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GROWTH OPTION AND DEBT MATURITY WITH EQUITY DEFAULT SWAPS IN A REGIME-SWITCHING FRAMEWORK

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We consider a firm with assets-in-place and a growth option. There is a funding gap for the expansion investment, which is covered by entering into an equity-for-guarantee swap or fee-for-guarantee swap. We explicitly derive all contingent claim prices with the pricing and timing of the growth option taking business cycle and debt maturity into account. For short-term loan, we produce an explanation why Chinese government suggests that guarantee fee rate should be approximately the fraction 50% of the interest rate of bank loans with the same maturity, but for long-term debt, we show this fraction is too small.

Keywords: Growth Option, Equity Default Swaps, Business Cycle

1. INTRODUCTION

Motivation: Small- and medium-sized enterprises (SMEs) usually face financing difficulties due to their limited wealth, lack of collaterals, and institutional frictions such as information asymmetry and agency costs. To overcome the financing constraints, credit guarantees are great financial innovations. In fact, guarantees of financial performance on loans and even real investment projects are widely used in China and many other countries. As argued by Merton and Bodie (1992), parent corporations frequently guarantee the debt obligations of their subsidiaries. Commercial banks, insurance companies, and sometimes sovereigns offer guarantees in return for fees on a long list of financial instruments. Currently, the credit guarantee market has developed very fast. In China, the amount of outstanding guaranteed debt arrived at 1.1503 trillion RMB at the end of 2010. The number increased to 1.9120 trillion in 2011, 2.1356 trillion in 2012, and 2.5376 trillion in 2013, reported by the web site www.chinabgao.com. There were 8,590

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credit guarantee companies in 2012, according to Chenghui Zhang, a researcher of the Chinese State Council. However, there is a major shortcoming in current guarantee contracts, i.e., credit guarantee companies are exposed to too much loss risk, whereas the benefits they obtain are very limited. As a result, many small businesses are difficult to get a guarantee service since a lot of credit guarantee companies refuse to take such unfair guarantee. There are also medium- and large-sized enterprises facing a similar problem in operating assets in place and growth option. To overcome the shortcoming, the equity-for-guarantee swap (EGS) has been invented since 2002 but there are no theories studying how its price varies with debt maturity, economic regime, and how it impacts on the pricing and timing of the growth option of a firm. On the other hand, for a long time, the most popular guarantee agreements have still been the fee-for-guarantee swaps (FGSs). Chinese government suggests that their guarantee fee rates had better be approximately the fraction 50% of the interest rate of bank loans with the same maturity. However, there are no papers in the literature telling us whether or why the fraction is appropriate, how it is related with debt maturity and the economic regime shift, and what the advantages or disadvantages of EGSs over FGSs are. In this paper, we aim to provide answers to the questions raised above.

Our contributions: We consider a firm (e.g., SME) with assets-in-place. The firm has an irreversible but delayable investment opportunity. Once the investment is undertaken, the firm's cash flow increases by a factor after paying investment cost I. In contrast to the classical model, we assume that the firm faces a funding gap, which is covered by bank loans after entering into an EGS or FGS, which is a new type of equity default swaps. We adopt stationary debt structure with finite maturity, and the cash flow follows a geometric Brownian motion but its drift depends on the state of the economic regime, which is described by a two-state Markov chain. Using a risk-neutral pricing method, we explicitly derive the prices of corporate securities and guarantee costs, and the pricing and timing of the growth option. We examine the impacts of EGSs and FGSs on the growth option. We find that the investment threshold in recession is higher than that in boom, and EGSs induce a postponement of investment in comparison with FGSs. The guarantee costs in boom are higher than those in recession. For short-term loans, we produce an explanation why Chinese government suggests that guarantee fee rate should be approximately the fraction 50% of the interest rate of the bank loans with the same maturity, but for long-term debt, we show this fraction is too small. EGSs can bring more tax shields than FGSs, and their advantage increases with the funding gap to be covered. EGSs induce somewhat more serious inefficiencies from debt overhang but a weaker risk-shifting incentive than FGSs. We believe these studies add to a highly relevant recent topic and are of significant relevance for policy makers.

Literature review: In their seminal work, Brennan and Schwartz (1985) use option pricing theory to fix the value and the optimal production policies of a natural resource investment. After that, real options theory, which captures the value of flexibility under uncertainty, has attracted growing attention following McDonald

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and Siegel (1986). Recently, Hackbarth and Mauer (2012) among others investigate the investment problems for a firm with growth options. Additionally, a vast literature is dedicated to the effect of business cycle, and most of the recent works presume the exogenous regime switching.[1] Chen (2010) and Bhamra et al. (2010) incorporate the business cycle into a consumption-based asset pricing model, and the diffusion regime of the state process is assumed to be switched via Markov chain. Based on their models, Arnold et al. (2013) further consider an irreversible growth option. Arnold et al. (2017) investigate when firms sell assets to fund investments across the business cycle. Luo and Yang (2017) consider a firm's real investment in a regime-switching economy after taking contingent convertibles into account. Guo et al. (2005) examine the problem of irreversible investment given the exogenous shift of diffusion regimes, but they consider only an all-equity firm. Hackbarth et al. (2006) examine the effect of cyclical cash flow with rollover debt on the default decision of a firm. But their model does not consider the growth option. Jeon and Nishihara (2015) consider switching from recession to boom as an investment in production facilities and examine the impact of business cycle and debt maturity on investment and financing. However, all these papers do not take EGSs or FGSs into account.

EGSs and FGSs are actually two new types of equity default swaps. Our paper is most closely related with Yang and Zhang (2013), Yang and Zhang (2015), Xiang and Yang (2015), Wang et al. (2015), and Luo et al. (2016). These papers study EGSs or similarly the option-for-guarantee swaps. However, all of them assume that debt is perpetual, i.e., it pays out a fixed coupon per unit time forever until default and never repays principal. Clearly, debt maturity is generally finite, and it is of course an important determinant of EGSs or FGSs. Further, it is common for many borrowers, suffering financing constraints, to undertake an expansion investment in a regime-switching environment. Nevertheless, all these papers on guarantee swaps do not take these factors into account.

The structure of the paper is as follows. Section 2 is model setup. Section 3 discusses the pricing and timing of a growth option under EGSs. Section 4 considers FGSs instead of EGSs. Section 5 provides numerical results and analysis. Section 6 concludes. Technical proofs are relegated to the appendix.

2. MODEL SETUP

Let $(\Omega; \mathcal{F}; \mathbb{P})$ be a probability space on which a standard Brownian motion B and a two-state continuous-time Markov chain L are defined. The Markov chain L is right-continuous with values in $\{0, 1\}$ without loss of generality, representing the bad (recession) and good (boom) states of the economy regime, respectively. We suppose the process L, starting at $i \in \{0, 1\}$, remains there for an exponentially distributed length of time with jump intensity parameter being λ_i, and then jumps to state $j \in \{0, 1\}(j \neq i)$. The exponentially distributed interregime times are independent, and independent of Brownian motion B.

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Consider a firm with assets-in-place and a growth option. Its cash flow X before expansion investment, which is earnings before interest and tax generated by assets-in-place of the firm, is described by the following geometric Brownian motion:

$$dX_t = \mu(L_t)X_t dt + \sigma X_t dB_t, \tag{1}$$

where $\mu(L_t)$ is the risk-adjusted drift parameter determined by the state L of economic environment (recession or boom), and $\sigma > 0$ is the constant volatility. Once the growth option is exercised, the firm's cash flow level increases immediately from X_t to aX_t all the time after paying investment cost I. We call the parameter "a" the growth scale, which is a constant satisfying $a > 1$.

We assume that the firm faces a funding gap P for the expansion investment that is covered by entering into a three-party agreement (EGS or FGS) with a bank and an insurer. Under the agreement, the bank lends a loan at a given interest rate to the firm and gets constant coupon payment c from the firm. Once the firm defaults, the insurer gets the salvage value of the firm and pays all the outstanding interest and principal to the bank instead of the firm. Under an EGS (FGS), in return for the guarantee, the firm allocates a fraction of equity (loan) to the insurer at the starting time. We call the fraction the *guarantee cost*.[2]

Following Jeon and Nishihara (2015), we adopt stationary debt structure with finite maturity. Namely, the firm issues debt with principal P, pays a coupon c constantly, and rolls over a fraction m of the total debt. The average maturity of the debt is therefore $1/m$ if bankruptcy never occurs, and the debt structure is completely characterized by a tuple (c, m, P). Thanks to the pecking order theory of debt, we naturally assume that the principal P equals the funding gap. As usual, debt is issued at par and the coupon c is accordingly determined.

Similar to Goldstein et al. (2001), we assume a simple tax structure that includes corporate and personal taxes, where interest payments are taxed at a personal rate τ_i, effective dividends are taxed at τ_d, and corporate profits are taxed at τ_c, with full loss offset provisions. Naturally, $\tau_f \equiv 1 - (1 - \tau_c)(1 - \tau_d) > 0$ represents the corporate effective tax rate.

3. THE PRICING AND TIMING OF THE GROWTH OPTION UNDER EGSs

In this section, we first derive the prices of corporate securities without a guarantee. Then, we compute the fair guarantee cost for the EGS and address the pricing and timing of the growth option.

3.1. The Pricing of Corporate Securities without a Guarantee

To this end, we first compute the unlevered firm's value without taking the growth option into account. Next, we utilize a backward induction method to derive the prices of corporate securities after and before expansion investment, respectively.

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The derivations are similar to those presented by Luo et al. (2017), and therefore, our exposition here is as short as possible to save space.

We assume that the current cash flow level is x and economic state is $l \in \{0, 1\}$. We emphasize that the real cash flow rate received by the firm is nevertheless ax after expansion investment instead of x before the investment. Clearly, all the prices depend on both x and l. Hence, we denote by $V_l(x)$ the unlevered firm's value. Following Luo et al. (2017), we have for $l \in \{0, 1\}$

$$V_l(x) = q_l x, \text{ and } q_l \equiv \frac{(1-\tau_f)(r+\lambda_0+\lambda_1-\mu_0-\mu_1+\mu_l)}{(r+\lambda_0-\mu_0)(r+\lambda_1-\mu_1)-\lambda_0\lambda_1}, \tag{2}$$

where the constant r is the risk-free interest rate.

After expansion investment, the firm issues debt, of which default thresholds are endogenously determined by shareholders. We denote by x_1^b the default threshold in boom and by x_0^b the threshold in recession. Naturally, we have $x_1^b < x_0^b$. Should the firm go default on its debt, the value of the firm's future cash flow is assigned to the debt holders but a fraction, denoted by α, of the value of the future cash flow will lost due to bankruptcy costs. We call α the bankruptcy loss rate.

Denoting by $J_l(x), l \in \{0, 1\}$ the firm value after expansion investment and letting $g_l(\gamma) = r + \lambda_l - \mu_l\gamma - \frac{\sigma^2}{2}\gamma(\gamma-1)$, γ_1 and γ_2 be the two negative roots of the equation $g_0(\gamma)g_1(\gamma) = \lambda_0\lambda_1$ and β_1 and β_2 be two roots of the equation $g_1(\gamma) = 0$, we conclude from Luo et al. (2017) the following proposition.

PROPOSITION 3.1. *For given default thresholds x_1^b in boom and x_0^b in recession, the firm value after expansion investment is shown below:*

$$J_0(x) = \begin{cases} A_1x^{\gamma_1} + A_2x^{\gamma_2} + q_0ax + \frac{(\tau_f-\tau_i)c}{r}, & \text{if } x > x_0^b, \\ (1-\alpha)q_0ax, & \text{if } x \le x_0^b; \end{cases}$$

and

$$J_1(x) = \begin{cases} B_1x^{\gamma_1} + B_2x^{\gamma_2} + q_1ax + \frac{(\tau_f-\tau_i)c}{r}, & \text{if } x > x_0^b, \\ E_1x^{\beta_1} + E_2x^{\beta_2} + \frac{(1-\tau_f)ax}{r+\lambda_1-\mu_1} + \frac{\lambda_1(1-\alpha)q_0ax}{r+\lambda_1-\mu_1} + \frac{(\tau_f-\tau_i)c}{r+\lambda_1}, & \text{if } x_1^b \le x \le x_0^b, \\ (1-\alpha)q_1ax, & \text{if } x < x_1^b, \end{cases}$$

where $B_1 = \frac{\lambda_1}{g_1(\gamma_1)}A_1$ and $B_2 = \frac{\lambda_1}{g_1(\gamma_2)}A_2$. The constants A_1, A_2, E_1, and E_2 constitute a column vector $(A_1, A_2, E_1, E_2)'$, which is the solution of the system of linear equations: $\Gamma_1 X = \Phi_1$, where

$$\Gamma_1 \equiv \begin{pmatrix} (x_0^b)^{\gamma_1} & (x_0^b)^{\gamma_2} & 0 & 0 \\ \frac{\lambda_1}{g_1(\gamma_1)}(x_0^b)^{\gamma_1} & \frac{\lambda_1}{g_1(\gamma_2)}(x_0^b)^{\gamma_2} & -(x_0^b)^{\beta_1} & -(x_0^b)^{\beta_2} \\ \gamma_1\frac{\lambda_1}{g_1(\gamma_1)}(x_0^b)^{\gamma_1} & \gamma_2\frac{\lambda_1}{g_1(\gamma_2)}(x_0^b)^{\gamma_2} & -\beta_1(x_0^b)^{\beta_1} & -\beta_2(x_0^b)^{\beta_2} \\ 0 & 0 & (x_1^b)^{\beta_1} & (x_1^b)^{\beta_2} \end{pmatrix}, \tag{3}$$

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$$\Phi_1 \equiv \begin{pmatrix} -\alpha q_0 a x_0^b - \frac{(\tau_f-\tau_i)c}{r} \\ \frac{(1-\tau_f)ax_0^b}{r+\lambda_1-\mu_1} + \frac{\lambda_1(1-\alpha)q_0 a x_0^b}{r+\lambda_1-\mu_1} - q_1 a x_0^b - \frac{\lambda_1(\tau_f-\tau_i)c}{(r+\lambda_1)r} \\ \frac{(1-\tau_f)ax_0^b}{r+\lambda_1-\mu_1} + \frac{\lambda_1(1-\alpha)q_0 a x_0^b}{r+\lambda_1-\mu_1} - q_1 a x_0^b \\ (1-\alpha)q_1 a x_1^b - \frac{(1-\tau_f)ax_1^b}{r+\lambda_1-\mu_1} - \frac{\lambda_1(1-\alpha)q_0 a x_1^b}{r+\lambda_1-\mu_1} - \frac{(\tau_f-\tau_i)c}{r+\lambda_1} \end{pmatrix}.$$

Following Guo et al. (2005), we call the sets $[x_i^b, \infty)$ and $(0, x_i^b]$ the continuation region and the stopping region for the firm value $J_i(x)$, respectively, whereas the set $[x_1^b, x_0^b]$ is called the transient region. Proposition 3.1 provides the firm value after expansion investment in a levered firm when the dynamics of cash flows from assets in place is governed by the Markov regime switching process (1). It says that the firm value after expansion investment in the continuation region $[x_i^b, \infty)$ has two components. First, the firm value incorporates the unlimited liability value $q_i a x + \frac{(\tau_f-\tau_i)c}{r}$ of a perpetual claim to the future cash flow. Second, it reflects the change in value that occurs either after a regime shift or when the state variable (cash flow level) reaches the boundary x_i^b. The firm value $J_1(x)$ in the transient region $[x_1^b, x_0^b]$ also has two components. First, it includes the unlimited liability value $\frac{(1-\tau_f)ax}{r+\lambda_1-\mu_1} + \frac{\lambda_1(1-\alpha)q_0 a x}{r+\lambda_1-\mu_1} + \frac{(\tau_f-\tau_i)c}{r+\lambda_1}$ of a perpetual claim to the future cash flow. Second, it captures the change arising in firm value after expansion investment when the cash flow level reaches either the default threshold or the upper boundary of the transient region. The remaining expressions are easier to follow.

In the same way, we denote by $D_l(x), l \in \{0, 1\}$ the value of the rollover debt and let $g_{ml}(\gamma) = r + m + \lambda_l - \mu_l\gamma - \frac{\sigma^2}{2}\gamma(\gamma-1)$, γ_{m1} and γ_{m2} be the negative roots of the equation $g_{m0}(\gamma)g_{m1}(\gamma) = \lambda_0\lambda_1$ and β_{m1} and β_{m2} be the two roots of the equation $g_{m1}(\gamma) = 0$. We derive the following conclusions.

PROPOSITION 3.2. *The value of the rollover debt after expansion investment is given below:*

$$D_0(x) = \begin{cases} A_3 x^{\gamma_{m1}} + A_4 x^{\gamma_{m2}} + \frac{(1-\tau_i)(c+mP)}{r+m}, & \text{if } x > x_0^b, \\ (1-\alpha)q_0 a x, & \text{if } x \le x_0^b, \end{cases}$$

and

$$D_1(x) = \begin{cases} B_3 x^{\gamma_{m1}} + B_4 x^{\gamma_{m2}} + \frac{(1-\tau_i)(c+mP)}{r+m}, & \text{if } x > x_0^b, \\ E_3 x^{\beta_{m1}} + E_4 x^{\beta_{m2}} + \frac{\lambda_1(1-\alpha)q_0 a x}{r+m+\lambda_1-\mu_1} + \frac{(1-\tau_i)(c+mP)}{r+m+\lambda_1}, & \text{if } x_1^b \le x \le x_0^b, \\ (1-\alpha)q_1 a x, & \text{if } x < x_1^b, \end{cases}$$

where $B_3 = \frac{\lambda_1}{g_{m1}(\gamma_{m1})}A_3$ *and* $B_4 = \frac{\lambda_1}{g_{m1}(\gamma_{m2})}A_4$. *The constants* A_3, A_4, E_3, *and* E_4 *constitute a column vector* $(A_3, A_4, E_3, E_4)'$, *which is the solution of the system*

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of linear equations: $\Gamma_2 X = \Phi_2$, *where*

$$\Gamma_2 \equiv \begin{pmatrix} (x_0^b)^{\gamma_{m1}} & (x_0^b)^{\gamma_{m2}} & 0 & 0 \\ \frac{\lambda_1}{g_{m1}(\gamma_{m1})}(x_0^b)^{\gamma_{m1}} & \frac{\lambda_1}{g_{m1}(\gamma_{m2})}(x_0^b)^{\gamma_{m2}} & -(x_0^b)^{\beta_{m1}} & -(x_0^b)^{\beta_{m2}} \\ \gamma_{m1}\frac{\lambda_1}{g_{m1}(\gamma_{m1})}(x_0^b)^{\gamma_{m1}} & \gamma_{m2}\frac{\lambda_1}{g_{m1}(\gamma_{m2})}(x_0^b)^{\gamma_{m2}} & -\beta_{m1}(x_0^b)^{\beta_{m1}} & -\beta_{m2}(x_0^b)^{\beta_{m2}} \\ 0 & 0 & (x_1^b)^{\beta_{m1}} & (x_1^b)^{\beta_{m2}} \end{pmatrix}, \tag{4}$$

$$\Phi_2 \equiv \begin{pmatrix} (1-\alpha)q_0 a x_0^b - \frac{(1-\tau_i)(c+mP)}{r+m} \\ \frac{\lambda_1(1-\alpha)q_0 a x_0^b}{r+m+\lambda_1-\mu_1} + \frac{(1-\tau_i)(c+mP)}{r+m+\lambda_1} - \frac{(1-\tau_i)(c+mP)}{r+m} \\ \frac{\lambda_1(1-\alpha)a q_0 x_0^b}{r+m+\lambda_1-\mu_1} \\ (1-\alpha)q_1 a x_1^b - \frac{\lambda_1(1-\alpha)q_0 a x_1^b}{r+m+\lambda_1-\mu_1} - \frac{(1-\tau_i)(c+mP)}{r+m+\lambda_1} \end{pmatrix}.$$

The expressions for the value of corporate debt reported in Proposition 3.2 are similar to those provided for the firm value after expansion investment, and thus they admit a similar interpretation. For example, the value of corporate debt is equal to the sum of the value of a perpetual entitlement to the future cash flow and the changes in value that occur either after a regime shift or when the cash flow level reaches a boundary x_i^b. There are three major differences, however, between debt and equity values. First, the value of corporate debt depends on the abandonment value $q_i a x_i^b$ of the firm's assets. Second, the promised payments to bondholders are state independent. Third, the default threshold is determined by shareholders and hence is an exogenous parameter for bondholders. That is, the change occurring to the value of corporate debt at the boundaries x_i^b is always exogenous.

Last, we denote by $E_l(x), l \in \{0, 1\}$ the value of equity, which is immediately derived by the obvious equality $E_l(x) = J_l(x) - D_l(x)$. Further, utilizing the well-known smooth-pasting conditions to maximize the value $E_l(x)$ of equity, we derive the optimal default thresholds, which are given by the following proposition.

PROPOSITION 3.3. *The optimal default thresholds* x_0^b *and* x_1^b *satisfy the following equations:*

$$\begin{cases} A_1\gamma_1(x_0^b)^{\gamma_1} + A_2\gamma_2(x_0^b)^{\gamma_2} - A_3\gamma_{m1}(x_0^b)^{\gamma_{m1}} - A_4\gamma_{m2}(x_0^b)^{\gamma_{m2}} + q_0 a(x_0^b) = 0, \\ E_1\beta_1(x_1^b)^{\beta_1} + E_2\beta_2(x_1^b)^{\beta_2} - E_3\beta_{m1}(x_1^b)^{\beta_{m1}} - E_4\beta_{m2}(x_1^b)^{\beta_{m2}} + \frac{(1-\tau_f)a(x_1^b)}{r+\lambda_1-\mu_1} \\ + \frac{\lambda_1(1-\alpha)q_0 a(x_1^b)}{r+\lambda_1-\mu_1} - \frac{\lambda_1(1-\alpha)q_0 a(x_1^b)}{r+m+\lambda_1-\mu_1} = 0. \end{cases} \tag{5}$$

The change in the value of equity that occurs at the boundaries x_i^b is either endogenous or exogenous, as Guo et al. (2005) argue. It is endogenous when the value of the state variable reaches the stopping region. In that case, shareholders

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exercise their option to default. It is exogenous when the state variable reaches the upper boundary of the transient region. In that case, the change in value arises from the fact that below the upper boundary of the transient region a switch in the process L leads to default whereas above this boundary it does not.

3.2. Guarantee Costs

We assume that once a borrower defaults, the insurer must pay the creditor all the default loss and thus loans are actually risk-free with the guarantee. For this reason, the value $D_l^{\text{guar}}(x)$ of the insurer's compensatory payment at investment time and economic state $l \in \{0, 1\}$ must satisfy the following equation:

$$D_l(x) + \frac{1-\tau_i}{1-\tau_f} D_l^{\text{guar}}(x) = \frac{(1-\tau_i)(c+mP)}{r+m}, \tag{6}$$

where the right side is the value of the rollover debt if bankruptcy never occurs (i.e., $x \to \infty$) according to Proposition 3.2.

We assume that the market is sufficiently competitive, and hence an insurer's compensatory payment $D_l^{\text{guar}}(x)$ to the creditor must equal the value of the granted equity, namely

$$D_l^{\text{guar}}(x) = \phi_l E_l(x), \text{ or } \phi_l = \frac{D_l^{\text{guar}}(x)}{E_l(x)}, \quad l \in \{0, 1\}, \tag{7}$$

where ϕ_l is the guarantee cost, i.e., the fraction of equity allocated to the insurer in return for the guarantee.

3.3. The Pricing and Timing of Growth Option

We denote by $u_l(x), l \in \{0, 1\}$ the total firm value after expansion investment and with the guarantee. In our model, the insurer does not harvest anything, i.e., the value of what he obtains is just that of what he pays. Therefore, we have

$$u_l(x) = \frac{(1-\tau_i)(c+mp)}{r+m} + (1-\phi_l)E_l(x) = J_l(x) + \left(\frac{1-\tau_i}{1-\tau_f} - 1\right)\phi_l E_l(x), \tag{8}$$

where $J_l(x)$ is determined by Proposition 3.1. For this reason, the firm value with a guarantee is similar but different from that ($J_l(x)$) without a guarantee and the former is in general much higher than the latter since the firm obtains much more tax shields thanks to the guarantee as implied by (8).

Now, we turn to the value of equity before expansion investment, which we denote by $G_l(x), l \in \{0, 1\}$. In contrast to the unlevered firm value determined by (2), we emphasize that we should take the growth option into account in computing the value $G_l(x)$. To achieve this goal, we denote by x_l^e the investment threshold. Naturally, we have $x_1^e < x_0^e$. Following Luo et al. (2017), we have for $x \in (0, x_1^e)$

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that

$$\begin{cases} rG_0(x) = (1-\tau_f)x + \mu_0 x G_0'(x) + \frac{\sigma^2 x^2}{2} G_0''(x) + \lambda_0(G_1(x) - G_0(x)), \\ rG_1(x) = (1-\tau_f)x + \mu_1 x G_1'(x) + \frac{\sigma^2 x^2}{2} G_1''(x) + \lambda_1(G_0(x) - G_1(x)); \end{cases} \tag{9}$$

for $x \in (x_1^e, x_0^e)$ that

$$\begin{cases} rG_0(x) = (1-\tau_f)x + \mu_0 x G_0'(x) + \frac{\sigma^2 x^2}{2} G_0''(x) + \lambda_0(u_1(x) - I - G_0(x)), \\ G_1(x) = u_1(x) - I; \end{cases} \tag{10}$$

and for $x \in (x_0^e, +\infty)$ that

$$\begin{cases} G_0(x) = u_0(x) - I, \\ G_1(x) = u_1(x) - I. \end{cases} \tag{11}$$

Within our framework, the equilibrium expected return on corporate securities is r. Thus, the left-hand side of these equations reflect the required rate of return for holding one share per unit of time. The right-hand side is the expected change in the value of equity in the region for the state where the firm does not invest. The expressions we obtain are similar to those of the diffusion case. However, they contain an additional term $\lambda_0(u_1(x) - I - G_0(x))$ in (10) that reflects the impact of the possibility of a regime shift on the value functions. We have the smooth-pasting conditions $G_0'(x_0^e) = u_0'(x_0^e)$ and $G_1'(x_1^e) = u_1'(x_1^e)$ ensuring that investment occurs along the optimal path by requiring a continuity of the slopes at the endogenous investment thresholds. We also have $\lim_{x \to x_1^{e+}} G_0^{'}(x_1^e) = \lim_{x \to x_1^{e-}} G_0'(x_1^e)$. Although optimal stopping rules involve an instantaneous jump, the resulting value functions should have certain smoothness, because the values are measured on average.

Letting γ_3 and γ_4 be the two positive roots of the equation $g_0(\gamma)g_1(\gamma) = \lambda_0\lambda_1$, and β_3 and β_4 be the two roots of the equation $g_0(\gamma) = 0$, we conclude the next proposition from Luo et al. (2017).

PROPOSITION 3.4. *For given investment thresholds x_0^e and x_1^e, the value of equity before expansion investment is*

$$G_0(x) = \begin{cases} A_5 x^{\gamma_3} + A_6 x^{\gamma_4} + q_0 x, & \text{if } x < x_1^e, \\ E_5 x^{\beta_3} + E_6 x^{\beta_4} + A_1 x^{\gamma_1} + A_2 x^{\gamma_2} & \\ \quad - \frac{\tau_f - \tau_i}{1-\tau_i}(A_3 x^{\gamma_{m1}} + A_4 x^{\gamma_{m2}}) & \\ + \frac{\lambda_0 q_1 a x}{r+\lambda_0-\mu_0} + \frac{(1-\tau_f)x}{r+\lambda_0-\mu_0} + \frac{\lambda_0}{r+\lambda_0}[\frac{(\tau_f-\tau_i)c}{r} - I], & \text{if } x_1^e \le x \le x_0^e, \\ u_0(x) - I, & \text{if } x > x_0^e; \end{cases}$$

and

$$G_1(x) = \begin{cases} B_5 x^{\gamma_3} + B_6 x^{\gamma_4} + q_1 x, & \text{if } x < x_1^e, \\ u_1(x) - I, & \text{if } x \ge x_1^e, \end{cases}$$

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where $B_5 = \frac{\lambda_1}{g_1(\gamma_3)}A_5$ *and* $B_6 = \frac{\lambda_1}{g_1(\gamma_4)}A_6$. *The constants* A_5, A_6, E_5, *and* E_6 *constitute a column vector* $(A_5, A_6, E_5, E_6)'$, *which is the solution of the system of linear equations:* $\Gamma_3 X = \Phi_3$, *where*

$$\Gamma_3 \equiv \begin{pmatrix} (x_1^e)^{\gamma_3} & (x_1^e)^{\gamma_4} & -(x_1^e)^{\beta_3} & -(x_1^e)^{\beta_4} \\ \gamma_3(x_1^e)^{\gamma_3} & \gamma_4(x_1^e)^{\gamma_4} & -\beta_3(x_1^e)^{\beta_3} & -\beta_4(x_1^e)^{\beta_4} \\ 0 & 0 & (x_0^e)^{\beta_3} & (x_0^e)^{\beta_4} \\ \frac{\lambda_1}{g_1(\gamma_3)}(x_1^e)^{\gamma_3} & \frac{\lambda_1}{g_1(\gamma_4)}(x_1^e)^{\gamma_4} & 0 & 0 \end{pmatrix}, \tag{12}$$

$$\Phi_3 \equiv \begin{pmatrix} A_1(x_1^e)^{\gamma_1} + A_2(x_1^e)^{\gamma_2} - \frac{\tau_f-\tau_i}{1-\tau_i}[A_3(x_1^e)^{\gamma_{m1}} + A_4(x_1^e)^{\gamma_{m2}}] \\ +\frac{\lambda_0 q_1 a x_1^e}{r+\lambda_0-\mu_0} - q_0 x_1^e + \frac{(1-\tau_f)x_1^e}{r+\lambda_0-\mu_0} + \frac{\lambda_0}{r+\lambda_0}[\frac{(\tau_f-\tau_i)c}{r} - I] \\ A_1\gamma_1(x_1^e)^{\gamma_1} + \gamma_2 A_2(x_1^e)^{\gamma_2} - \frac{\tau_f-\tau_i}{1-\tau_i}[A_3\gamma_{m1}(x_1^e)^{\gamma_{m1}} + A_4\gamma_{m2}(x_1^e)^{\gamma_{m2}}] \\ +\frac{\lambda_0 q_1 a x_1^e}{r+\lambda_0-\mu_0} - q_0 x_1^e + \frac{(1-\tau_f)x_1^e}{r+\lambda_0-\mu_0} \\ q_0 a x_0^e - \frac{\lambda_0 q_1 a x_0^e}{r+\lambda_0-\mu_0} - \frac{(1-\tau_f)x_0^e}{r+\lambda_0-\mu_0} + \frac{r}{r+\lambda_0}[\frac{(\tau_f-\tau_i)c}{r} - I] \\ B_1(x_1^e)^{\gamma_1} + B_2(x_1^e)^{\gamma_2} - \frac{\tau_f-\tau_i}{1-\tau_i}[B_3(x_1^e)^{\gamma_{m1}} + B_4(x_1^e)^{\gamma_{m2}}] \\ +q_1(a-1)x_1^e + \frac{(\tau_f-\tau_i)c}{r} - I \end{pmatrix}.$$

The optimal investment thresholds x_0^e *and* x_1^e, *which maximize the value of equity, satisfy the following equations:*

$$\begin{cases} B_5\gamma_3(x_1^e)^{\gamma_3} + B_6\gamma_4(x_1^e)^{\gamma_4} = B_1\gamma_1(x_1^e)^{\gamma_1} + B_2\gamma_2(x_1^e)^{\gamma_2} \\ -\frac{\tau_f-\tau_i}{1-\tau_i}[B_3\gamma_{m1}(x_1^e)^{\gamma_{m1}} + B_4\gamma_{m2}(x_1^e)^{\gamma_{m2}}] + q_1(a-1)x_1^e, \\ E_5\beta_3(x_0^e)^{\beta 3} + E_6\beta_4(x_0^e)^{\beta_4} = q_0 a x_0^e - \frac{\lambda_0 q_1 a x_0^e}{r+\lambda_0-\mu_0} - \frac{(1-\tau_f)x_0^e}{r+\lambda_0-\mu_0}. \end{cases} \tag{13}$$

The expressions for the value of equity before expansion investment reported in Proposition 3.4 are similar to those provided for the values of firm and equity after expansion investment, and thus, they admit a similar interpretation.

4. THE CASE OF THE FEE-FOR-GUARANTEE SWAP

In this section, we consider the same problem with the preceding section, but we examine the FGS instead of the EGS. Actually, FGSs are more popular than EGSs in China.

Under an FGS, an insurer receives a guarantee fee rather than a fraction of equity in return for his guarantee when the swap agreement is signed. The fee is a fraction of the loan guaranteed, which is denote by $g_l, l \in \{0, 1\}$.

For the same reason with the discussion in the preceding section, to fully protect the debt, the value $D_l^{\text{guar}}(x)$ of the insurer's compensatory payment satisfies

$$D_l(x) + \frac{1-\tau_i}{1-\tau_i}D_l^{\text{guar}}(x) \equiv D_l(x) + D_l^{\text{guar}}(x) = \frac{(1-\tau_i)(c+mP)}{r+m}, \tag{14}$$

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where the denominator at the second term in the left-hand side of the equation is $1-\tau_i$ instead of $1-\tau_f$ as in (6) since the fee the insurer receives is taxed at the rate τ_i rather than τ_f. In addition, the guarantee cost naturally satisfies

$$D_l^{\text{guar}}(x) = g_l \frac{(1-\tau_i)(c+mP)}{r+m} \text{ or } g_l = \frac{D_l^{\text{guar}}(x)(r+m)}{(1-\tau_i)(c+mP)}. \tag{15}$$

Denote by $v_l(x)$ the total firm value under the FGS. According to (8) and the fact that τ_f is replaced with τ_i, here, we obtain

$$v_l(x) = J_l(x), \quad l \in \{0,1\}. \tag{16}$$

In addition, under the FGS, we denote by $F_l(x)$ the value of equity before expansion investment and by x_l^f the investment threshold for $l \in \{0,1\}$. In the same way with the case of the EGS, we derive the following conclusions.

PROPOSITION 4.1. *If the firm enters into the FGS, then for given investment thresholds x_0^f and x_1^f, the value of equity before expansion investment is*

$$F_0(x) = \begin{cases} A_7 x^{\gamma_3} + A_8 x^{\gamma_4} + q_0 x, & \text{if } x < x_1^f, \\ E_7 x^{\beta_3} + E_8 x^{\beta_4} + A_1 x^{\gamma_1} + A_2 x^{\gamma_2} + \frac{\lambda_0 q_1 a x}{r+\lambda_0-\mu_0} & \\ + \frac{(1-\tau_f)x}{r+\lambda_0-\mu_0} + \frac{\lambda_0}{r+\lambda_0}[\frac{(\tau_f-\tau_i)c}{r} - I], & \text{if } x_1^f \le x \le x_0^f, \\ v_0(x) - I, & \text{if } x > x_0^f; \end{cases}$$

and

$$F_1(x) = \begin{cases} B_7 x^{\gamma_3} + B_8 x^{\gamma_4} + q_1 x, & \text{if } x < x_1^f, \\ v_1(x) - I, & \text{if } x \ge x_1^f, \end{cases}$$

where $B_7 = \frac{\lambda_1}{g_1(\gamma_3)} A_7$ and $B_8 = \frac{\lambda_1}{g_1(\gamma_4)} A_8$. The constants A_7, A_8, E_7, and E_8 constitute a column vector $(A_7, A_8, E_7, E_8)'$, which is the solution of the system of linear equations, $\Gamma_4 X = \Phi_4$, where

$$\Gamma_4 \equiv \begin{pmatrix} (x_1^f)^{\gamma_3} & (x_1^f)^{\gamma_4} & -(x_1^f)^{\beta_3} & -(x_1^f)^{\beta_4} \\ \gamma_3 (x_1^f)^{\gamma_3} & \gamma_4 (x_1^f)^{\gamma_4} & -\beta_3 (x_1^f)^{\beta_3} & -\beta_4 (x_1^f)^{\beta_4} \\ 0 & 0 & (x_0^f)^{\beta_3} & (x_0^f)^{\beta_4} \\ \frac{\lambda_1}{g_1(\gamma_3)} (x_1^f)^{\gamma_3} & \frac{\lambda_1}{g_1(\gamma_4)} (x_1^f)^{\gamma_4} & 0 & 0 \end{pmatrix}, \tag{17}$$

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$$\Phi_4 \equiv \begin{pmatrix} A_1(x_1^f)^{\gamma_1} + A_2(x_1^f)^{\gamma_2} + \frac{\lambda_0 q_1 a x_1^f}{r+\lambda_0-\mu_0} - q_0 x_1^f \\ + \frac{(1-\tau_f)x_1^f}{r+\lambda_0-\mu_0} + \frac{\lambda_0}{r+\lambda_0}\left[\frac{(\tau_f-\tau_i)c}{r} - I\right] \\ A_1\gamma_1(x_1^f)^{\gamma_1} + A_2\gamma_2(x_1^f)^{\gamma_2} + \frac{\lambda_0 q_1 a x_1^f}{r+\lambda_0-\mu_0} - q_0 x_1^f + \frac{(1-\tau_f)x_1^f}{r+\lambda_0-\mu_0} \\ q_0 a x_0^f - \frac{\lambda_0 q_1 a x_0^f}{r+\lambda_0-\mu_0} - \frac{(1-\tau_f)x_0^f}{r+\lambda_0-\mu_0} + \frac{r}{r+\lambda_0}\left[\frac{(\tau_f-\tau_i)c}{r} - I\right] \\ B_1(x_1^f)^{\gamma_1} + B_2(x_1^f)^{\gamma_2} + q_1(a-1)x_1^f + \frac{(\tau_f-\tau_i)c}{r} - I \end{pmatrix}.$$

The optimal investment threshold x_0^f and x_1^f, which maximize the value of equity after entering into the FGS, satisfy the following equations:

$$\begin{cases} B_7\gamma_3(x_1^f)^{\gamma_3} + B_8\gamma_4(x_1^f)^{\gamma_4} = B_1\gamma_1(x_1^f)^{\gamma_1} + B_2\gamma_2(x_1^f)^{\gamma_2} + q_1(a-1)x_1^f, \\ E_7\beta_3(x_0^f)^{\beta 3} + E_8\beta_4(x_0^f)^{\beta_4} = q_0 a x_0^f - \frac{\lambda_0 q_1 a x_0^f}{r+\lambda_0-\mu_0} - \frac{(1-\tau_f)x_0^f}{r+\lambda_0-\mu_0}. \end{cases} \quad (\mathbf{18})$$

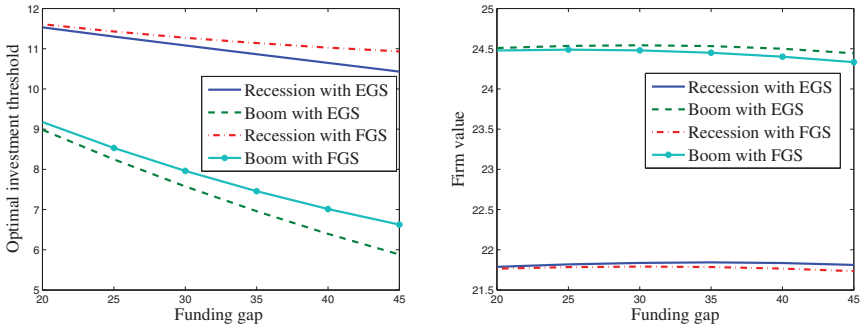
Last, we emphasize that the most important function of the two swaps is to overcome the financing constraints experienced by many borrowers (e.g., SMEs) all over the world. Further, according to (8) and (16), entering into EGSs instead of FGSs, the borrowers will harvest much more tax shields since as a matter of fact, the corporate effective tax rate is always greater than the personal rate, i.e., $\tau_f > \tau_i$.

5. NUMERICAL RESULTS AND ANALYSIS

Baseline parameters: To make a reliable analysis, we select model parameters as real as possible but we do not consider trivial situations, say the case where the growth option should be exercised at once. Specifically, we take following baseline parameter values unless otherwise stated: the annualized risk-free rate of interest $r = 0.08$,[3] the low drift rate $\mu_0 = 0.01$, the high drift rate $\mu_1 = 0.04$, the volatility $\sigma = 0.6$, and the current cash flow level $x = 1$. Following Yang and Zhang (2015), we select the personal rate $\tau_i = 0.05$ and the effective tax rate $\tau_f = 0.4$. The bankruptcy loss rate $\alpha = 0.5$, the rollover debt parameter $m = 0.2$ (i.e., 5 years of average debt maturity), the growth scale $a = 3$, and the investment cost $I = 50$. Following Hackbarth et al. (2006), we select the jump intensity $\lambda_0 = 0.15$ and $\lambda_1 = 0.1$. In addition, according to the pecking order theory of debt, a firm borrows only for bridging the funding gap. Thus, we do not consider optimal capital structure or the optimal loan amount, i.e., the principal P the firm borrows is just equal to its funding gap, which is therefore given exogenously. We take $P = 45$ and assume that debt is issued at par, i.e., $(1-\tau_i)(c+mP)/(r+m) = P$ and so we get the coupon rate $c = P(r+\tau_i m)/(1-\tau_i) = 15.632$ under the baseline parameter values.

Investment threshold and the firm value versus funding gaps: Figure 1(a) demonstrates the investment threshold decreases with the funding gap. This is in line with

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(a) Investment threshold vs. funding gaps (b) The firm value vs. funding gaps

FIGURE 1. The figure demonstrates (a) optimal investment threshold and (b) the firm value before expansion investment for different funding gaps.

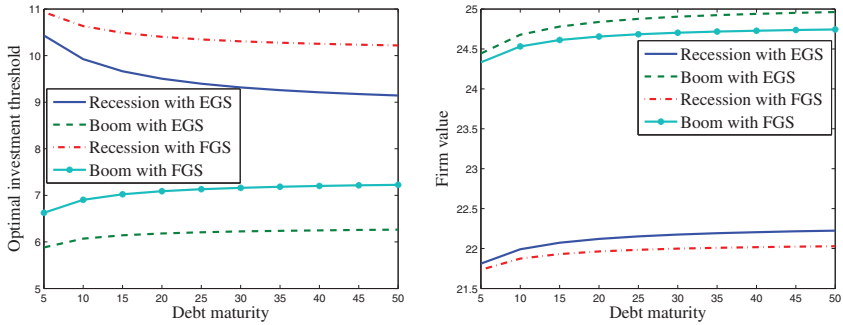
Hackbarth and Mauer (2012) resulting from the tax shield effect, since the wider the gap, the more the debt issued and thus the more the tax shields. Additionally, the investment threshold in recession is higher than that in boom and EGSs induce earlier investment than FGSs. The former is obvious and is in agreement with Jeon and Nishihara (2015). The latter occurs since EGSs gets more tax shields than FGSs as said before. Figure 1(b) depicts that the firm value (i.e., the equity value) before expansion investment decreases slowly with the funding gap. The firm value in boom is much higher than that in recession. This is evident and is similar to Arnold et al. (2013). The firm value under the EGS is higher than that under the FGS as predicted in our model and the advantage increases with the funding gap.

Investment threshold and the firm value versus debt maturity: Figure 2(a) shows that investment threshold decreases (increases) with the debt maturity in recession (boom). Figure 2(b) demonstrates that the firm value before expansion investment increases with the debt maturity. As before, an EGS lead to a higher firm value than an FGS and the difference increases with the maturity.

Investment threshold and the firm value versus jump intensity: Our model has two jump intensity parameters, and clearly, the fraction of time on average for the economic regime staying at boom is $\lambda_0/(\lambda_0 + \lambda_1)$. Thus, we study how the investment threshold and firm value vary in jump intensity λ_1 for a given λ_0. Figure 3(a) plots that the investment threshold increases slowly with λ_1, whereas Figure 3(b) shows that the firm value before expansion investment decreases quickly with the parameter. These phenomena appear, since the larger the intensity λ_1, the less the probability for the economic regime being in boom.

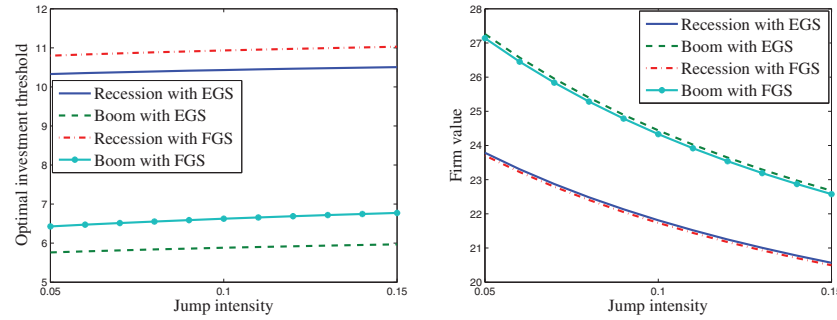
Guarantee costs versus funding gaps: Figure 4 states that guarantee costs under both EGSs and FGSs increase with the funding gap as we expected. However, to our surprise at first sight, the costs in boom are higher than those in recession. It

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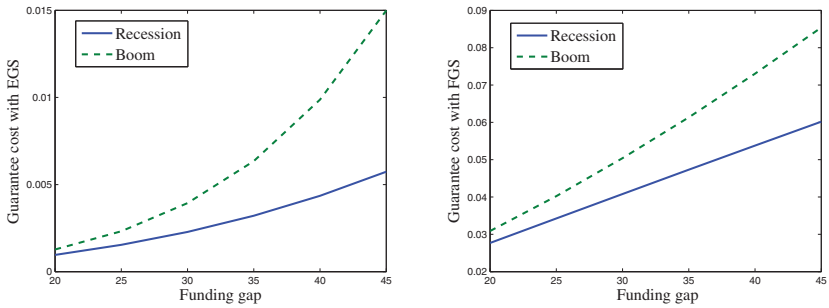
(a) Investment threshold vs. debt maturity (b) The firm value vs. debt maturity

FIGURE 2. The figure demonstrates (a) optimal investment threshold and (b) the firm value before expansion investment for different debt maturities.



(a) Investment threshold vs. jump intensity (b) The firm value vs. jump intensity

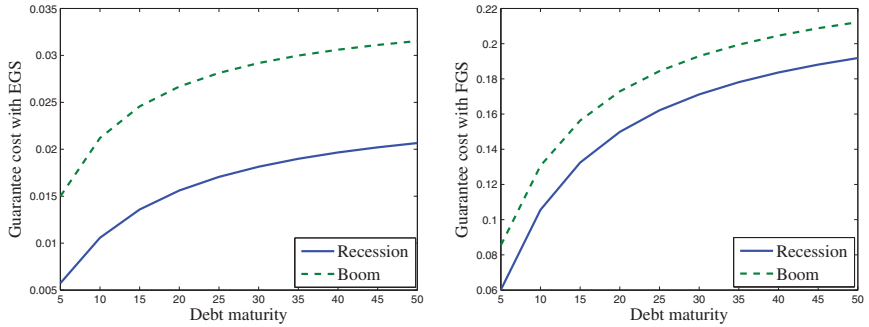
FIGURE 3. The figure demonstrates (a) optimal investment threshold and (b) the firm value before expansion investment for different jump intensity parameter λ_1.



(a) Guarantee cost of EGS vs. funding gaps (b) Guarantee cost of FGS vs. funding gaps

FIGURE 4. The figure demonstrates (a) the guarantee cost of an EGS and (b) the guarantee cost of an FGS for different funding gaps.

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(a) Guarantee cost vs. debt maturity (b) Guarantee cost vs. debt maturity

FIGURE 5. The figure demonstrates (a) the guarantee cost with EGS and (b) the guarantee cost with FGS for different values of debt maturity.

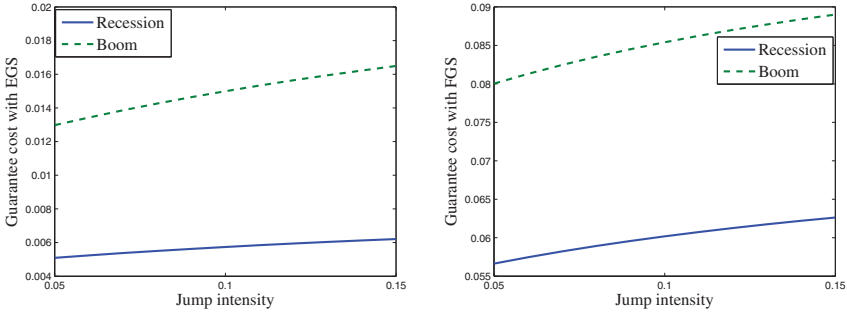
turns out that the investment threshold in boom is much less than that in recession and all swaps are signed at investment time.

Guarantee costs versus debt maturities: Figure 5(a) explains that as we expected, guarantee costs increase with debt maturities. This is because the longer the debt maturity, the higher the debt risk. In particular, with regard to short-term debt under FGSs, our findings are in line with the stipulation of Chinese government, which demands that guarantee cost should be approximately the fraction 50% of the interest rate of the bank loan with the same maturity. Fox example, if debt has a maturity of 1 year, the guarantee cost is 0.5% in recession and is 1.9% in boom according to our model. However, with regard to long-term debt under FGSs, our model states that the suggested fraction is too low. Fox example, if debt has a maturity of 10 years, the guarantee cost is 10.6% in recession and is 12.9% in boom as displayed in Figure 5(b). In other words, we have actually explained why the guarantees provided by the government do not meet the guarantee demand in the market particularly for long-term debt.

Guarantee costs versus jump intensities: Figure 6 depicts that guarantee costs increase with jump intensity λ_1. This is obvious, since according to the previous text, the higher the parameter λ_1, the bigger the probability of the economic regime being in recession.

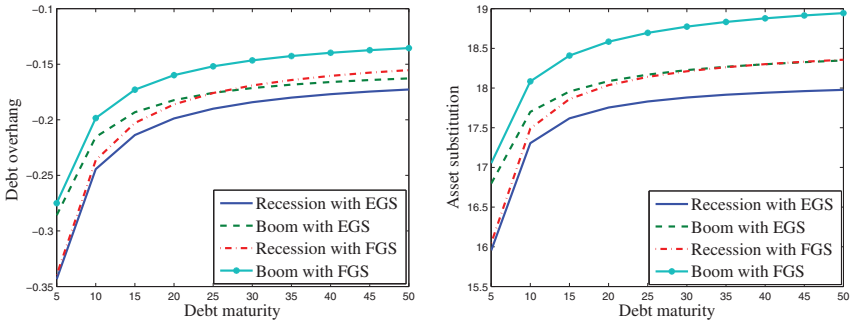
Debt overhang: To this end, noting there is no debt in the firm before expansion investment, we only consider the situation where the investment is already undertaken. Similar to Pennacchi et al. (2014), we compute the net increment of the equity value as the unlevered firm's value adds one unit. Namely, it is defined by $\frac{(1-\phi_l)\partial E_l(x)}{a\partial V_l(x)} - 1, l \in \{0, 1\}$ under EGSs, and by $\frac{\partial E_l(x)}{a\partial V_l(x)} - 1$ under FGSs. A negative value of them means that the amount shareholders ultimately harvest is less than what they originally invest, and therefore it indicates debt overhang distortion. Naturally, the less the value, the worse the debt overhang and the less incentive the shareholders have to replenish equity. Figure 7(a) says the debt overhang problem

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(a) Guarantee costs of EGS vs. intensities (b) Guarantee costs of FGS vs. intensities

FIGURE 6. The figure demonstrates (a) the guarantee cost under an EGS and (b) the guarantee cost under an FGS for different jump intensity λ_1 from boom to recession.



(a) Debt overhang vs. debt maturity (b) Asset substitution vs. debt maturity

FIGURE 7. The figure demonstrates (a) the debt overhang and (b) the asset substitution for different values of debt maturity m. The cash flow $x = 1$.

under short-term debt is more serious. This phenomenon happens since the coupon rate is bigger under short-term debt than long-term one. As we expected, it gets more severe in recession than in boom or under an EGS than an FGS. For example, under EGSs, the firm must allocate a fraction of equity to an insurer, and thus, when the borrowers inject cash into the firm, the insurer harvests free the benefits brought by the fraction of equity granted to him.

Asset substitution: To measure the risk-shifting incentive, following Pennacchi et al. (2014), we take the value of equity as a function of the volatility rate and compute its derivative, i.e., $(1-\phi_l)\frac{\partial E_l(x)}{\partial \sigma}, l \in \{0,1\}$ under the EGS and $\frac{\partial E_l(x)}{\partial \sigma}$ under the FGS. If it is positive, then an increase in business risk leads to a growth of the value of the shareholders' claim, and accordingly, there is a risk-shifting incentive of the shareholders. Naturally, the larger the value, the stronger the incentive. Figure 7(b) depicts that the degree of asset substitution increases with

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the debt maturity. The short-term debt can mitigate asset substitution, which is in line with Leland and Toft (1996). In particular, the asset substitution problem in boom is more serious than that in recession, and EGSs suffer such inefficiency less than FEGs.

6. CONCLUSION

SMEs play a vital role in promoting economic growth over the world. However, it is difficult for them to get bank loans, and as a result, many of them have to give up investing in a project even it is extremely profitable. This situation gets even more severe due to the recent global financial crisis.

Chinese government has been paying much attention to this problem for a long time. To overcome the financing constraints, the government has founded or supported a lot of credit guarantee companies and many financial innovations are designed. Among them, EGSs and in particular FGSs are very popular in China, but there are rarely quantitative studies on these swaps.

In this paper, we consider a firm with assets-in-place, which are financed by pure equity, and a growth option, of which investment cost is financed by entering into an EGS or FGS. We examine the effects of EGSs and FGSs on the growth option taking business cycle and debt maturity into account. We provide a method to determine guarantee costs, which are approximate to the ones suggested by Chinese government for short-term debt but significantly higher than the latter for long-term debt. Our model demonstrates that EGSs can improve the firm value and mitigate asset substitution, but make debt overhang more serious than FGSs. The expansion investment accelerates as the funding gap rises but it is postponed if the economy is more likely to be in recession.

NOTES

1. For example, please refer to Korenok et al. (2009), Branch et al. (2013), Nason and Tallman (2015), and Morier and Teles (2016).

2. In China, there are mainly five steps to enter into a credit default swap: (i) A potential borrower submits an application to a qualified insurer for a loan guarantee. (ii) The potential insurer reviews the application by checking the applicant's identity, credit record, financial state and if the draft agreement is legitimate and practical. (iii) The swap agreement is signed if there are no problems. (iv) The agreement is registered in the Chinese credit guarantee association. (v) A bank cooperating with the insurer lends the money to the borrower after receiving the guarantee agreement and finding no problems.

3. This rate is approximate to the historical average treasury rate, see Huang and Huang (2012).

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APPENDIX A: TECHNICAL PROOFS

A.1. PROOF OF PROPOSITION 3.1

Thanks to Luo et al. (2017), the firm value $J_l(x), l \in \{0, 1\}$ after expansion investment is given by

$$\begin{cases} rJ_0(x) = (1-\tau_f)ax + (\tau_f - \tau_i)c + \mu_0 x J_0'(x) + \frac{\sigma^2 x^2}{2} J_0''(x) + \lambda_0(J_1(x) - J_0(x)), \\ rJ_1(x) = (1-\tau_f)ax + (\tau_f - \tau_i)c + \mu_1 x J_1'(x) + \frac{\sigma^2 x^2}{2} J_1''(x) + \lambda_1(J_0(x) - J_1(x)), \end{cases} \tag{A.1}$$

for $x \in (x_0^b, +\infty)$; by

$$\begin{cases} J_0(x) = (1-\alpha)q_0 ax, \\ rJ_1(x) = (1-\tau_f)ax + (\tau_f - \tau_i)c + \mu_1 x J_1'(x) + \frac{\sigma^2 x^2}{2} J_1''(x) \\ \qquad +\lambda_1((1-\alpha)q_0 ax - J_1(x)), \end{cases} \tag{A.2}$$

for $x \in (x_1^b, x_0^b]$; and by

$$\begin{cases} J_0(x) = (1-\alpha)q_0 ax, \\ J_1(x) = (1-\alpha)q_1 ax, \end{cases} \tag{A.3}$$

for $x \in (0, x_1^b]$. Next, the proposition holds according to Luo et al. (2017).

A.2. PROOF OF PROPOSITION 3.2

From Luo et al. (2017), we derive the value $D_l(x), l \in \{0, 1\}$ of the rollover debt after expansion investment satisfying

$$\begin{cases} (r+m)D_0(x) = (1-\tau_i)(c+mP) + \mu_0 x D_0'(x) + \frac{\sigma^2 x^2}{2} D_0''(x) + \lambda_0(D_1(x) - D_0(x)), \\ (r+m)D_1(x) = (1-\tau_i)(c+mP) + \mu_1 x D_1'(x) + \frac{\sigma^2 x^2}{2} D_1''(x) + \lambda_1(D_0(x) - D_1(x)), \end{cases} \tag{A.4}$$

for $x \in (x_0^b, +\infty)$;

$$\begin{cases} D_0(x) = (1-\alpha)q_0 ax, \\ (r+m)D_1(x) = (1-\tau_i)(c+mP) + \mu_1 x D_1'(x) + \frac{\sigma^2 x^2}{2} D_1''(x) \\ \qquad +\lambda_1((1-\alpha)q_0 x - D_1(x)), \end{cases} \tag{A.5}$$

for $x \in (x_1^b, x_0^b]$; and

$$\begin{cases} D_0(x) = (1-\alpha)q_0 ax, \\ D_1(x) = (1-\alpha)q_1 ax, \end{cases} \tag{A.6}$$

for $x \in (0, x_1^b]$. Hence, the proposition is proved from Luo et al. (2017).

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