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RTK-GPS Correction Generation Technique For Low-Rate Data-Link

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This paper investigates an RTK-GPS correction generation technique suitable for low-rate (<600 bps) data-link. We combined correction filtering algorithm with an efficient transmission technique which is based upon correction scheduling. The correction scheduling technique can reduce the required amount of correction data as well as the latency effect on RTK-GPS positioning accuracy. We processed real time GPS data to evaluate the performance of the RTK-GPS correction generator for low-rate data-link. The experimental results demonstrate that the proposed techniques can lower the required bandwidth of data-link up to 500 bps, maintaining the high accuracy of RTK-GPS.

KEY WORDS

1. RTK-GPS. 2. correction. 3. protocol. 4. SNUR-2000.

1. INTRODUCTION. As a first step toward GPS modernization, the Selective Availability (SA) was discontinued. The SA termination has brought tangible benefits to GPS users, especially to stand alone GPS users. Additionally, it enables Real Time Kinematic (RTK)-GPS users to receive correction data with low-rate data-link (Kalafus, 1983; Kremer, 1990). To minimize the required bandwidth of the data-link, we can minimize the amount of data or maximize the robustness of corrections to latency. Several observation based protocols have been designed to minimize the amount of data (Talbot, 1996; RTCM, 2002). However, these protocols are susceptible to latency; therefore, they require a latency compensator such as a reference phase predictor to mitigate the latency effect (Lawrence, 1996). Correction based protocols contain RTK-GPS corrections rather than raw observations. Corrections are robust to latency and independent of a latency compensator. Therefore, the correction based protocols have great potential to reduce the bandwidth of data-link; however, the RTK-GPS RS (reference station) should generate continuous and quasi-linear corrections to ensure the accuracy and reliability of RTK-GPS. Generating continuous and quasi-linear corrections is the key to reducing the bandwidth of data-link.

One of the most important functions of the RTK-GPS correction generator is to filter out random errors unique to the RTK-GPS RS (Loomis, 1989). The main obstacle to lowering the bandwidth of data-link is the short-term instability of the RS GPS receiver's clock. When the RS GPS receiver is not equipped with a stable atomic oscillator, all of the corrections are contaminated by clock phase error.

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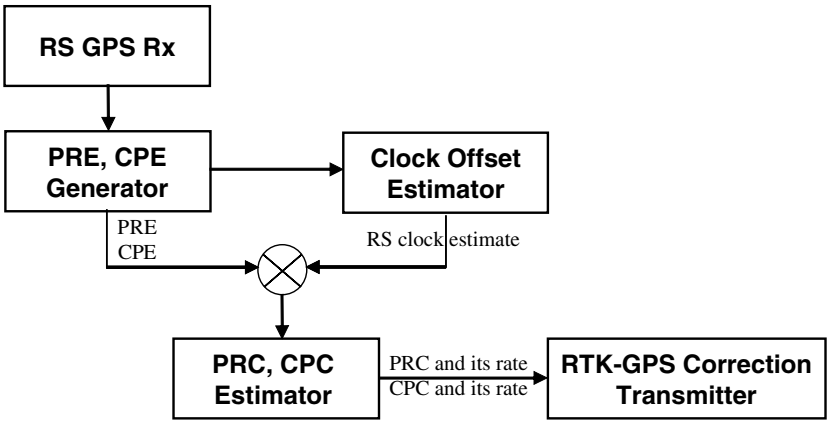


Figure 1. RTK-GPS correction generator.

An improved RS clock estimator can eliminate the clock phase error and sudden jumps from the clock offset of the RS GPS receiver, although the oscillator of the RS GPS receiver is not as stable as an atomic oscillator. We also introduce an efficient correction scheduling technique to reduce the latency effect on RTK-GPS positioning accuracy. It is anticipated that these techniques combined with an appropriate correction based protocol can lower the bandwidth of data-link significantly (Kee, 2002; Kim, 2002).

2. RTK-GPS CORRECTION GENERATING ALGORITHM.

2.1. *RTK-GPS corrections.* Figure 1 shows the construction of the RTK-GPS correction generator. First, it collects the real-time pseudorange, carrier phase observations, and ephemeris data. Second, it computes pseudorange error (PRE) and carrier phase error (CPE) for each satellite. It also monitors the integrity of PRE and that of CPE and detects the IOD (Issue of Data) change. Third, it estimates the clock offset of the RS GPS receiver. Fourth, it removes random noise from pseudorange correction (PRC) and carrier phase correction (CPC). Finally, it transmits the filtered PRC and CPC in an optimized sequence ensuring minimum latency effect.

The pseudorange and carrier phase observations are as follows;

$$\begin{aligned}\rho^{SV_i} &= d^{SV_i} - b^{SV_i} + B + I^{SV_i} + T^{SV_i} + \delta R^{SV_i} - \delta b^{SV_i} + \varepsilon^{SV_i} \\ \phi^{SV_i} &= d^{SV_i} - b^{SV_i} + B - I^{SV_i} + T^{SV_i} + \delta R^{SV_i} - \delta b^{SV_i} + \mu^{SV_i} + \lambda \cdot N^{SV_i}\end{aligned} \tag{1}$$

Where, the subscript SV_i refers to the i-th GPS satellite, ρ and ϕ are the pseudorange and carrier phase observation respectively. d is the computed distance between RS GPS receiver and satellite; b is the computed satellite clock offset; B the GPS receiver clock offset; I the Ionospheric and T the tropospheric errors; δR is the residual range error caused by ephemeris error; δb the residual satellite clock error; ε and μ are the pseudorange and carrier phase random errors and λ the wavelength for carrier phase; N is the integer ambiguity cycle. All measurements, with the exception of N, are in metres.

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The pseudorange error ρ_e (PRE) and carrier phase error ϕ_e (CPE) can be defined as (Loomis, 1989);

$$\begin{aligned} \rho_e^{SV_i} &= \rho^{SV_i} - d^{SV_i} + b^{SV_i} \\ \phi_e^{SV_i} &= [\phi^{SV_i} - \lambda \cdot \hat{N}^{SV_i}] - d^{SV_i} + b^{SV_i} \end{aligned} \tag{2}$$

where $\hat{N}^{SV_i}$ is the estimated integer ambiguity for i-th GPS satellite (cycle).
Let us define the PRC (ρ_c) and CPC (ϕ_c) (Loomis, 1989; Chou, 1991);

$$\begin{aligned} \rho_c^{SV_i} &= \hat{B} - \rho_e^{SV_i} \\ \phi_c^{SV_i} &= \hat{B} - \phi_e^{SV_i} \end{aligned} \tag{3}$$

Where, $\hat{B}$ is the estimated clock offset of the RS GPS receiver (metres).
To reduce the dynamic ranges of the CPC, the integer ambiguity of the CPC can be initialized as follows;

$$\hat{N}^{SV_i} = Round\left(\frac{\phi^{SV_i}(k_0) - \rho^{SV_i}(k_0) + 2 \cdot \hat{I}^{SV_i}(k_0)}{\lambda}\right) \tag{4}$$

Where, k_0 is the epoch of the ambiguity initialization and $\hat{I}^{SV_i}$ the computed ionospheric delay using Klobuchar model.

2.2. *Precise estimation of the clock offset of the RS GPS receiver.* The RS clock-offset can be estimated using pseudorange observations (Brown, 1997; Chou, 1991; Farrell, 1998). It is very important to reserve high precision of the estimated RS clock-offset, because we should remove the clock phase error from corrections; the clock phase error weakens the robustness of corrections to latency. The other crucial factor affecting the quality of the RS clock offset is the discontinuity caused by sudden jump (Loomis, 1989). Sudden jumps in the estimated RS clock offset appear directly in the broadcast PRC and CPC. These can apparently impair the RTK-GPS positioning accuracy in some RTK-GPS users implementing carrier-aided smoothing like the Hatch filter. For this reason, it is necessary to remove all sudden jumps from the estimate of RS clock-offset.

The factors of the sudden jumps can be classified into two groups; oscillator instability and change of the observation set. We can implement a highly stable oscillator as a clock of the RS GPS receiver to preclude the occurrence of a sudden jump caused by the former factor. Low pass filter with heavy damping can make the effect of the sudden jump caused by the latter factor negligible (Loomis, 1989; Chou, 1991; Zhang, 1997). However, when the higher order terms of the estimated RS clock offset are not negligible, we cannot implement a low pass filter to eliminate the sudden jumps caused by the change of observation set. The observation set changes when a new GPS satellite rises or an old GPS satellite sinks. We can use highly precise carrier phase observations to detect, identify and isolate the sudden jumps.

The RS clock offset can be estimated precisely using L1 carrier phase observation as follows:

$$\hat{B}(k) = \frac{\sum_{i=1}^{m(k)} \phi_e^{SV_i}(k)}{m(k)} - \Delta\hat{B}(l) \tag{5}$$

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Where, $\hat{B}(k)$ is the estimated RS clock offset; $m(k)$ is the number of the L1 CPE; $\Delta\hat{B}(l)$ the cumulative sum of the clock jumps and l, the number of occurrences of the sudden jumps. The estimated RS clock offset in equation (5) can be influenced by a sudden jump as follows:

$$\delta\hat{B}(k) = \frac{\sum_{j=1}^{m_{fs}(k)} \phi_e^{SV_j}(k)}{m_f(k)+m_s(k)} - \frac{\sum_{i=1}^{m_{nr}(k)} \phi_e^{SV_i}(k)}{m_n(k)+m_r(k)} \tag{6}$$

Where,

$m_n(k)$: The number of the newly risen GPS satellites
$m_r(k)$: The number of the L1 CPE whose ambiguities were re-initialized
$m_f(k)$: The number of the set GPS satellites
$m_s(k)$: The number of the L1 CPE whose cycles are slipped

The condition that there is no sudden jump is,

$$m_{nr}(k)=0,\ m_{fs}(k)=0 \tag{7}$$

Where, $m_{nr}(k)=m_n(k)+m_r(k)$ and $m_{fs}(k)=m_f(k)+m_s(k)$.
The basic idea to exclude sudden jumps from the estimated RS clock offset is that the sudden jump has the same effect on the CPC for each satellite. When there is a sudden jump, all of the predicted residuals of the CPC filters are identical. *A priori* L1 CPC including the sudden jump ($\phi_c^{SV_i}(k_)$) can be defined as follows:

$$\begin{aligned}\phi_c^{SV_i}(k_) &= \hat{B}(k_) - \phi_e^{SV_i}(k) \\ &= \delta B(k) + \phi_c^{SV_i}(k)\end{aligned} \tag{8}$$

Where, $\hat{B}(k_)$ is *a priori* RS clock offset including clock jump; $\phi_c^{SV_i}(k)$ *a posteriori* L1 CPC for i-th GPS satellite and $\delta B(k)$ a sudden jump. We can estimate the sudden jump using predicted L1 CPC whose age is less than several seconds:

$$\delta\hat{B}(k) = \frac{\sum_{i=1}^{m_c(k)} [\phi_c^{SV_i}(k_) - \hat{\phi}_c^{SV_i}(k\,|\,k-1)]}{m_c(k)} \tag{9}$$

Where, $m_c(k)$ is the number of the L1 CPC's that are commonly visible and cycle-slip-free; $\hat{\phi}_c^{SV_i}(k\,|\,k-1)$ is the predicted L1 CPC.

After estimating sudden jump of the estimated RS clock offset, we can replace *a priori* clock offset with *a posteriori* clock offset and update the cumulative sum of clock jumps as follows;

$$\begin{aligned}\hat{B}(k) &= \hat{B}(k_) - \delta\hat{B}(k) \\ \Delta\hat{B}(l) &= \Delta\hat{B}(l_) + \delta\hat{B}(k)\end{aligned} \tag{10}$$

Where, $\hat{B}(k)$ is the *a posteriori* RS clock offset after removing sudden jump; $\Delta\hat{B}(l_)$ the cumulative sum of the clock jumps before update and $\Delta\hat{B}(l)$ the updated cumulative sum of the clock jumps.

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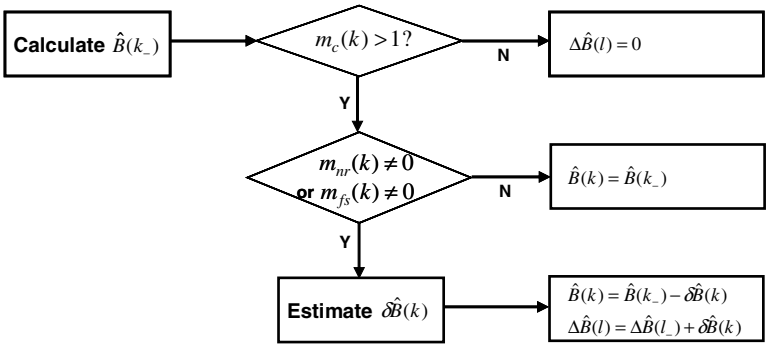


Figure 2. The estimator of the RTK-GPS RS clock offset.

Figure 2 describes the process of the RS clock offset estimator. First, estimate *a priori* RS clock offset using equation (5). Next, count the number of valid, commonly visible, and cycle-slip-free channels. If there is a sudden jump, we estimate and remove it from *a priori* estimate of the RS clock offset using equations (9) and (10).

2.3. *Filtering of the PRC and CPC.* We can implement a bank of independent Kalman filters to estimate the PRC and CPC as follows (Gelb, 1974; Loomis, 1989; Jin, 1995):

$$\begin{aligned} \hat{\mathbf{x}}(k+1|k) &= \Phi(k)\hat{\mathbf{x}}(k) \\ \mathbf{P}(k+1|k) &= \Phi(k)\mathbf{P}(k)\Phi(k)^T + \mathbf{Q}(k) \end{aligned} \tag{11}$$

$$\begin{aligned} \hat{\mathbf{x}}(k) &= \hat{\mathbf{x}}(k|k-1) + \mathbf{K}(k)[\mathbf{z}(k) - \mathbf{H}(k)\hat{\mathbf{x}}(k|k-1)] \\ \mathbf{P}(k) &= [\mathbf{I} - \mathbf{K}(k)\mathbf{H}(k)]\mathbf{P}(k|k-1) \\ \mathbf{K}(k) &= \mathbf{P}(k|k-1)\mathbf{H}(k)^{\mathbf{T}}[\mathbf{H}(k)\mathbf{P}(k|k-1)\mathbf{H}(k)^{\mathbf{T}} + \mathbf{R}(k)]^{-1} \end{aligned} \tag{12}$$

Where,

$$\hat{\mathbf{x}}(k) = [\,\hat{\rho}_c^{SV_i} \quad \hat{\dot{\rho}}_c^{SV_i} \quad \hat{I}^{SV_i} \quad \hat{\phi}_c^{SV_i} \quad \hat{\dot{\phi}}_c^{SV_i}\,]^T$$

$$\mathbf{z}(k) = [\,\rho_c^{SV_i} \quad \Delta\phi_c^{SV_i} \quad \phi_c^{SV_i}\,]^T$$

$$\Phi(k) = \left[\begin{array}{ccc|cc} 1 & \Delta t & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ \hline 0 & 0 & 0 & 1 & \Delta t \\ 0 & 0 & 0 & 0 & 1 \end{array}\right]$$

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$$\mathbf{Q}(k)=\begin{bmatrix} \frac{q_\rho \Delta t^3}{3} & \frac{q_\rho \Delta t^2}{2} & 0 & 0 & 0 \\ \frac{q_\rho \Delta t^2}{2} & q_\rho \Delta t & 0 & 0 & 0 \\ 0 & 0 & q_I \Delta t & 0 & 0 \\ \hline 0 & 0 & 0 & \frac{q_\phi \Delta t^3}{3} & \frac{q_\phi \Delta t^2}{2} \\ 0 & 0 & 0 & \frac{q_\phi \Delta t^2}{2} & q_\phi \Delta t \end{bmatrix}$$

$$\mathbf{H}(k)=\left[\begin{array}{ccc|cc} 1 & 0 & 0 & 0 & 0 \\ 0 & \Delta t & -2\Delta t & 0 & 0 \\ \hline 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

$$\mathbf{R}(k)=\begin{bmatrix} \sigma_\rho^2 & 0 & 0 \\ 0 & 2\sigma_\phi^2 & 0 \\ \hline 0 & 0 & \sigma_\phi^2 \end{bmatrix}$$

with Δt the sampling time (second), q_ρ, q_ϕ, q_I the spectral density for the random walk process; σ_ρ, σ_ϕ the standard deviations of the PRC and CPC; and $\Delta\phi_c^{SV_i}=\phi_c^{SV_i}(k)-\phi_c^{SV_i}(k-1)$ the time difference of the CPC.

In equation (11) and (12), we ignored temporal, inter-channel, and inter-frequency correlations (Jonkman, 1998). The parameters (q_ρ, q_ϕ) are tuned to the dynamics of the ephemeris and atmospheric errors. The parameter (q_I) is tuned to low-frequency fluctuation of the ionospheric error.

3. RTK-GPS CORRECTION SCHEDULING TECHNIQUE.

An RTK-GPS RS generates each PRC and CPC for every visible GPS satellite and broadcasts correction messages. Since SA has been terminated, the latency effect on DGPS is very low, even if the latency reaches as far as several minutes. However, the performance of RTK-GPS is quite sensitive to latency because the required positioning accuracy is several centimetres. This means that the CPC error caused by latency should be below the noise level of carrier phase observation. The CPC latency is directly proportional to the amount of the CPC data. Therefore, it is necessary to minimize the amount of the CPC data to minimize the effect of latency on the RTK-GPS performance.

In general, the RTK-GPS RS transmits the CPC's for all visible GPS satellites in the same data block (Goguen, 1995; RTCM, 2002; Talbot, 1996). As the latency grows, each CPC has different value of latency error, even though the latency is the same. This is because the linearity of the CPC is different one from another. Transmitting linear CPC less frequently can increase the efficiency of the correction broadcasting and reduce the effect of latency on RTK-GPS positioning accuracy. The latency error of each CPC should be estimated precisely prior to transmitting CPC asynchronously at the RTK-GPS RS.

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Table 1. The summary of GPS data processing.

	Reference Station	User
Location	SNU, Seoul	SNU, Seoul
Latitude[a]	37° 26′ 58·903″ N	37° 27′ 03·603″ N
Longitude[a]	126° 57′ 09·887″ N	126° 57′ 06·006″ N
Height[a]	281·925 m	217·704 m
Date	March 3, 2003 ~ March 4, 2003	
GPS Receiver	Trimble 4000SSi (L1/L2 Dual-frequency)	
GPS Receiver clock	Receiver internal (High-quality CXO)	
Sampling Time	1 sec	
Elevation Mask	10°	
Baud-rate	500 bps	

[a] WGS-84 based latitude, longitude, and height.

The RTK-GPS RS can predict the expected latency error of the CPC as (Kee, 2002),

$$\delta\hat{\boldsymbol{\phi}}_c^{SV_i}(t) = \hat{\boldsymbol{\phi}}_c^{SV_i}(t) - [\hat{\boldsymbol{\phi}}_c^{SV_i}(t_{tr}^{SV_i}) + \dot{\hat{\boldsymbol{\phi}}}_c^{SV_i}(t_{tr}^{SV_i}) \cdot \Delta t_r^{SV_i}] \tag{13}$$

Where, t is GPS time of current epoch; $t_{tr}^{SV_i}$ the recent transmission time of the CPC for i-th GPS satellite; $\hat{\boldsymbol{\phi}}_c^{SV_i}(t)$ is the estimated CPC for i-th GPS satellite; $\delta\hat{\boldsymbol{\phi}}_c^{SV_i}(t)$ the CPC error caused by latency; and $\Delta t_r^{SV_i} = t - t_{tr}^{SV_i}$ the expected latency computed at the RTK-GPS RS.

We can transmit the CPC satisfying the following condition:

$$\left|\delta\hat{\boldsymbol{\phi}}_c^{SV_i}(t)\right| > \sigma_\phi \tag{14}$$

Where σ_ϕ is the standard deviation of the CPC. Using this strategy, we can reduce the required amount of correction data as well as the CPC error caused by latency (Kee, 2002).

4. EXPERIMENTAL RESULTS. We analyzed the performance of the RTK-GPS correction generator and the positioning accuracy of the RTK-GPS using GPS data. Table 1 shows a summary of the GPS data processing. We collected 1-Hz real-time GPS data during the 24 hours at the RTK-GPS RS and the RTK-GPS user station simultaneously. We used two sets of Trimble 4000 SSi to collect the GPS data. Figure 3 shows the sky-plot of all visible GPS satellites at the RTK-GPS RS during the collection of the GPS data. The length of baseline is about 185 metres and the altitude difference 64 metres. Using the collected GPS data, we generated RTK-GPS corrections for all visible satellites and transmitted the dual-frequency corrections to the RTK-GPS user through a virtual 500 bps data-link. We used the SNUR-2000 v. 2.2 as a RTK-GPS protocol (Kee, 2002; Kim, 2002) which is optimized for low-rate data-link.

Figure 4 shows the estimated offset, drift, and acceleration of the RS GPS receiver clock and Figure 5 shows the Allan standard deviation. The short-term stability is about $8{\cdot}2 \times 10^{-11}$ second when the averaging time is 1 second, which is equivalent to that of high-quality quartz crystal oscillator (Sturza, 1983).

Figures 6 and 7 show the L1 PRC and its rate, L1/L2 CPC and their rates for PRN 16. Since the SA has been terminated, the dynamic ranges of the PRC rate and the

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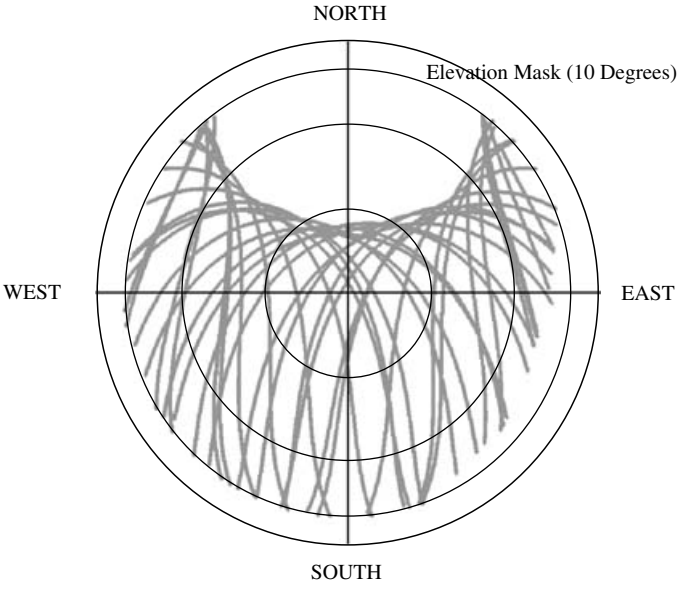


Figure 3. Sky plot of all visible GPS satellites at the RTK-GPS RS.

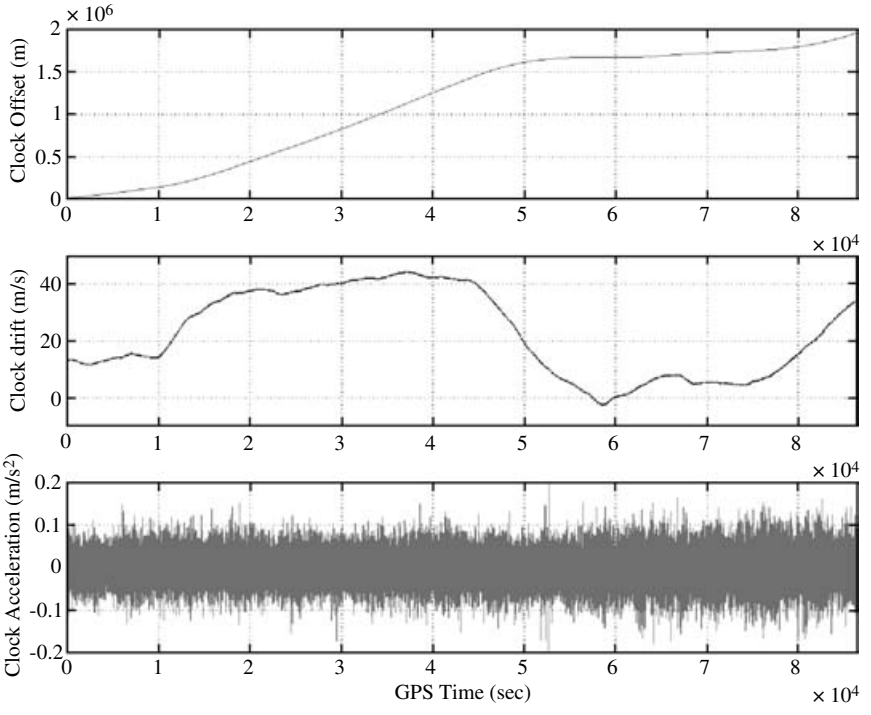


Figure 4. Estimated clock offset, drift and acceleration.

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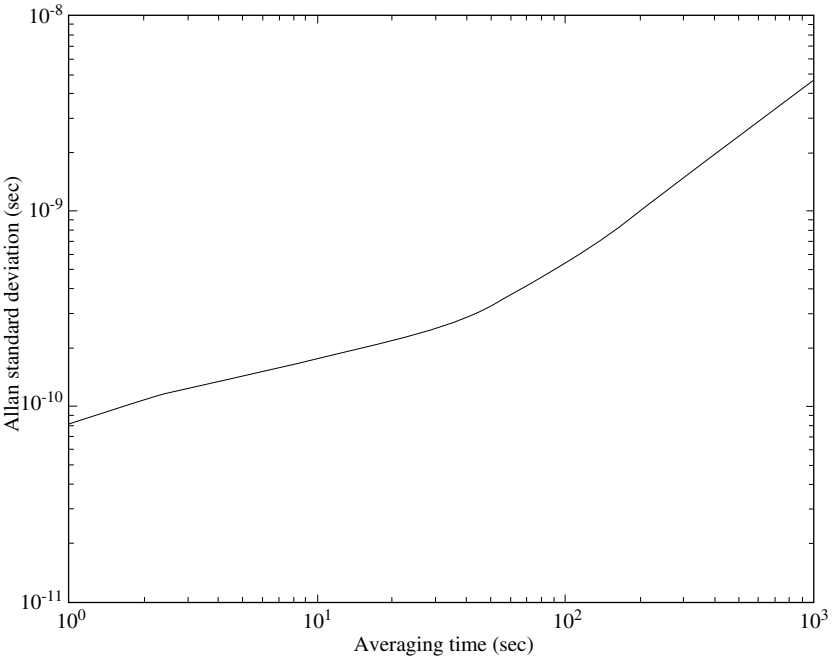


Figure 5. Allan standard deviation of the clock of the RTK-GPS RS receiver.

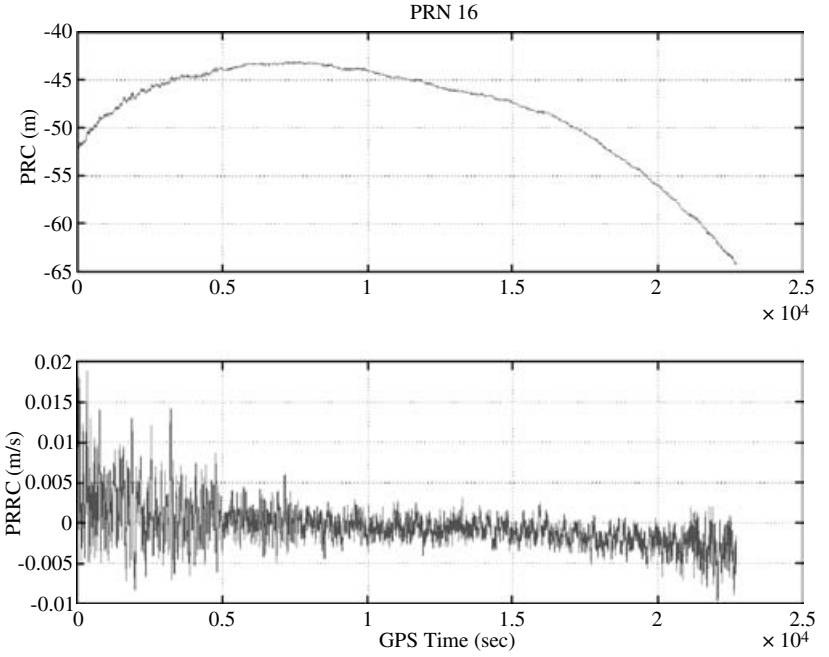


Figure 6. L1/L2 PRC and its rate for PRN 16.

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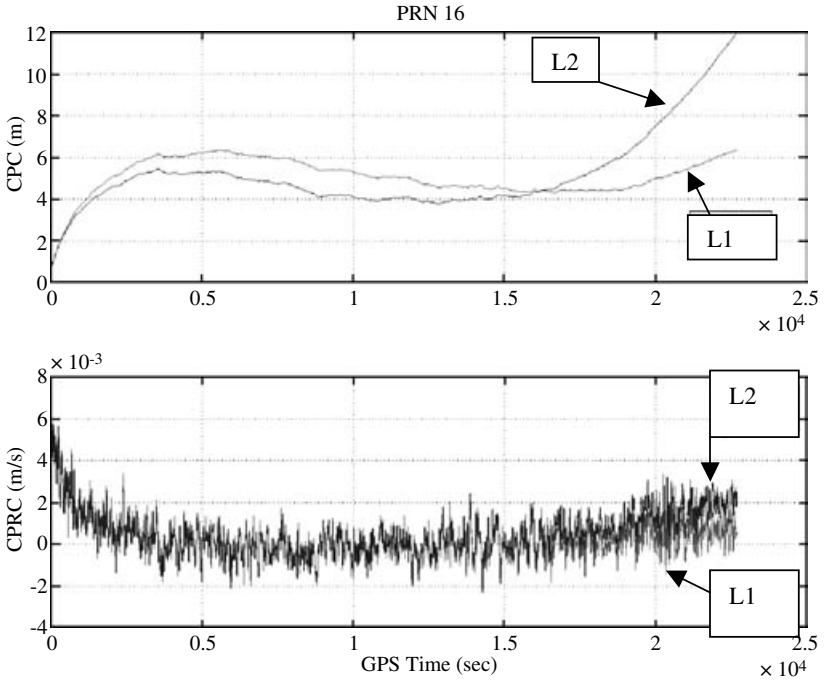


Figure 7. L1/L2 CPC and their rates for PRN 16.

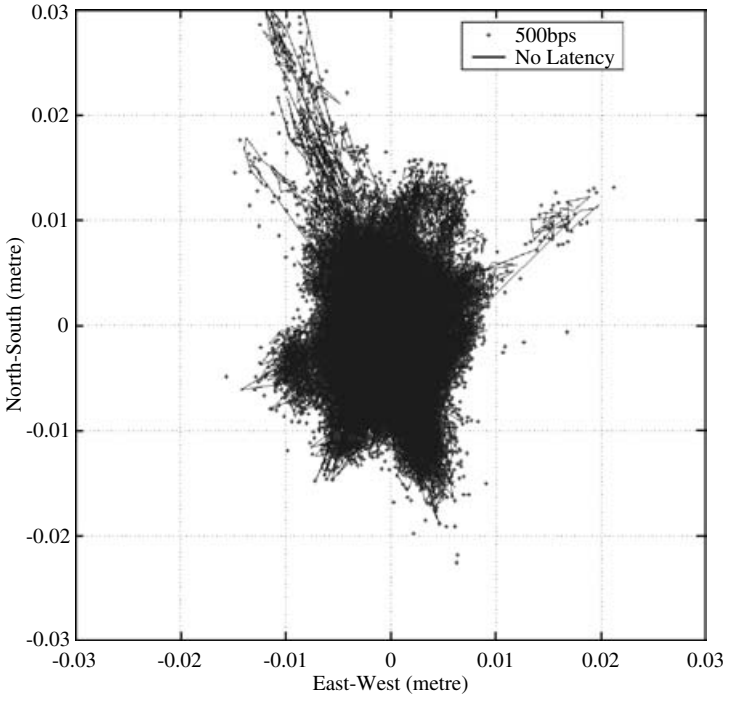


Figure 8. Horizontal positioning error of RTK-GPS.

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Table 2. RMS positioning error analysis.

	No latency	500 bps
East	0·265 cm	0·272 cm
North	0·390 cm	0·398 cm
Up	0·966 cm	0·984 cm

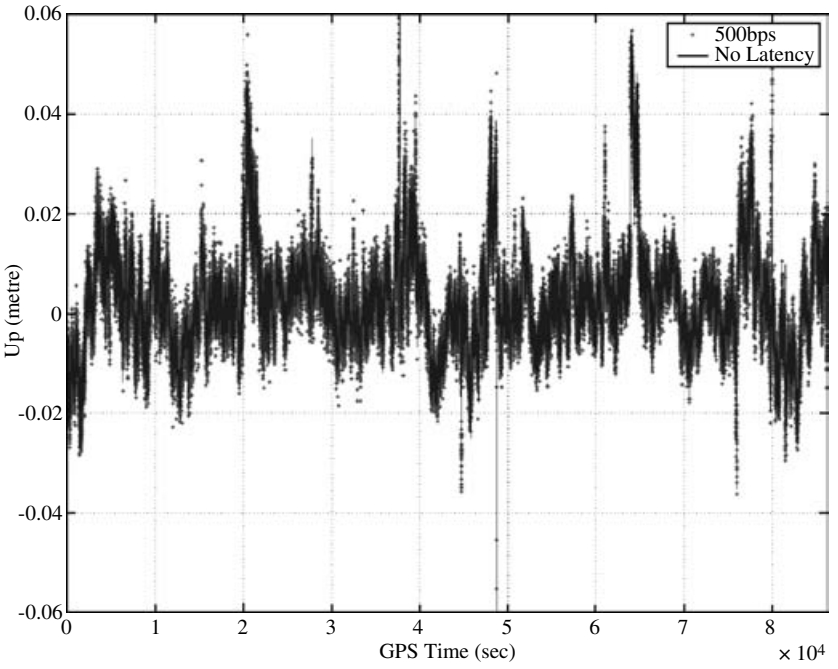


Figure 9. Vertical positioning error of RTK-GPS.

CPC rate are within several centimetres for low elevation angles and several millimetres for high elevation angles. In Figure 7, we can find that short period coloured noise remains, this is small multi-path error and clock phase error of GPS satellite. It is the short period random errors that are the main obstacles to reducing the bandwidth of data-link. Since L1/L2 GPS antenna with ground plane was used as a GPS antenna at the RTK-GPS RS, most of the multi-path signals were blocked. The short-term stability of GPS satellite clock is on the level of $10^{-11} \sim 10^{-12}$ second (Bowen, 1985; Lutwak, 2001). When the older GPS II/IIA satellites are replaced with new GPS III satellites, the short-term stability of GPS satellite clock can be enhanced up to $10^{-12} \sim 10^{-13}$ second (Lutwak, 2001). The RTK-GPS RS transmitted every PRC and CPC asynchronously using the correction scheduler. The time-average of CPC latency is 1·7 seconds, and that of PRC latency 8·8 seconds.

Figure 8, 9 and Table 2 show the RMS positioning errors of the RTK-GPS user both for no latency and 500 bps. As a method for resolving double difference (DD) cycle ambiguities, we implemented LAMBDA (Least squares AMBiguity

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Decorrelation Adjustment) (De Jonge, 1998). The additional horizontal and vertical RMS errors due to latency do not exceed 0·1 mm and 0·2 mm, respectively. No kind of compensator for reducing the latency error, such as the reference phase predictor (Lawrence, 1996; Neumann, 1996), was implemented at the RTK-GPS user station. The RTK-GPS user station reduced the latency effect by first order extrapolation using CPC rate. These experimental results demonstrate that the accuracy of RTK-GPS with 500 bps data-link is equivalent to that of RTK-GPS with high-rate data-link. And there is no performance degradation for resolving DD ambiguities, even though only 500 bps data-link is used (Kee, 2002).

5. CONCLUSIONS. To realize precise and reliable RTK-GPS using low-rate data-link, we improved RTK-GPS correction generating algorithms. Without resorting to a highly stable atomic clock, we can isolate the clock phase error and sudden jumps of the estimated clock offset of the RS GPS receiver. The experimental results show that the additional positioning error of RTK-GPS caused by latency is less than 1 mm, even though the bandwidth of data-link is as low as 500 bps. When the GPS satellites are upgraded, we expect to realize precise and reliable RTK-GPS with a lower-rate data-link, like the Eurofix system (Offermans, 1999).

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