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BINDING MINIMUM WAGE AS AN EQUILIBRIUM SELECTION DEVICE

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This paper investigates the effects of a binding minimum wage in an economy which exhibits multiple unemployment equilibria. For this purpose, we develop a theoretical model based on the simple imperfectly competitive model of Manning [In Conference Papers, *Economic Journal* 100, 151–162 (1990)], in which we introduce labor heterogeneity and knowledge spillovers in the individual production technology. Then, using numerical simulations, we show that a binding minimum wage rules out the occurrence of an inefficient equilibrium. Last, we analyze the effects of a minimum wage increase on the labor market's outcomes.

Keywords: Multiple Equilibria, Binding Minimum Wage, Decentralized Wage Bargaining, Monopolistic Competition

1. INTRODUCTION

The minimum wage and its repeated increases have been actively discussed among economists. The primary goal of such government intervention is to improve the welfare of low-paid workers. However, a large debate about its employment effects exists. Indeed, two views confront each other. On one hand, the vast majority of economists think that an increase in the minimum wage leads to employment losses, especially for workers at the bottom of the wage distribution, such as young workers or low-skilled workers [(see Neumark and Wascher (2008) for a review of the main contributions on this topic)]. On the other hand, on the basis of econometric investigations, some economists claim that a higher minimum wage can have positive effects on employment [Card and Krueger (1995); Wolfson and Belman (2004)]. Most papers deal with the consequences for low-skilled workers or those who are at the bottom of the wage distribution, i.e., workers widely affected by such increases. Studies of the effects of a minimum wage on workers who earn more than the minimum wage and on total unemployment are less frequent in the literature.

From a theoretical point of view, two models are mainly used. The neoclassical model and its alternative specifications [Neumark and Wascher (2008)], in which wages are equal to the marginal productivity of labor, always conclude that there is a negative effect on low-wage workers employment. According to the nature

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of the production process, the employment effect on workers who earn more than the minimum wage can be either positive or negative. The monopsony model, in which firms have market power on wage determination,[1] sometimes concludes that there is a positive employment effect from a higher minimum wage [Machin and Manning (1994); Card and Krueger (1995); Dickens et al. (1999)]. In this kind of model, firms consent to pay higher wages in order to reduce quit rates and make recruitment easier. However, as mentioned by Cahuc et al. (2001), only a few papers rely on models using wage bargaining with trade unions, even though this form of wage determination appears as an important feature of European labor markets.

Furthermore, the sharp increase in European unemployment rates at the end of the seventies and their persistence at high levels have induced economists to explain the European unemployment experience as resulting from shifts between distinct unemployment equilibria [Manning (1990)]. Nowadays, the possibility of multiple unemployment equilibria in an economy is widely accepted. Multiple equilibria result from the presence of strategic complementarities between agents. In his book, Cooper (1999) reports and analyzes various sources of complementarities: external or internal returns to scale in the production technology, imperfect competition, and the way in which agents come together to trade (search and matching) or make their decisions (timing of choices). When multiple equilibria exist, a coordination problem can emerge. Agents can coordinate on an inefficient equilibrium although a better one exists (this situation is called coordination failure). In this case, an appropriate policy can be implemented to help agents to coordinate on the best equilibrium, or to eliminate inefficient equilibria [Cooper (2005)].

The purpose of the paper is to analyze the effects of a binding minimum wage and its increase on labor market outcomes in an economy with likely multiple equilibria and decentralized wage negotiation. Thus, we try to find whether a binding minimum wage can help to select an efficient equilibrium when multiple unemployment equilibria exist. If so, we make a significant contribution to the debate on the minimum wage by pointing out a new role for it. The model is derived from Manning's model (1990), to which we add two key assumptions. First, as we focus our analysis on the effects of a binding minimum wage, we assume the heterogeneity of labor. Therefore, we consider that the labor force employed by the firm consists of low-skilled workers who are paid the minimum wage and high-skilled workers who are paid a negotiated wage. The latter results from bilateral bargaining between the firm and a trade union, which is assumed not to be dominated by high-skilled workers. Next, we introduce the presence of knowledge spillovers in our economy. This assumption represents a convenient way to produce increasing returns to scale at the aggregate level despite having constant returns to scale at the firm level. In Manning's model, the possibility of multiple equilibria is due to increasing returns to scale at the firm level, although this assumption has no strong theoretical justification. However, for several years, knowledge has been considered as a fundamental source of increasing returns to

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scale (Romer, 1986; Lucas, 1988). Thus, knowledge spillovers give theoretical support to increasing returns to scale. As high-skilled labor is both the engine and the carrier of knowledge, we model knowledge spillovers thanks to the effect of average high-skilled employment on individual production efficiency.

The paper is organized as follows. In the next section, we present the model: the firm's price-setting behavior, the wage determination at the firm level, and the general symmetric equilibrium. Then we deal with the equilibrium issue. Numerical simulations help to identify three different cases: no equilibrium, a range of equilibria that can be reduced to a unique equilibrium, and two distinct equilibria. In the last section, we focus on the multiplicity case and show how a mandatory minimum wage rules out the occurrence of the inefficient equilibrium. We also run a comparative statics analysis to investigate the effects of a minimum wage hike on the labor market's outcomes. Briefly, the contribution of our findings is twofold. First, they confirm the neoclassical view according to which an increase in the minimum wage always has a negative effect on the total unemployment, whereas its effect on the other wages can be either positive or negative under alternative circumstances. Nonetheless, our findings also give support to a welfare-enhancing effect of a binding minimum wage when multiple equilibria exist.

2. THE MODEL

We use Manning's model (1990), broadly based on Layard–Nickell's model (1985, 1986), in which we introduce the heterogeneity of labor and the presence of knowledge spillovers. The sequence of events is as follows: First, the firms fix prices in order to maximize their profits, and their demand for labor is deduced from this price-setting behavior. Then a wage bargaining game begins between the firm and a trade union that takes care for the welfare of all workers in the firm. Finally, when the two parties agree, production takes place.

2.1. The Firm's Behavior

We assume monopolistic competition on the goods market between a continuum of identical imperfectly competitive firms uniformly distributed on the interval [0, 1]. Each produces a single good that is imperfectly substitutable with others. The firm i has a production technology of the form[2]

$$y_i = A n_{1i}^{\alpha} n_{2i}^{\beta} \qquad \text{with} \qquad \alpha + \beta = 1, \tag{1}$$

where n_{1i} represents its employment of high-skilled labor, n_{2i} its employment of low-skilled labor, and A a labor efficiency parameter. Because $\alpha + \beta = 1$, we have constant returns to scale at the firm level. A is assumed to be a function of the average employment level of high-skilled labor in the economy n_1 as follows:

$$A = n_1^{\sigma}. \tag{2}$$

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Thus, knowledge spillovers influence the firm's production efficiency through that parameter A. $\sigma > 0$ represents the size of knowledge spillovers; that is, σ is a measure of the externality degree. It is usual to consider knowledge as a source of new technologies and as mainly produced and spread by high-skilled labor [Lucas (1988)], so our modeling assumption seems relevant. In this way, the more the economy employs high-skilled labor, the more the labor employed by one firm, and so its production, is efficient. However, a firm is assumed to be too small to have any influence on the aggregate state of our economy, so that it takes as given the parameter A. In other words, the positive externality of high-skilled employment is not internalized by firms that gain advantages from the other firms' high-skilled employment without having to pay for it.

Monopolistic competition and knowledge spillovers represent two sources of strategic complementarities in our economy. On one hand, the production decision of each firm has a positive effect on the other firms' production level through the aggregate demand channel (a consequence of the monopolistic competition). On the other hand, involving more high-skilled labor employment, it also has a positive effect on the other firms' production efficiency through the knowledge spillovers channel. These complementarities between firms make possible the existence of multiple unemployment equilibria in our economy.[3] Focusing on the effects of a binding minimum wage on the aggregate state of the economy, we need to compute the optimal share of each kind of labor in the firm's labor demand according to the wage dispersion.

Optimal shares of high-skilled and low-skilled labor in the firm's employment. The total employment of firm i is equal to the sum of its employment of high-skilled labor n_{1i} and low-skilled labor n_{2i}:

$$n_i = n_{1i} + n_{2i}. \tag{3}$$

Let $\gamma_i = n_{2i}/n_i \in]0; 1[$ be the share of low-skilled labor employment in its total employment and $[1 - \gamma_i] = n_{1i}/n_i \in]0; 1[$ the share of high-skilled labor in its total employment. Let $\theta_i = \frac{\underline{w}}{w_{1i}} \in]0; 1[$ be a measure of the wage dispersion between high-skilled and low-skilled labor, where $\underline{w}$ represents the real minimum wage that is earned by all low-skilled workers and is strictly inferior to the real wage earned by high-skilled workers w_{1i}. Thus, the lower θ_i, the larger the gap between low-skilled workers' wage and high-skilled workers' wage. The real total wage cost of the firm i can be written as

$$w_i n_i = w_{1i} n_{1i} + \underline{w} n_{2i}. \tag{4}$$

Thus, the firm i chooses the relative share of each labor type that minimizes its wage cost under the constraint of its production technology and given the wage

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levels. It solves the following program:

$$\begin{aligned} \min_{n_{1i},n_{2i}} \; & w_{1i}n_{1i} + \underline{w}n_{2i} \\ \text{s.t.} \quad & y_i = An_{1i}^{\alpha}n_{2i}^{\beta} \\ & \underline{w} < w_{1i} \text{ given.} \end{aligned} \tag{5}$$

From the first-order conditions[4], we deduce the relative share of each labor type as a function of the wage dispersion, θ_i,[5] and the features of the production technology, α and β:

$$\frac{n_{1i}}{n_{2i}} = \frac{\alpha}{\beta}\frac{\underline{w}}{w_{1i}} = \frac{\alpha}{\beta}\theta_i. \tag{6}$$

Given (3), we infer the share of each labor type in the total employment of the firm:

$$\gamma_i(\theta_i) = \frac{n_{2i}}{n_i} = \frac{\beta w_{1i}}{\alpha\underline{w} + \beta w_{1i}} = \frac{\beta}{\alpha\theta_i + \beta}, \tag{7}$$

$$[1 - \gamma_i(\theta_i)] = \frac{n_{1i}}{n_i} = \frac{\alpha\underline{w}}{\alpha\underline{w} + \beta w_{1i}} = \frac{\alpha\theta_i}{\alpha\theta_i + \beta}. \tag{8}$$

Consequently, we can rewrite the production technology of the firm and its real total wage cost as functions of total employment n_i:

$$y_i(n_i, \theta_i) = AB(\theta_i)n_i \qquad \text{with} \qquad B(\theta_i) = [1 - \gamma_i(\theta_i)]^{\alpha}\gamma_i(\theta_i)^{\beta} \tag{9}$$

$$\begin{aligned} w_i n_i &= w_{1i}n_{1i} + \underline{w}n_{2i} = w_{1i}[1 - \gamma_i(\theta_i)]n_i + \underline{w}\gamma_i(\theta_i)n_i \qquad (10) \\ &= w_{1i}[1 - \gamma_i(\theta_i)]n_i + \theta_i w_{1i}\gamma_i(\theta_i)n_i \\ &= \frac{(\alpha + \beta)\theta_i}{\alpha\theta_i + \beta}w_{1i}n_i = \frac{\theta_i}{\alpha\theta_i + \beta}w_{1i}n_i \\ &= C(\theta_i)w_{1i}n_i, \qquad (11) \end{aligned}$$

with $C(\theta_i) \in]0; 1[$ and where $w_i = C(\theta_i)w_{1i}$ represents a wage index paid by the firm to its workers.

Price-setting behavior and labor demand. Monopolistic competition on the good market implies that the demand for the firm i's output is given by[6]

$$y_i^d = Y(p_i)^{-s}, \text{ with } s > 1, \tag{12}$$

where Y is the total aggregate demand, p_i the firm i's relative price, and s the substitution elasticity between differentiated goods.

The real profit of the firm is equal to the return from the sold output minus the total wage cost:

$$\pi_i(w_{1i}, n_i) = p_i y_i(n_i, \theta_i) - C(\theta_i)w_{1i}n_i. \tag{13}$$

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Thus, each firm i chooses p_i in order to maximize its real profit. We assume that it takes the aggregate state of the economy and real wages as given, and produces exactly the demanded output. Thus, it solves the following program:

$$\begin{aligned} \max_{p_i} \quad & \pi_i(w_{1i}, n_i) \\ \text{s.t.} \quad & y_i = AB(\theta_i)n_i \\ & y_i^d = Y\,(p_i)^{-s} \\ & y_i = y_i^d \\ w_{1i} \quad & \text{and} \quad \theta_i = \frac{\underline{w}}{w_{1i}} \in]0;\,1[\quad \text{given.} \end{aligned} \tag{14}$$

After substitution, the first-order condition gives the following partial-equilibrium pricing equation:[7]

$$p_i^* = \left(\frac{s}{s-1}\right)\frac{C(\theta_i)}{AB(\theta_i)}w_{1i}. \tag{15}$$

This shows that the price fixed by the firm is an increasing function of the wage cost, i.e., of the wage index $C(\theta_i)w_{1i}$. Then, from the equations (9), (12), and (15), we deduce the optimal level of total employment for the firm i, which can be interpreted as its total demand for labor:

$$n_i^* = \left[\left(\frac{s}{s-1}\right)C(\theta_i)w_{1i}\right]^{-s}[AB(\theta_i)]^{s-1}\,Y. \tag{16}$$

This has the common features found in the literature: namely, it is increasing in the total aggregate demand Y and decreasing in the wage cost.[8]

2.2. Wage Determination

The wage bargaining is decentralized and takes place between the firm and a trade union. We assume a "right-to-manage" model, so the wage is the only variable that is negotiated and the employment level is unilaterally fixed by the firm. Furthermore, all low-skilled workers earn the minimum wage, which is exogenously fixed by law and not negotiated between firms and unions. As a consequence, the bargaining concerns the high-skilled real wage only.

We assume that the trade union has no insider behavior and is not dominated by high-skilled workers. As a result, it gives as much importance to the employment level as to the wage level during the bargaining[9] and cares about not only the welfare of high-skilled workers, but also that of low-skilled workers. When the welfare of all workers is weighted equally, the utility function of the trade union in the firm i is given by

$$\begin{aligned} V\,(w_i, n_i) &= (w_i - w_R)\,n_i \\ \leftrightarrow V\,(w_{1i}, n_i) &= [C(\theta_i)w_{1i} - w_R]\,n_i, \end{aligned} \tag{17}$$

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where w_R represents the real reservation wage, which is exogenous at the firm level. This reservation wage represents the alternative utility of workers when the bargaining leads to no agreement. The firm's utility during the bargaining is given by its real profits.

The high-skilled real wage is negotiated in order to solve the following Nash bargaining program:

$$\max_{w_{1i}} \quad \{\pi_i(n_i, \theta_i)\}^{(1-\rho)} \{[C(\theta_i) w_{1i} - w_R] n_i\}^{\rho} \qquad \textbf{(18)}$$
$$\text{s.t.} \quad n_i = n_i^* \quad \text{and} \quad \theta_i = \theta_i(w_{1i})$$
$$w_R \quad \text{given},$$

where $\rho \in]0; 1[$ and $(1-\rho) \in]0; 1[$ represent the bargaining power of the trade union and that of the firm, respectively. The constraint $n_i = n_i^*$ reflects that the level of employment is unilaterally chosen by the firm. The first-order condition gives the high-skilled real wage that results from the negotiation:[10]

$$w_{1i}^* = \frac{\chi(\rho, s, \theta_i)}{C(\theta_i)} w_R \leftrightarrow C(\theta_i) w_{1i}^* = \chi(\rho, s, \theta_i) w_R \qquad \textbf{(19)}$$

$$\text{with} \quad \chi(\rho, s, \theta_i) = \left(\frac{\rho}{s-1}\right) \frac{[1 - \gamma_i(\theta_i)]}{\alpha} + 1 \geq 1. \qquad \textbf{(20)}$$

$\chi(\rho, s, \theta_i)$ is the markup of the wage index over the reservation wage. This partial equilibrium wage equation also is consistent with the existing literature and shows that the wage index paid by the firm to its workers $C(\theta_i) w_{1i}$ is marked up over the reservation wage with a markup greater than one and increasing in the bargaining power of the union, ρ. Furthermore, we can also conclude from equations (19) and (20) that the high-skilled real wage is also marked up over the reservation wage, with a markup superior to that of the wage index.[11] Now, we turn to the general equilibrium of our economy.

2.3. The General Equilibrium

At the general symmetric equilibrium, all firms and trade unions are identical and make the same decisions. As a consequence, we have the following equalities: $p_i = 1$[12], $w_{1i} = w_1, n_{1i} = n_1, n_{2i} = n_2, n_i = n$, and we can write

$$AB(\theta) = [1 - \gamma(\theta)]^{\alpha+\sigma} \gamma(\theta)^{\beta} n^{\sigma}. \qquad \textbf{(21)}$$

As the labor force consists of a continuum of workers of size normalized to one, the unemployment rate of our economy is given by $u = 1 - n$. As we have a continuum of firms uniformly distributed on the interval [0, 1], the aggregate production technology is now given by

$$Y = [1 - \gamma(\theta)]^{\alpha+\sigma} \gamma(\theta)^{\beta} n^{(1+\sigma)}. \qquad \textbf{(22)}$$

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Thus, at the aggregate level, we have increasing returns to scale of labor (hereafter referred to as IRS) because of the presence of knowledge spillovers ($1 + \sigma \geqslant 1$). Note that their extent depends on the knowledge spillovers' size σ. These IRS are external to the firm[13] and correspond to social IRS. Consequently, the assumption of knowledge spillovers allows IRS at the aggregate level although there are constant returns to scale at the firm level. In our economy, social IRS reinforce the strategic complementarities between firms already introduced by monopolistic competition.

The insertion of (21) and (22) into (16) yields the price-setting (PS) curve, which implicitly relates the real wage of high-skilled workers to the unemployment rate and other parameters[14]:

$$(\text{PS}) : w_1 = \frac{[1 - \gamma(\theta)]^{\alpha+\sigma} \gamma(\theta)^{\beta}}{\frac{s}{s-1} C(\theta)} (1-u)^{\sigma}. \tag{23}$$

To determine the wage-setting (WS) curve, we need to model the reservation wage, which is exogenous at the firm level. Like Manning (1990), we use the following specification:

$$\begin{aligned} w_R &= uB + (1-u)w \\ &= uB + (1-u)C(\theta)w_1, \end{aligned} \tag{24}$$

where u is the unemployment rate, B the real unemployment benefits, and w the real wage index paid by firms to their workers. The unemployment benefits are assumed to be identical for all unemployed workers whatever their skill level, and lower than the minimum wage. Such an unemployment benefits system is not a general feature of labor markets but corresponds to that of the United Kingdom. Indeed, in the United Kingdom, unemployment benefits are fixed, do not depend on the previous wage level, and are inferior to the minimum wage. We also assume that the high-skilled workers can be employed for high-skilled jobs only (no downgrading). Thus, the reservation wage is an average of the workers' earnings from employment and unemployment weighted according to the unemployment rate. We introduce (24) into (19) and we obtain the following (WS) curve, which implicitly relates the high-skilled real wage to the unemployment rate and other parameters:

$$(\text{WS}) : w_1 = \frac{\chi(\rho, s, \theta) Bu}{C(\theta)[1 - \chi(\rho, s, \theta)(1-u)]}. \tag{25}$$

We note that (WS) admits a vertical asymptote $\tilde{u}$ in the space (u, w_1) such that $\tilde{u} = \frac{\chi(\rho,s,\theta)-1}{\chi(\rho,s,\theta)} < 1$. This asymptote implies an inferior bound for the definition interval of the unemployment rate in our economy, which leads to the restricted definition interval $]\tilde{u}, 1[$. The lower bound $\tilde{u}$ represents the minimum unemployment rate that the economy can achieve given the bargaining power of trade unions and the degree of competition in the goods market. Indeed, if firms set wages by themselves

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($\rho = 0$) or if there is perfect competition in the goods market ($s \to \infty$), the wage index markup over the reservation wage is equal to one and the minimum likely unemployment rate is null. However, we must keep in mind that high-skilled workers always earn more than the reservation wage even if $\chi(\rho, s, \theta) = 1$.[15]

3. MULTIPLE EQUILIBRIA

To deal with the existence of multiple equilibria in the economy, we first analyze the properties of the (PS) and (WS) curves. Then we verify the existence of multiple equilibria and characterize them thanks to numerical simulations. Indeed, given that both curves correspond to complex implicit functions, we use numerical simulations to determine their shape in the space (u, w_1) and the likely equilibrium cases.

First and foremost, we have to define what an equilibrium means here. In our model, an equilibrium corresponds to any pair of variables (u, w_1) for which the optimal behavior of firms and of unions are consistent with each other. In other words, it gives the total unemployment rate and the high-skilled real wage for which the wage aspirations of unions and the willingness to pay of firms are consistent, given the labor demand of firms and the minimum wage in force.

3.1. Curves' Properties

For both general equilibrium equations, we analytically determine the sign of their first derivative only and, thus, whether they are increasing or decreasing in the space (u, w_1). Next, we need to run numerical simulations to pronounce on their concavity or convexity.

The first derivatives of (PS) and (WS) allow stating the following proposition:

PROPOSITION 1. *As* $\chi(\rho, s, \theta) > 1$ *and* $u \in]\tilde{u}, 1[$, *the (PS) and (WS) curves are always downward-sloping in the space* (u, w_1).

Proof. We show in Appendix E that the first derivatives of (PS) and (WS) are always negative when $\chi(\rho, s, \theta) > 1$ and $u \in]\tilde{u}, 1[$. ■

To give an economic interpretation of a downward-sloping (PS) curve, we need to understand the consequences of knowledge spillovers and social IRS on the firms' behavior. When a firm increases its production, it increases its employment level, which often results in an increase of its high-skilled employment level. This behavior leads to a rise in the aggregate production and aggregate income, which involves an increase in the aggregate demand. Consequently, the other firms are incited to increase their own output (demand externality resulting from monopolistic competition) and, in turn, to increase their own high-skilled employment level. Finally, that rise makes the efficiency of labor used by each firm higher (knowledge spillovers/IRS effect). With more efficient labor, firms produce more output with a given employment level, which implies a higher labor productivity. Thus, the

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higher the employment level, the more firms are willing to pay a higher wage. That is why the (PS) curve is downward-sloping in the space (u, w_1). Concerning the (WS) curve, its economic interpretation is as usual. A higher unemployment rate decreases the reservation wage, given the lack of outside options. As a consequence, the high-skilled real wage resulting from bargaining is decreasing in the unemployment rate.[16]

Limits computations allow the conclusion that

$$\text{(PS)} : \lim_{u \to \tilde{u}^+} w_1 = cste > 0 \quad \text{and} \quad \lim_{u \to 1^-} w_1 = 0,$$
$$\text{(WS)} : \lim_{u \to \tilde{u}^+} w_1 = +\infty \quad \text{and} \quad \lim_{u \to 1^-} w_1 = cste > 0.$$

The first limit of (PS) can be interpreted as the maximum real wage that firms consent to pay if the economy achieves its minimum unemployment rate. The second one clearly shows that if all workers are unemployed, there is no production process and so a null wage. Note that the limit of (WS) when u tends toward $\tilde{u}^+$ highlights the vertical asymptote of the curve.

Numerical simulations allow stating the following proposition about the shape of (PS) and (WS).

PROPOSITION 2. *The (PS) curve is strictly concave when $\sigma < 1$, whatever the values of other parameters, and becomes concave–convex when $\sigma \geqslant 1$. The (WS) curve is always convex.*

Proof. See Appendix F. ■

Thus, the (PS) curve can be either concave or concave–convex according to the extent of social IRS and the (WS) curve is always convex. Consequently, as both are downward-sloping, they can cut themselves once, twice, or never in the interval $]\tilde{u}, 1[$. At curve intersections, the optimal behavior of firms and of trade unions are consistent with each other, so an intersection gives an equilibrium unemployment rate and an equilibrium high-skilled real wage given the minimum wage which prevails in the economy. We discuss the different possible cases in the next subsection.

3.2. Equilibrium Cases

In accordance with the previous properties of the curves and the numerical simulations presented in Appendix F, we can state the following:

PROPOSITION 3. *According to the parameters' values, we have either non equilibrium or a range of equilibria, which can be reduced to one equilibrium or two distinct equilibria.*

Proof. See Appendix F. ■

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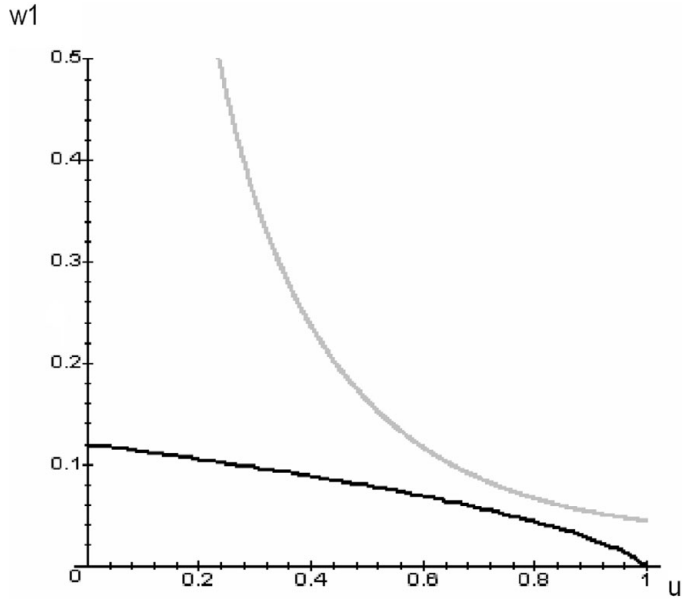


FIGURE 1. No equilibrium.

Thus, we can find all equilibrium possibilities in our model. As we show in Appendix F, the presence and number of equilibria mainly depend on the degree of competition on the goods market s. Indeed, when the product market is too monopolistic, no equilibrium exists. Afterwards, if s increases (namely, if the product market becomes more competitive), one equilibrium appears, followed by a range of equilibria and finally two distinct equilibria. These different cases are illustrated in Figures 1–3, where the black curves and the gray curves represent respectively the (PS) and the (WS) curves given by the implicit functions (23) and (25).

In what follows we focus our analysis on the case of multiple equilibria (Figure 3), which seems the most relevant according to the value of s. Indeed, multiple equilibria exist as soon as the degree competition on the goods market is slightly superior to one (see Appendix F), which reflects weak competition between firms. When there are two distinct equilibria, we can characterize them as follows: one named the "low equilibrium" with a high unemployment rate and a low high-skilled real wage (u_L, w_{1L}) and another named the "high equilibrium" with a low unemployment rate and a high high-skilled real wage (u_H, w_{1H}).

4. BINDING MINIMUM WAGE

In this section, we first investigate the implications of the implementation of a binding minimum wage and then the effects of its increase on the economy's

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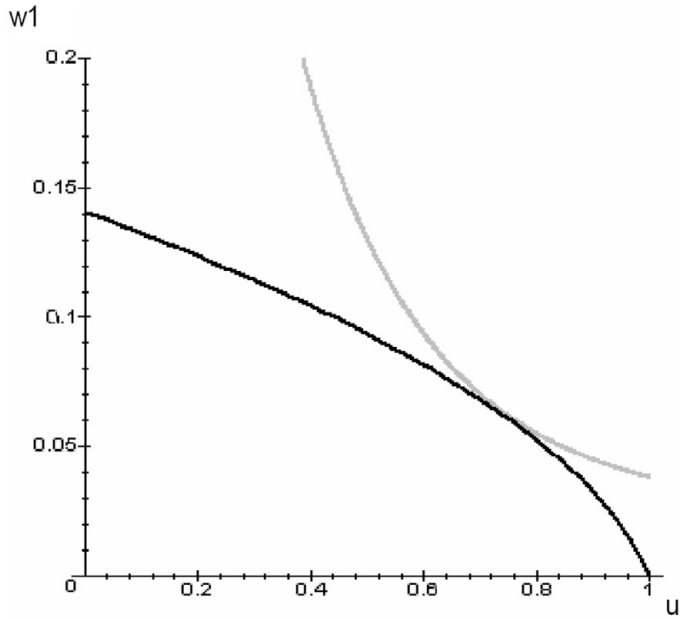


FIGURE 2. Range of equilibria.

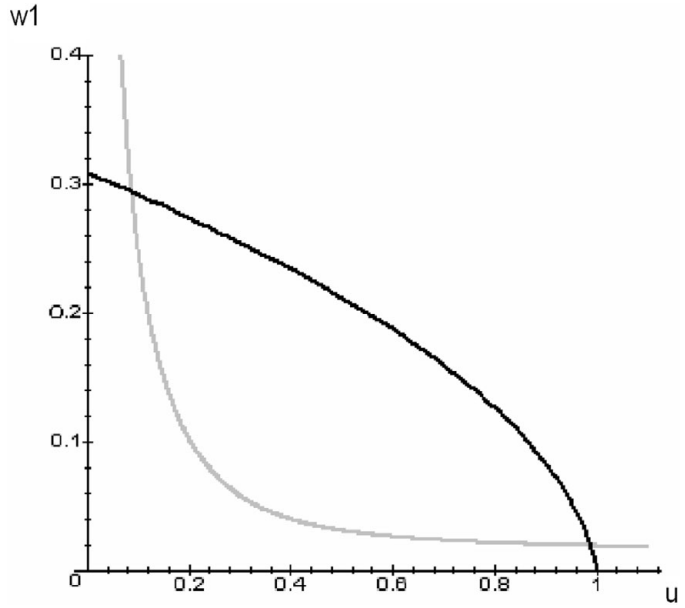


FIGURE 3. Two distinct equilibria.

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outcomes. Because of the complexity of our general equilibrium equations, we have to run numerical simulations to analyze our economy. Consequently, with a view to covering most possible cases, we run numerical simulations on ranges of values for each parameter (see Table F.1 in Appendix F) and set the real minimum wage at various levels subject to $\underline{w} > B$.

4.1. Equilibrium Selection Device

Thanks to our extensive numerical exploration, we can state with confidence that the low equilibrium always displays a high-skilled real wage inferior to the minimum wage,[17] which is in contradiction to the constraint $w_{1i} > \underline{w}$. Consequently, we establish the following proposition.

PROPOSITION 4. *When the economy exhibits two distinct equilibria, the low equilibrium never occurs with a binding minimum wage and the high equilibrium is the only feasible equilibrium in the economy.*

Proof. See Appendix F. ■

Thus, even if our economy exhibits two equilibria, the imposition of a minimum wage makes the occurrence of the low equilibrium unlikely. Consequently, a binding minimum wage appears as an adequate device to single out the Pareto-superior equilibrium when multiple equilibria exist.[18] This results from the link that exists between the aggregate demand level and the firms' production and employment decision. In our economy, the imposition of a minimum wage makes certain a minimum level of aggregate demand whatever the share of low-skilled and high-skilled workers. Firms face a minimum demand level for their good and employ enough to produce it. As a consequence, the low equilibrium with a high unemployment rate cannot occur. In the reverse case, because of the strategic complementarities between firms, the absence of a binding minimum wage can lead the economy to the low equilibrium if firms expect a low aggregate demand level.

Van der Berg (2003) also shows both a welfare-enhancing effect and a selection device feature of a binding minimum wage in an economy that exhibits multiple equilibria. He analyzes an economy with search frictions and heterogenous production technologies in which multiple equilibria exist because of mutual dependence in the search behavior of unemployed workers and low-productivity firms. In such an economy, a binding minimum wage may improve social welfare by eliminating the equilibrium at which low-productivity firms are active.

4.2. Minimum Wage Hike

As the low equilibrium is ruled out by a binding minimum wage, we focus our comparative statics analysis of a minimum wage increase on the high equilibrium. Thanks to our numerical simulations (see Table F.2 in Appendix F), we find that the effect on the unemployment rate is always the same: it rises. However, the

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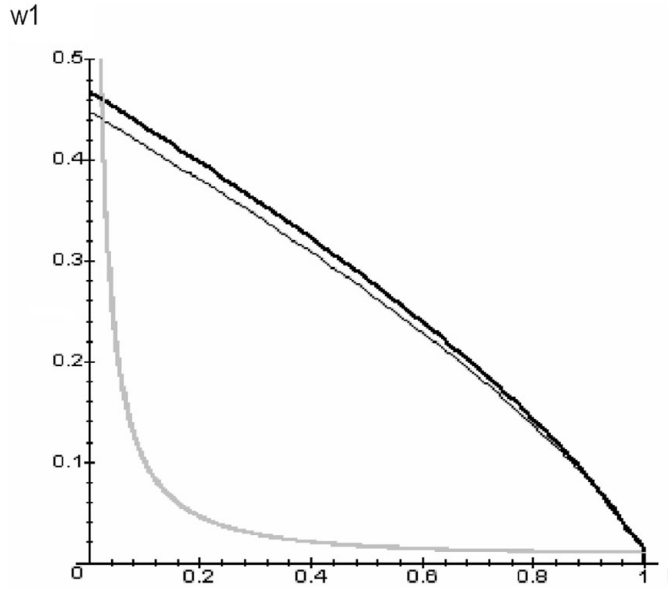


FIGURE 4. Lower high-skilled real wage.

effect on the high-skilled real wage is either positive or negative according to some parameters' values.

PROPOSITION 5. *An increase in the real minimum wage implies both a higher unemployment rate and a higher high-skilled real wage if and only if* $\gamma(\theta) > \dfrac{\beta}{\sigma}$.

Proof. Numerical evidence and the analytical proof of the second statement are given in Appendix F. ■

Figures 4–7[19] present the two likely behaviors of our economy following an increase of 10% in the real minimum wage.

As we can see in Figures 5 and 7, an increase in $\underline{w}$ always shifts the (WS) curve to the right. However, the (PS) curve shifts either to the right when $\gamma(\theta) > \beta/\sigma$ (see Figures 6 and 7) or to the left when $\gamma(\theta) < \beta/\sigma$ (see Figures 4 and 5).

Before economic interpretations of these findings are given, it seems useful to outline more general effects of an increase in the minimum wage in our economy. According to the numerical investigation,[20] we establish the following statements:

(i) The greater the share of low-skilled labor employment, the more the unemployment rate increases.
(ii) The wage dispersion is always reduced (i.e., θ increases) even if the high-skilled real wage increases.

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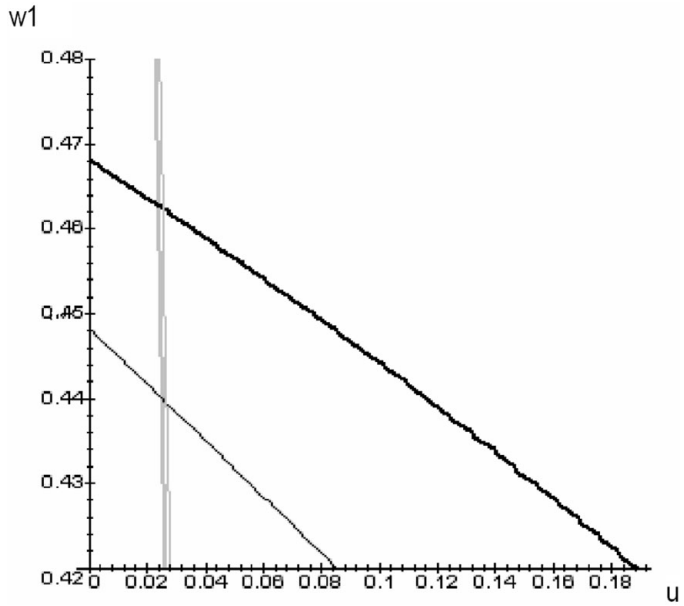


FIGURE 5. Zoom on the high equilibrium.

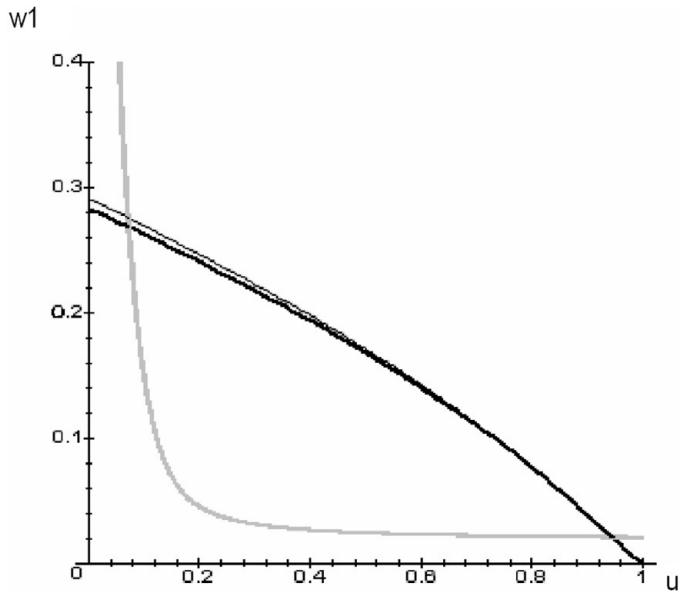


FIGURE 6. Higher high-skilled real wage.

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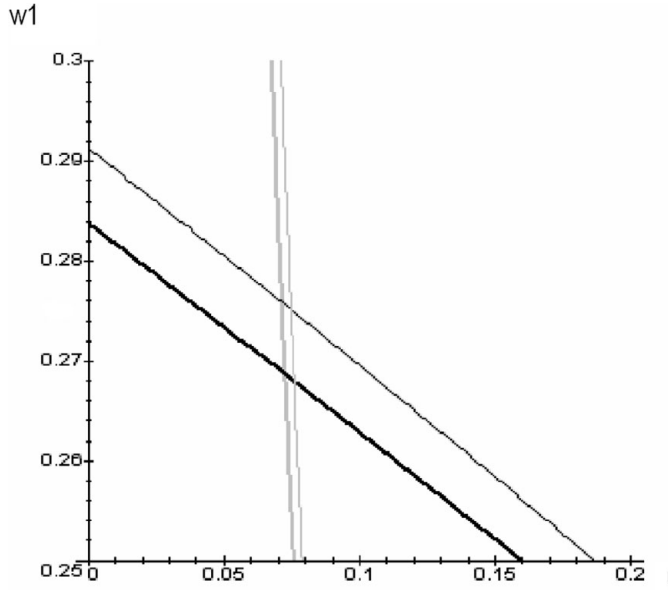


FIGURE 7. Zoom on the high equilibrium.

(iii) The share of low-skilled labor employment $\gamma(\theta)$ diminishes and that of high-skilled labor employment $[1 - \gamma(\theta)]$ increases (substitution effect in favor of high-skilled workers).
(iv) The more firms use high-skilled labor, the smaller its share in total employment increases and the more that of low-skilled labor decreases.
(v) The more firms use low-skilled labor, the smaller its share in total employment decreases and the more that of high-skilled labor increases.

Minimum wage hikes have an effect on both the price-setting and wage-setting behaviors. The effect on wage-setting behavior is always the same. By increasing the share of high-skilled labor (substitution effect), a higher minimum wage raises the markup over the wage index[21] obtained by unions as well as the reservation wage, which implies a shift of the (WS) curve to the right [see equation (25)]. In other words, trade unions claim higher wages for a given unemployment level.

The effect on the price-setting behavior depends on the firms' production technology and on the social IRS extent. Firms are willing to pay higher high-skilled wages following an increase in the minimum wage if and only if they use high-skilled production technologies when social IRS are weak (as in the case of low β and σ) or if the social IRS are great enough to offset the use of low-skilled technologies (as in the case of high β and σ).[22] In the first case, the effect of a minimum wage hike on the wage bill is so weak that firms are willing to pay a higher high-skilled wage because of extra profits from social IRS.[23] In the second one, the extent of social IRS is so great, that the additional cost from a higher

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minimum wage is offset. The reverse reasoning can be made concerning the case plotted in Figures 4 and 5, i.e., the case of a lower high-skilled wage. In this case, a higher minimum wage is too costly for firms to dispose them to pay higher wages to high-skilled workers[24] because they use too much low-skilled labor compared to the extent of social IRS. Note that in the absence of social IRS[25] the high-skilled real wage always decreases following a minimum wage hike, whatever the production technology.

5. CONCLUSION

In this paper we develop an imperfectly competitive model with social increasing returns to scale in which multiple equilibria can occur. Then we show how a binding minimum wage rules out the occurrence of the inefficient equilibrium that exhibits a high unemployment rate. Furthermore, we analyze the effect of minimum wage hikes on the total unemployment level and show how they make it worse. We also show that the effect on the wages of workers who earn more than the minimum wage depends on the production technology's features compared with the extent of social IRS. Finally, our numerical simulations allow distinguishing between the effects on the employment levels of high-skilled and low-skilled workers. We particularly show that an increase in the minimum wage always induces a substitution effect at the expense of minimum wage earners, whatever the change in the high-skilled real wage. However, the disemployment effect on low-skilled labor is less strong when the economy uses low-skilled production technology.

In brief, although our findings do not support the positive employment effect of a higher minimum wage brought to light in the monopsony model, they show how a binding minimum wage avoids the occurrence of an inefficient equilibrium when multiple equilibria exist. In this way, our work gives a convincing argument for the implementation of a binding minimum wage in an economy, but mixed ones for its repeated increases.

NOTES

1. Machin and Manning (1994, p. 326). "A monopsonistic labor market is one in which an employer possesses some market power in setting wages, so that the supply of labor to the firm is a positive function of the wage paid."
2. As in Manning's model, the number of firms is assumed fixed and the capital is ignored.
3. The presence of strategic complementarities is a necessary (but not sufficient) condition for multiple equilibria existence [Cooper (1999)].
4. See Appendix A.
5. We must keep in mind that the measure of the wage dispersion is always a function of the high-skilled real wage, i.e., $\theta_i\ (w_{1i})$. However, for convenience, we will write it as θ_i in what follows.
6. This specification of the demand function is derived from CES specification of preferences [see Blanchard and Kiyotaki (1987) and Julien and Sanz (2007)].
7. See Appendix B.
8. Indeed, we have $\partial n_i^* / \partial [C(\theta_i) w_{1i}] < 0$.
9. Here, the union's preferences correspond to the utilitarian model of Oswald (1982).
10. See Appendix C for detailed calculations.

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11. Indeed, as $C(\theta_i) \in]0, 1[$, the markup of the real high-skilled wage over the real reservation wage is superior to that of the real wage index: $\chi(\rho, s, \theta_i)/C(\theta_i) > \chi(\rho, s, \theta_i)$.

12. As all firms fix the same price, the individual price is equal to the general price and the relative price to one.

13. Keep in mind that each firm is assumed to take the decisions of other firms as given and to have no influence on the aggregate state of the economy.

14. Indeed, (PS) only gives an implicit expression for the high-skilled real wage, given that the parameter θ is a function of w_1.

15. Indeed, the markup of the high-skilled wage over the reservation wage is equal to $\chi(\rho, s, \theta)/C(\theta)$, which is always superior to 1 even if $\chi(\rho, s, \theta) = 1$.

16. This reasoning remains true whatever the extent of knowledge spillovers in our economy.

17. See Appendix F.

18. Although no welfare analysis is carried out, we can intuitively conclude that the high equilibrium is Pareto-superior to the low equilibrium in our economy.

19. Figures 4 and 5 illustrate the case A(1) of Table F.2 in Appendix F and, Figures 6 and 7 the case B(1).

20. See Table F.2 in Appendix F.

21. $\chi(\rho, s, \theta)$ increases when $(1 - \gamma(\theta))$ increases.

22. In this case the (PS) curve moves to the right.

23. Indeed, because of the substitution effect, a higher minimum wage involves a more efficient production technology for each firm and a higher labor productivity in the aggregate state.

24. In this case, the (PS) curve moves to the left.

25. In this case $\sigma \to 0$, which implies the unrealistic constraint $\gamma(\theta) > +\infty$ for a positive effect of a higher minimum wage on the high-skilled wage (see Appendix F).

26. The other parameters' values are the same than those of the previous plots with $\sigma = 0.5$.

27. Other numerical simulations have been run but are not given in the paper.

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APPENDIX A: OPTIMAL RELATIVE SHARE OF HIGH-SKILLED AND LOW-SKILLED LABOR

The firm i solves the following program:

$$\begin{aligned} \min_{n_{1i},n_{2i}} \; & w_{1i}n_{1i} + \underline{w}n_{2i} \\ \text{s.t.} \quad & y_i = An_{1i}^{\alpha}n_{2i}^{\beta} \\ & \underline{w} < w_{1i} \text{ given.} \end{aligned}$$

The Lagrangian of this program is given by

$$L = -w_{1i}n_{1i} - \underline{w}n_{2i} - \lambda\left[y_i - An_{1i}^{\alpha}n_{2i}^{\beta}\right].$$

Thus, we have

$$\begin{aligned} \frac{\partial L}{\partial n_{1i}} = 0 &\Longleftrightarrow w_{1i} = \frac{\lambda\alpha y_i}{n_{1i}}, \\ \frac{\partial L}{\partial n_{2i}} = 0 &\Longleftrightarrow \underline{w} = \frac{\lambda\beta y_i}{n_{2i}}, \\ \frac{\partial L}{\partial \lambda} = 0 &\Longleftrightarrow y_i = An_{1i}^{\alpha}n_{2i}^{\beta}. \end{aligned}$$

As a consequence, the relative parts of high-skilled labor and low-skilled labor are given by

$$\frac{n_{1i}}{n_{2i}} = \frac{\alpha}{\beta}\frac{\underline{w}}{w_{1i}} = \frac{\alpha}{\beta}\theta_i \quad \text{with } \theta_i = \frac{\underline{w}}{w_{1i}} \in]0;1[\,.$$

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APPENDIX B: THE FIRM *i*'S PROGRAM

The firm i solves the following program:

$$\begin{aligned} \max_{p_i} \quad & \pi_i(n_i, \theta_i, w_{1i}) \\ \text{s.t} \quad & y_i = AB(\theta_i) n_i \\ & y_i^d = Y\,(p_i)^{-s} \\ & y_i = y_i^d \\ & w_{1i} \quad \text{and} \quad \theta_i = \theta_i(w_{1i}) \quad \text{given.} \end{aligned}$$

According to the expression for the production technology and the demand for the firm's output, we can rewrite the real profit as a function of the real price of the firm:

$$\pi_i\,(p_i, w_{1i}) = Y\,(p_i)^{1-s} - \frac{C\,(\theta_i)\,w_{1i}}{AB\,(\theta_i)} Y\,(p_i)^{-s}\,. \tag{B.1}$$

Then we deduce its optimal price:

$$\frac{\partial \pi_i}{\partial p_i} = 0 \Leftrightarrow p_i^* = \left(\frac{s}{s-1}\right) \frac{C(\theta_i)}{AB(\theta_i)} w_{1i}. \tag{B.2}$$

As $y_i(n_i, \theta_i) = y_i^d$, we can write

$$p_i = \left[\frac{AB(\theta_i)}{Y} n_i\right]^{-1/s} \tag{B.3}$$

and the optimal employment demand of the firm is as follows:

$$n_i^* = \left[\left(\frac{s}{s-1}\right) C(\theta_i) w_{1i}\right]^{-s} [AB(\theta_i)]^{s-1}\, Y.$$

APPENDIX C: THE WAGE BARGAINING

During the bargaining, the firm and the trade union negotiate the high-skilled real wage in order to solve the following program:

$$\begin{aligned} \max_{w_{1i}} \quad & S\,(n_i, w_{1i}) = \{\pi_i(n_i, w_{1i})\}^{(1-\rho)}\,\{[C(\theta_i) w_{1i} - w_R]\, n_i\}^{\rho} \\ \text{s.t.} \quad & n_i = n_i^* \\ & \theta_i = \theta_i\,(w_{1i}) \\ & w_R \quad \text{given.} \end{aligned}$$

For convenience, we log linearize the objective function of the program:

$$\log S\left(n_i^*, w_{1i}\right) = (1-\rho) \log\left[\pi_i(n_i^*, w_{1i})\right] + \rho \log\left[C(\theta_i) w_{1i} - w_R\right] + \rho \log n_i^*.$$

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Consequently, w_{1i}^* solves

$$\frac{\partial \log S(n_i^*, w_{1i})}{\partial w_{1i}} = 0,$$

which gives

$$(1-\rho)\frac{\partial\left[\pi_i(n_i^*, w_{1i})\right]}{\partial w_{1i}}\left[\pi_i(n_i^*, w_{1i})\right]^{-1} + \rho\frac{\partial\left[C(\theta_i)w_{1i} - w_R\right]}{\partial w_{1i}}\left[C(\theta_i)w_{1i} - w_R\right]^{-1} + \rho\frac{\partial n_i^*}{\partial w_{1i}}\left(n_i^*\right)^{-1} = 0.$$

After rearrangement, we can rewrite this expression as a function of the profit elasticity and total labor demand elasticity with respect to the high-skilled real wage, respectively $E_{w_{1i}}^{\pi_i^*}$ and $E_{w_{1i}}^{n_i^*}$:

$$(1-\rho)\frac{E_{w_{1i}}^{\pi_i^*}}{w_{1i}} + \rho\frac{\partial\left[C(\theta_i)w_{1i} - w_R\right]}{\partial w_{1i}}\left[C(\theta_i)w_{1i} - w_R\right]^{-1} + \rho\frac{E_{w_{1i}}^{n_i^*}}{w_{1i}} = 0.$$

Given that

$$E_{w_{1i}}^{\pi_i^*} = \frac{\partial\left[\pi_i(n_i^*, w_{1i})\right]}{\partial w_{1i}}\frac{w_{1i}}{\pi_i(n_i^*, w_{1i})} = \alpha(1-s),$$

$$E_{w_{1i}}^{n_i^*} = \frac{\partial n_i^*}{\partial w_{1i}}\frac{w_{1i}}{n_i^*} = \alpha(1-s) - [1-\gamma_i(\theta_i)],$$

and

$$\frac{\partial\left[C(\theta_i)w_{1i} - w_R\right]}{\partial w_{1i}} = C(\theta_i)\left[1-\gamma_i(\theta_i)\right],$$

the high-skilled real wage that solves the Nash bargaining program is given by

$$w_{1i}^* = \left\{\left(\frac{\rho}{s-1}\right)\frac{[1-\gamma_i(\theta_i)]}{\alpha} + 1\right\}\frac{w_R}{C(\theta_i)} = \frac{\chi(\rho, s, \theta_i)}{C(\theta_i)}w_R \qquad \text{with} \quad \chi(\rho, s, \theta_i) \geq 1,$$

where $\chi(\rho, s, \theta_i)$ represents the wage index markup over the reservation wage and $\frac{\chi(\rho,s,\theta_i)}{C(\theta_i)}$ the high-skilled real wage markup over the reservation wage. As $C(\theta_i) \in]0, 1[$, we have $\frac{\chi(\rho,s,\theta_i)}{C(\theta_i)} > \chi(\rho, s, \theta_i) \geqslant 1$.

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APPENDIX D: DERIVATIVES SUMMARY

$$\frac{\partial \theta}{\partial w_1} = \frac{-\theta}{w_1} < 0,$$

$$\frac{\partial C(\theta)}{\partial w_1} = \frac{\partial C(\theta)}{\partial \theta}\frac{\partial \theta}{\partial w_1} = \frac{-\gamma(\theta) C(\theta)}{w_1} < 0,$$

$$\frac{\partial [1-\gamma(\theta)]}{\partial w_1} = \frac{\partial [1-\gamma(\theta)]}{\partial \theta}\frac{\partial \theta}{\partial w_1} = \frac{-\gamma(\theta)[1-\gamma(\theta)]}{w_1} < 0,$$

$$\frac{\partial \gamma(\theta)}{\partial w_1} = \frac{\partial \gamma(\theta)}{\partial \theta}\frac{\partial \theta}{\partial w_1} = \frac{\gamma(\theta)[1-\gamma(\theta)]}{w_1} > 0,$$

$$\frac{\partial \chi(\rho, s, \theta)}{\partial w_1} = \frac{\partial \chi(\rho, s, \theta)}{\partial \theta}\frac{\partial \theta}{\partial w_1} = \frac{\gamma(\theta)[\chi(\rho, s, \theta) - 1]}{w_1} < 0.$$

APPENDIX E: FIRST DERIVATIVES OF (PS) AND (WS)

To show whether the (PS) and (WS) curves are upward or downward sloping in the space (u, w_1), we compute their first derivatives after a log linearization of the implicit functions (23) and (25):

$$\text{(PS)}: w_1 = (\alpha+\sigma)\log[1-\gamma(\theta)] + \beta\log[\gamma(\theta)] + \sigma\log(1-u) - \log\left(\frac{s}{s-1}\right) - \log[C(\theta)],$$

$$\text{(WS)}: w_1 = \log[\chi(\rho, s, \theta)] + \log u + \log B - \log[C(\theta)] - \log[1-\chi(\rho, s, \theta)(1-u)].$$

Given the derivatives in Appendix D and given that $\chi(\rho, s, \theta) > 1$ and $u \in]\tilde{u}, 1[$, we find

$$\text{(PS)}: \frac{\partial w_1}{\partial u} = \left[\frac{-\sigma}{[\alpha+\sigma\gamma(\theta)](1-u)}\right] w_1 < 0,$$

$$\text{(WS)}: \frac{\partial w_1}{\partial u} = \left[\frac{-\chi(\rho, s, \theta)[\chi(\rho, s, \theta) - 1]}{\{\chi(\rho, s, \theta)[1-\gamma(\theta)][1-\chi(\rho, s, \theta)(1-u)] + \gamma(\theta)[\chi(\rho, s, \theta) - 1]\}}\right] w_1 < 0.$$

Thus, (PS) and (WS) are always downward sloping in the space (u, w_1).

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APPENDIX F: NUMERICAL SIMULATIONS

To check the shape of the curves, to show whether multiple unemployment equilibria exist, and to manage a comparative statics analysis, we run a large number of numerical simulations. These are performed thanks to the developed expressions for (23) and (25), namely the expressions for (PS) and (WS) as functions of w_1, $\underline{w}$, u and other parameters such as $\alpha, \beta, \sigma, \rho$. Given the expressions for $C(\theta)$, $\gamma(\theta)$, $[1-\gamma(\theta)]$, and $\chi(\rho, s, \theta)$, we have

$$(\text{PS}) : w_1 = \left[\alpha^{\alpha+\beta}\beta^{\beta}\left(\frac{s-1}{s}\right)\right]^{\frac{1}{\alpha}} \underline{w}^{1+\frac{\sigma-1}{\alpha}} \left[\frac{1-u}{\alpha\underline{w}+\beta w_1}\right]^{\frac{\sigma}{\alpha}}, \tag{F.1}$$

$$(\text{WS}) : w_1 = \frac{\left[\dfrac{\rho}{s-1}\dfrac{\underline{w}}{\alpha\underline{w}+\beta w_1}+1\right]\left(\alpha\underline{w}+\beta w_1\right)Bu}{\underline{w}\left[1-\left(\dfrac{\rho}{s-1}\dfrac{\underline{w}}{\alpha\underline{w}+\beta w_1}+1\right)(1-u)\right]}. \tag{F.2}$$

Concerned with analyzing most of the cases, we use ranges of values that are summarized in Table F.1.

In these ranges of parameter values, we can find all equilibrium cases: no equilibrium, or an interval of equilibria that can be reduced to one equilibrium or two distinct equilibria. We also find that the (PS) curve can be either concave or convex according to the size of social IRS. Indeed, (PS) is strictly concave when $\sigma < 1$ and becomes concave–convex when σ grows.

F.1. EQUILIBRIUM CASES AND THE IMPLICATIONS OF A BINDING MINIMUM WAGE

Hereafter, we give examples of the different possible cases. The functions (F.1) and (F.2) are plotted using Maple software in the space (u, w_1). The (PS) and (WS) curves are represented by black and gray curves, respectively.

Figures F.1–F.3 show how the (PS) curve is distorted when σ grows. The values of the other parameters are $s = 3$, $\alpha = \beta = \rho = 0.5$, $\underline{w} = 0.05$, and $B = 0.02$.

As the minimum wage level is fixed to 0.05, we can deduce from our figures that the high-skilled real wage is always inferior to the minimum wage at the low equilibrium,

TABLE F.1. Range of parameter values

Parameter	Value
α	$]0.1, 0.9[$
σ	$]0, 2[$
s	$]1, 10[$
ρ	$]0.1, 0.9[$

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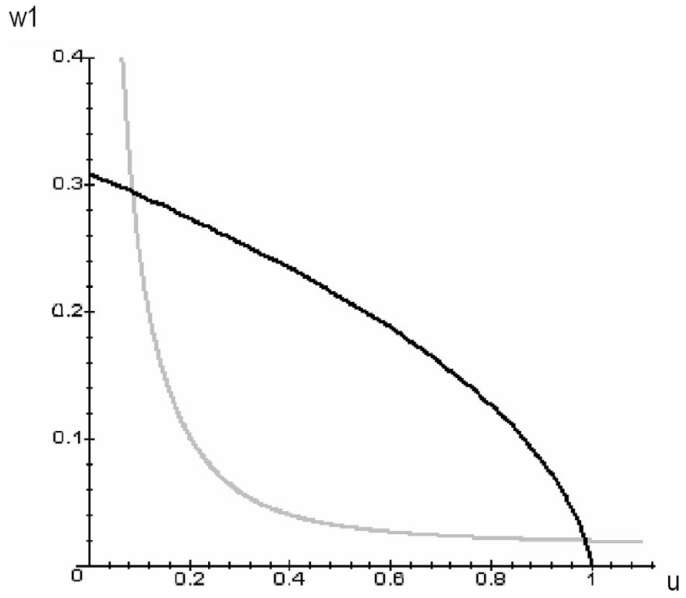


FIGURE F.1. $\sigma = 0.5$.

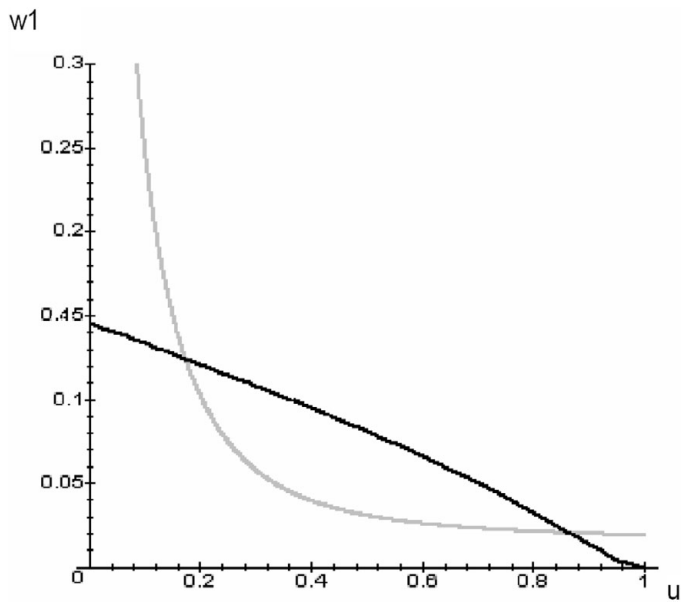


FIGURE F.2. $\sigma = 1$.

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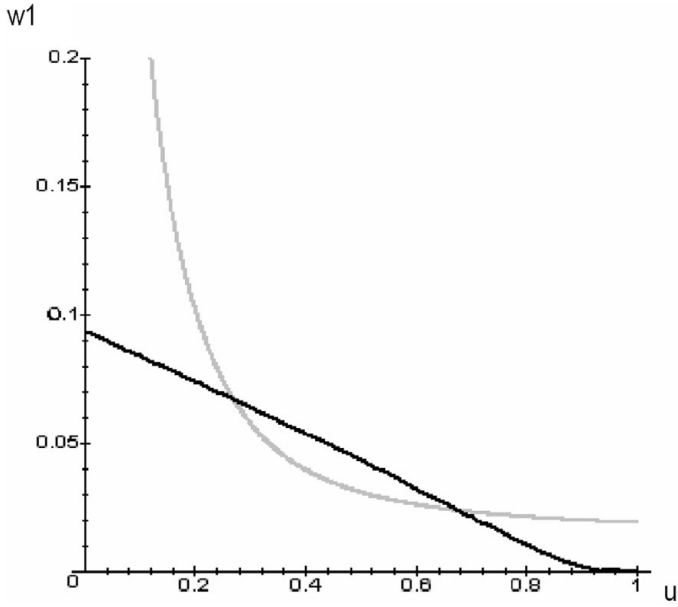


FIGURE F.3. $\sigma = 1.5$.

i.e., at the equilibrium with a high unemployment rate. As a consequence, the constraint $w_{1i} > \underline{w}$ is not verified at this equilibrium. We ran numerous numerical simulations with different minimum wage levels and always arrived at the same conclusion: the occurrence of the low equilibrium is ruled out by the presence of a mandatory minimum wage.

In the previous figures, we always have two distinct equilibria. However, in some cases, a range of equilibria can exist, or none. These cases can be highlighted by varying the degree of competition on the good market. Indeed, if the competition is too monopolistic (s near one in our example[26]), we do not find two distinct equilibria in spite of social IRS. See Figures F.4 and F.5.

F.2. EFFECTS OF A HIGHER MINIMUM WAGE

To gauge the role of some parameters following a minimum wage increase, we ran numerical simulations for different values of (α, β) and σ. As the low equilibrium is unlikely, we focus our investigation on the high equilibrium (u_H, w_{1_H}). For the results given in Table F.2,[27] the values of other parameters are $s = 5$, $\rho = 0.5$, $B = 0.02$, and $\underline{w} = 0.05$. They represents the variations of the equilibrium unemployment rate u_H, the high-skilled real wage w_{1_H}, the wage dispersion θ, and the share of low-skilled labor γ and high-skilled labor $(1 - \gamma)$ in total employment after an increase of 10% in the real minimum wage (its value increases from 0.05 to 0.055).

We note that the effect on the high-skilled real wage is either negative or positive. To determine what parameters entail this reversed effect, we compute the total differential of

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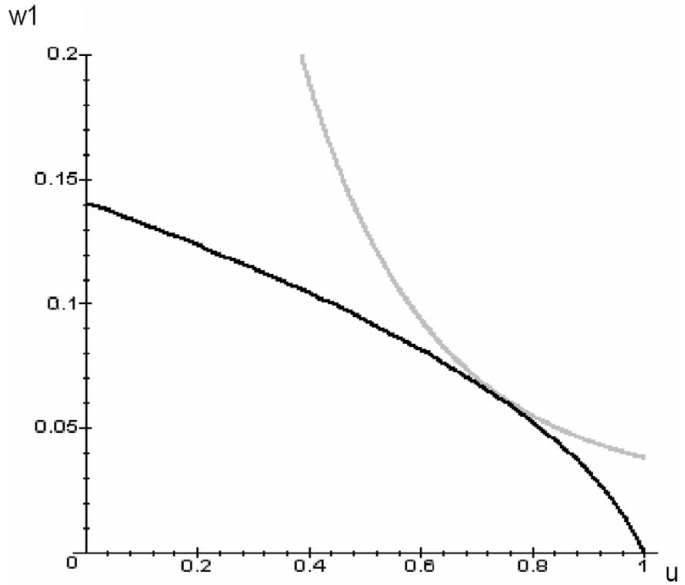


FIGURE F.4. Range of equilibria ($s = 1.488$).

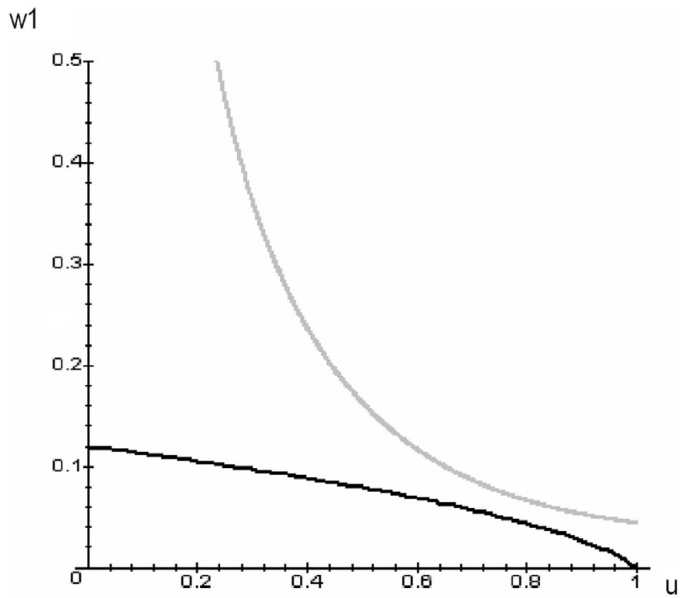


FIGURE F.5. No equilibrium ($s = 1.4$).

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TABLE F.2. Effects of an increase of 10% in the real minimum wage

	$\alpha = 0.2 \quad \beta = 0.8$	$\alpha = 0.5 \quad \beta = 0.5$	$\alpha = 0.8 \quad \beta = 0.2$
variation in %	(1)	(2)	(3)
	Case A: $\sigma = 0.5$		
$\triangle u$	+9.69	+6.52	+4.62
$\triangle w_1$	−4.44	−0.77	+1
$\triangle \theta$	+15.08	+10.85	+8.91
$\triangle \gamma$	−0.39	−1.27	−2.83
$\triangle (1-\gamma)$	+14.33	+9.36	+5.83
	Case B: $\sigma = 1$		
$\triangle u$	+2.96	+3	+2.82
$\triangle w_1$	+0.16	+1.73	+2.44
$\triangle \theta$	+9.83	+8.14	+7.36
$\triangle \gamma$	−1.25	−1.92	−3.07
$\triangle(1-\gamma)$	+8.40	+6.02	+4.10

the log linearized (PS) curve given by the relation (F.1), from which we deduce that

$$\frac{dw_1}{d\underline{w}} = \frac{\left[\dfrac{1}{\underline{w}} + \dfrac{\sigma-1}{\alpha\underline{w}} - \dfrac{\sigma}{(\alpha\underline{w}+\beta w_1)}\right]}{\left[\dfrac{1}{w_1} + \dfrac{\sigma}{\alpha}\dfrac{\beta}{(\alpha\underline{w}+\beta w_1)}\right]}. \tag{F.3}$$

Thus, the sign of the effect on the high-skilled real wage depends on the sign of the numerator, given that the denominator is always positive. After rearrangements, we find that an increase in the minimum wage always has a positive effect on the high-skilled real wage if and only if

$$\gamma(\theta) > \frac{\beta}{\sigma}. \tag{F.4}$$

Furthermore, according to numerical simulations, we can also report the following statements:

(i) The share of low-skilled labor in total employment is decreasing in α (i.e., increasing in β): $\gamma_{(1)} > \gamma_{(2)} > \gamma_{(3)}$.
(ii) The share of high-skilled labor in total employment is increasing in α (i.e., decreasing in β): $(1-\gamma)_{(1)} < (1-\gamma)_{(2)} < (1-\gamma)_{(3)}$.
(iii) The share of high-skilled labor in total employment is increasing in σ: $(1-\gamma)_{(A)} < (1-\gamma)_{(B)}$.
(iv) The share of low-skilled labor in total employment is decreasing in σ: $\gamma_{(A)} > \gamma_{(B)}$.

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