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Dipolar radiation from spinning dust grains coupled to an electromagnetic wave

A. GUERREIRO[1], M. ELOY[2], J. T. MENDONÇA[3] and R. BINGHAM[4]

[1]Physics Department, Faculdade de Ciências da Universidade do Porto, Rua do Campo Alegre, 687, 4169-007 Porto, Portugal

[2]Faculdade de Engenharia da Universidade Católica Portuguesa, Estrada Octávio Pato, 2635-631 Rio de Mouro, Portugal

[3]Instituto Superior Técnico, Av. Rovisco Pais, 1000 Lisboa, Portugal

[4]Rutherford Appleton Laboratory, Chilton, Didcot, Oxfordshire OX11 OQX, UK

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Abstract. In this paper we investigate how the complex rotation and quivering motion of an elongated polarized dust grain in the presence of a monochromatic electromagnetic (EM) wave can produce dipolar emission with two distinct spectral components. We present a model for the emission of radiation by elongated polarized dust grains under the influence of both an external EM wave and a constant background magnetic field. The dust, exhibiting rotational motion at the external EM field frequency ω_0 as well as quivering motion at a frequency Ω_0, proportional to the EM field amplitude, will radiate with frequencies that will depend on the external field wavelength and amplitude. The radiated spectra exhibits a frequency around ω_0, and sidebands at $\omega_0 \pm \Omega_0$ and $\omega_0 \pm 2\Omega_0$. Since the amplitude and the frequency of the background EM field are independent parameters, this model establishes a correlation between different spectral components of galactic dipolar emission, which may help to explain the correlation between a component of the Galactic microwave emission and the 100 μm thermal emission from interstellar dust that has been recently measured.

1. Introduction

Observations on cosmic background radiation have demonstrated the existence of a correlation between a component of the Galactic microwave emission and the 100 μm thermal emission from interstellar dust, and several models have been proposed to justify the results measured. Leitcht et al. [1] suggested free–free emission from heated gas to be the cause for this correlation but the model could not account for the expected H_α emission unless the temperature would be in excess of 10^6 K, as occurs in shock-heated gas from a supernova remnant. Drain and Lazarain [2] showed that for the free–free emission mechanism to be able to explain the observed microwave excess it would require an energy input of at least two orders of magnitude larger than that provided by supernovae, and they proposed an alternative mechanism to explain the excess of microwave radiation observed, based on electric dipole emission from rotating dust grains, which has been able to account for many aspects of the experimental measurements.

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More recently, data received from the Wilkinson Microwave Anisotropy Probe (WMAP) has lead to the claim that the model proposed by Drain and Lazarain could only account for 5% of the Galactic microwave emission, and, once again rekindled the debate about the origin of 'anomalous' dust correlated microwave emission [3]. Also, Lazarain and Prunet [4] have analyzed the importance of the thermal emission of magnetized dust, locally aligned by the Galactic magnetic field in the context of cosmic microwave background (CMB) contamination by Galactic dust emission, while Ponthieu et al. [5] and Baccigalupi [6] have stressed the importance of the polarization effects by measuring a 3–5% polarized dust signal on the Galactic plane.

In parallel, several studies of levitation and dynamics of charged dust grains have been carried out in both space and laboratory environments [8], to investigate the motion of charged dust particles in low-temperature dusty plasmas discharges [9]. It has been observed that in a dust plasma sheath dust grains levitate owing to a balance between gravitational and electrostatic forces, presenting a bouncing motion between electrodes and a quivering motion across the electric field of the sheath.

Several processes have, since then, been proposed to explain dust dynamics, with special emphasis on rotational excitation and damping of the dust grains, which, in both astrophysical and laboratory plasmas, are known to possess a rather elongated shape [10, 11] exhibiting a non-zero dipole moment. Examples include recoil from thermal collisions and/or evaporation [12], collisions with gas atoms or plasma and plasma drag [13], absorption and emission of radiation [14,15], random H_2 formation [12, 16] and systematic torques [12, 17]. However, the influence of interstellar electromagnetic (EM) background radiation on dust dynamics has not yet, to our knowledge, been fully addressed.

Tskhakaya and Shukla [18] have proposed a simple analytical model to describe the dynamics of elongated dust grains in the presence of circularly polarized EM waves and have concluded that the grains not only rotate—or spin—with the frequency of EM fields but also exhibit a complex quivering motion. Radiative processes from such spinning dusts have not, however, been included in the model.

This paper addresses the emission of radiation from a single rotating or spinning dust grain under the influence of an external EM wave and a constant background magnetic field, correlating the external EM field amplitude as well as its wavelength with the dust-emitted frequency spectra and predicting the radiated power due to both rotation and quivering motions of the dust grain. The work presented is organized as follows: starting from the same grain equations of motion as derived in [18] in Sec. 2, we address the emission of dipolar radiation from spinning dust in Sec. 3, show the results of numerical simulations of the emission spectrum of dust grains in Sec. 4 and present the conclusions in Sec. 5.

2. Spinning dust dynamics

We start by considering the propagation along the z-direction of a circularly polarized EM beam in a medium composed of neutral elongated dust grains, such as a stellar dust cloud, in the presence of a constant background magnetic field $\mathbf{B} = (0, 0, B_0)$.

The electric field can be written as

$$\mathbf{E}(\mathbf{r}, t) = E_0 \exp(i\mathbf{k} \cdot \mathbf{r})\mathbf{e}(t), \tag{1}$$

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where E_0 is the electric field amplitude and $\mathbf{e}(t) = (\cos(\omega_0 t), \sin(\omega_0 t), 0)$ is the polarization unit vector.

We will assume that the dust magnetic moment of each grain is along the direction of the background magnetic field (z) and neglect the precession motion around this axis, meaning that the dust will rotate in the xy plane. The dipole moment of the grain is expressed as $\mathbf{d} = d(\cos\phi, \sin\phi, 0)$ with ϕ the orientation of the grain relative to the x-direction.

The equation of motion describing the rotation of the grain will be [18]

$$I_z \frac{d^2\phi}{dt^2} = -dE_0 \sin(\phi - \omega_0 t), \tag{2}$$

which can be written as

$$\frac{d^2\phi}{dt^2} = -\Omega_0^2 \sin(\phi - \omega_0 t), \tag{3}$$

where $\Omega_0^2 = dE_0/I_z$ and I_z is the z component of the principal moment of inertia of the dust grain.

Under the influence of the electromagnetic wave E_0, the grain will rotate with the same angular frequency, ω_0 [18–20]. Besides this spinning motion, the grain may also exhibit a quivering motion corresponding to fluctuations on the mean spinning motion. It is therefore convenient to write the angle of orientation of the grain as $\phi = \omega_0 t + \delta\phi$, which is equivalent to considering the quivering motion of the grain in a reference frame which rotates with the polarization of the field E_0. The equation of motion then becomes

$$\ddot{\delta\phi} = -\Omega_0^2 \sin(\delta\phi). \tag{4}$$

As demonstrated in [18], (4) can be integrated and, imposing the initial conditions

$$\delta\phi(t=0) = \delta\phi_0, \tag{5}$$

$$\dot{\delta\phi}(t=0) = \dot{\delta\phi}_0, \tag{6}$$

results in the following equation:

$$\frac{1}{2\Omega_0^2}(\dot{\delta\phi})^2 - \cos(\delta\phi) = \frac{1}{2\Omega_0^2}(\dot{\delta\phi}_0)^2 - \cos(\delta\phi_0) \equiv \varepsilon. \tag{7}$$

The constant of integration ε plays the role of an effective energy and for values in the range $[-1, 1]$ the variation of $\delta\phi$ is bounded, resulting in a quivering motion of the grain with a frequency roughly equal to Ω_0 [18].

In this paper we are interested in situations when $\varepsilon \simeq -1$ and the quivering motion of the grain is approximately harmonic:

$$\delta\phi(t) \simeq \delta\phi_0 \cos(\Omega_0 t) + \frac{\dot{\delta\phi}_0}{\Omega_0} \sin(\Omega_0 t). \tag{8}$$

In the original frame of reference, the dust grains rotate with an angular velocity given by $\omega = \omega_0 + \dot{\delta\phi}$, and thus they exhibit a rotation motion with the frequency of the external EM field ω_0, and a quivering motion across the direction of the electric field with a frequency Ω_0, proportional to the external EM field amplitude. The quivering motion can be identified as the fluctuations of the mean rotation motion.

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3. Radiation from spinning dusts

As the grain rotates about the z-axis, the grain emits EM radiation according to the following equation [21]:

$$\mathbf{E}_{\rm rad}(\mathbf{r},t) = d^3r \frac{\rho(\mathbf{r})}{c^2|\mathbf{R}+\mathbf{r}|}[\mathbf{n}\times[\mathbf{n}\times\mathbf{a}]], \tag{9}$$

where the retardation effects have been neglected, $\mathbf{r}$ is the position vector of each element of volume of the grain relative to the center of mass of the grain, $\mathbf{R}$ is the position vector of the center of mass of the grain relative to the observer, $\mathbf{n}$ is the direction of observation, $\rho(\mathbf{r})$ is the charge density and $\mathbf{a}$ is the acceleration of each element of the volume of the grain.

Assuming that the center of mass of the grain remains stationary, the velocity of each element of the volume of the grain is

$$\mathbf{v} = \boldsymbol{\omega}\times\mathbf{r}, \tag{10}$$

with $\boldsymbol{\omega} = (0,0,\omega_0+\dot{\delta\phi})$ is the angular velocity vector of the grain; hence, the corresponding acceleration is

$$\mathbf{a} = \boldsymbol{\delta\alpha}\times\mathbf{r} + \boldsymbol{\omega}\times[\boldsymbol{\omega}\times\mathbf{r}], \tag{11}$$

where $\boldsymbol{\delta\alpha} = (0,0,\ddot{\delta\phi})$. The first term in (11) corresponds to a tangential acceleration $\mathbf{a}_{\rm t}$, whereas the second term describes a radial or centripetal acceleration $\mathbf{a}_{\rm r}$, which are mutually orthogonal.

Using simple textbook algebra, we can derive the following equality:

$$[\mathbf{n}\times[\mathbf{n}\times\mathbf{a}]] = \mathbf{a}_{\rm t} - (\mathbf{a}_{\rm t}\cdot\mathbf{n})\mathbf{n} + \mathbf{a}_{\rm r} - (\mathbf{a}_{\rm r}\cdot\mathbf{n})\mathbf{n}.$$

Integrating (9) over the volume of the grain and using the fact that $R \gg r$, we obtain

$$\mathbf{E}_{\rm rad}(\mathbf{r},t) \simeq \frac{d}{c^2|\mathbf{R}|}[\ddot{\delta\phi}(t)\mathbf{O}_{\mathbf{n}}[\mathbf{u}_{\rm t}] + \omega^2(t)\mathbf{O}_{\mathbf{n}}[\mathbf{u}_{\rm r}]], \tag{12}$$

where we have used the following definitions:

$$\mathbf{u}_{\rm t} \equiv (\cos(\phi),\sin(\phi),0), \tag{13}$$

$$\mathbf{u}_{\rm r} \equiv (-\sin(\phi),\cos(\phi),0), \tag{14}$$

$$\mathbf{O}_{\mathbf{n}}[\mathbf{x}] \equiv \mathbf{x} - (\mathbf{x}\cdot\mathbf{n})\mathbf{n}, \tag{15}$$

and where $\mathbf{d} = d^3r\rho(\mathbf{r})\mathbf{r}$ is the dipole moment of the grain.

Using the fact that $\omega^2 = \omega_0^2 + 2\omega_0\delta\phi + \delta\dot{\phi}^2$, yields

$$\mathbf{E}_{\rm rad}(\mathbf{r},t) \simeq \frac{d}{c^2|\mathbf{R}|}\left[\ddot{\delta\phi}(t)\mathbf{O}_{\mathbf{n}}[\mathbf{u}_{\rm t}] + \left[\omega_0^2 + 2\omega_0\dot{\delta\phi}(t) + \delta\dot{\phi}(t)^2\right]\mathbf{O}_{\mathbf{n}}[\mathbf{u}_{\rm r}]\right]. \tag{16}$$

Since the CMB has a low intensity, we will investigate the case of weak field amplitude, corresponding to small quivering frequencies and $\Omega_0 \ll \omega_0$, so that the dust-radiated electric field can be separated into three sets of spectral lines around ω_0. The first corresponds to the emission of the standard dipolar radiation with frequency ω_0 (remember that $\mathbf{u}_{\rm t}$ and $\mathbf{u}_{\rm r}$ rotate approximately with frequency ω_0) associated with the rotational motion of the grain:

$$\mathbf{E}_{\rm rad,1}(\mathbf{r},t) \simeq \frac{d}{c^2|\mathbf{R}|}\omega_0^2\mathbf{O}_{\mathbf{n}}[\mathbf{u}_{\rm r}] \propto \frac{d}{c^2|\mathbf{R}|}\omega_0^2 e^{i\omega_0 t}. \tag{17}$$

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The second set of spectral lines is associated with the quivering motion, resulting in a spectral broadening of the standard dipole radiation with the generation of two sidebands with frequencies $\omega_0 + \Omega_0$ (anti-Stokes) and $\omega_0 - \Omega_0$ (Stokes):

$$\mathbf{E}_{\mathrm{rad},2}(\mathbf{r},t) \simeq \frac{d}{c^2|\mathbf{R}|}[\delta\ddot{\phi}(t)\mathbf{O}_{\mathbf{n}}[\mathbf{u}_{\mathrm{t}}] + 2\omega_0\delta\dot{\phi}(t)\mathbf{O}_{\mathbf{n}}[\mathbf{u}_{\mathrm{r}}]] \tag{18}$$

$$\propto \frac{d}{c^2|\mathbf{R}|}[\Omega_0^2 + 2\omega_0\Omega_0]\arg\cos(\varepsilon/2)e^{i(\omega_0\pm\Omega_0)t}. \tag{19}$$

Finally, the third set of spectral lines is also associated with the quivering motion, but the spectral broadening produces essentially two sidebands with frequencies $\omega_0 + 2\Omega_0$ (double anti-Stokes) and $\omega_0 - 2\Omega_0$ (double Stokes):

$$\mathbf{E}_{\mathrm{rad},3}(\mathbf{r},t) \simeq \frac{d}{c^2|\mathbf{R}|}[\delta\dot{\phi}(t)^2\mathbf{O}_{\mathbf{n}}[\mathbf{u}_{\mathrm{r}}]] \tag{20}$$

$$\propto \frac{d}{c^2|\mathbf{R}|}[\Omega_0^2]\arg\cos^2(\varepsilon/2)e^{i(\omega_0\pm2\Omega_0)t}. \tag{21}$$

The instantaneous energy flux is given by the Poynting vector

$$\mathbf{S} = \frac{c}{4\pi}\mathbf{E}_{\mathrm{rad}} \times \mathbf{B}_{\mathrm{rad}} = \frac{c}{4\pi}|E_{\mathrm{rad}}|^2\mathbf{n}, \tag{22}$$

and the power radiated per unit solid angle can be written as

$$\begin{aligned}\frac{dP}{d\Omega} &= \frac{c}{4\pi}|\mathbf{R}|^2|E_{\mathrm{rad}}|^2 \\ &= \frac{d^2}{4\pi c^3}\{|\delta\ddot{\phi}(t)|^2\sin^2(\Theta_{\mathrm{t}}) + |\omega_0^2 + 2\omega_0\delta\dot{\phi}(t) + \delta\dot{\phi}(t)^2|^2\sin^2(\Theta_{\mathrm{r}}) \\ &\quad - 2\delta\ddot{\phi}[\omega_0^2 + 2\omega_0\delta\dot{\phi}(t) + \delta\dot{\phi}(t)^2]\cos(\Theta_{\mathrm{t}})\cos(\Theta_{\mathrm{r}})\},\end{aligned} \tag{23}$$

where Θ_{t} and Θ_{r} are respectively the angle between $\mathbf{u}_{\mathrm{t}}$ and $\mathbf{u}_{\mathrm{r}}$, and the direction of observation $\mathbf{n}$.

By separating the radiation into the two spectral components we recover the standard result for the power radiated per unit solid angle emitted by a dipole rotating with constant angular velocity ω_0,

$$\frac{dP_1}{d\Omega} = \frac{d^2}{4\pi c^3}\omega_0^4\sin^2(\Theta_{\mathrm{r}}), \tag{24}$$

and we identify the power radiated per unit solid angle owing to the quivering:

$$\begin{aligned}\frac{dP_2}{d\Omega} &= \frac{d^2}{4\pi c^3}\{|\delta\ddot{\phi}|^2\sin^2(\Theta_{\mathrm{t}}) + |2\omega_0\delta\dot{\phi}(t)|^2\sin^2(\Theta_{\mathrm{r}}) \\ &\quad - 4\omega_0\delta\ddot{\phi}\delta\dot{\phi}(t)\cos(\Theta_{\mathrm{t}})\cos(\Theta_{\mathrm{r}})\}\end{aligned} \tag{25}$$

and

$$\frac{dP_3}{d\Omega} = \frac{d^2}{4\pi c^3}|\delta\dot{\phi}(t)|^4\sin^2(\Theta_{\mathrm{r}}). \tag{26}$$

Since the quivering motion is much slower than the rotation of the grain, the average radiated power per unit solid angle over a period of rotation of the grain

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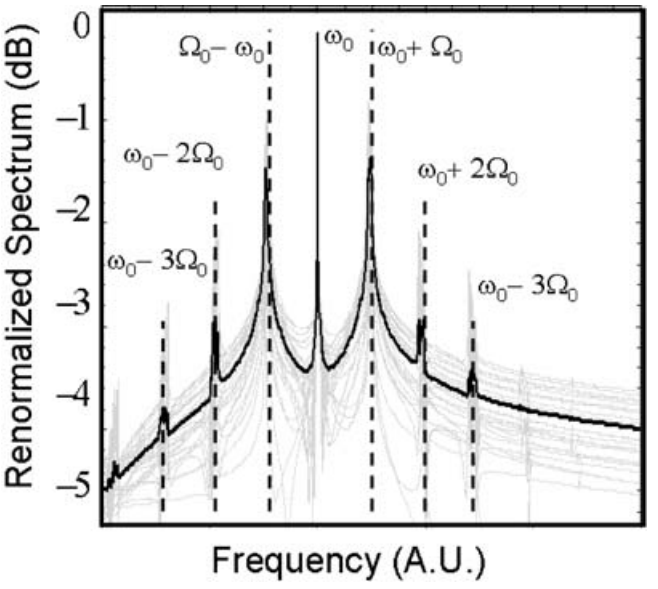


Figure 1. Scheme of the mean angular emission of a rotating dust grain: the emission is larger along the direction of the background magnetic field and null along the plane of rotation of the grain.

yields

$$\frac{d\langle P_1\rangle}{d\Omega} \approx \frac{d^2}{4\pi c^3}\omega_0^4[1-\cos^2(\Phi)], \tag{27}$$

$$\frac{d\langle P_2\rangle}{d\Omega} \approx \frac{d^2}{4\pi c^3}[\Omega_0^4+4\omega_0^2\Omega_0^2][1-\cos^2(\Phi)], \tag{28}$$

$$\frac{d\langle P_3\rangle}{d\Omega} \approx \frac{d^2}{4\pi c^3}\Omega_0^4[1-\cos^2(\Phi)], \tag{29}$$

where the theorem of equipartion of energy, $\langle\delta\phi(t)\rangle = \arg\cos(\varepsilon/2)$, has been used and Φ represents the angle between the direction of observation $\mathbf{n}$ and the plane of rotation of the grain, or the direction of the background magnetic field.

The angular emission distribution is represented in Fig. 1, showing that the emission is larger along the direction of the background magnetic field and null along the plane of rotation of the grain, which indicates that each dust grain behaves as a small probe sensing the local electromagnetic environment—both the frequency and intensity of the electromagnetic field as well as the direction of the background magnetic field—and imprinting them in the emitted dipolar radiation.

4. Simulation

The analysis of the dynamics of a single dust grain presented in the previous section has provided much insight into the dipolar emission spectrum in a dust cloud, enabling the identification of two regimes (for $\Omega_0 \gg \omega_0$ and $\Omega_0 \ll \omega_0$), as well as the dominant frequencies of the emitted spectrum. A more real model can be devised by averaging the emission spectrum over an ensemble of identical grains with the

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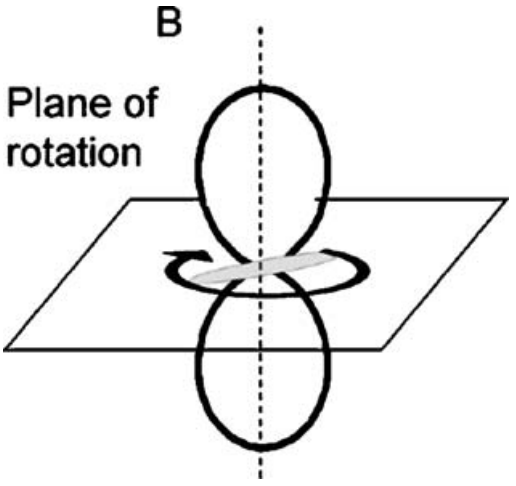


Figure 2. Emission spectrum of the grain: the quivering frequency Ω_0 is much smaller than the rotation frequency ω_0. Sidebands at $\omega_0 \pm \Omega_0$ and $\omega_0 \pm 2\Omega_0$ are clearly identifiable.

quivering temperature T:

$$I(\omega) = \int\limits_{[-1,1]} d\varepsilon \; n(\varepsilon) \langle I(\omega, \varepsilon) \rangle, \tag{30}$$

where $\langle I(\omega, \varepsilon) \rangle$ is the Gibbs average of the emission spectrum over the ensemble, $n(\varepsilon) \simeq n_0 \exp[-(\varepsilon + 1) I \Omega_0^2 / 2 k_{\rm B} T]$ the Boltzmann distribution for the probabilities of occupancy of the rotational energies in the large angular momentum limit [22] (notice that the quivering angular momentum of the grain is $\varepsilon I \Omega_0$) and $k_{\rm B}$ the Boltzmann constant. Since the grains are quasi-periodic systems, the Gibbs average $\langle I(\omega, \varepsilon) \rangle$ in (30) can be replaced by a time sampling according to the Ergodic hypothesis, which states that the evolution of a complex classical dynamical system takes it, with equal probability, through all states which are accessible from the starting point subject to the constrain of energy conservation [23, 24].

The emission spectrum of each dust grain has been simulated using a simple fourth-order Runge–Kutta integrator to numerically solve the following equation:

$$\frac{d^2 \delta\phi}{d\tau^2} = -\sin(\delta\phi), \tag{31}$$

where $\tau = \Omega_0 t$ is the natural time scale of the process. To sample over different energy configurations, a method based on the Metropolis algorithm [24] has been used, in which a random configuration of initial conditions is generated and the effective energy ε is calculated. Should the energy change relative to the previous configuration ε' be negative, the new configuration is automatically accepted; otherwise the new configuration is accepted with probability $\exp[(\varepsilon - \varepsilon') I \Omega_0^2 / 2 k_{\rm B} T]$.

Simulation results are shown in Figure 2 for both regimes, in which the quivering motion is assumed to be in equilibrium with the background at temperature $T =$ 3 K. Also, a cutoff of the effective energy at $\varepsilon = 0.5$ has been imposed, in order to maintain the results close to the domain of validity of the model. The emission pertaining to each single grain is shown in grey (which can be interpreted as the sampling fluctuations) and the mean radiated spectrum in black. In both regimes

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the thermal sampling leads to the expected broadening of all the emission spectral picks, although it is still possible to identify the predicted features.

Since all nonlinear terms of (4) have been included in the simulations, it is possible to detect the signature, although weak, of higher-order sidebands in the fluctuation spectrum (grey lines) at $3\omega_0$, $4\omega_0, \ldots$ for the low-intensity regime, corresponding to nonlinear wave mixing of the external EM wave through the coupling with the grain.

5. Conclusions

In summary, we have presented a model for the emission of radiation by elongated polarized dust grains under the influence of both an external EM wave and a constant background magnetic field. The emission spectrum depends on the wavelength of the external EM wave but—and most importantly—also on the amplitude, or intensity, of the external EM field, which is, in fact, the parameter that determines the frequencies of much of the components of the emitted radiation.

The emission spectrum of the grain exhibits a spectral line centered at the external EM field wavelength, or frequency, ω_0, and Raman-like sidebands at $\omega_0 \pm \Omega_0$ and $\omega_0 \pm 2\Omega_0$. We have also identified that the optimum direction of emission is parallel to the background magnetic field, rendering dust grains exceptionally useful for local probing of magnetic fields near far astrophysical objects.

Numerical simulations indicate that, as expected, thermal averaging of the spectrum, and the inclusion of the nonlinear dust dynamics, can lead to the broadening and smoothing of the emitted spectrum.

Since the amplitude and the frequency of the background EM field are independent parameters, this model allows the prediction of a correlation between different spectral components of galactic dipolar emission, which may explain the correlation between a component of the Galactic microwave emission and the 100 μm thermal emission from interstellar dust.

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