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Computer-calculated explicit forms for representations of the three-dimensional pure rotation group

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Abstract. Explicit forms are given for the Wigner $\mathscr{D}^J_{MN}$'s for values of J from $J = 0$ to $J = \frac{7}{2}$. The tedious labour involved in evaluating available formulas is reduced by use of a computer.

1. *Introduction.* Formulas are available for the matrix elements of the $(2J+1)$-dimensional matrix $\mathscr{D}^J(\alpha, \beta, \gamma)$, which represents a rotation defined by the Euler angles α, β and γ (1). However, these formulas can lead to complicated expressions which are difficult to simplify (2, 3, 4). As a result, tabulations are readily available only for elements having $J = \frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}$ ($\mathscr{D}^0_{00} = 1$).

In this note, the tables for $\mathscr{D}^J_{MN}(\alpha, \beta, \gamma)$ are extended to $J = \frac{7}{2}$. The Condon and Shortley phase choice is used to ensure that the reduced matrix element $d^J_{MN}(\beta)$ is real (2), where

$$d^J_{MN}(\beta) = \langle JM | e^{i\beta \mathscr{J}_Y} | JN \rangle$$

and

$$\mathscr{D}^J_{MN}(\alpha, \beta, \gamma) = e^{-iM\alpha} d^J_{MN}(\beta)\, e^{-iN\gamma}.$$

The straightforward but tedious and error-prone explicit evaluation of the matrix elements has been handled by a computer.

2. *Results.* The formula used is (2):

$$d^J_{MN}(\beta) = \sum_t (-1)^t \frac{[(J+M)!\,(J-M)!\,(J+N)!\,(J-N)!]^{\frac{1}{2}}}{(J+M-t)!\,(J-N-t)!\,t!\,(t+N-M)!} \times (\cos \tfrac{1}{2}\beta)^{2J+M-N-2t} (\sin \tfrac{1}{2}\beta)^{2t+N-M}.$$

Combination of like powers of $\cos \frac{1}{2}\beta$ is achieved by the computer as is reduction of the half-angle expressions to whole-angle expression for integer values of J.

To conserve space, some symmetry properties of the $d^J_{MN}(\beta)$'s are used to minimize the number of entries. These relevant properties are

$$d^J_{MN}(\beta) = (-1)^{M-N} d^J_{NM}(\beta),$$

$$d^J_{MN}(\beta) = (-1)^{M-N} d^J_{-M-N}(\beta).$$

With the aid of these two properties, the expression for any $d^J_{MN}(\beta)$ for J up to $\frac{7}{2}$ can be obtained from the entries in the table.

The results illustrate how a computer can ease the burden of algebraic manipulation.

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Table of explicit forms for d^J_{MN}

$$J = \tfrac{1}{2}$$

$$d^{\frac{1}{2}}_{\frac{1}{2}\frac{1}{2}} = \cos(\tfrac{1}{2}\beta)$$
$$d^{\frac{1}{2}}_{\frac{1}{2}-\frac{1}{2}} = -\sin(\tfrac{1}{2}\beta)$$

$$J = 1$$

$$d^1_{1\,1} = \cos^2(\tfrac{1}{2}\beta) \quad \text{or} \quad (\tfrac{1}{2}) + (\tfrac{1}{2})\cos\beta$$
$$d^1_{1\,0} = -\sqrt{2}\cos(\tfrac{1}{2}\beta)\sin(\tfrac{1}{2}\beta) \quad \text{or} \quad -\sqrt{2}(\tfrac{1}{2})\sin\beta$$
$$d^1_{1\,-1} = 1-\cos^2(\tfrac{1}{2}\beta) \quad \text{or} \quad (\tfrac{1}{2}) + (-\tfrac{1}{2})\cos\beta$$
$$d^1_{0\,0} = -1+2\cos^2(\tfrac{1}{2}\beta) \quad \text{or} \quad \cos\beta$$

$$J = \tfrac{3}{2}$$

$$d^{\frac{3}{2}}_{\frac{3}{2}\frac{3}{2}} = \cos^3(\tfrac{1}{2}\beta)$$
$$d^{\frac{3}{2}}_{\frac{3}{2}\frac{1}{2}} = -\sqrt{3}\cos^2(\tfrac{1}{2}\beta)\sin(\tfrac{1}{2}\beta)$$
$$d^{\frac{3}{2}}_{\frac{3}{2}-\frac{1}{2}} = \sqrt{3}\cos(\tfrac{1}{2}\beta) - \sqrt{3}\cos^3(\tfrac{1}{2}\beta)$$
$$d^{\frac{3}{2}}_{\frac{3}{2}-\frac{3}{2}} = -\sin(\tfrac{1}{2}\beta) + \cos^2(\tfrac{1}{2}\beta)\sin(\tfrac{1}{2}\beta)$$
$$d^{\frac{3}{2}}_{\frac{1}{2}\frac{1}{2}} = -2\cos(\tfrac{1}{2}\beta) + 3\cos^3(\tfrac{1}{2}\beta)$$
$$d^{\frac{3}{2}}_{\frac{1}{2}-\frac{1}{2}} = \sin(\tfrac{1}{2}\beta) - 3\cos^2(\tfrac{1}{2}\beta)\sin(\tfrac{1}{2}\beta)$$

$$J = 2$$

$$d^2_{2\,2} = \cos^4(\tfrac{1}{2}\beta) \quad \text{or} \quad (\tfrac{1}{4}) + (\tfrac{1}{2})\cos\beta + (\tfrac{1}{4})\cos^2\beta$$
$$d^2_{2\,1} = -2\cos^3(\tfrac{1}{2}\beta)\sin(\tfrac{1}{2}\beta) \quad \text{or} \quad (-\tfrac{1}{2})\sin\beta + (-\tfrac{1}{2})\cos\beta\sin\beta$$
$$d^2_{2\,0} = \sqrt{6}\cos^2(\tfrac{1}{2}\beta) - \sqrt{6}\cos^4(\tfrac{1}{2}\beta) \quad \text{or} \quad \sqrt{6}(\tfrac{1}{4}) + \sqrt{6}(-\tfrac{1}{4})\cos^2\beta$$
$$d^2_{2\,-1} = -2\cos(\tfrac{1}{2}\beta)\sin(\tfrac{1}{2}\beta) + 2\cos^3(\tfrac{1}{2}\beta)\sin(\tfrac{1}{2}\beta) \quad \text{or} \quad (-\tfrac{1}{2})\sin\beta + (\tfrac{1}{2})\cos\beta\sin\beta$$
$$d^2_{2\,-2} = 1-2\cos^2(\tfrac{1}{2}\beta) + \cos^4(\tfrac{1}{2}\beta) \quad \text{or} \quad (\tfrac{1}{4}) + (-\tfrac{1}{2})\cos\beta + (\tfrac{1}{4})\cos^2\beta$$
$$d^2_{1\,1} = -3\cos^2(\tfrac{1}{2}\beta) + 4\cos^4(\tfrac{1}{2}\beta) \quad \text{or} \quad (-\tfrac{1}{2}) + (\tfrac{1}{2})\cos\beta + \cos^2\beta$$
$$d^2_{1\,0} = \sqrt{6}\cos(\tfrac{1}{2}\beta)\sin(\tfrac{1}{2}\beta) - \sqrt{6}(2)\cos^3(\tfrac{1}{2}\beta)\sin(\tfrac{1}{2}\beta) \quad \text{or} \quad -\sqrt{6}(\tfrac{1}{2})\cos\beta\sin\beta$$
$$d^2_{1\,-1} = -1+5\cos^2(\tfrac{1}{2}\beta) - 4\cos^4(\tfrac{1}{2}\beta) \quad \text{or} \quad (\tfrac{1}{2}) + (\tfrac{1}{2})\cos\beta - \cos^2\beta$$
$$d^2_{0\,0} = 1-6\cos^2(\tfrac{1}{2}\beta) + 6\cos^4(\tfrac{1}{2}\beta) \quad \text{or} \quad (-\tfrac{1}{2}) + (\tfrac{3}{2})\cos^2\beta$$

$$J = \tfrac{5}{2}$$

$$d^{\frac{5}{2}}_{\frac{5}{2}\frac{5}{2}} = \cos^5(\tfrac{1}{2}\beta)$$
$$d^{\frac{5}{2}}_{\frac{5}{2}\frac{3}{2}} = -\sqrt{5}\cos^4(\tfrac{1}{2}\beta)\sin(\tfrac{1}{2}\beta)$$
$$d^{\frac{5}{2}}_{\frac{5}{2}\frac{1}{2}} = \sqrt{10}\cos^3(\tfrac{1}{2}\beta) - \sqrt{10}\cos^5(\tfrac{1}{2}\beta)$$
$$d^{\frac{5}{2}}_{\frac{5}{2}-\frac{1}{2}} = -\sqrt{10}\cos^2(\tfrac{1}{2}\beta)\sin(\tfrac{1}{2}\beta) + \sqrt{10}\cos^4(\tfrac{1}{2}\beta)\sin(\tfrac{1}{2}\beta)$$
$$d^{\frac{5}{2}}_{\frac{5}{2}-\frac{3}{2}} = \sqrt{5}\cos(\tfrac{1}{2}\beta) - \sqrt{5}(2)\cos^3(\tfrac{1}{2}\beta) + \sqrt{5}\cos^5(\tfrac{1}{2}\beta)$$
$$d^{\frac{5}{2}}_{\frac{5}{2}-\frac{5}{2}} = -\sin(\tfrac{1}{2}\beta) + 2\cos^2(\tfrac{1}{2}\beta)\sin(\tfrac{1}{2}\beta) - \cos^4(\tfrac{1}{2}\beta)\sin(\tfrac{1}{2}\beta)$$
$$d^{\frac{5}{2}}_{\frac{3}{2}\frac{3}{2}} = -4\cos^3(\tfrac{1}{2}\beta) + 5\cos^5(\tfrac{1}{2}\beta)$$

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$$J = \tfrac{5}{2} \text{ (cont.)}$$

$$d^{\frac{5}{2}}_{\frac{3}{2}\frac{1}{2}} = \sqrt{2}\,(3)\cos^2(\tfrac{1}{2}\beta)\sin(\tfrac{1}{2}\beta) - \sqrt{2}\,(5)\cos^4(\tfrac{1}{2}\beta)\sin(\tfrac{1}{2}\beta)$$

$$d^{\frac{5}{2}}_{\frac{3}{2}-\frac{1}{2}} = -\sqrt{2}(2)\cos(\tfrac{1}{2}\beta) + \sqrt{2}(7)\cos^3(\tfrac{1}{2}\beta) - \sqrt{2}\,(5)\cos^5(\tfrac{1}{2}\beta)$$

$$d^{\frac{5}{2}}_{\frac{3}{2}-\frac{3}{2}} = \sin(\tfrac{1}{2}\beta) - 6\cos^2(\tfrac{1}{2}\beta)\sin(\tfrac{1}{2}\beta) + 5\cos^4(\tfrac{1}{2}\beta)\sin(\tfrac{1}{2}\beta)$$

$$d^{\frac{5}{2}}_{\frac{1}{2}\frac{1}{2}} = 3\cos(\tfrac{1}{2}\beta) - 12\cos^3(\tfrac{1}{2}\beta) + 10\cos^5(\tfrac{1}{2}\beta)$$

$$d^{\frac{5}{2}}_{\frac{1}{2}-\frac{1}{2}} = -\sin(\tfrac{1}{2}\beta) + 8\cos^2(\tfrac{1}{2}\beta)\sin(\tfrac{1}{2}\beta) - 10\cos^4(\tfrac{1}{2}\beta)\sin(\tfrac{1}{2}\beta)$$

$$J = 3$$

$$d^3_{3\,3} = \cos^6(\tfrac{1}{2}\beta) \quad \text{or} \quad (\tfrac{1}{8}) + (\tfrac{3}{8})\cos\beta + (\tfrac{3}{8})\cos^2\beta + (\tfrac{1}{8})\cos^3\beta$$

$$d^3_{3\,2} = -\sqrt{6}\cos^5(\tfrac{1}{2}\beta)\sin(\tfrac{1}{2}\beta)$$
$$\text{or} \quad -\sqrt{6}\,(\tfrac{1}{8})\sin\beta - \sqrt{6}\,(\tfrac{1}{4})\cos\beta\sin\beta - \sqrt{6}\,(\tfrac{1}{8})\cos^2\beta\sin\beta$$

$$d^3_{3\,1} = \sqrt{15}\cos^4(\tfrac{1}{2}\beta) - \sqrt{15}\cos^6(\tfrac{1}{2}\beta)$$
$$\text{or} \quad \sqrt{15}\,(\tfrac{1}{8}) + \sqrt{15}\,(\tfrac{1}{8})\cos\beta - \sqrt{15}\,(\tfrac{1}{8})\cos^2\beta - \sqrt{15}\,(\tfrac{1}{8})\cos^3\beta$$

$$d^3_{3\,0} = -\sqrt{5}\,(2)\cos^3(\tfrac{1}{2}\beta)\sin(\tfrac{1}{2}\beta) + \sqrt{5}\,(2)\cos^5(\tfrac{1}{2}\beta)\sin(\tfrac{1}{2}\beta)$$
$$\text{or} \quad -\sqrt{5}\,(\tfrac{1}{4})\sin\beta + \sqrt{5}\,(\tfrac{1}{4})\cos^2\beta\sin\beta$$

$$d^3_{3\,-1} = \sqrt{15}\cos^2(\tfrac{1}{2}\beta) - \sqrt{15}\,(2)\cos^4(\tfrac{1}{2}\beta) + \sqrt{15}\cos^6(\tfrac{1}{2}\beta)$$
$$\text{or} \quad \sqrt{15}\,(\tfrac{1}{8}) - \sqrt{15}\,(\tfrac{1}{8})\cos\beta - \sqrt{15}\,(\tfrac{1}{8})\cos^2\beta + \sqrt{15}\,(\tfrac{1}{8})\cos^3\beta$$

$$d^2_{3\,-2} = -\sqrt{6}\cos(\tfrac{1}{2}\beta)\sin(\tfrac{1}{2}\beta) + \sqrt{6}\,(2)\cos^3(\tfrac{1}{2}\beta)\sin(\tfrac{1}{2}\beta) - \sqrt{6}\cos^5(\tfrac{1}{2}\beta)\sin(\tfrac{1}{2}\beta)$$
$$\text{or} \quad -\sqrt{6}\,(\tfrac{1}{8})\sin\beta + \sqrt{6}\,(\tfrac{1}{4})\cos\beta\sin\beta - \sqrt{6}\,(\tfrac{1}{8})\cos^2\beta\sin\beta$$

$$d^3_{3\,-3} = 1 - 3\cos^2(\tfrac{1}{2}\beta) + 3\cos^4(\tfrac{1}{2}\beta) - \cos^6(\tfrac{1}{2}\beta)$$
$$\text{or} \quad (\tfrac{1}{8}) + (-\tfrac{3}{8})\cos\beta + (\tfrac{3}{8})\cos^2\beta + (-\tfrac{1}{8})\cos^3\beta$$

$$d^3_{2\,2} = -5\cos^4(\tfrac{1}{2}\beta) + 6\cos^6(\tfrac{1}{2}\beta) \quad \text{or} \quad (-\tfrac{1}{2}) + (-\tfrac{1}{4})\cos\beta + \cos^2\beta + (\tfrac{3}{4})\cos^3\beta$$

$$d^3_{2\,1} = \sqrt{10}\,(2)\cos^3(\tfrac{1}{2}\beta)\sin(\tfrac{1}{2}\beta) - \sqrt{10}\,(3)\cos^5(\tfrac{1}{2}\beta)\sin(\tfrac{1}{2}\beta)$$
$$\text{or} \quad \sqrt{10}\,(\tfrac{1}{8})\sin\beta - \sqrt{10}\,(\tfrac{1}{4})\cos\beta\sin\beta - \sqrt{10}\,(\tfrac{3}{8})\cos^2\beta\sin\beta$$

$$d^3_{2\,0} = -\sqrt{30}\cos^2(\tfrac{1}{2}\beta) + \sqrt{30}\,(3)\cos^4(\tfrac{1}{2}\beta) - \sqrt{30}\,(2)\cos^6(\tfrac{1}{2}\beta)$$
$$\text{or} \quad \sqrt{30}\,(\tfrac{1}{4})\cos\beta - \sqrt{30}\,(\tfrac{1}{4})\cos^3\beta$$

$$d^3_{2\,-1} = \sqrt{10}\cos(\tfrac{1}{2}\beta)\sin(\tfrac{1}{2}\beta) - \sqrt{10}\,(4)\cos^3(\tfrac{1}{2}\beta)\sin(\tfrac{1}{2}\beta) + \sqrt{10}\,(3)\cos^5(\tfrac{1}{2}\beta)\sin(\tfrac{1}{2}\beta)$$
$$\text{or} \quad -\sqrt{10}\,(\tfrac{1}{8})\sin\beta - \sqrt{10}\,(\tfrac{1}{4})\cos\beta\sin\beta + \sqrt{10}\,(\tfrac{3}{8})\cos^2\beta\sin\beta$$

$$d^3_{2\,-2} = -1 + 8\cos^2(\tfrac{1}{2}\beta) - 13\cos^4(\tfrac{1}{2}\beta) + 6\cos^6(\tfrac{1}{2}\beta)$$
$$\text{or} \quad (\tfrac{1}{2}) + (-\tfrac{1}{4})\cos\beta - \cos^2\beta + (\tfrac{3}{4})\cos^3\beta$$

$$d^3_{1\,1} = 6\cos^2(\tfrac{1}{2}\beta) - 20\cos^4(\tfrac{1}{2}\beta) + 15\cos^6(\tfrac{1}{2}\beta)$$
$$\text{or} \quad (-\tfrac{1}{8}) + (-\tfrac{11}{8})\cos\beta + (\tfrac{5}{8})\cos^2\beta + (\tfrac{15}{8})\cos^3\beta$$

$$d^3_{1\,0} = -\sqrt{3}\,(2)\cos(\tfrac{1}{2}\beta)\sin(\tfrac{1}{2}\beta) + \sqrt{3}(10)\cos^3(\tfrac{1}{2}\beta)\sin(\tfrac{1}{2}\beta) - \sqrt{3}\,(10)\cos^5(\tfrac{1}{2}\beta)\sin(\tfrac{1}{2}\beta)$$
$$\text{or} \quad \sqrt{3}(\tfrac{1}{4})\sin\beta - \sqrt{3}\,(\tfrac{5}{4})\cos^2\beta\sin\beta$$

$$d^3_{1\,-1} = 1 - 11\cos^2(\tfrac{1}{2}\beta) + 25\cos^4(\tfrac{1}{2}\beta) - 15\cos^6(\tfrac{1}{2}\beta)$$
$$\text{or} \quad (-\tfrac{1}{8}) + (\tfrac{11}{8})\cos\beta + (\tfrac{5}{8})\cos^2\beta + (-\tfrac{15}{8})\cos^3\beta$$

$$d^3_{0\,0} = -1 + 12\cos^2(\tfrac{1}{2}\beta) - 30\cos^4(\tfrac{1}{2}\beta) + 20\cos^6(\tfrac{1}{2}\beta) \quad \text{or} \quad (-\tfrac{3}{2})\cos\beta + (\tfrac{5}{2})\cos^3\beta$$

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$$J = \tfrac{7}{2}$$

$$d^{\frac{7}{2}}_{\frac{7}{2}\frac{7}{2}} = \cos^7(\tfrac{1}{2}\beta)$$

$$d^{\frac{7}{2}}_{\frac{7}{2}\frac{5}{2}} = -\sqrt{7}\cos^6(\tfrac{1}{2}\beta)\sin(\tfrac{1}{2}\beta)$$

$$d^{\frac{7}{2}}_{\frac{7}{2}\frac{3}{2}} = \sqrt{21}\cos^5(\tfrac{1}{2}\beta) - \sqrt{21}\cos^7(\tfrac{1}{2}\beta)$$

$$d^{\frac{7}{2}}_{\frac{7}{2}\frac{1}{2}} = -\sqrt{35}\cos^4(\tfrac{1}{2}\beta)\sin(\tfrac{1}{2}\beta) + \sqrt{35}\cos^6(\tfrac{1}{2}\beta)\sin(\tfrac{1}{2}\beta)$$

$$d^{\frac{7}{2}}_{\frac{7}{2}-\frac{1}{2}} = \sqrt{35}\cos^3(\tfrac{1}{2}\beta) - \sqrt{35}\,(2)\cos^5(\tfrac{1}{2}\beta) + \sqrt{35}\cos^7(\tfrac{1}{2}\beta)$$

$$d^{\frac{7}{2}}_{\frac{7}{2}-\frac{3}{2}} = -\sqrt{21}\cos^2(\tfrac{1}{2}\beta)\sin(\tfrac{1}{2}\beta) + \sqrt{21}\,(2)\cos^4(\tfrac{1}{2}\beta)\sin(\tfrac{1}{2}\beta) - \sqrt{21}\cos^6(\tfrac{1}{2}\beta)\sin(\tfrac{1}{2}\beta)$$

$$d^{\frac{7}{2}}_{\frac{7}{2}-\frac{5}{2}} = \sqrt{7}\cos(\tfrac{1}{2}\beta) - \sqrt{7}(3)\cos^3(\tfrac{1}{2}\beta) + \sqrt{7}(3)\cos^5(\tfrac{1}{2}\beta) - \sqrt{7}\cos^7(\tfrac{1}{2}\beta)$$

$$d^{\frac{7}{2}}_{\frac{7}{2}-\frac{7}{2}} = -\sin(\tfrac{1}{2}\beta) + 3\cos^2(\tfrac{1}{2}\beta)\sin(\tfrac{1}{2}\beta) - 3\cos^4(\tfrac{1}{2}\beta)\sin(\tfrac{1}{2}\beta) + \cos^6(\tfrac{1}{2}\beta)\sin(\tfrac{1}{2}\beta)$$

$$d^{\frac{7}{2}}_{\frac{5}{2}\frac{5}{2}} = -6\cos^5(\tfrac{1}{2}\beta) + 7\cos^7(\tfrac{1}{2}\beta)$$

$$d^{\frac{7}{2}}_{\frac{5}{2}\frac{3}{2}} = \sqrt{3}\,(5)\cos^4(\tfrac{1}{2}\beta)\sin(\tfrac{1}{2}\beta) - \sqrt{3}\,(7)\cos^6(\tfrac{1}{2}\beta)\sin(\tfrac{1}{2}\beta)$$

$$d^{\frac{7}{2}}_{\frac{5}{2}\frac{1}{2}} = -\sqrt{5}\,(4)\cos^3(\tfrac{1}{2}\beta) + \sqrt{5}(11)\cos^5(\tfrac{1}{2}\beta) - \sqrt{5}\,(7)\cos^7(\tfrac{1}{2}\beta)$$

$$d^{\frac{7}{2}}_{\frac{5}{2}-\frac{1}{2}} = \sqrt{5}\,(3)\cos^2(\tfrac{1}{2}\beta)\sin(\tfrac{1}{2}\beta) - \sqrt{5}(10)\cos^4(\tfrac{1}{2}\beta)\sin(\tfrac{1}{2}\beta) + \sqrt{5}\,(7)\cos^6(\tfrac{1}{2}\beta)\sin(\tfrac{1}{2}\beta)$$

$$d^{\frac{7}{2}}_{\frac{5}{2}-\frac{3}{2}} = -\sqrt{3}\,(2)\cos(\tfrac{1}{2}\beta) + \sqrt{3}\,(11)\cos^3(\tfrac{1}{2}\beta) - \sqrt{3}\,(16)\cos^5(\tfrac{1}{2}\beta) + \sqrt{3}\,(7)\cos^7(\tfrac{1}{2}\beta)$$

$$d^{\frac{7}{2}}_{\frac{5}{2}-\frac{5}{2}} = \sin(\tfrac{1}{2}\beta) - 9\cos^2(\tfrac{1}{2}\beta)\sin(\tfrac{1}{2}\beta) + 15\cos^4(\tfrac{1}{2}\beta)\sin(\tfrac{1}{2}\beta) - 7\cos^6(\tfrac{1}{2}\beta)\sin(\tfrac{1}{2}\beta)$$

$$d^{\frac{7}{2}}_{\frac{3}{2}\frac{3}{2}} = 10\cos^3(\tfrac{1}{2}\beta) - 30\cos^5(\tfrac{1}{2}\beta) + 21\cos^7(\tfrac{1}{2}\beta)$$

$$d^{\frac{7}{2}}_{\frac{3}{2}\frac{1}{2}} = -\sqrt{15}\,(2)\cos^2(\tfrac{1}{2}\beta)\sin(\tfrac{1}{2}\beta) + \sqrt{15}\,(8)\cos^4(\tfrac{1}{2}\beta)\sin(\tfrac{1}{2}\beta) - \sqrt{15}\,(7)\cos^6(\tfrac{1}{2}\beta)\sin(\tfrac{1}{2}\beta)$$

$$d^{\frac{7}{2}}_{\frac{3}{2}-\frac{1}{2}} = \sqrt{15}\cos(\tfrac{1}{2}\beta) - \sqrt{15}\,(7)\cos^3(\tfrac{1}{2}\beta) + \sqrt{15}\,(13)\cos^5(\tfrac{1}{2}\beta) - \sqrt{15}\,(7)\cos^7(\tfrac{1}{2}\beta)$$

$$d^{\frac{7}{2}}_{\frac{3}{2}-\frac{3}{2}} = -\sin(\tfrac{1}{2}\beta) + 13\cos^2(\tfrac{1}{2}\beta)\sin(\tfrac{1}{2}\beta) - 33\cos^4(\tfrac{1}{2}\beta)\sin(\tfrac{1}{2}\beta) + (21\cos^6\tfrac{1}{2}\beta)\sin(\tfrac{1}{2}\beta)$$

$$d^{\frac{7}{2}}_{\frac{1}{2}\frac{1}{2}} = -4\cos(\tfrac{1}{2}\beta) + 30\cos^3(\tfrac{1}{2}\beta) - 60\cos^5(\tfrac{1}{2}\beta) + 35\cos^7(\tfrac{1}{2}\beta)$$

$$d^{\frac{7}{2}}_{\frac{1}{2}-\frac{1}{2}} = \sin(\tfrac{1}{2}\beta) - 15\cos^2(\tfrac{1}{2}\beta)\sin(\tfrac{1}{2}\beta) + 45\cos^4(\tfrac{1}{2}\beta)\sin(\tfrac{1}{2}\beta) - 35\cos^6(\tfrac{1}{2}\beta)\sin(\tfrac{1}{2}\beta)$$

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