation of P_1 as a function of c/v_b for two different values of the effective electron collision frequency. The dashed (respectively solid) line corresponds to the power P_1 (respectively $P_1/3$) emitted by the beam injected into a plasma column characterized by a collision frequency ν_e (respectively $\nu_e/10$). It should be noted that the damping rate of the modes guided by the column increases with ν_e. As expected, when the collision frequency increases, the power radiated at the guided modes decreases essentially while the width of the peak corresponding to each mode increases. So, a sufficiently high rate of collisions inside the plasma column can reduce the beam radiation at the resonance peaks to negligible levels.

As final remark, note that the model of the step-shaped radial density profile used to describe the plasma column works rather well for realistic situations if the characteristic perpendicular length $L_\perp$ which scales the density fall-off near the column boundary satisfies the condition $L_\perp \ll 2\pi(k_0|q_2|)^{-1}$. The more complex case of a smoothly varying density profile needs a separate consideration that will be achieved in forthcoming works.

5. Conclusion

In this paper we have studied the whistler wave radiation by a density modulated and thin electron beam of finite length injected parallel with respect to the constant ambient magnetic field into an axisymmetrical cylindrical plasma column surrounded by a uniform isotropic dielectric medium. A rigorous analytical derivation of the wave fields excited by the beam in the plasma column is presented. The time-averaged power radiated by the beam at the modulation frequency is determined as a function of the surrounding medium dielectric permittivity and of the plasma column's and beam's characteristics. For physical conditions typical of relevant laboratory magnetized plasmas, numerical calculations show that, when Cherenkov resonance between the beam and the whistler modes guided in the plasma column is realized, the time-averaged power radiated by the beam at the guided modes is significantly increased compared to the case when the beam is injected in a homogeneous unbounded magnetoplasma.

Forthcoming works will be devoted to the generalization of the theoretical model (i) by considering more realistic density profiles inside the plasma column, (ii) by treating the case of a beam spiraling in a magnetoplasma column and (iii) by including the nonlinear description of the wave–particle interactions.

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