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Coupled azimuthal modes propagating in current-carrying plasma waveguides

V. GIRKA, I. GIRKA, I. PAVLENKO, O. GIRKA and A. GIRKA

Kharkiv National University, Svobody sq., 4 Kharkiv 61077, Ukraine
(v.girka@gmail.com)

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Abstract. The paper is devoted to the theory of electromagnetic surface waves propagating along the azimuthal direction in cylindrical metal waveguides, which are filled with current-carrying plasmas. The problem is solved by the method of successive approximation. Adequacy of this method application is proved here. To study the coupling of ordinary (O-) and extraordinary (X-) azimuthal modes, the linear theory of the eigenazimuthal X- and O-modes is applied as zero approximation. Plasma particles are described in the framework of magneto-hydrodynamics, electromagnetic fields of the coupled azimuthal modes are determined from Maxwell equations. Spatial distribution of electromagnetic field of these coupled modes and their damping caused for different reasons are studied. Possibility to observe experimentally the phenomena, which accompany propagation of these coupled modes, is estimated numerically. Branches of their possible utilization are discussed as well.

1. Introduction

Plasma-filled waveguides are actively studied both theoretically and experimentally for the purposes of radiophysics and electronics [1]. Experimental investigation of plasmas' influence on the efficiency of antennas' operation and testing various antennas with plasma coating prove that power of their radiation enhances and their frequency characteristics become better. Design of antennae (circular or cylindrical shape, presence or absence of dielectric coating, etc.), operating frequency range, applications, either magnetoactive or isotropic plasma affect on the field of radiation of both azimuthally symmetrical and azimuthally non-symmetrical waves, increase their amplitude and enhance quality of antenna. It is found that power of radiation obtained from the antennas that are immersed into plasma depends on the utilized plasma density in a resonant manner [2].

The waves being excited in restricted plasma-waveguide structures should be the eigenwaves of such structures. Among the eigenwaves, there are also electromagnetic waves in which amplitude decreases with moving away from plasma interface. These waves are usually referred to as surface waves (SW). These can propagate not only in the case of gaseous plasma but also in the case of solid-state plasmas, see, for example [1, 3]. Moreover, SW can propagate in multi-layered planar waveguide structures, where the layers could be produced from materials with different electro-dynamical properties. Possibility of SW propagation along flat plasma-metal boundary was indicated for the first time by Hirota [4] and after that their properties have been actively studied by many authors. Review devoted to the properties of SW propagating along planar plasma-metal boundary across an external magnetic field is presented in [5]. However, one often deals with plasma boundary that is characterized by curvature radius of finite value in experimental practice and these circumstances strongly affect the properties of SW propagating in such plasma-waveguide structures. Namely, it leads to the changing of frequency ranges within which these waves can propagate, spatial distribution of their fields, their polarization, and so on (see for example [6]). Propagation of SW is also widely used in various plasma technologies, which utilize gas discharge for cleaning and polishing solids, pumping lasers and plasma production, elaboration of fullerenes, etc. It is explained by the fact that these possess a lot of advantages over bulk waves; for instance, they can produce plasma with more spatially uniform density profile, efficiency of their interaction with plasma particles is also higher [7].

Experimental investigation of various phenomena in plasmas confined in different devices and properties of various waves that can propagate there under different regimes are rather expensive, so the role of theoretical investigation in the various branches of plasma physics is the actual one. Development of plasma-based technologies initiates studying the more and more complicated models of plasma and waves propagating in it. Primarily, for instance, it could be enough to investigate electrostatic waves in non-magnetized plasma [8], but not long after that technological progress puts new tasks and investigation of electromagnetic waves in magnetized plasma becomes an actual problem [9]. Construction of

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more complicated plasma devices unavoidably leads to the appearance of more complicated plasma phenomena, for example, to coupling between different modes. In the paper by Rama and Schultz [10], conditions for optimized mode conversion to the electron Bernstein waves are defined analytically and numerically using cold plasma and approximate kinetic models. It is proved there that conventional O- and X-modes from the electron cyclotron range of frequencies are not suitable for core heating in modern spherical tokamak plasmas, as they damp weakly at high harmonics in this range. A two-dimensional integral full-wave model was used in [11] to calculate poloidal forces driven by mode conversion in tokamak plasmas. It was found that mode conversion near ion–ion hybrid resonance is dominated by transition from the fast magnetosonic wave to the slow ion cyclotron wave in the presence of steady poloidal magnetic field. Experimentalists often deal with plasma, which is either sustained by the flow of electric current, or flow of current in confined plasma is necessary because of some other reasons. For instance, in tokamak, plasma is confined by both toroidal and poloidal magnetic fields, the last one is produced just by axial electric current. So, conversion of O- and X-modes is a typical phenomenon, which is actively used, for instance, for plasma heating. The technique, referred to as the O–X–B mode conversion, has been utilized successfully on stellarator Wendelstein 7-AS. Experimental measurements and modeling the mode conversion at ion cyclotron range of frequencies in the Alcator C-Mod tokamak were under consideration in [12]. Nonlinear interaction between upper-hybrid and the Alfven modes in magnetized plasma is studied in [13].

This paper is devoted to theoretical investigations of interaction between the electromagnetic eigenmodes, which propagate along azimuthal direction in cylindrical current-carrying plasma-filled waveguides. As these propagate along azimuthal direction (with zero axial wave vector), these have been entitled to be azimuthal waves [14]. Often value of axial electric current is small enough to consider azimuthal magnetic field produced by it as a small parameter of the considered task. Thus, here we have restricted our research by the small value of this azimuthal magnetic field. To solve the problem of coupled azimuthal waves we have applied here the method of successive approximation and used results of the linear theory of azimuthal X- and O-modes as zero-th approach.

The paper is arranged as follows: the first section is devoted to reviewing the properties of azimuthal surface waves (ASW) with different polarizations that can propagate in cylindrical metal waveguides filled with either isotropic or gyrotropic plasma. Interaction between ordinarily polarized surface or bulk waves and extraordinarily polarized ASW in waveguides filled with current-carrying plasma is studied in the second section; finally, results obtained are summarized in Conclusions.

1.1. Azimuthal surface waves (ASW) in cylindrical metal waveguides with plasma filling

Let us consider here properties of electromagnetic waves propagating across axial direction in cylindrical metal waveguides filled with either isotropic or gyrotropic plasma. Influence of waveguides parameters, such as radius of plasma column, value of external magnetic field, constructional peculiarities, etc. on the azimuthal waves' properties, is examined in this section under the condition of uniform plasma filling. This assumption is valid both for the case of solid-state plasma (plasma of semi-conductors) and for the case of gaseous plasma with a thin transient plasma layer (its thickness is small as compared to penetration depth of azimuthal modes). It is assumed that external constant magnetic field is uniform and it has only axial component in the case of waveguides filled with magnetoactive plasma.

Dependence of ASW electromagnetic fields on coordinates and time is assumed in the following form: $f(r)\exp(im\varphi - i\omega t)$, where φ is azimuthal angle, m is azimuthal wave number, and ω is ASW frequency. In approach to cold magnetoactive plasma, Maxwell equations can be divided in two independent sets of equations, which describe extraordinary and ordinary polarized waves (or, in other words, X- and O-modes, correspondingly). As spatial distribution of O-modes is often of bulk character, and they don't depend on an external axial magnetic field but strongly depend on features of dielectric layer between plasma column and metal wall of the waveguide, the main attention is given to the properties of X-modes [14]. Thus, the next five subsections are devoted to studying the properties of azimuthal X-modes, and properties of the azimuthal O-modes are studied in the last subsection. Nevertheless, knowledge on the O-modes is necessary for studying the case of coupled azimuthal modes, which is under consideration in the second section and is the main content of this paper.

1.1.1. Extraordinary polarized ASW propagating in magnetoactive plasma waveguides. Let us consider the properties of ASW, which propagate along the interface of plasma column of radius R_1, and are separated by a dielectric layer from metal wall of cylindrical waveguide of radius $R_2 > R_1$. An external constant magnetic field is oriented along the axis of the waveguide $\vec{B}_0 \| \vec{z}$. We consider in this subsection the case of dense plasma, when the inequality, $\Omega_e^2 > \omega_e^2$, is valid; here ω_e and Ω_e are electron cyclotron and the Langmuir frequencies, respectively.

In the region of plasma column, the set of Maxwell equations derived for azimuthal X-modes can be reduced to the modified Bessel equation for the ASW magnetic field. Consequently, its solution can be represented in the following general form:

$$H_z = A_1 I_m(k_\perp r) + A_2 K_m(k_\perp r), \qquad (1)$$

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where $k_\perp = k\sqrt{(\mu^2-1)\varepsilon_1}$, $\mu = \varepsilon_2/\varepsilon_1$, $k = \omega/c$, A_1 and A_2 are constants of integration, $I_m(z)$ and $K_m(z)$ are modified Bessel function and MacDonald function, respectively [15], and ε_{ik} is a tensor of dielectric permeability of a cold magnetoactive plasma [16].

Then in the region $R_2 > r > R_1$, ASW fields are described by a superposition of Neumann $N_n(z)$ and the first kind Bessel $J_n(z)$ functions [15]:

$$H_z = B_1 J_m(kr) + B_2 N_m(kr), \tag{2}$$

where B_1 and B_2 are constants of integration.

In order to determine integration constants, one can apply the following boundary conditions. First, ASW fields should be restricted inside the whole waveguide and, in particular, on the waveguide axis; second, tangential components of the ASW fields should be continuous on the plasma-dielectric interface; third, tangential components of the ASW electric field should be equal to zero on the metal wall of the waveguide.

Azimuthal X-modes exist in the form of surface-type waves if the following inequality is valid: $k_\perp^2 > 0$. Frequency ranges, within which this inequality is valid, are determined by the following expressions:

$$\omega_{LH} < \omega < |\omega_e|, \quad |\omega_e| < \omega < \omega_1, \tag{3}$$

$$\omega_{UH} < \omega < \omega_2, \tag{4}$$

where $\omega_{1,2} = \mp|\omega_e|/2 + \sqrt{\Omega_e^2 + \omega_e^2/4}$ are cut-off frequencies for bulk modes, ω_{LH} and ω_{UH} are lower and upper hybrid frequencies, respectively. Let us entitle frequency ranges (3) and (4) as low-frequency (LF) and high-frequency (HF) ranges, respectively. Applying the above-indicated boundary conditions, one can derive the following dispersion equation $D_o(\omega) = 0$ for azimuthal X-modes:

$$D_0 = \frac{I'_m(k_\perp R_1)}{I_m(k_\perp R_1)}\sqrt{\frac{\varepsilon_1}{\varepsilon_2^2-\varepsilon_1^2}} + \frac{m\varepsilon_2}{kR_1\left(\varepsilon_2^2-\varepsilon_1^2\right)} + \frac{J'_m(kR_1)N'_m(kR_2) - J'_m(kR_2)N'_m(kR_1)}{J_m(kR_1)N'_m(kR_2) - J'_m(kR_2)N_m(kR_1)}, \tag{5}$$

here and in the following text, prime symbol denotes derivative of the corresponding cylindrical function over its argument.

Let us consider the most interesting case, when thickness of a vacuum layer that separates the plasma column from the inner surface of metal wall of the waveguide is small enough: $\Delta = 1 - R_1 R_2^{-1} \ll 1$, and effect of plasma is most pronounced. In this case the dispersion equation (5) can be rewritten in a more simple form:

$$\frac{I'_m(k_\perp R_1)}{I_m(k_\perp R_1)}\sqrt{\frac{\varepsilon_1}{\varepsilon_2^2-\varepsilon_1^2}} + \frac{m\varepsilon_2}{kR_1\left(\varepsilon_2^2-\varepsilon_1^2\right)} \approx \frac{m^2 - k^2R_1^2}{kR_1}\Delta. \tag{6}$$

Analyzing expression (6) can show that azimuthal wave number m cannot be equal to zero if $\Delta \neq 0$; and only in a special case of plasma-metal boundary (when $\Delta = 0$) one can realize oscillations with $m = 0$ [17]. One can obtain simple expression for the solutions of dispersion equation (6) in some limiting cases.

In the HF range (4) the sign of azimuth number can only be negative, and for $|m| \gg 1$, frequency of azimuthal X-modes can be represented in the following form:

$$\omega \approx \omega_1\left(1 + \frac{m^2\delta^2}{2R_1^2} - \Delta m\frac{\omega_e}{\omega_1}\right), \tag{7}$$

here inequality $1 \gg m^2\delta^2 R_1^{-2} \gg m\omega_e\Delta\omega_1^{-1}$ is assumed to be true. In the limiting case of the large value of waveguide radius ($k_\perp R_1 \gg |m|$) the azimuthal X-modes frequency can be expressed in another form:

$$\omega \approx \omega_1 + \Omega_e\frac{m^2\delta^2}{2R_1^2}\left(1 - 2m\Delta\frac{\omega_e}{\omega_1}\right), \tag{8}$$

here $R_1^2 > \delta^2 m^2 \Omega_e/|\omega_e|$ and $\omega_1 > m\omega_e\Delta$. For azimuthal X-modes, in which frequency belongs to the LF range (3), the dispersion equation (6) has the following solution:

$$\omega^2 \approx \frac{|m|\,\Delta\Omega_e^2}{1+|m|\,\Delta}\left\{1 + \frac{\Delta R_1^2[1+2\Delta(1+|m|)]}{2(1+|m|\Delta)^2\delta^2(1+|m|)}\right\}, \tag{9}$$

which is derived in the limiting case, $R_1 \ll \delta$ and $\delta|m| < 1|$. If inequality $|m| \gg 1$ is valid, then one can find the following expression for the azimuthal X-modes frequency:

$$\omega \approx \frac{\pm|\omega_e|}{2\psi_1} + \sqrt{\frac{\Delta\,|m|\,\Omega_e^2}{\psi_1} + \frac{\omega_e^2}{4\psi_1^2}}, \tag{10}$$

here $\psi_1 \approx 1 + \Delta R_1^2\delta^{-2}|m|^{-1} + 2.7R_1\delta^{-1}|m|^{-1}$, upper sign relates to the case of positive m, and bottom one relates to the case $m < 0$. In the limiting case, $k_\perp R_1 \gg |m|$, analysis of dispersion equation (6) allows one to find the following solution:

$$\omega \approx \frac{-m\omega_e}{2\psi_2} + \sqrt{\frac{m^2\Omega_e^2\Delta}{\psi_2} + \frac{m^2\omega_e^2}{4\psi_2^2}}, \tag{11}$$

here $\psi_2 \approx R_1\delta^{-1} + R_1^2\delta^{-2}\Delta$ and $R_1 \gg \delta|m|$. Thus, azimuth wave number m can be both positive and negative in the LF range unlike in the case of HF azimuthal X-modes. Analyzing the analytical expressions derived for the LF azimuthal X-modes, one can make the following inferences: Their dispersion is normal; their frequency increases with increase in vacuum layer thickness.

Results of numerical analysis of expression (5) show [14] that increase in radius R_1 of the plasma column, external magnetic field B_0, and thickness Δ of vacuum layer leads to decrease in the frequency of HF azimuthal X-modes. Unlike that, the LF azimuthal X-modes depend on these parameters in a different way: their frequency increases with increasing Δ and with diminishing of R_1 and B_0. Change in the sign of azimuth wave number makes a substantial effect on the azimuthal X-modes dispersion properties. HF branch of X-modes is absent in the case $m > 0$, and frequency of their LF branch with $m > 0$ considerably grows in comparison with the respective values of their frequencies in the case $m < 0$. Consequently, X-modes in such plasma structures are non-mutual: Frequencies of the waves that are characterized by azimuthal wave numbers with identical

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absolute value, but with opposite signs, are different under the same other conditions. This is typical for SW propagating across an external magnetic field oriented along plasma boundary [1, 3, 5].

1.1.2. Extraordinary polarized ASW propagating along magnetoactive plasma-metal boundary. Let us consider the properties of ASW that propagate along the interface between cold magnetoactive plasma and the metal wall [17] of corresponding waveguide. Such structure can be realized for the following cases: Cylindrical metal waveguide is completely filled with plasma or cylindrical metal bar is immersed into plasma. Thus, one can apply the coordinate system that is same as in previous subsection, but the radii of metal and plasma surfaces are equal to each other $R_1 = R_2 = R$. Other initial assumptions, which have been done in the Sec. 1.1, are also valid here. That is why spatial distribution of ASW fields in the plasma region is described in general by expression (1). But one can understand that in the case of plasma cylinder that is surrounded by metal wall with ideal conductivity the second constant of integration is equal to zero, i.e., $A_2 = 0$, and in the case of cylindrical metal bar that is immersed into magnetoactive plasma the constant $A_1 = 0$. Starting from the Maxwell equations, one can derive equations that establish connections between magnetic and electric components of X-modes:

$$E_r = \frac{\omega}{ck_\perp^2}\left[\mu\frac{\partial H_z}{\partial r} + \frac{m}{r}H_z\right],$$

$$E_\varphi = \frac{i\omega}{ck_\perp^2}\left[\frac{\partial H_z}{\partial r} + \mu\frac{m}{r}H_z\right]. \qquad (12)$$

As on the plasma-metal interface the boundary condition $E_\varphi(r = R) = 0$ is valid, one can find from (1) and (12) the following dispersion relation for azimuthal X-modes propagating along plasma-metal:

$$k_\perp R F_m'(k_\perp R) + m\mu F_m(k_\perp R) = 0. \qquad (13)$$

In the case of metal waveguide, which is completely filled with plasma, $F_m(\xi) = I_m(\xi)$, and $F_m(\xi) = K_m(\xi)$ for the case of cylinder metal bar that is immersed into magnetoactive plasma. In the approach of large value of azimuthal wave number, $|m| \gg 1$, (13) transforms into dispersion relation for SW, which propagate exactly across the external magnetic field along a planar plasma-metal boundary [1, 5]. In this case the ratio m/R plays a role of a wave vector. Analysis of the dispersion relation (13) proves that in the case under consideration in this subsection, the frequency range of azimuthal X-modes diminishes as compared with the case of three-component waveguide structure: metal-dielectric plasma. Equation (13) has solutions in the shortened frequency range (3) (namely, only in the range $\omega_{LH} < \omega < |\omega_e|$) and in the HF range (4).

Unlike the case that has been investigated in Sec. 1.1, ASWs propagate along plasma-metal boundary across axial external constant magnetic field only in one direction in all frequency ranges and they can be only X-polarized modes. This corresponds to the general conception of the SW theory, in the case of their propagation along plasma-metal interface [5] across external magnetic field. The sign of azimuth wave number m correlates uniquely with the type of plasma surface curvature. Namely, in the case $F_m = I_m$, wave number is positive, $m > 0$ in the frequency range $\omega < |\omega_e|$, and in the range (15) ASW number is negative, $m < 0$. For the case of cylindrical metal bar that is immersed into plasma, the situation is opposite: $m < 0$ for LF ASW ($\omega < |\omega_e|$) and $m > 0$ for HF ASW ($\omega_{UH} < \omega$).

Let us derive analytical solutions of dispersion relation (13) in some limiting cases. Axial-symmetric ($m = 0$) solution of (13) is realized under the condition $\varepsilon_1^2 = \varepsilon_2^2$. It describes surface oscillations of plasma cylinder at cut-off frequencies of bulk modes $\omega = \omega_{1,2}$. In the case $|m| \geqslant 1$ in the LF range spectrum of extraordinary polarized ASW can be represented by the following expressions:

$$\omega \approx \begin{cases} |m\omega_e|\,\delta R^{-1},\ R > |m|\,\delta,\ \delta = c/\Omega_e \\ |\omega_e|\left(1 + R^2\omega_1^2 m^{-2}c^{-2}\right)^{-1/2},\ |m| \gg 1 \\ |\omega_e|\left[1 + 0.5R^2\delta^{-2}(|m|)^{-1}(|m| \pm 1)^{-1}\right]^{-1},\ R < \delta \end{cases}. \qquad (14)$$

Here and in the next formula of this subsection, application of double signs means that upper sign relates to the case $F_m = I_m$, and bottom sign relates to the case $F_m = K_m$. In the case of HF ASW, dispersion relation (13) has the following solution:

$$\omega \approx \sqrt{\omega_1^2 + \frac{m^2c^2}{R^2}},\quad \frac{R^2\,|\omega_e|}{\delta^2\Omega_e} \gg m^2 \gg 1. \qquad (15)$$

Frequencies of HF and LF ASW increase with increasing azimuthal mode number $|m|$, these tend to the maximum values of LF and HF ranges with increasing B_0 value. Difference between ASW frequencies in the cases $F_m = K_m$ and $F_m = I_m$ is negligibly small.

Thus, account of finite value of curvature radius of plasma interface changes SW features as compared with the case of planar plasma-metal [5] structures. First, the azimuthal X-modes frequency has two limiting values in both HF and LF ranges. Second, unique correspondence exits between the sign of azimuth wave number and sign of curvature radius of plasma surface. Third, spatial distributions of these transverse modes propagating along planar and curved plasma-metal interfaces are different, namely, ASW are extraordinary polarized waves, which have two electrical components and one magnetic component, while in the case of planar waveguide, tangential electric field of SW is absent in all volumes of plasma.

1.1.3. Extraordinary polarized ASW propagating in isotropic plasma waveguides. Under the condition of absence of an external magnetic field, ASW cannot

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propagate along the boundary between cold plasma and metal: this is evident from the analysis of (13). Solutions of dispersion relation in the form of azimuthal X-modes can exist if metal wall of waveguide is separated from isotropic plasma by dielectric layer. That is why let us consider cylindrical metal waveguide that is filled with cold isotropic plasma with radius R_1 that is separated by dielectric layer with thickness $R_2 - R_1$ from a metal wall of the waveguide. Metal wall is assumed to be an ideal conductor, ε_p is dielectric permeability of cold isotropic plasma [16], and the dielectric layer is characterized by permeability ε_d. Then, in the plasma region the ASW magnetic field is described by the modified Bessel functions with the argument $kr\sqrt{|\varepsilon_p|}$, and in the dielectric region these fields are described by the superposition of the first type Bessel functions, $J_n(kr\sqrt{\varepsilon_d})$, and Neumann functions, $N_n(kr\sqrt{\varepsilon_d})$ [15].

Using the above-mentioned boundary conditions, one can derive the following dispersion relation [18] for extraordinary polarized ASW:

$$\sqrt{\frac{\varepsilon_0}{|\varepsilon|}}\frac{I'_m(x_3)}{I_m(x_3)} = \frac{J'_m(x_1)N'_m(x_2) - J'_m(x_2)N'_m(x_1)}{J'_m(x_2)N_m(x_1) - J_m(x_1)N'_m(x_2)}, \quad (16)$$

where $x_{1,2} = kR_{1,2}\sqrt{\varepsilon_d}$ and $x_3 = kR_1\sqrt{|\varepsilon_p|}$. The considered electromagnetic perturbations are of surface-type if the inequality $\varepsilon_p < 0$ is valid. Using this one can find the frequency range of ASW in isotropic plasma waveguide, $\omega < \Omega_e$, which differs from the SW frequency range in the case of planar plasma border [1]. One can analytically solve dispersion equation (16) in approach of enough thin dielectric layer, namely if inequality $\Delta = R_2/R_1 - 1 \ll 1$ is valid. Then the right-hand side of (16) can be simplified and one can rewrite it approximately in the form similar to the right-hand side of (6).

One can find the following approximate solutions for the below-indicated limiting cases:

$$\omega \approx \begin{cases} \Omega_e m\sqrt{\Delta/(\zeta\sqrt{\varepsilon_0})}, & \zeta \gg m, \quad \Delta\zeta < 1 \\ \Omega_e\sqrt{\Delta m/(1+\Delta m)}, & m \gg 1, \quad kR_1 \ll 1 \end{cases}, \quad (17)$$

where $\zeta = R_1/\delta$.

Results of the numerical analysis of (16) allow to find that the ASW frequency diminishes with increase in the radius of the plasma column and decrease in the thickness of dielectric layer. It also proves that the ASW frequency depends strongly on the azimuthal wave number, thickness of dielectric layer, and the radius of the plasma column. Thus, considered modes can be used in antenna devices of plasma electronics. For example, to excite extraordinary polarized ASW one could utilize either metal spiral with electric current or alternating electric field at the ASW frequency (in this case plasma can be grounded). Energy from such type of sources can be transformed into the energy of ASW by mechanism of linear conversion. Further, radiation of the ASW energy can be carried out through dielectric "windows" in metal wall of the waveguide. As wavelength of the ASW radiation is determined by the relation $\lambda \approx 6 \cdot 10^6 n_{pl}^{-1/2}$ cm, for the plasma density $n_{pl} \sim 10^{12}$ cm^{-3} one can obtain radiation in a centimeter range, which can be gradually controlled.

1.1.4. ASW propagating in strongly magnetized plasma waveguides. Only the case of enough dense $\Omega_e^2 \gg \omega_e^2$ plasma has been examined up to this subsection. Thus, frequency of the above-considered electromagnetic perturbations belongs to the range that is higher than the low hybrid resonance; that is why motion of plasma ions was neglected there. Indicated correlation between Langmuir and electron cyclotron frequencies can be easily satisfied, for instance, in the plasma of semiconductors [1] because it is characterized by the high concentration of free electrons. However, under laboratory conditions to produce plasma one can often utilize a strong magnetic field so the other inequality $\Omega_e^2 < \omega_e^2$ can be realized. Thus, let us consider just this case [19] when propagation of ASW is connected with perturbations of plasma ions' motion in waveguides of cylindrical geometry. The account of plasma ions' motion changes expressions for the frequency ranges where extraordinarily polarized ASW exist as compared with the case of the weak external magnetic field application:

$$\omega_{LH} < \omega < \omega_1 \quad \text{and} \quad \omega_{UH} < \omega < \omega_2, \quad (18)$$

which are referred to as LF and HF ranges, respectively, in this subsection. Here $\omega_{1,2} = \pm 0.5|\omega_e| + \sqrt{0.25\omega_e^2 + \omega_i|\omega_e| + \Omega_e^2}$ are cut-off frequencies of bulk waves [16] in the case of strongly magnetized plasma. In this case the HF range becomes very narrow and it is located near-by electron cyclotron frequency; therefore, ASW frequencies in this range depend very weakly on plasma parameters. Under these conditions, low hybrid resonance is located in the range of ion cyclotron frequency and therefore there exists a possibility of propagation of plasma ion perturbations across an external magnetic field.

Applying the above-mentioned boundary conditions one can derive the dispersion relation for extraordinary polarized ASW in strongly magnetized plasma in the form of (5). Analytically, it can be examined in the limiting case of narrow dielectric layer, ($m\Delta \ll 1$, $(R_2 - R_1)k \ll |m|$), when the influence of plasma parameters on dispersion properties of ASW is the strongest. It has been found that unlike the case of ASW propagating in waveguides, which are partly filled with weakly magnetized dense plasma, extraordinarily polarized ASW cannot propagate in wide cylindrical metal waveguides with thin dielectric coating, which are filled with strongly magnetized plasma. This can be proved by mathematical, analysis of both parts of (5). In the case of strongly magnetized plasma left-hand side of (5) is positive and right-hand side is negative, therefore the dispersion relation has no solutions [19].

So, let us consider the limiting case of a narrow waveguide, which is partly filled with strongly magnetized plasma: $k_\perp R_1 \ll 1$. Then the analytical expression for

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the azimuthal X-modes frequency can be written in the following form:

$$\omega \approx \omega_i \left[1 + \left|\frac{R_2^{2m} - R_1^{2m}}{R_2^{2m} + R_1^{2m}}\right| \Omega_i^2 \omega_i^{-2} \left(\left|\frac{R_2^{2m} - R_1^{2m}}{R_2^{2m} + R_1^{2m}}\right| + \varepsilon_d\right)^{-1}\right]. \quad (19)$$

In this case ASW have unique direction of propagation, namely, azimuthal wave number can be negative. Thus, direction of their propagation coincides with the direction of cyclotron rotation of positively charged ions near plasma surface, and the ASW frequency is approximately equal to ion cyclotron frequency. Analysis of spatial distribution of ASW fields proves that in this case ASW can be considered as quasi-potential waves because magnetic field of ASW is much less than their electric fields. Amplitude of ASW electric fields with azimuth number $m = -1$ remains practically independent upon radial coordinate even in plasma region.

In waveguides with a thick dielectric layer, the properties of azimuthal X-modes depend weakly on the plasma parameters and are mainly determined by the properties of dielectric layer that separate plasma column from the metal wall of the waveguide. Thus, for this case high radial harmonics of ASW can propagate, and solution of dispersion equation (5) exists if the following inequality is valid:

$$(n + 0.5)\pi < (R_2 - R_1)k < (n+1)\pi;\ n = 1, 2, 3 \ldots; ka \gg |m|. \quad (20)$$

In this case ASW become substantially un-potential waves unlike the previous case of the thin dielectric layer. In wide waveguides, which are immersed into strong magnetic fields, $k_\perp R_1 \gg |m|$, ASW penetration depth in plasma is small, and the analytical expression for their frequency of nth radial harmonic approximately equals $\omega_{0n} \approx 0.5\pi\, n\, c/(R_2 - R_1)$.

Analyzing the dispersion relation, one can make the following conclusions: Dependence of ASW frequency on an azimuth number is very weak in this limiting case; this dependence becomes weaker and weaker with increasing value of plasma concentration; frequency correction $\delta\omega$ is larger for positive wave number m than for negative m; difference between the values of ASW frequencies with mutually opposite values of azimuthal wave numbers is maximal near the upper band of the LF range (18) where the equality $\varepsilon_1 + \varepsilon_2 = 0$ is valid [19].

Results of numerical analysis prove that the frequency of the zero-th radial mode lies approximately between 20 and 40 ion cyclotron harmonics. The limiting values of the frequency range of the first radial mode are approximately equal to 60 ω_i and 80 ω_i. As an example, let us calculate the eigenfrequency of ASW with $m = -3$ that propagates in the fully ionized helium plasma under the following conditions: $R_1 = 10\,\text{cm}$, $B_0 = 3\,\text{kg}$, $T = 1\,\text{eV}$, $n_{pl} = 10^{10}\,\text{cm}^{-3}, \Delta = 0.1$. Under these conditions the azimuthal X-modes frequency is as follows: $\omega \approx 19\omega_i = 1.4 \cdot 10^8\,\text{s}^{-1}$.

1.1.5. Ordinary polarized azimuthal electromagnetic waves. Results of studying dispersion properties of extraordinary polarized ASW propagating in uniform plasma-filled waveguides were under consideration in the four previous subsections. But, as it was indicated at the beginning of the Sec. 1, azimuthal electromagnetic perturbations with ordinary polarization can also propagate in such cylindrical waveguides. In the limiting case of uniform plasma-filled waveguides, these O-modes are not interesting from the practical point of view in comparison with extraordinarily polarized ASW because these cannot propagate along plasma-metal boundary and their properties are mostly determined by the properties of dielectric layer that separates plasma column from the metal wall of waveguide. But in the current-carrying plasma-filled waveguides, the ASW cannot propagate as two independent X- and O-modes. Therefore, let us consider the dispersion properties of the ordinary polarized ASW propagating in waveguides filled with uniform plasma. In the general case, axial component of electric field of azimuthal O-modes is described by the following equation:

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dE_z^{(m)}}{dr}\right) + \left(k^2\varepsilon_3 - \frac{m^2}{r^2}\right)E_z^{(m)} = \widehat{M}H_z^{(m)}, \quad (21)$$

where $\varepsilon_3 \approx 1 - \Omega_e^2(r)\omega^{-2}$; operator $\widehat{M}H_z^{(m)} = 0$ if external azimuthal magnetic field is absent, i.e., $B_{0\varphi} = 0$. If external azimuthal magnetic field is applied, $B_{0\varphi} \neq 0$, then the explicit form of the operator $\widehat{M}H_z^{(m)}$ is indicated for each specific case in Sec. 2. Regardless of the values of both B_{0z} and $B_{0\varphi}$, other components of electromagnetic field of azimuthal O-modes are expressed through $E_z^{(m)}$ in the following way:

$$H_r^{(m)} = mE_z^{(m)}(kr)^{-1}, \quad H_\varphi^{(m)} = -k^{-1}dE_z^{(m)}/dr. \quad (22)$$

Solution of (21) in a dielectric region, which satisfies the boundary condition for the waves' tangential electric field on the metal wall of waveguide, $E_z^{(m)}(R_2) = 0$, is written in terms of the Bessel and Neumann functions [15]. Application of boundary conditions on plasma-dielectric interface, which determines continuity of tangential (in respect to the plasma interface) wave fields allows one to derive the dispersion relation in the form $D_o = 0$, where

$$D_o = \frac{\varphi_o}{d\varphi_o/dR_1} - \frac{1}{k}\frac{J_m(kR_1)N_m(kR_2) - J_m(kR_2)N_m(kR_1)}{J'_m(kR_1)N_m(kR_2) - J_m(kR_2)N'_m(kR_1)}. \quad (23)$$

In expression (23), $\varphi_0(r)$ is a solution of (21) that is restricted on the waveguide axis, $r = 0$. By analyzing (21) and (23), one can conclude that properties of the azimuthal O-modes do not depend on the value of an external axial magnetic field. This is typical for all ordinary polarized modes [16].

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In the case of cylindrical metal waveguide that is partially filled with uniform plasma column, azimuthal O-modes can propagate in the form of ordinary polarized ASW (OASW) within the frequency range $\omega < \Omega_e$. Solution of (21) in this case looks as follows: $\varphi_0 = I_m(kr\sqrt{-\varepsilon_3})$ (compare it with the case of azimuthal X-modes propagating in waveguides partially filled with isotropic plasma, see Sec. 1.3). OASW could not propagate with any frequency that is higher than the Langmuir frequency: In this frequency range ordinary polarized electromagnetic azimuthal modes acquire bulk nature, $\varphi_0 = J_m(kr\sqrt{\varepsilon_3})$; here argument of the first kind of the Bessel function is real because $\varepsilon_3 > 0$. If thickness of dielectric (to simplify our consideration, let us assume that dielectric constant is equal to unit) coating is small enough, which means that inequality $k(R_2-R_1) \ll 1$ is valid, then dispersion relation of OASW can be reduced and written is the following approximate form:

$$\frac{\varphi_o}{d\varphi_o/dR_1} \approx -(R_2 - R_1). \tag{24}$$

It becomes clear from the analysis of (24) that OASW cannot propagate in this limiting case because inequality $d(\ln|\varphi_0|)/dR_1 < 0$ could be valid only for some azimuthal bulk O-modes.

In the opposite limiting case, namely, for a waveguide with a wide dielectric layer, $k(R_2 - R_1) \gg |m|$, analytical solution of the dispersion relation (24) could be found by the method of successive approximations. Then, analytical expression for OASW frequency has the form of sum $\omega^{(0)} + \omega^{(1)}$, where $\omega^{(0)}$ is the wave frequency calculated in the zero-th approximation and $\omega^{(1)}$ is a small correction to it. Since OASW propagate within the frequency range $\omega < \Omega_e$ (in this case $\varepsilon_3 < 0$), in the limiting case of large radius of the plasma column ($kR_1 \gg |m|$) one can obtain the following expressions for $\omega^{(0)}$ and $\omega^{(1)}$:

$$\omega^{(0)} = \frac{\pi nc}{2(R_2 - R_1)}\left[1 + \sqrt{1 + \frac{2m^2(R_2 - R_1)^2}{\pi^2 n^2 R_2 R_1}}\right], \tag{25}$$

$$\omega^{(1)} = \frac{\omega^{(0)}\delta_o}{R_2 - R_1}\left(1 - \frac{m^2}{2k^2R_1^2}\right)\left[1 + \frac{m^2c^2}{2\left(\omega^{(0)}\right)^2 R_2 R_1}\right]^{-1}. \tag{26}$$

Influence of plasma density on OASW properties becomes weaker with the diminishing of plasma column radius, i.e., if inequality $kR_1 < 1$ is valid:

$$\omega^{(0)} = \frac{\pi nc}{R_2}\sqrt{\left(1 + \frac{|m|}{2n} - \frac{1}{4n}\right)^2 - \left(\frac{m}{\pi n}\right)^2}, \tag{27}$$

$$\omega^{(1)} = \frac{\pi|m|c}{R_2(|m|!)^2}\left(\frac{kR_1}{2}\right)^{2|m|}\left(1 - \frac{|m|\,\delta_o}{R_1}\right). \tag{28}$$

Thus, dispersion properties of OASW in cylindrical metal waveguides filled partially with uniform plasma are mainly determined by the geometry of the utilized waveguide, i.e., by the thickness of the dielectric layer (in the dielectric layer they have features of bulk waves). OASW frequency increases with diminishing dielectric layer thickness and increasing azimuthal and radial wave numbers of these modes. The Langmuir frequency is a high band of the OASW frequency range, i.e., it is $\sqrt{2}$ times larger than maximum frequency of SW propagating along planar interface of plasma [1]. At the frequency range $\omega > \Omega_e$, azimuthal O-modes propagate in the form of bulk waves.

1.2. Coupled ASW in current-carrying waveguides

In order to produce and sustain plasma in experimental device one can often utilize the flowing of electric current. In the previous section a possibility of such flowing was not allowed. But here we study the propagation of the coupled azimuthal electromagnetic modes in cylindrical metal waveguides with plasma filling and axial flow of electric current by method of successive approximations. Plasma is assumed to be non-uniform in radial direction, it is separated from metal wall by vacuum layer located in the region $R_2 > r > R_1$. Waveguide is assumed to be uniform along axial direction, $\partial/\partial z = 0$, plasma density is assumed to be large, $\Omega_e^2 \gg \omega_e^2$, and the external magnetic field is assumed to have the following two components: $\vec{B}_0 = B_{0z}\vec{e}_z + B_{0\vartheta}\vec{e}_\vartheta$ and $B_0^2 = B_{0z}^2 + B_{0\vartheta}^2$. Our consideration is restricted by the case when azimuthal component, $B_{0\vartheta}\vec{e}_\vartheta$, of the external magnetic field caused by axial flow of electric current is relatively small as compared with the axial component, $B_{0z}\vec{e}_z$.

Presence of two components of an external constant magnetic field does not allow one to divide the set of Maxwell equations for azimuthal electromagnetic perturbations into two independent sets, which describe extraordinarily polarized ASW (XASW) with fields components E_r, E_φ, and H_z, and OASW with the fields components E_z, H_r, H_φ.

1.2.1. Coupled low-frequency ASW in current-carrying plasma waveguide. Let us start our consideration with the case when ASW propagate in the plasma-filled waveguide, which is immersed into an external azimuthal magnetic field, $\vec{B}_0 = B_{0\vartheta}\vec{e}_\vartheta$ (the axial component of the magnetic field is absent, $B_{0z} = 0$). Value of azimuthal magnetic field is supposed to be small, so that the inequality $\beta = |\omega_e|/\omega \ll 1$ is valid. Ignoring summands, in which values are less than summands of the first order of smallness over small parameter β, one can derive from Maxwell equations a set of differential equations for ASW fields that consists of (21) in which the operator

$$\widehat{M} = im\varepsilon_2 k/(r\varepsilon_1), \tag{29}$$

and equation for the axial component of ASW magnetic field $H_z^{(m)}$:

$$\frac{1}{r}\frac{d}{dr}\left(\frac{r}{\varepsilon_1}\frac{dH_z^{(m)}}{dr}\right) + \left(k^2 - \frac{m^2}{r^2\varepsilon_1}\right)H_z^{(m)} = -\widehat{M}E_z^{(m)}. \tag{30}$$

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Other components of ASW field are expressed through axial components $E_z^{(m)}$ and $H_z^{(m)}$. In the present case, components of dielectric permeability tensors can be approximately written in the following form: $\varepsilon_1 \approx \varepsilon_3$, $\varepsilon_2 \approx |\omega_e|\Omega_e^2\omega^{-3}$. These equalities are satisfied with accuracy up to the summands of the second order of smallness over parameter β. Equations (21) and (30) can be solved using the theory of successive approximations.

Boundary conditions for ASW consist of the following: restrictions of wave fields inside the considered waveguide, specifically on the axis of the waveguide; continuity of tangential components of ASW fields on the plasma-dielectric surface, $r = R_1$; and equality to zero of tangential component of ASW electric field on the metal wall ($r = R_2$) of the waveguide. Application of these boundary conditions allows one to derive dispersion relation that has a typical form for the case of coupled modes:

$$D_o D_x = D(\beta), \tag{31}$$

where D_o is determined by (23), $D_x = 0$ is dispersion relation of XASW in the case of $B_{0z} = 0$ (see Sec. 1.3), and $D(\beta)$ is a coefficient of coupling between XASW and OASW, in which value is of the second order of smallness over parameter β:

$$D(\beta) = \frac{\varepsilon_1}{\psi_1^2 R_1^2 (d\varphi_o/dR_1)^2}\left[\int_0^{R_1} r\psi_1 \widehat{M}\varphi_o dr\right]^2. \tag{32}$$

Solution of the dispersion relation (31) can be found out in the form of sum $\omega = \omega_{x,o} + \delta w_{x,o}$. Here $\omega_{x,o}$ is ASW frequency calculated in the case of zero axial steady magnetic field and $\delta\omega_{x,o}$ is the correction caused by the coupling between two modes. Assuming that the values of ASW eigenfrequencies are known (they are from the Sec. 1), one can find out the frequency corrections for XASW and/or OASW, in which values are caused by an external azimuthal magnetic field in the following form:

$$\delta\omega_{x,o} = \frac{D(\beta)}{D_{o,x}}\left(\frac{\partial D_{x,o}}{\partial\omega}\right)^{-1}_{|\omega=\omega_{x,o}}. \tag{33}$$

The dispersion curves of OASW and XASW, which propagate in uniform plasma along the external azimuthal magnetic field, do not intersect with each other in most of the cases. But under definite conditions such crossing is possible; in that case influence of the external field $B_{0\vartheta}$ grows and the ratio $\delta\omega_{x,o}/\omega_{x,o}$ becomes of the first order of β:

$$(\delta\omega_{x,o})^2 = D(\beta)\left(\frac{\partial D_o}{\partial\omega}\right)^{-1}_{|\omega=\omega_o}\left(\frac{\partial D_x}{\partial\omega}\right)^{-1}_{|\omega=\omega_x}. \tag{34}$$

Analyzing explicit expression (33), it is possible to examine the influence of field $B_{0\vartheta}$ on the dispersion properties of XASW, which propagate along surface of uniform plasma cylinder separated by narrow vacuum layer $(R_2 - R_1)k \ll 1$ from metal wall of cylindrical waveguide. In this case expression for the eigenfrequency of XASW is same as in the case of isotropic plasma waveguide (see Sec. 1.3), and the frequency correction caused by field $B_{0\vartheta}$ strongly depends on the radius of the waveguide

$$\delta\omega_x \approx \beta^2\omega_x k^2\phi/(8R_2\Delta), \tag{35}$$

where $\phi = \delta^4(R_2 - R_1 + \delta)^{-1}$ if inequality$R_1 \gg |m|\delta$ is valid (it means the limiting case of a wide plasma waveguide), and in the opposite limiting case of narrow waveguide $R_1 \ll \delta$ multiplier ϕ has the following form: $\phi = R^4(R_2 - R_1 + R_1/|m|)^{-1}(m^2 + |m|^3)^{-2}$. Taking into account the dispersion properties of XASW propagating in plasma-filled waveguide under the condition $\vec{B}_{0z} = 0$ [18], one can conclude from the analysis of expression (35) that influence of external azimuth magnetic field diminishes with increasing thickness of vacuum layer and azimuth wave number.

In the case of non-uniform plasma, region of rarefied plasma can be located near plasma surface. Then the equality, $\mathrm{Re}\varepsilon_1(r_0) = 0$, could be satisfied. Such condition leads to the appearance of additional damping of XASW. In this case, during the calculation of a integral that is a multiplier in expression (32), one has to take into account the small imaginary part of the ε_1, which is caused by collisions between plasma electrons. One can approximately estimate the damping rate γ_r of XASW under the condition of this resonant damping associated with the external azimuthal magnetic field:

$$\gamma_r \approx \frac{\pi\beta^2 k^2\omega_x m^2}{2R_1^2 k_*^5 (R_2 - R_1)(1 + k_*(R_2 - R_1))}, \tag{36}$$

where $k_*^3 = k^2 d\varepsilon_1/dR_1 > 0$. Let us underline that XASW resonant damping caused by conversion in rarefied peripheral plasma, where $\mathrm{Re}\varepsilon_1(r_0) = 0$ becomes stronger under the condition of applying external azimuthal magnetic field than under the condition $B_{0\vartheta} = 0$.

Now, let us consider the influence of azimuthal magnetic field on ASW frequency under the condition of their propagation in magnetoactive plasma waveguide, assuming that the ratio $\kappa = B_{0\vartheta}/B_{0z} \ll 1$ is a small parameter of the task. Taking into account summands of the first order of smallness in the respect of parameter κ, one can derive from Maxwell equations the set of differential equations for axial fields of the ASW $E_z^{(m)}$ and $H_z^{(m)}$. This set consists of (21), in which now

$$\widehat{M} = -ik\kappa\frac{d}{dr} - \frac{k^2\kappa\varepsilon_3}{\varepsilon_1^2 - \varepsilon_2^2}\left(\frac{\varepsilon_1}{ik}\frac{d}{dr} + \frac{m\varepsilon_2}{ikr}\right), \tag{37}$$

and equation

$$\widehat{L}H_z^{(m)} = \widehat{K}E_z^{(m)}, \tag{38}$$

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where

$$\widehat{L} = \frac{d^2}{dr^2} + \left(\frac{1}{r} + \frac{1}{\varepsilon_1}\frac{d\varepsilon_1}{dr} - \frac{d\ln\left|\varepsilon_1^2 - \varepsilon_2^2\right|}{dr}\right)\frac{d}{dr} + \frac{m}{\varepsilon_1 r}\frac{d\varepsilon_2}{dr} - \frac{m\varepsilon_2}{r\varepsilon_1}\frac{d\ln\left|\varepsilon_1^2 - \varepsilon_2^2\right|}{dr} + k^2\frac{\varepsilon_1^2 - \varepsilon_2^2}{\varepsilon_1} - \frac{m^2}{r^2}, \tag{39}$$

$$\widehat{K} = \frac{ik}{\varepsilon_1}\left[\frac{-\kappa}{r}\left(\varepsilon_1^2 - \varepsilon_1\varepsilon_3 - \varepsilon_2^2\right) - \kappa\left(\varepsilon_1^2 - \varepsilon_1\varepsilon_3 - \varepsilon_2^2\right)\frac{d}{dr} - \frac{\kappa m\varepsilon_2\varepsilon_3}{r} - \frac{d\left(\kappa\varepsilon_1^2 - \kappa\varepsilon_1\varepsilon_3 - \kappa\varepsilon_2^2\right)}{dr} + \kappa\left(\varepsilon_1^2 - \varepsilon_1\varepsilon_2 - \varepsilon_2^2\right)\frac{d\ln\left|\varepsilon_1^2 - \varepsilon_2^2\right|}{dr}. \tag{40}$$

One can express the components $H_r^{(m)}$ and $H_\vartheta^{(m)}$ (which are the components of OASW in the case $B_{0\vartheta} = 0$) through the component $E_z^{(m)}$ by (22). Components $E_r^{(m)}$ and $E_\vartheta^{(m)}$ of wave fields are connected with both axial components $E_z^{(m)}$ and $H_z^{(m)}$. Taking into account the small right-hand sides of (21) and (38), one can find out their solutions by the aid of the theory of successive approximation.

Application of boundary conditions for ASW fields on the plasma–vacuum interface allows one to derive the dispersion relation for coupled OASW and XASW in the form of (31), in which one has to change expression for D_x by the following:

$$D_x = \frac{\varepsilon_1 d\psi_o/dR_1 + \varepsilon_2 m\psi_o/R_1}{\left(\varepsilon_1^2 - \varepsilon_2^2\right)\psi_o} - k\frac{J'_m(kR_2)N'_m(kR_1) - J'_m(kR_1)N'_m(kR_2)}{J'_m(kR_2)N_m(kR_1) - J_m(kR_1)N'_m(kR_2)}, \tag{41}$$

and coefficient $D(\beta)$ should be replaced as well by the following:

$$D(\kappa) = \frac{-(d\varphi_o/dR_1)^{-2}}{R_1}\int_0^{R_1} r\varphi_o\widehat{M}\psi_o dr \left[\frac{\varepsilon_1 W(\psi_o,\tilde{\psi}_o)}{(\varepsilon_1^2 - \varepsilon_2^2)\psi_o^2}\int_0^{R_1}\frac{\psi_o\widehat{K}\varphi_o}{W(\psi_o,\tilde{\psi}_o)}dr + ik\kappa\frac{\varepsilon_1^2 - \varepsilon_1\varepsilon_3 - \varepsilon_2^2}{\varepsilon_1^2 - \varepsilon_2^2}\frac{\varphi_o}{\psi_o}\right], \tag{42}$$

where the Wronskian $W(\psi_0,\tilde{\psi}_0) = k^2(\varepsilon_1^2 - \varepsilon_2^2)/r\varepsilon_1$, and all the summands represented in expressions (41) and (42), which depend on radial coordinate and are not integrants, should be calculated on the plasma surface $r = R_1$.

One can derive simple analytical expressions for the frequency corrections of XASW, which propagate along the interface of uniform plasma cylinder under the condition of rather narrow vacuum layer and large plasma density, $\Omega_e^2 \gg \omega_e^2$, by applying the (31), (33), (41), and (42). If plasma cylinder is narrow one, $\delta_x^2 = k^{-2}\psi^{-1} \gg R_1^2$, then the expression for the frequency correction is as follows:

$$\delta\omega_x = \frac{\kappa^2\omega_e^2 R_1^2}{4(|m|+1)^2c^2}\left(R_2 - R_1 + \frac{R_1}{|m|}\right)^{-1} \times \left[\frac{m\delta_x^2|\omega_e|}{c^2R_1} + \frac{2(R_2 - R_1)m^2}{\omega_x R_1^2}\right]^{-1}, \tag{43}$$

where the value of the azimuthal component of magnetic field $B_{0\vartheta}$, which determines the magnitude of the parameter κ, should be calculated at the point $r = R_1$. If the radius of the plasma cylinder is large enough ($R_1 \gg \delta_x$), then

$$\delta\omega_x \approx \frac{\kappa^2\delta_x^4\omega_e^2m^2}{4R_1^2c^2(R_2 - R_1 + \delta_0)}\left(1 - \frac{\omega|\omega_e|R_1\delta_x}{mc^2}\right)^2 \times \left[\frac{m\delta_x^2|\omega_e|}{c^2R_1} + \frac{2(R_2 - R_1)m^2}{\omega_x R_1^2}\right]^{-1}. \tag{44}$$

Carrying out the calculation of (43) and (44), the field $B_{0\vartheta}$ was considered as a linear function of radial coordinate. It should be outlined that the participation of electrons in axial motion does not lead to the Doppler shift of modes frequency because of assumption about axial homogeneity of the considered waveguide.

One can make the following conclusions by comparing expressions (43) and (44). First, influence of the azimuthal magnetic field on XASW frequency is stronger in a wide plasma cylinder than in a narrow plasma cylinder. Second, frequency correction $\delta\omega_x$ is larger in the case of plasma-metal interface $R_2 = R_1$ than in the case of vacuum layer separating plasma column from metal wall, $R_2 \neq R_1$. Third, under same conditions, the XASW eigenfrequency is larger in the presence of external azimuthal magnetic field than in its absence, i.e., $B_{0\vartheta} = 0$.

The eigenfrequency of the nth radial mode of ASW, which propagates in a wide plasma cylinder ($k\Delta \gg |m|R_1^{-1}$), can be represented in the form of sum $\omega_x = \omega^{(0)} + \omega_x^{(1)}$, where summand $\omega_x^{(1)}$ is much smaller than $\omega^{(0)}$. In this limiting case, frequency $\omega^{(0)}$ is determined by the vacuum layer and is described by (27). The frequency correction describes the influence of magnetized plasma column and is determined by the following formula:

$$\omega_x^{(1)} \approx \frac{-\omega_x\delta}{R_2 - R_1}\left(1 - \frac{m\delta|\omega_e|}{\omega_x R_1}\right)\left(1 - \frac{m^2}{2k^2R_1^2}\right)^{-1} \times \left(1 + \frac{m^2}{2k^2R_1R_2}\right)^{-1}_{|\omega=\omega^{(0)}}. \tag{45}$$

By equating the frequencies of OASW and XASW, $\omega_= \omega_x$, it is possible to find out the condition for crossing of their dispersion curves:

$$\frac{R_2 - R_1}{R_1} = \Delta = \frac{\pi n|\omega_e|}{m\Omega_e}\left(1 - \frac{\omega_e^2R_1}{2\Omega_e^2R_2}\right)^{-1}. \tag{46}$$

Equation (46) can be satisfied only for XASW that propagates along the direction of the plasma electrons

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rotation along the Larmor orbits near plasma surface. To calculate frequency corrections of azimuthal electromagnetic perturbations in this case, one can apply (35) under the indicated condition (46):

$$(\delta\omega)_x^2 \approx \frac{\kappa^2 m^4 c^4 \Omega_e^{-6} \omega_e^4}{16\pi^2 n^2 R_1^2 R_2^2} \left(1 - \frac{m^2}{k^2 R_1 R_2}\right). \qquad (47)$$

In this case, influence of field $B_{0\vartheta}$ on ASW frequency is enhanced with increasing azimuthal wave number and also with diminishing radial wave number, plasma density, and waveguide radius.

One can conclude that transversal component (in the respect of direction of ASW propagation) B_{0z} of external magnetic field influences strongly the dispersion properties of XASW than longitudinal $B_{0\vartheta}$ component of the magnetic field. Such dependence of eigenfrequency on the direction of external magnetic field coincides with the one obtained in [5] for surface magnetoplasma waves, which propagate along planar plasma-metal
boundary.

1.2.2. Coupled high-frequency ASW, which propagate around a metal current-carrying cylinder that is immersed into magnetoactive plasma. Let us consider an antenna system modeled by metal cylinder of radius a placed in a cold magnetoactive plasma with uniform density profile, in which density is large enough so that inequality $\Omega_e^2 \gg \omega_e^2$ is valid. An external constant magnetic field, $\vec{B}_0 = \vec{B}_{0\vartheta} + \vec{B}_{oz}$. Our consideration is restricted by the case of small value of an external azimuthal magnetic field, thus there is a small parameter of the problem: $\kappa = B_{0\vartheta}/B_{0z} \ll 1$.

Ignoring the summands, in which value is higher than the first order of smallness over small parameter κ, one can derive the set of differential equations for the axial components of electric and magnetic fields of ASW [20] from Maxwell equations. Equation for the axial electric field looks like (21). In the considered case, operator $\widehat{M}$ located on the right-hand side of this equation is as follows:

$$\widehat{M} = -ik\kappa \frac{d}{dr} - ik\kappa \frac{k_o^2}{k_x^2} \left(\frac{d}{dr} + \frac{m\mu}{r}\right), \qquad (48)$$

where the following notations are applied: $k_o^2 = k^2\varepsilon_3$, $k_x^2 = k_\perp^2$ (see notations to (1)). For the axial magnetic field of coupled ASW one can obtain the following equation:

$$\frac{k_x^2}{r}\frac{\partial}{\partial r}\frac{r}{k_x^2}\frac{\partial H_z}{\partial r} - k_x^2\left[1 + \frac{m^2}{r^2 k_x^2} - \frac{m}{r}\frac{\partial}{\partial r}\left(\frac{\mu}{k_x^2}\right)\right] H_z = \widehat{K} E_z, \qquad (49)$$

where operator $\widehat{K}$ looks as follows:

$$\widehat{K} = \frac{ik}{\varepsilon_1}\left(\varepsilon_1^2 - \varepsilon_1\varepsilon_3 - \varepsilon_2^2\right) \times \left[\frac{-\kappa}{r} - \kappa\frac{d}{dr} - \frac{\kappa\varepsilon_2\varepsilon_3 m r^{-1}}{\varepsilon_1^2 - \varepsilon_1\varepsilon_3 - \varepsilon_2^2} - \frac{d\kappa}{dr}\right]. \qquad (50)$$

Let us remind that only wave number k_o is present in (21), which describes azimuthal O-modes in the zero-th approximation over the small parameter κ. As $k_o^2 < 0$ under present conditions, fields of O-modes are determined by cylindrical functions of first and second types [15]. Consequently, field E_z describes the bulk component of these coupled azimuthal perturbations, and parameter k_o is their radial wave vector. Equation (50) in the zero-th approximation can be solved in the terms of modified cylindrical functions; It determines a surface type of spatial distribution of field H_z. That is why parameter k_x^{-1} (here $k_x = k_\perp$) determines the penetration depth of surface component of these coupled azimuthal modes into plasma. Value of k_x is real magnitude and its analysis allows one to calculate the limiting values of the frequency ranges, where XASW can exist under the condition of zero steady azimuthal magnetic field, i.e., $B_{0\vartheta} = 0$.

XASW with negative azimuthal wave number m propagate in the LF range and that with positive azimuthal wave number propagate in the HF range. Analysis of k_o^2 proves that in the LF range it is negative, and in the HF range it is positive. That is why parameter k_o can be referred as radial wave number of bulk component of these coupled azimuthal waves in the HF range. High band of the HF range is determined by the equation $\varepsilon_1 = \varepsilon_2$; that is why penetration depth of XASW into plasma near this band is considerably larger than the radius of metal cylinder: $k_x a \ll 1$. Low band of the HF range is determined by the equation $\varepsilon_1 = 0$; that is why near this band the opposite inequality is realized: $k_x a \gg 1$. Results of numerical analysis of dependence k_x^{-1} upon the value of metal cylinder radius testify that increase in the metal cylinder radius leads to decrease in the penetration depth of these azimuthal waves into plasma. Diminishing of the cylinder radius leads to sharp growing up of k_x^{-1}, this growth begins sooner for the waves that have larger azimuthal wave number m: $k_x^{-1} \approx 2\delta$; 4δ; 6δ for $m = 1, 2, 3$ (respectively).

In order to determine the spatial distribution of ASW fields, we applied the method of variation of a constant for solving (21) and (49). It allows us to find out the following expressions for axial components of ASW electric and magnetic fields [20]. Application of boundary conditions, mentioned in Sec. 1, allows one to make a conclusion that OASW cannot propagate along the boundary between magnetoactive plasma and the metal. Instead of these, under the condition of utilization, an external azimuthal magnetic field, the ordinary polarized azimuthal bulk waves (OABW), propagate in such plasma–antenna structures. OABW electromagnetic fields are described by the first type of cylindrical functions, their amplitudes are of the first order smallness over small parameter κ as compared with the XASW amplitudes.

Using the boundary condition on the absence of the axial electric field on the interface of the considered plasma–antenna system, one can derive the following

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equation:

$$A_1 N_m(k_o a) + A_2 J_m(k_o a) = 0. \quad (51)$$

Boundary condition, which means absence of the signal reflected from infinity, allows one to derive a relation between the constants of integration:

$$A_1 + B_1 \frac{\pi}{2} \int_a^\infty r J_m(k_o r)\, \widehat{M} K_m(k_x r)\, dr = i \left[A_2 - B_1 \frac{\pi}{2} \int_a^\infty r N_m(k_o r)\, \widehat{M} K_m(k_x r)\, dr \right]. \quad (52)$$

Application of (51) and (52) makes it possible to express integration constants A_1 and A_2 through constant B_1. After this we applied the boundary condition on the equality of azimuthal component of the electric field of the coupled ASW to zero on the interface of the metal antenna. In the long run it leads to the derivation of the following dispersion relation:

$$D_x^{(0)} + D^{(2)} = 0. \quad (53)$$

Here, $D_x^{(0)} = 0$ is a dispersion relation of XASW under the condition of zero external constant azimuthal magnetic field [17] (see (13), where one can put $F_n(x) = K_n(x)$). $D^{(2)}$ is a correction to the XASW dispersion relation, which is determined by an external constant azimuthal magnetic field, its value is of the second order smallness over the small parameter κ:

$$D^{(2)} = [k_x K_m'(ak_x) + m\mu K_m(ak_x) a^{-1}] \cdot \frac{1}{J_m(k_o a) + i N_m(k_o a)} \int_\infty^a r K_m(k_x r) [N_m(k_o r)\, \widehat{K} J_m \times (k_o r) - J_m(k_o a)\, \widehat{K} N_m(k_o r)] dr + \frac{\pi}{2} \int_a^\infty r [J_m(k_o r) + i N_m(k_o r)] \widehat{M} K_m(k_x r)\, dr. \quad (54)$$

It is suitable to search solution for (54) in the form $\omega = \omega^{(0)} + \Delta\omega$, where $\omega^{(0)}$ is the eigenfrequency of XASW under the condition $B_{0\vartheta} = 0$, and $|\Delta\omega| \ll \omega^{(0)}$ is a small correction to the XASW eigenfrequency. Then, $\Delta\omega$ can be found by solving dispersion relation (53), where $D_x^{(0)}$ can be expanded into the Taylor series over $\Delta\omega$. Its imaginary part is negative, since propagation of XASW in this case is characterized by losses of electromagnetic energy caused by their conversion into radiated bulk waves. Damping rate $\gamma_{\text{rad}} = \text{Im}(\Delta\omega)$ of XASW, which is caused by this process, is proportional: $\gamma_{\text{rad}} \propto D^{(2)}$.

Let us assume as further consideration within this subsection that the external azimuthal magnetic field depends on the radial coordinate in the following way: $B_{0\vartheta}(r) = aB_{0\vartheta}(a)/r$. This assumption relates to the case where electric current flows only along the surface of metal antenna immersed into plasma, and there is no electric current in plasma volume.

As it was proved in [17], XASW propagate in plasma-filled waveguides in the HF range under the condition of absence of external azimuthal magnetic field if inequality $a > m\delta$ is valid. Then, the eigenfrequency of HF XASW can be approximately written in the following form [20]:

$$\omega^{(0)} \approx \sqrt{\Omega_e^2 + \omega_e^2 + m^2 c^2 / a^2}. \quad (55)$$

Asymptotical expression for damping rate of these coupled azimuthal waves caused by the generation of electromagnetic radiation can be represented in the following form:

$$\gamma_{\text{rad}} \approx \kappa(a)^2 m^3 \delta^3 a^{-3} \omega^{(0)}. \quad (56)$$

Frequency of XASW propagating in the plasma-filled waveguides in the HF range in the absence of external azimuthal magnetic field is characterized by both imaginary part (see expression (56)) and real part, $\text{Re}(\Delta\omega)$, of the frequency correction. Under this condition $\text{Re}(\Delta\omega)$ for coupled azimuthal waves can be calculated from expression (34):

$$\text{Re}(\Delta\omega) = a \cdot \gamma_{\text{rad}} / \delta. \quad (57)$$

In order to calculate the damping rate of the coupled azimuthal waves that is caused by collisions between plasma particles, one can take into consideration the effective value of collisional frequency ν in the components of dielectric permeability tensor ε_{jk} applied in the dispersion relation (53). It allows one to find the expression for collisional damping rate of these coupled waves, $\gamma_{\text{col}} \approx \nu/2$. Comparing this value with (56), it is possible to determine the conditions when collisional damping rate is larger than the damping rate caused by radiation of the coupled azimuthal waves' electromagnetic energy:

$$\nu / \omega^{(0)} \gg \kappa^2 (m\delta / a)^3. \quad (58)$$

Let us estimate whether collisional damping rate can be larger than the damping rate caused by radiation of the coupled azimuthal waves' electromagnetic energy on the basis of inequality (58) for the case of antenna with radius that is twice greater than product $m\delta$. In the case where ν is five orders less than the eigenfrequency of the coupled azimuthal waves, for $\kappa = 0.01$, the collisional damping rate will be less than the damping rate caused by ASW power radiation due to bulk modes' emission. Reminded that if plasma density is $n_e \approx 10^{10}\text{cm}^{-3}$, then frequency of collisions between particles is about $n \approx 10^4 \text{s}^{-1}$ [21].

The carried out numerical analysis of dispersion relation (53) testifies that the dispersion curves of XASW with different azimuthal wave numbers weakly differ from each other in the case of antennas with large radius that increasing azimuthal wave number leads to narrowing of the range of waveguides' radius, which is possible in XASW propagation. One can see results of the carried out analysis in the following four figures. In Fig. 1 results of numerical research into the value of coupled azimuthal waves damping rate caused by conversion of XASW into OABW are presented. It is shown that the decreasing of azimuthal wave number leads to the enhancing of damping of the coupled azimuthal waves, which is related to the presence of

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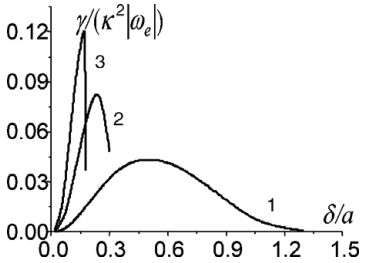


Figure 1. Damping rates of the coupled azimuthal waves versus reversed radius of metal antenna immersed into plasma; numbers indicate the values of azimuthal wave numbers *m*.

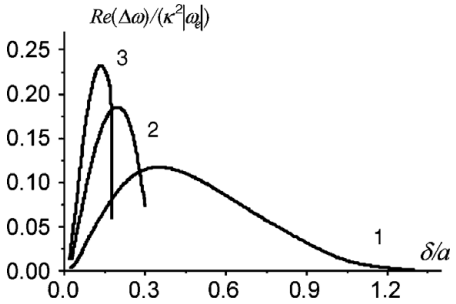


Figure 2. Real parts of the correction to the frequency of coupled azimuthal waves propagating around the metal antenna immersed into plasma; numbers indicate the values of azimuthal wave numbers *m*.

azimuthal component in the external magnetic field. The maximal value of this damping rate grows linearly with increasing *m*.

Results presented on Fig. 2 prove that larger the value of *m*, stronger is the influence caused by application of field $B_{0\vartheta}$ on the dispersion properties of coupled azimuthal waves. The maximal absolute value of the real part of the frequency correction is larger than the maximal absolute value of corresponding damping rate.

Numerical analysis shows that radial distribution of the real part of the first-order (over parameter κ) correction to the azimuthal magnetic field of the coupled waves, which is determined by the application of field $B_{0\vartheta}$, is approximately similar to graphic expression of the first type Bessel function, $J_n(x)$ [15]. Absolute value of the axial electric field of the coupled azimuthal waves is approximately of the same order as correction to the waves' azimuthal magnetic field, ΔH_z, near antenna up to a distance $r \approx 10a$.

In order to make conclusions about the possibility to observe the coupled azimuthal waves at experimental practice, let us make some estimation. If gaseous plasma is sustained in experimental device under the conditions, $n_e \approx 10^{10}\,\mathrm{cm}^{-3}$, $B_{0z} \approx 64\,\mathrm{G}$, then for the coupled azimuthal waves with $m = 1$ one can obtain $\Omega_e = 5|\omega_e|$, which is a little bit less than the eigenfrequency $\omega^{(0)} \approx 6 \cdot 10^9 \mathrm{s}^{-1}$. Under the condition of utilizing the axial current $I \approx 24$ A, which produces azimuthal magnetic field, $B_{0\vartheta} \approx 0.45\,\mathrm{G}$, the small parameter of the problem is $\kappa \approx 7 \cdot 10^{-3}$, then one can find the frequency correction, $\mathrm{Re}(\Delta\omega) \approx 5700\ \mathrm{s}^{-1}$, and damping rate, $\gamma \approx 2400\ \mathrm{s}^{-1}$. This means that the value of γ_{rad} is maximum for $a \approx 2\delta \approx 10.6$ sm. To estimate the frequency of ion–electron collisions, we have applied the formulae obtained in [21]: $\nu \approx 25\, n_e T_e^{-3/2}$ (here concentration of plasma particles is calculated in cm^{-3} and electron temperature T_e is calculated in degrees Kelvin), so for the temperature $T_e \approx 5\,eV$ one can obtain $\gamma_{\mathrm{col}} \approx 9000\ \mathrm{s}^{-1}$, which is larger than both γ_{rad} and frequency correction $\mathrm{Re}(\Delta\omega)$. Q-factor of such oscillations is enough large: $Q = \omega^{(0)}/(2\gamma_{\mathrm{col}}) \approx 7 \cdot 10^5$. But increasing of plasma temperature up to $T_e \approx 50$ eV leads to decreasing of collisional frequency, it becomes approximately 30 times less as compared with the above-indicated value, then the damping rate γ_{rad} is about $10\gamma_{\mathrm{col}}$ and the value of Q-factor also increases by 3.5 times. Therefore, experimental research in coupling between XASW and OABW can be carried out without utilizing some special apparatus or conditions.

1.2.3. Coupled high-frequency ASW propagating in a waveguide that is completely filled with current-carrying plasma. Let us consider a cylindrical metal ideally conducting waveguide with radius *a*, which is completely filled with cold plasma with non-uniform radial density profile. Plasma density is assumed to be uniform along axial and azimuthal directions within this subsection. Here, we consider the case of small external azimuthal magnetic field so that inequality $\kappa = B_{0\vartheta}/B_{0z} \ll 1$ is valid. Absence of dielectric layer between metal wall of the waveguide and plasma column makes propagation of OASW impossible and ordinary polarized azimuthal waves propagate as bulk waves. To find out spatial distribution of azimuthal waves' electromagnetic fields, we used a set of Maxwell equations. Presence of an azimuth component $B_{0\vartheta}(r)$ in the external constant magnetic field leads to the situation where, in cylindrical coordinates $(\vec{e}_r, \vec{e}_\vartheta, \vec{e}_z)$, all nine elements of dielectric permeability tensor ε_{ik} of cold magnetoactive plasma become non-zero. That is why solving Maxwell equations becomes a complicated problem.

In order to simplify the task, one can choose a special local coordinate system. Let us apply here the orthonormal basis of coordinate vectors $(\vec{e}_1, \vec{e}_2, \vec{e}_3)$, which are connected with force lines of magnetic field $\vec{B}_0$. Its first vector coincides with the radial basis vector in cylindrical coordinates, $\vec{e}_1 = \vec{e}_r$; the third vector is parallel to the magnetic force lines, $\vec{e}_3 = \vec{B}_0/|\vec{B}_0|$; the second vector adds these three vectors to the right-hand coordinate system, $\vec{e}_2 = \vec{e}_3 \times \vec{e}_1$. Within accuracy up to the summands of the second order over a small parameter $\kappa \ll 1$ of the problem, one can obtain the following approximate relations between the proposed coordinate vectors and coordinate vectors of cylindrical system, $\vec{e}_3 \approx \kappa\vec{e}_\vartheta + \vec{e}_z$, $\vec{e}_2 \approx \vec{e}_\vartheta - \kappa\vec{e}_z$. Using these local coordinates, one can write down the set of Maxwell's equations, incorporating the simplest relation between the components of the displacement vector $\vec{D}$ and the electric field $\vec{E}$ of the wave:

$$\vec{D} = \varepsilon_1 (E_1\vec{e}_1 + E_2\vec{e}_2) + \varepsilon_3 E_3\vec{e}_3 - i\varepsilon_2\vec{e}_3 \times \vec{E}. \qquad (59)$$

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In this case the tensor of dielectric permeability tensor ε_j of cold magnetoactive plasma with five non-zero components [16] is applied. Value of cyclotron frequency in the expressions for components of the tensor ε_j is determined by the complete constant magnetic field $\vec{B}_0$, in which value with accuracy up to small summands of the second order over κ is equal to the axial magnetic field, $B_0 \approx B_{0z}$. In the applied right-hand coordinate system $(\vec{e}_1, \vec{e}_2, \vec{e}_3)$, components of a rotor within accuracy up to the summands of the second order of smallness $\sim \kappa^2$ can be expressed through the components of rotor in cylindrical coordinates in the following way:

$$rot_1 = rot_r, \quad rot_2 \approx rot_\vartheta - \kappa\, rot_z, \quad rot_3 \approx \kappa\, rot_\vartheta - rot_z. \tag{60}$$

Taking into account all the above-mentioned coordinates, one can obtain the following three scalar equations from Maxwell equations, which in zero approximation describe the ordinary polarized electromagnetic waves:

$$-mH_z/kr = \varepsilon_1 E_r + i\varepsilon_2(E_\vartheta - \kappa E_z), \tag{61}$$

$$\frac{idH_z}{kdr} + \frac{i\kappa}{kr}\frac{d(rH_\vartheta)}{dr} + \frac{\kappa m}{kr}H_r = i\varepsilon_2 E_r - \varepsilon_1 E_\vartheta + \kappa\varepsilon_1 E_z, \tag{62}$$

$$-\frac{i\kappa}{k}\frac{dH_z}{dr} + \frac{i}{kr}\frac{d(rH_\vartheta)}{dr} + \frac{m}{kr}H_r = \varepsilon_3(E_z + \kappa E_\vartheta). \tag{63}$$

Solving the set of (61)–(63) for the axial electric field, one can derive the non-uniform differential equation of the second order (21) with small right-hand part. It looks like expression (48). The other three from Maxwell equations in zero approximation describe extraordinary polarized SWs. They can also be reduced to non-uniform differential equation of the second order like (49), where the influence of the constant azimuthal magnetic field shows up by itself by the appearance of a small right-hand part such as expression (50). Radial wave number of OABW and penetration depth of XASW are determined in Sec. 2.2 (see (48) and notations for k_o^{-1} and k_x^{-1} after it).

As these waves are described by weakly coupled equations, for their study it is suitable to apply the method of successive approximations, using the theory of XASW as zero approximation. Solutions of the right-hand sides of (21) and (49) as indicated above can be found by the method of variation of constants. The fields of the investigated waves should satisfy the boundary conditions, which are indicated in the Sec. 1. Their application leads to the derivation of dispersion relation for the coupled azimuthal modes likes (31), where D_o is determined by formula (23), where one should put $R_1 = R_2 = a$; $D_x = 0$ is the dispersion equation of XASW in the absence of an external constant azimuthal magnetic field; it can be written as follows:

$$D_x = -\frac{k^2}{k_x^2}\left(\frac{1}{\psi_x}\frac{d\psi_x}{dr} + \frac{m\mu}{a}\right). \tag{64}$$

Term $D(\kappa)$, which is located in the (31), is the coupling coefficient that describes the interaction between OABW and XASW, its value is quadratic over the small parameter κ:

$$D(\kappa) = \left(\frac{d\varphi_o}{dr}\right)^{-2} \int_0^a \frac{r}{a}\varphi_o \widehat{M}\psi_x dr \times \left[\frac{k^2 W(\psi_x, \tilde{\psi}_x)}{k_x^2 \psi_x^2} \int_0^a \frac{\psi_x \widehat{K}\varphi_o dr}{W(\psi_x, \tilde{\psi}_x)} - ik\left(1 + \frac{k_o^2}{k_x^2}\right)\frac{\varphi_o}{\psi_x}\right]. \tag{65}$$

Here functions φ_o and ψ_x, which are utilized in formulae (64) and (65), are the solutions of (21) and (49) with zero right-hand sides, they should be of limited values within all the volumes of the waveguide, particularly in the center of plasma column ($r = 0$). Parameters, which depend on the radial coordinate but are not integrand in expression (65), should be calculated at the point $r = a$.

Solutions of this dispersion relation can be found in the form $\omega = \omega_{O,X} + \delta\omega_{O,X}$, where $\omega_{O,X}$ is the eigenfrequency of OABW and XASW in absence of an external azimuthal magnetic field, $|\delta\omega_{O,X}| \ll \omega_{O,X}$. Correction $\delta\omega_{O,X}$ to eigenfrequencies are determined by formula (36). Simple analytical solutions of the dispersion relation can be obtained in the limiting case of uniform dense plasmas. In this case (21) and (49) for E_z and H_z are the non-uniform Bessel equations. Solutions of the corresponding uniform equations, which are limited in the center of plasma column, can be expressed in terms of the Bessel functions of the first type with the argument $k_o r$ and the modified Bessel functions with the argument $k_x r$, respectively. Analysis of expressions for arguments of these functions allows one to determine frequency ranges, where XASW can exist. Analysis of k_o^2 testifies that it has different signs in HF and LF ranges, namely, in the LF range $k_o^2 < 0$, and in the HF range it becomes positive, $k_o^2 > 0$. Thus, linear interaction between OABW and XASW is possible only in the HF range.

Let us consider the case of linear dependence of the external constant azimuthal magnetic field on radial coordinate, $B_{0\vartheta}(r) = rB_{o\vartheta}(a)/a$. Then integrals, which are included in expression (65) for $D(\kappa)$, can be calculated in an explicit form [22]. Analysis of the XASW dispersion relation proves that in narrow waveguides, HF XASW cannot propagate. In wide waveguides ($a \gg \delta|m|$) eigenfrequency of HF XASW ω_X is approximately determined by expression (55) indicated in the previous subsection.

If the values of the utilized magnetic fields and/or waveguide radii are small enough so that the following inequality takes place,

$$a^2\omega_e^2/c^2 < g_{|m|,s}^2 - m^2, \tag{66}$$

then OABW cannot propagate in the HF range. Here $g_{|m|,S}$ is the meaning of the sth root of the first type

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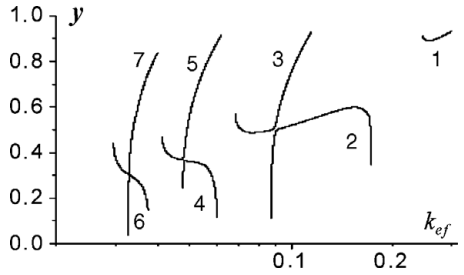


Figure 3. Spectral interaction between the XASW and three radial modes of OABW propagating in cylindrical waveguide that is completely filled with plasma.

Bessel function of the order $|m|$ [15]. With increasing utilized waveguide radius and/or value of the external constant magnetic field, inequality (66) becomes invalid and one can obtain corresponding equality instead of inequality (66). Validity of this equality means that frequencies of XASW and OABW are equal under such condition. In this case correction to the XASW eigenfrequency has the following form:

$$\delta\omega_X \approx \omega\, c^2 a^{-2}\sqrt{m^3\omega_e\Omega_e^{-3}} \approx \kappa\,|m|^{3/2}\,Z^{-5/2}\Omega_e/g^2_{|m|,s}, \tag{67}$$

where $Z = \Omega_e/|\omega_e| > 1$. From analysis of expression (67), one can conclude that influence of $B_{0\vartheta}$ on the frequency of XASW increases with increasing axial constant magnetic field and also with reducing plasma density, absolute value of azimuthal wave number, and radius of the waveguide [23].

In order to investigate the dispersion properties of the coupled azimuthal waves for the arbitrary values of parameters of the waveguide system one can apply numerical method. It allows one to calculate, for instance, cross-point of XASW with azimuthal wave number, $m = -1$, and first three radial modes of OABW. For the following values of the waveguide parameters, $\kappa = 0.1$, $Z = 3$, they can be found out approximately from expression (66): $k_{ef} \approx |m|/(Z\, g_{|m|,S}) \approx 0.087$, 0.048, 0.033... To see the fine structure of the frequency spectrum, which illustrates interaction between XASW and OABW, one can apply dimensionless variable $y = \sqrt[4]{\omega^2\omega_e^{-2} - 1 - Z^2}$ in Fig. 3, where level of $y = 0$ indicates the low band of the HF range.

Let us consider variation of the fine structure of spectral interaction between XASW and OABW with decreasing effective azimuthal waver number, k_{ef}. It is seen from Fig. 3 that the dispersion curve for XASW (right-hand branch of curve 1) comes from the range of maximal effective wave numbers and function, $y(k_{\mathrm{ef}})$, experiences a discontinuity near $k_{\mathrm{ef}} \approx 0.21$. On the left of this discontinuity, the dispersion curve for XASW emerges from the lower boundary of the frequency range (Fig. 3, right-hand branch of curve 2). Near $k_{\mathrm{ef}} \approx 0.087$, XASW transforms into OABW and the dispersion curve abruptly falls on the boundary of the frequency range (Fig. 3, left-hand branch of curve 2). The dispersion curve for the first radial OABW mode (Fig. 3, right-hand branch of curve 3) comes from the range of relatively high effective wave numbers. Near $k_{\mathrm{ef}} \approx 0.087$, OABW transforms into XASW and the curve runs almost horizontally to the left and rises rapidly when approaching $k_{\mathrm{ef}} \approx 0.064$ (Fig. 3, left-hand branch of curve 3) and so on. Therefore, if radial wave number of OABW grows, then first, differentiation of OABW and XASW dispersion curves becomes less and less conspicuous; second, the range of k_{ef} between two neighboring points of azimuthal modes conversion becomes more narrow; third, diminishing of the k_{ef} value leads to the decreasing of XASW frequency to the lower band of the HF range.

Thus, interaction between XASW and OABW results in the appearance of forbidden stripes in the azimuthal waves frequency spectrum, which are typical for the waves propagating in media with periodic spatial non-uniformity (see, for example, [24]). Appearance of forbidden stripes in the frequency spectrum, which is represented in Fig. 3, is equivalent to the originating of the ranges of the k_{ef} forbidden values. It is explained by the fact that for these ranges of values of plasma cylinder radius, OABW stop to be the eigenmodes and approaching to these forbidden ranges of the k_{ef} leads to sharp growing of correction of OABW frequency. One can see the appearance of the ranges of k_{ef}, within which XASW are characterized by reverse dispersion ($d\omega/dk_{\mathrm{ef}} < 0$), which is caused by the interaction between XASW and OABW. It means that in waveguides with corresponding values of parameters, azimuthal waves' energy is transported in the direction that is opposite to that of their phase velocity.

It should be underlined that points, $k_{\mathrm{ef}} \approx 0.260, 0.160, 0.075, \ldots$, on the dispersion curve presented in Fig. 3, where direct dispersion changes by reverse dispersion (or vice versa) and where $d\omega/dk_{\mathrm{ef}} = 0$, are of particular interest. Dependence of the azimuthal waves frequency $\omega = \omega(k_{\mathrm{ef}})$ near these points can be presented in the following form:

$$\omega \approx \omega_0\,(1 + \alpha^2 k_{\mathrm{ef}}^2/2). \tag{68}$$

Let at the initial time instant an electromagnetic pulse has the form of a modulated sinusoidal function with the Gaussian envelope:

$$H_z(\vartheta, 0) = H^{(0)}\cos(m\vartheta)\exp[-\vartheta^2/(2\beta^2)]. \tag{69}$$

Then, at any moment of time this electromagnetic pulse has a modulated sine-wave form, but effective width β_t of the electromagnetic pulse envelope, which is determined by the Gaussian curve, depends on time in the following way:

$$\beta_t^2 = \beta^2 + \omega_0\alpha^2\delta^2 t a^{-2}. \tag{70}$$

Thus, the electromagnetic pulse can be considered as the one in which form coincides with its initial form only during certain interval of time of $t \ll a^2\beta^2\alpha^{-2}\delta^{-2}$ [25].

Let us estimate the possibility of experimental realization of the investigated phenomenon [25]. For laboratory

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gaseous plasma with the density $n_e \approx 10^{11}\text{cm}^{-3}$, it is necessary to utilize the axial magnetic field, $B_{0z} \approx 0.048\ T$ (then $z = 3$). Coming from condition (66), one can find that minimum radius of plasma is necessary for achieving resonant interaction between XASW and OABW with azimuthal wave number $m = -1$, it is equal to $a \approx 12$ cm. To satisfy the condition $B_{0\vartheta} = 0.1\ B_{0z}$, one has to apply the axial electric current, $I \approx 3$ kA. Then, difference between frequencies near the cross point of dispersion curves of XASW and the first radial mode of OABW is equal to $1.4 \times 10^7\ \text{s}^{-1}$.

Summarizing the results of numerical analysis of coupled azimuthal modes, one can see that if condition of linear interaction between OABW and XASW is not satisfied, then frequency correction to the XASW eigenfrequency is proportional to squared external azimuthal magnetic field, while corresponding correction determined by constant axial magnetic field B_{0z} is proportional to its value (in the first power). Linear resonant interaction between XASW and OABW is realized near the cross points of their dispersion curves. It is accompanied by appearance on ranges of the forbidden values of their frequencies. In this case, frequency correction becomes larger: It is directly proportional to the external azimuthal magnetic field.

It was, however, found that the constant azimuthal magnetic field, $B_{0\vartheta}$, exerts the greatest effect on the dispersion properties of XASW by no means at the points of intersection between the dispersion curves of XASW and OABW. It should be reminded that the radius accommodates an almost integer number of OABW half-lengths near these points. Therefore, as in the case of linear XASW–OABW interaction, the boundary conditions are satisfied owing to a small variation of frequency because the field $B_{0\vartheta}$ has a strong effect on the spatial distributions of both waves. Since the OABW field distribution is volumetric, one can select sets of waveguide parameters such that the radius can accommodate an odd number of quarters of the induced OABW in the zero-th approximation. Such conditions are observed almost in the middle between the points where the dispersion curve for XASW intersects with successive radial OABW harmonics. In this situation, the plot of the dispersion curve for the coupled XASW and OABW exhibits bands of forbidden wave numbers. Then, to satisfy the boundary conditions, the frequency correction must be so high that the corresponding increase (or decrease) in the wavelength removes (or adds) an excessive (or missing) quarter wavelength. This also results in a considerable change in XASW dispersion in wave number ranges adjacent to the forbidden k_{ef} bands. In these ranges, the XASW dispersion becomes reverse, i.e., the energy of these waves is transferred in the direction opposite to the phase velocity vector. It illustrates the qualitative difference between XASW–OABW linear interaction and interaction of XASW with ordinarily polarized ASWs.

Let us compare the present results with those obtained in Sec. 2.2. Values of XASW frequencies, which propagate in metal waveguides completely filled with plasma, are very close to frequencies of XASW, which propagate along the boundary between cold magnetoactive plasma and metal bar immersed into it [17]. But influence of a weak, external constant azimuthal magnetic field on their dispersion properties appears to be different in these two cases. In the case of metal waveguide, which is completely filled with plasma [22], resonant linear interaction between XASW and OABW can be realized, which is not observed in the case of the coupled azimuthal waves that propagate along the boundary of a metal bar immersed into plasma (that is the case studied in Sec. 2.2). On the other hand, linear interaction between XASW and OABW in a metal waveguide, which is completely filled with cold magnetoactive plasma, is not accompanied by damping of the coupled azimuthal waves. Their interaction under the condition of their propagation along the boundary of metal bar immersed into cold magnetoactive plasma results in azimuthal waves damping caused by conversion of XASW into OABW that are radiated out of antenna–plasma system. Thus, amplitude of the waves, which are radiated from the plasma-coated antenna, is linear over the strength of electric current flowing along the antenna, and the damping rate of azimuthal waves is quadratic over the strength of electric current.

1.2.4. Coupled high-frequency ASW in coaxial plasma waveguide, which is completely filled with current-carrying plasma. In order to increase the efficiency of operation of HF electronic devices, coaxial waveguide structures are utilized more and more frequently because their characteristics are found to be better than the best ones of cylindrical structures. Therefore, let us examine coaxial metal waveguide with respective internal and external radii of a and b. Space between metal walls of coaxial waveguide is completely filled with cold magnetoactive plasma with uniform density profile. Let us consider the case when $\kappa = B_{0\vartheta}/B_{0z} \ll 1$. This allows one to carry out not only numerical research but also analytical research in the properties of coupled azimuthal waves, which can propagate in such waveguides.

Ignoring the summands, in which order of smallness is higher than κ, one can derive the set of differential equations for axial components of the electric field E_z of OABW and magnetic field H_z of XASW from Maxwell equations. The tangential components of the electric field of the considered electromagnetic perturbations are assumed to be equal to zero on the metal surfaces of the coaxial line. Frequency range of existence of these azimuthal waves and signs of their azimuthal wave numbers are determined in the Sec. 1. To study the dispersion properties of such coupled azimuthal electromagnetic waves let us use the theory of successive approximations, applying the theory of XASW (see sec. 2.1) as the zero-th approximation. In this case

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one can find solutions of Maxwell equations for axial electric and magnetic fields of azimuthal electromagnetic oscillations by the method of variation of constants [26]. Other components of the electromagnetic field of coupled azimuthal waves can be expressed through E_z and H_z from Maxwell equations.

Application of boundary conditions indicated in Sec. 1 allows one to derive the dispersion relation in the form (31), where $D_o = J_m(k_o a)N_m(k_o b) - N_m(k_o r)J_m(k_o r) = 0$ is dispersion relation of OABW and $D_x = 0$ is dispersion relation of XASW, both obtained for zero azimuthal eternal magnetic field, $B_{0\vartheta} = 0$, which is as follows:

$$D_x = a_{23}a_{44} - a_{24}a_{43}. \tag{71}$$

Term $D(\kappa)$ in this equation is a coefficient of coupling between OABW and XASW that describes influence of azimuthal component of external magnetic field on propagation of these waves:

$$\begin{aligned} D(\kappa) = \frac{\pi}{2}\Bigg\{\Bigg[a_{23}\int_a^b rI_m(k_x r)\widehat{K}N_m(k_o r)\,dr \\ - a_{24}\int_a^b rK_m(k_x r)\widehat{K}N_m(k_o r)\,dr\Bigg]J_m(k_o r) \\ -N_m(k_o r)\Bigg[a_{23}\int_a^b rI_m(k_x r)\widehat{K}J_m(k_o r)\,dr \\ - a_{24}\int_a^b rK_m(k_x r)\widehat{K}J_m(k_o r)\,dr\Bigg]\Bigg\} \\ \times J_m(k_o r)\int_a^b rJ_m(k_o r)\widehat{M}K_m(k_x r)\,dr]a_{44} \\ -\{[N_m(k_o r)\int_a^b rJ_m(k_o r)\widehat{M}K_m(k_x r)\,dr \\ - a_{43}[N_m(k_o b)\int_a^b rJ_m(k_o r)\widehat{M}I_m(k_x r)dr \\ - J_m(k_o b)\int_a^b rN_m(k_o r)\widehat{M}I_m(k_x r)\,dr]\}. \end{aligned} \tag{72}$$

Here the factors, which determine both expressions for D_x and $D(\kappa)$, have the following form:

$$\begin{aligned} a_{23} &= \left[k_x K'_m(k_x a) + \frac{m\mu}{a}K_m(k_x a)\right], \\ a_{24} &= \left[k_x I'_m(k_x a) + \frac{m\mu}{a}I_m(k_x a)\right], \\ a_{43} &= \left[k_x K'_m(k_x b) + \frac{m\mu}{b}K_m(k_x b)\right], \\ a_{44} &= \left[k_x I'_m(k_x b) + \frac{m\mu}{b}I_m(k_x b)\right]. \end{aligned} \tag{73, 74}$$

As one can see from the analysis of expression (72), the value of coupling coefficient $D(\kappa) \propto \widehat{M}\widehat{K} \propto \kappa^2$ is the value of the second order of smallness.

Let us discuss in detail the structure of XASW dispersion relation (see expression (71)) in the case of zero external azimuthal magnetic field. Condition $a_{44} = 0$ coincides with dispersion relation for XASW [17] propagating in the cylindrical metal waveguide of radius b, which is completely filled with plasma. The condition $a_{23} = 0$ is the dispersion relation of XASW propagating along the interface of metal cylinder with radius a that is immersed into uniform plasma and is oriented just along the external axial magnetic field.

From the formal point of view, the structure of equation $D_x = 0$ is similar to the structure of dispersion equation of coupling waves with linear conversion. Thus, one can make the following conclusion: First, $a_{44} = 0$ plays the role of dispersion relation of the wave that propagates along the external metal surface of the considered coaxial waveguide; second, condition $a_{23} = 0$ plays the role of dispersion relation of the other wave that propagates along the internal metal surface of waveguide; third, the product $a_{43}a_{24}$ plays the role of parameter of weak coupling between them. However, it should be underlined here that the studied waves cannot interact linearly because they cannot propagate with the same value of azimuthal wave number m simultaneously. In other words, for a fixed sign of number m, the solution exists either only for the equation $a_{44} = 0$ or only for the equation $a_{23} = 0$. This is the property of HF XASW: These propagate along a convex metal surface in wide coaxial waveguides only with positive azimuthal wave numbers, $m > 0$, and along a concave metal surface only with the negative wave numbers, $m < 0$ [17].

The Eigenfrequency of HF XASW, which propagate in coaxial waveguide completely filled with cold magnetoactive plasma with $m > 0$, can be approximately calculated from (71):

$$\omega_X \approx \sqrt{\Omega_e^2 + \omega_e^2 + \frac{m^2c^2}{a^2} + 4\frac{m^2c^2}{a^2}\frac{b-a}{b+a}\exp[-2k_x(b-a)]}. \tag{75}$$

Presence of external metal surface with radius of finite value leads to some increase in HF XASW frequency with $m > 0$ propagating in the considered coaxial waveguide as compared with the frequency of HF XASW propagating around a metal bar immersed into infinite uniform magnetoactive plasma ceteris paribus.

Along an external metal surface, HF XASW propagate in wide coaxial waveguides only with negative value of the wave number, $m < 0$. In this case, value of the XASW frequency is less than the frequency of HF XASW, which propagate in a cylindrical metal waveguide completely filled with magnetoactive plasma under the fixed conditions. It is evident from comparison of expressions for frequencies of HF XASW with opposite meanings of azimuthal wave numbers that for the fixed absolute value of azimuthal wave number $|m|$, frequency of the wave propagating along an internal surface is higher than the frequency of the wave propagating along

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external surface with radius $r = b > a$ in the opposite direction. For the case $m < 0$, expression of HF XASW frequency is similar to (75); there are only two differences: first, the fourth addendum over sign of square root is minus; and second, in this addendum instead a^2one can write b^2. Thus, in a coaxial resonator, XASW have an un-mutual character of propagation and dependence of their frequency on effective wave-number $k_{\text{ef}}=|m|\delta/R$ (where $R = a$ or $R = b$, respectively) is different for positive and negative meanings of the azimuthal wave number m. As HF XASW propagate only in wide plasma waveguides, presence of the second radial border of waves' localization influences their dispersion properties weakly. That is why difference between frequencies of HF XASW propagating in non-coaxial structures (see Sec. 1) is also weak.

The considered coaxial waveguides can be used for designing the waveguide phase shifters. Changes in electromagnetic signal phases take place on different interfaces of the coaxial plasma-filled resonator in different directions: With increasing azimuthal angle, signal phase diminishes on the external surface $r = b$ and increases on the internal surface $r = a$. Selection of the coaxial waveguide frequency corresponding to the frequency of input signal to be changed can be done by means of changing the external axial magnetic field. Application of such type of plasma-filled resonators allows one to get a signal with any necessary phase and to take off simultaneously a set of signals with phase changes assigned in advance.

High frequency OABW propagate in a wide coaxial waveguide with the following frequency:

$$\omega_o \approx \sqrt{\Omega_e^2 + \pi^2 c^2 j^2 (b-a)^{-2}}, \quad \text{where } j = 1, 2, 3. \quad (76)$$

If the external axial magnetic field and size of waveguide are large enough so that the following equality takes place

$$\frac{\omega_e^2}{c^2} \approx \frac{\pi^2 j^2}{(b-a)^2} - \frac{m^2}{b^2}, \quad (77)$$

then dispersion curves of XASW and OABW can intersect. In this case, their linear conversion becomes possible under the condition of utilization of the external magnetic field, which has both large and small azimuthal components. Therefore, frequency corrections to the eigenvalues of the frequencies of ordinary and extraordinary modes can be calculated by the aid of formula (33) and its values are linear over the small parameter κ.

More information can be obtained by numerical research in the properties of the coupled azimuthal waves propagating in coaxial waveguides. In Fig. 4, radial distribution of the axial magnetic and electric fields of ASW propagating with $m = -1$ in coaxial waveguide, in which internal radius is four times less than external radius, is represented. The following values of the coaxial waveguide parameters are applied for numerical analysis: $Z = 3$, $k_{\text{ef}} = 0,1$, $y = 0,537$. The

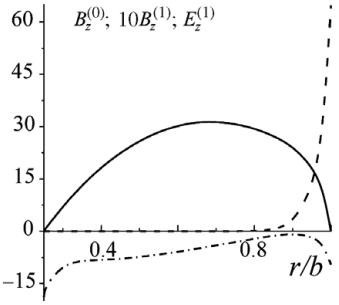


Figure 4. Radial distribution of the axial electromagnetic fields for coupled azimuthal waves propagating in coaxial plasma-filled waveguide for the case: $m = -1$, $\kappa = 0.3$, $b = 4a$.

dashed line indicates their radial dependence on ASW axial magnetic field in zero-th approximation. It has typical surface character. This means that the wave amplitude is mainly localized near the external border of coaxial waveguide. A bold line shows radial distribution of the axial electric field of ASW (that equals to zero in the absence of external azimuthal magnetic field), this dependence is normalized by the small parameter of the problem to the visible one. One can see that the presented radial distribution is the first radial mode of OABW. Then the phase difference between internal and external surfaces of coaxial waveguide makes up $k_o(b-a) \approx 2.602$. The chain line presents correction of radial dependence of ASW axial magnetic field caused by the presence of external constant azimuthal magnetic field.

Let us make some common remarks on the results of numerical analysis. Frequency spectrum of the coupled azimuthal waves propagating in coaxial waveguides has some peculiarities as compared with the spectrums of coupled azimuthal waves studied in the previous subsections. First, growth of radial wave number of bulk azimuthal mode results in the fact that splitting of the frequencies of azimuthal modes becomes less expressed near the point of their conversion. Second, the range of k_{ef} between two neighboring points of their transformation becomes narrower. Third, diminishing of k_{ef} value leads to decrease in XASW frequency, its mean approaches to the low band of frequency range (dimensionless parameter $y \to 0$). Fourth, for XASW with $m = +1$, which propagate along the internal metal surface, there appear the areas of non-transparency (this happens after the second intersection of dispersion curves). Fifth, frequencies of OABW diminish with increasing distance between the walls of coaxial waveguide, and intersections of dispersion curves move toward greater values of k_{ef}. Sixth, the minimum size of coaxial line (wherein linear interaction between XASW and OABW can be realized) is related just for the waves with the minimum value of azimuthal wave number. Seventh, increase of Z parameter leads to the diminishing (in units of electron cyclotron frequency) of HF range where XASW can exist.

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In order to estimate the possibility of experimental observation of this phenomenon, let us assume that plasma density, $n = 10^{13}\,\mathrm{cm}^{-3}$ and magnetic field, $B_{0z} \approx 4831\ G$. Then, for realization of linear interaction between XASW and the first radial mode of OABW one has to utilize the waveguide with $b = 3.12\,\mathrm{cm}$ and $a = 1.56\,\mathrm{cm}$. In this case [26] the splitting of frequencies of XASW and OABW is equal to $2\Delta\omega_X \approx 3 \cdot 10^8 \mathrm{s}^{-1}$ for axial electric current of 90 kA.

2. Conclusions

Results of theoretical investigations in the properties of electromagnetic azimuthal waves propagating in different plasma-filled waveguides are represented here for the case of axial electric current flowing in plasma column. Carried out investigations allow one to determine bands of frequency ranges, within which ordinary and extraordinary azimuthal waves can propagate, to analyze their dispersion properties and spatial distribution of their fields, and to calculate their damping rates, which are caused by the following physical mechanisms: collisions between plasma particles, resonant conversion of extraordinary polarized surface azimuthal modes into ordinary polarized bulk azimuthal modes, and radiation of their energy out of waveguides. It is proved that in the presence of axial external magnetic field, azimuthal waves can propagate both along plasma-dielectric and plasma-metal boundaries and in the absence of external magnetic field azimuthal waves propagate along plasma-dielectric boundary only. The theory of ASW developed in the Sec. 1 is applied in Sec. 2 for studying linear interaction between their ordinary and extraordinary modes in current-carrying plasma waveguides.

Results of studying the propagation of ASW in current-carrying waveguides with plasma filling seem to be interesting for various practical applications for plasma electronics and technologies. Under different conditions, flowing of axial electric current leads to coupling between ordinary and extraordinary modes and forced ordinary polarized modes can be of either surface or bulk type. In different plasma-filled waveguides, coupling between ordinary and extraordinary polarized modes can lead to different phenomena: appearance of non-reciprocal frequency ranges in the frequency spectrum of coupled azimuthal modes; change in the sign of their group velocity; damping of extraordinary azimuthal modes caused by conversion into radiated forced ordinary bulk mode; and so on.

Phenomenon of coupling between azimuthal X- and O-modes studied here is liner in respect of temporal evolution of these modes' mutual power transfer. Studied power transfer between these modes is characterized only by nonlinear dependence upon small parameter of the problem, $\kappa = B_{0\vartheta}/B_{0z} \ll 1$. In spite of this fact, it is similar to such strongly nonlinear effect as coupled plasma flows studied in [27, 28], which were studied numerically by the aid of choosing suitable spatial variations for plasma particle velocities and electromagnetic fields without application of the method of successive approximations. It was found in [27] that disturbances of azimuthal gaseous plasma flows didn't affect other components of the flow. Thus, energy of azimuthal flow oscillations could not be converted into radial and axial degrees of freedom, but the energy of latter could be transferred into azimuthal component. As it was proved in [28], cylindrically symmetric nonlinear flows and oscillations in rotating and expanding electron–positron plasma also tended to concentrate their energy just into the azimuthal flow component. The results obtained in [27, 28] can be applied for describing the experiments with rotating plasma in fusion devices, understanding solar dynamos, observing the plasma crystal formation, etc. Therefore, results of our paper can be useful not only for plasma electronics but also for above-mentioned branches of plasma physics. That is why it is planed to study nonlinear regimes of coupled azimuthal modes' propagation in future works.

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