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Direct position analysis of parallel manipulators which generate SP-2PS structures

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SUMMARY

The determination of the assembly modes of the parallel structures with three legs of type PS or SP (P and S stand for prismatic pair and spherical pair, respectively) consists of solving the direct position analysis of all the three-legged parallel manipulators which have, in each leg, one not actuated prismatic pair, one not actuated spherical pair and one or two one-dof actuated pairs of any type, placed along the leg in any order. There are two types of such structures: (i) 3PS structures and (ii) SP-2PS structures. The procedure to determine the assembly modes of the SP-2PS structures has not been presented yet, in the literature. This paper presents the analytic form determination of the assembly modes of the SP-2PS structures. In particular, the closure equations of a generic SP-2PS structure will be written and their solution will be reduced to the solution of an eight-degree univariate polynomial equation with real coefficients. Finally, the proposed algorithm will be applied to a real case. The result of this study is that the assembly modes of any SP-2PS structure are at most eight, and the end-effector poses, which solve the direct position analysis of the parallel manipulators that generate those structures, are also eight.

KEYWORDS: Kinematics; Position analysis; Parallel mechanisms; Parallel structure.

I. INTRODUCTION

Parallel manipulators (PMs) are closed-loop mechanisms where the end effector is connected to the frame through a number of kinematic chains (legs). Nowadays, PMs are used in nearly all the fields that, once, were reserved to serial manipulators. This increase in the PM applications was possible because of the progressive solution of a set of theoretical problems regarding the spatial closed-loop mechanisms. One out of these problems is the position analysis. The position analysis of a manipulator involves the solution of two sub-problems: the direct position analysis (DPA) and the inverse position analysis (IPA). The DPA is the determination of the end-effector poses (positions and orientations) for given values of the actuated-joint variables. The IPA is the determination of the values of the actuated-joint variables for a given end-effector pose. Both these problems are involved in the design of the manipulator controller.

In the study of the PMs' position analysis, great attention[1–5] has been paid to the Stewart platforms that are PMs with six degrees of freedom (dof) and six legs of type UPS (U, P and S stand for universal joint, prismatic pair and spherical pair respectively) where the prismatic pair is actuated. The interest for these types of PMs is due to the fact that the first PM applications[6,7] used them. The inverse position analysis of the Stewart platforms is straightforward (the unknowns can be immediately written as explicit functions of the data), whereas their direct position analyses involve the solution of non-linear equation systems that are very difficult to solve.[2–5]

Actually, many parallel manipulators do not belong to the family of the Stewart platforms and, often, both the direct and the inverse position analyses of these other PMs involve the solution of non-linear equation systems[8,9] whose solution techniques are not presented yet.

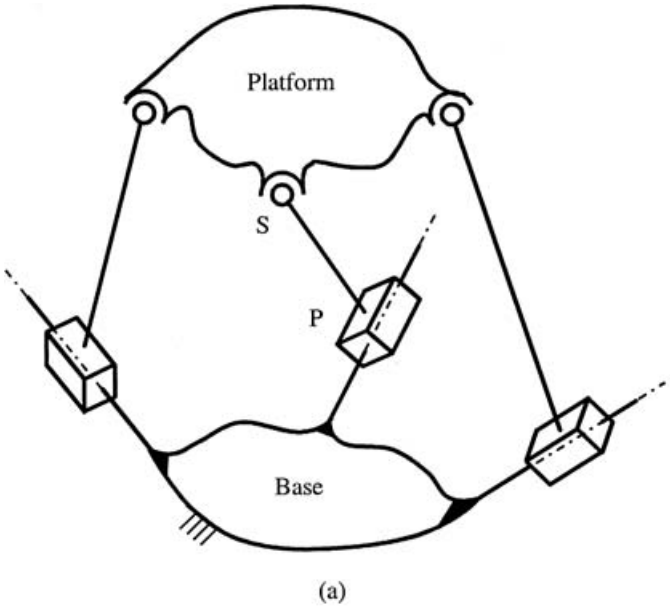
A set of PMs which do not belong to the Stewart-platforms' family is the one collecting all the PMs that, when the actuators are locked, become parallel structures constituted of two rigid bodies (platform and base) connected by three legs of type PS or SP (Fig. 1). Such structures can have two topologies: (i) 3PS topology (3PS structures), where the three spherical pairs lie on the platform (base) (Fig. 1a), and (ii) SP-2PS topology (SP-2PS structures) where only two spherical pairs lie on the platform (base) (Fig. 1b).

The assembly modes of a structure, without link permutations, correspond one-to-one to the end-effector poses that are solutions of the DPA of the manipulator which generates such a structure when the actuators are locked. Different manipulators can generate structures that are topologically equal. Therefore, the determination of the assembly modes of a structure with a given topology brings to find the solutions of the DPA of a wide family of manipulators.

In particular, the determination of the assembly modes of the parallel structures with three legs of type PS correspond to solve the DPA of all the three-legged PMs which have, in each leg, one not actuated prismatic pair, one not actuated spherical pair and one or two one-dof actuated pairs of any type placed in any order along the leg.

The method to determine the assembly modes of the 3PS structures has been presented by Parenti-Castelli and Innocenti,[10] whereas the determination of the assembly modes of the SP-2PS structures has not been presented yet, in the literature.

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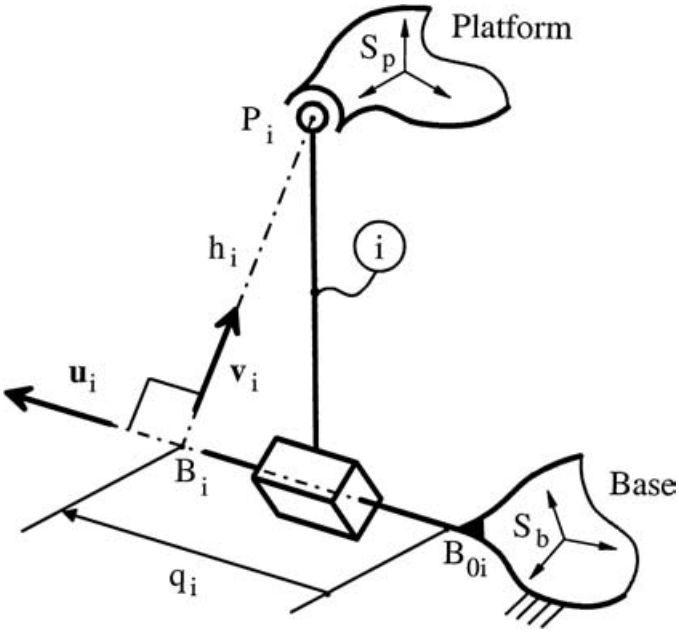


Fig. 2. The i-th leg of type PS (i = 1, 2).

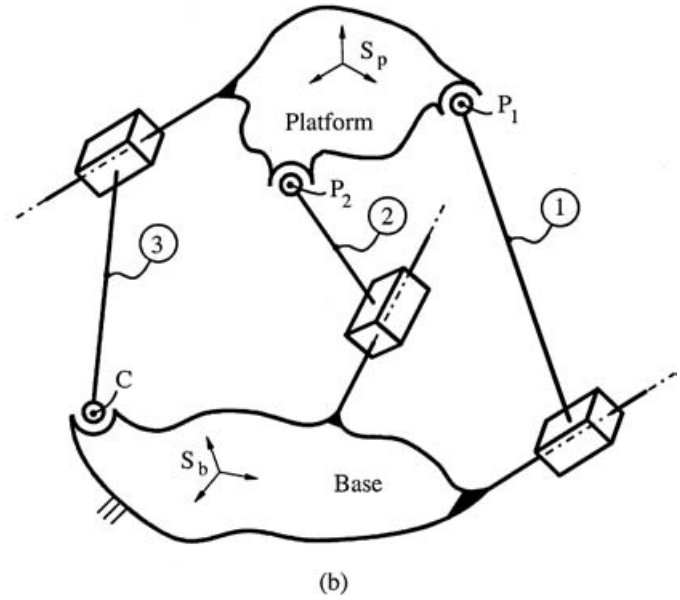


Fig. 1. (a) 3PS structure, (b) SP-2PS structure.

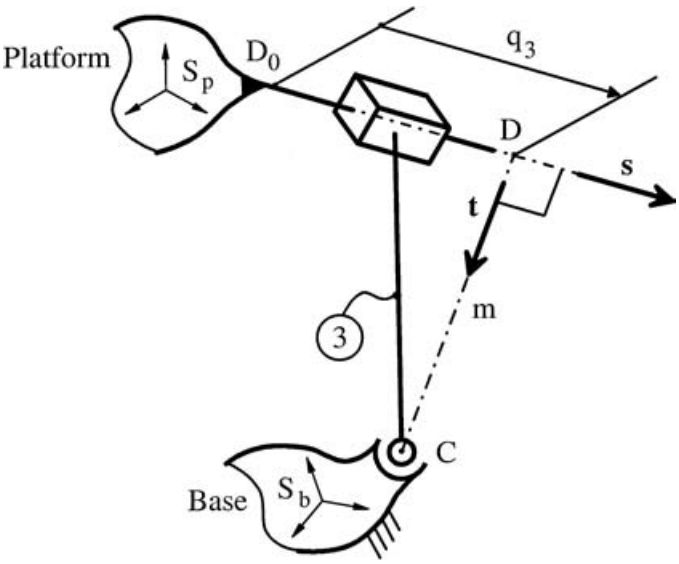


Fig. 3. The leg of type SP.

This paper presents the analytic form determination of the assembly modes of the SP-2PS structures.

In particular, the closure equations of a generic SP-2PS structure are written and their solution is reduced to the solution of an eight-degree univariate polynomial equation with real coefficients. The proposed algorithm is applied to a real case.

It is worth noting that the elimination technique used to obtain the eight-degree-univariate polynomial equation is similar to the techniques used by other authors[10,11] to solve the DPA of PMs that generate other types of three-legged structures with one spherical pair per leg.

The result of this study is that the assembly modes of any SP-2PS structure (without link permutations) are at most eight and the end-effector poses, which solve the direct position analysis of the parallel manipulators that generate those structures, are as many.

II. CLOSURE EQUATIONS

An SP-2PS structure has two legs of type PS (Fig. 2) and one leg of type SP (Fig. 3).

Figure 2 shows the i-th leg of type PS with i = 1, 2. With reference to Fig. 2, P_i is the center of the spherical pair that joins the leg to the platform. $\mathbf{u}_i$ is the unit vector of the sliding direction of the prismatic pair. B_{0i} is a point of the base. B_i is the foot of the perpendicular through P_i to the line parallel to $\mathbf{u}_i$ and passing through B_{0i}. q_i is the joint variable of the prismatic pair. h_i is the length of the segment B_iP_i and $\mathbf{v}_i$ is the unit vector $(\mathbf{P}_i - \mathbf{B}_i)/h_i$. Link i is the link between the prismatic pair and the spherical pair. S_b and S_p are two Cartesian reference systems fixed to the base and the platform respectively.

Figure 3 shows the third leg which is of type SP. With reference to Fig. 3, C is the center of the spherical pair that joins the leg to the base. **s** is the unit vector of the sliding

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direction of the prismatic pair. D_0 is a point of the platform. D is the foot of the perpendicular through C to the line parallel to **s** and passing through D_0. q_3 is the joint variable of the prismatic pair. m is the length of the segment CD and **t** is the unit vector $(\mathbf{C}-\mathbf{D})/m$. Link 3 is the link between the spherical pair and the prismatic pair.

Hereafter, a vector symbol with a leading superscript b or p (e.g. ${}^{b}\mathbf{a}$ or ${}^{p}\mathbf{a}$) will mean that the vector is projected on S_b or S_p respectively. Moreover, a bold capital letter will indicate the position vector associated to a point, if that capital letter has been already used to denote a point; otherwise, it will indicate a matrix.

With these notations, the following relationships hold (see Figs. 2 and 3)

$$ {}^{b}\mathbf{P}_i = q_i\,{}^{b}\mathbf{u}_i + h_i\,{}^{b}\mathbf{v}_i + {}^{b}\mathbf{B}_{0i} \quad i = 1, 2 \tag{1a} $$

$$ {}^{p}\mathbf{C} = q_3\,{}^{p}\mathbf{s} + m\,{}^{p}\mathbf{t} + {}^{p}\mathbf{D}_0 \tag{1b} $$

where ${}^{b}\mathbf{B}_{0i}$, h_i, ${}^{b}\mathbf{u}_i$ and ${}^{b}\mathbf{v}_i$ are either vector or scalar geometric constants of the base, whereas ${}^{p}\mathbf{D}_0$, m, ${}^{p}\mathbf{s}$ and ${}^{p}\mathbf{t}$ are either vector or scalar geometric constants of the platform.

Moreover, ${}^{b}\mathbf{C}$ is a vector constant of the base, whereas ${}^{p}\mathbf{P}_i$ with $i = 1, 2$ and $p_{12} \equiv \|\mathbf{P}_2 - \mathbf{P}_1\|$ are either vector or scalar geometric constants of the platform.

Relationships (1) allow the following additional relationships to be deduced

$$ {}^{b}\mathbf{C} - {}^{b}\mathbf{P}_i = {}^{b}\mathbf{w}_i - q_i\,{}^{b}\mathbf{u}_i \qquad i = 1, 2 \tag{2a} $$

$$ {}^{p}\mathbf{C} - {}^{p}\mathbf{P}_i = {}^{p}\mathbf{r}_i + q_3\,{}^{p}\mathbf{s} \qquad i = 1, 2 \tag{2b} $$

$$ {}^{b}\mathbf{P}_2 - {}^{b}\mathbf{P}_1 = q_2\,{}^{b}\mathbf{u}_2 - q_1\,{}^{b}\mathbf{u}_1 + {}^{b}\mathbf{n}_{21} \tag{2c} $$

where

$$ {}^{b}\mathbf{w}_i = {}^{b}\mathbf{C} - h_i\,{}^{b}\mathbf{v}_i - {}^{b}\mathbf{B}_{0i} \qquad i = 1, 2 \tag{3a} $$

$$ {}^{p}\mathbf{r}_i = m\,{}^{p}\mathbf{t} + {}^{p}\mathbf{D}_0 - {}^{p}\mathbf{P}_i \qquad i = 1, 2 \tag{3b} $$

$$ {}^{b}\mathbf{n}_{21} = h_2\,{}^{b}\mathbf{v}_2 + {}^{b}\mathbf{B}_{02} - h_1\,{}^{b}\mathbf{v}_1 - {}^{b}\mathbf{B}_{01} \tag{3c} $$

The vectors ${}^{b}\mathbf{w}_i$ and ${}^{p}\mathbf{r}_i$ are vector constants of the base and the platform respectively.

The closure equations of a generic SP-2PS structure (see Fig. 1b) can be written as follows:

$$ \left({}^{b}\mathbf{C} - {}^{b}\mathbf{P}_i\right)^2 = \left({}^{p}\mathbf{C} - {}^{p}\mathbf{P}_i\right)^2 \quad i = 1, 2 \tag{4a} $$

$$ \left({}^{b}\mathbf{P}_2 - {}^{b}\mathbf{P}_1\right)^2 = p_{12}^2 \tag{4b} $$

The introduction of relationships (2) into Eqs. (4) yields

$$ {}^{b}\mathbf{w}_i^2 + q_i^2 - 2q_i\,{}^{b}\mathbf{u}_i \cdot {}^{b}\mathbf{w}_i = {}^{p}\mathbf{r}_i^2 + q_3^2 + 2q_3\,{}^{p}\mathbf{s} \cdot {}^{p}\mathbf{r}_i \quad i = 1, 2 \tag{5a} $$

$$ q_2^2 + q_1^2 + {}^{b}\mathbf{n}_{21}^2 + 2q_2\,{}^{b}\mathbf{u}_2 \cdot {}^{b}\mathbf{n}_{21} - 2q_1\,{}^{b}\mathbf{u}_1 \cdot {}^{b}\mathbf{n}_{21} - 2q_1q_2\,{}^{b}\mathbf{u}_1 \cdot {}^{b}\mathbf{u}_2 = p_{12}^2 \tag{5b} $$

System (5) is a system of three quadratic equations in three unknowns: q_1, q_2 and q_3.

Since the position of the platform points P_1, P_2 and D are uniquely determined both in S_b and in S_p if the values of q_1, q_2 and q_3 are known, an one-to-one correspondence exists between the triplets of q_1, q_2 and q_3 values, that are solutions of system (5), and the poses of the platform with respect to the base, that identify the assembly modes of the SP-2PS structure.

III. SOLUTION OF THE CLOSURE EQUATIONS

System (5) can be rewritten as follows:

$$ q_3^2 + a_i q_3 + b_i = 0 \quad i = 1, 2 \tag{6a} $$

$$ q_2^2 + c\,q_2 + d = 0 \tag{6b} $$

where

$$ a_i = 2\,{}^{p}\mathbf{s} \cdot {}^{p}\mathbf{r}_i, \qquad i = 1, 2 \tag{7a} $$

$$ b_i = -q_i^2 + q_i b_{i1} + b_{i0}, \qquad i = 1, 2 \tag{7b} $$

$$ b_{i0} = {}^{p}\mathbf{r}_i^2 - {}^{b}\mathbf{w}_i^2, \qquad i = 1, 2 \tag{7c} $$

$$ b_{i1} = 2\,{}^{b}\mathbf{u}_i \cdot {}^{b}\mathbf{w}_i, \qquad i = 1, 2 \tag{7d} $$

$$ c = q_1 c_1 + c_0, \tag{7e} $$

$$ c_0 = 2\,{}^{b}\mathbf{u}_2 \cdot {}^{b}\mathbf{n}_{21}, \tag{7f} $$

$$ c_1 = -2\,{}^{b}\mathbf{u}_1 \cdot {}^{b}\mathbf{u}_2, \tag{7g} $$

$$ d = q_1^2 + q_1 d_1 + d_0, \tag{7h} $$

$$ d_0 = {}^{b}\mathbf{n}_{21}^2 - p_{12}^2, \tag{7i} $$

$$ d_1 = -2\,{}^{b}\mathbf{u}_1 \cdot {}^{b}\mathbf{n}_{21}, \tag{7j} $$

In order to solve system (6), firstly, the eliminant of Eqs. (6a) with respect to q_3 will be found, then the eliminant with respect to q_2 of the system constituted by Eq. (6b) and the eliminant of Eqs. (6a) with respect to q_3 will be determined. The resulting equation will be an univariate polynomial equation in q_1 which is the Sylvester's eliminant[12] of system (6).

III.1. Elimination of q_3

The product of Eqs. (6a) by q_3 yields two additional nonlinear equations that, together with Eqs. (6a), constitute the following system of four equations which still has the solutions of the equation system (6a):

$$ \mathbf{H}\mathbf{x} = \mathbf{0} \tag{8} $$

where

$$ \mathbf{H} = \begin{bmatrix} 1 & a_1 & b_1 & 0 \\ 0 & 1 & a_1 & b_1 \\ 1 & a_2 & b_2 & 0 \\ 0 & 1 & a_2 & b_2 \end{bmatrix} \tag{9a} $$

$$ \mathbf{x} = \left\{q_3^3, \quad q_3^2, \quad q_3, \quad 1\right\}^{T} \tag{9b} $$

System (8) is a linear and homogeneous system and it admits solutions different from the null vector for **x** (note that the

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fourth component of **x** must be equal to 1) if and only if matrix **H** is singular, which implies that the following equation must be satisfied:

$$\det(\mathbf{H}) = 0 \tag{10}$$

The introduction of expression (9a) for **H** into Eq. (10) yields the explicit form of Eq. (10):

$$(a_1 - a_2)(a_1 b_2 - a_2 b_1) + (b_2 - b_1)^2 = 0 \tag{11}$$

The analysis of Eq. (11) (see definitions (7)) reveals that Eq. (11) is a fourth-degree polynomial equation in q_1 and q_2. Equation (11) is the eliminant of Eqs. (6a) with respect to q_3.

III.2. Elimination of q_2

By using relationships (7) to expand the left-hand side of Eq. (11), Eq. (11) can be written as follows

$$q_2^4 + e q_2^3 + f q_2^2 + g q_2 + k = 0 \tag{12}$$

where

$$e = -2b_{21} \tag{13a}$$

$$f = -a_1^2 + a_1 a_2 + 2(b_{10} - b_{20}) + b_{21}^2 + 2b_{11} q_1 - 2q_1^2 \tag{13b}$$

$$g = -b_{21}\left(f - b_{21}^2\right) \tag{13c}$$

$$k = -a_1 a_2 b_{10} + a_2^2 b_{10} + b_{10}^2 + a_1^2 b_{20} - a_1 a_2 b_{20} - 2b_{10} b_{20} + b_{20}^2 + \left(-a_1 a_2 b_{11} + a_2^2 b_{11} + 2b_{10} b_{11} - 2b_{11} b_{20}\right) q_1 + \left(a_1 a_2 - a_2^2 - 2b_{10} + b_{11}^2 + 2b_{20}\right) q_1^2 - 2b_{11} q_1^3 + q_1^4 \tag{13d}$$

The system constituted by Eqs. (6b) and (12) has the same solutions of system (6) for q_1 and q_2.

The product of Eq. (12) by q_2 and the products of Eq. (6b) by q_2, q_2^2 and q_2^3 yield four additional non-linear equations that, together with Eqs. (6b) and (12), yield the following system of six non-linear equations, which has the same solution of the system constituted by Eqs. (6b) and (12) (i.e. the same solution of system (6) for q_1 and q_2):

$$\mathbf{M}\mathbf{y} = \mathbf{0} \tag{14}$$

where

$$\mathbf{M} = \begin{bmatrix} 1 & e & f & g & k & 0 \\ 0 & 1 & e & f & g & k \\ 0 & 0 & 0 & 1 & c & d \\ 0 & 0 & 1 & c & d & 0 \\ 0 & 1 & c & d & 0 & 0 \\ 1 & c & d & 0 & 0 & 0 \end{bmatrix} \tag{15a}$$

$$\mathbf{y} = \left\{q_2^5, \quad q_2^4, \quad q_2^3, \quad q_2^2, \quad q_2, \quad 1\right\}^T \tag{15b}$$

System (14) is a linear and homogeneous system and it admits solutions different from the null vector for **y** (note that the sixth component of **y** must be equal to 1) if and only if the following equation is satisfied:

$$\det(\mathbf{M}) = 0 \tag{16}$$

By expanding the determinant of matrix **M** with the aid of an algebraic manipulator, Eq. (16) becomes:

$$d^4 + d^3(e^2 - ce - 2f) + k[c^3(c - e) + c(cf - g) + k] + d^2[cf(c - e) + f^2 + g(3c - 2e) + 2k] + d[g(g - c^3) + c^2(eg - 4k) - 2fk + c(3ek - fg)] = 0 \tag{17}$$

The analysis of the terms appearing in Eq. (17) (see definitions (13)) reveals that Eq. (17) is a univariate eighth-degree polynomial equations in q_1. The solutions of Eq. (17) coincide with the solutions of system (6) for q_1. Equation (17) is the Sylvester's eliminant of system (6).

III.3. Solution procedure

The solution of the direct position analysis of a parallel manipulator which generates an SP-2PS structure when the actuators are locked can be implemented as follows:

(a) by using given values of the actuated-joint variables of the manipulator, the geometric data of the SP-2PS structure obtained when the actuators are locked are computed;
(b) by using the geometric data of the SP-2PS structure, the coefficients of the polynomial Eq. (17) are computed;
(c) the values of q_1 that solve Eq. (17) are determined by using a standard software package that calculates all the roots of a polynomial equation;
(d) for each value of q_1 (calculated in the previous step) the corresponding value q_2 is determined by computing the entries of matrix **M** and by solving the linear system (14);
(e) for each couple of values (q_1, q_2), computed in the previous step, the corresponding value of q_3 is determined by computing the entries of matrix **H** and by solving the linear system (8);
(f) for each triplet of values (q_1, q_2, q_3), computed in the previous step, the pose of the platform with respect to the base is determined.

The platform poses determined at the end of the above procedure are the end-effector poses that solve the direct position analysis of the manipulator. Since at most eight real values for q_1 can be determined in step (c), at most eight platform poses will be determined by the procedure. Therefore, the direct position analysis of any manipulator, that generates an SP-2PS structure when the actuators are locked, has at most eight solutions. In the next section, an example of SP-2PS structure that has exactly eight solutions will be given.

IV. NUMERICAL EXAMPLE

With reference to the introduced notations (Figs. 1b and 2), the proposed procedure has been applied to an SP-2PS structure defined by the following data (the lengths are measured in an arbitrary length unit): ${}^b\mathbf{C} \equiv \{0,\ 0,\ 0\}^T$, ${}^b\mathbf{u}_1 \equiv \{1,\ 0,\ 0\}^T$, ${}^b\mathbf{v}_1 \equiv \{0,\ 0,\ 1\}^T$, $h_1 = 10$, ${}^b\mathbf{B}_{01} \equiv {}^b\mathbf{B}_{02} \equiv$

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Table I. Results of the numerical example: assembly modes of the SP-2PS structure (the lengths are measured in an arbitrary length unit (l.u.)).

No.	q_1 (l.u.)	q_2 (l.u.)	q_3 (l.u.)	${}^b\mathbf{P}_1$ (l.u.)	${}^b\mathbf{P}_2$ (l.u.)	${}^b\mathbf{D}_0$ (l.u.)
1	20	20	$-5\sqrt{2}$	$\{20,\ 0,\ 10\}^T$	$\{0,\ 20,\ 10\}^T$	$\{5,\ 5,\ 10\}^T$
2	20	−20	$-5\sqrt{2}$	$\{20,\ 0,\ 10\}^T$	$\{0,\ -20,\ 10\}^T$	$\{5,\ -5,\ 10\}^T$
3	−20	20	$-5\sqrt{2}$	$\{-20,\ 0,\ 10\}^T$	$\{0,\ 20,\ 10\}^T$	$\{-5,\ 5,\ 10\}^T$
4	−20	−20	$-5\sqrt{2}$	$\{-20,\ 0,\ 10\}^T$	$\{0,\ -20,\ 10\}^T$	$\{-5,\ -5,\ 10\}^T$
5	20	20	$15\sqrt{2}$	$\{20,\ 0,\ 10\}^T$	$\{0,\ 20,\ 10\}^T$	$\{15,\ 15,\ 10\}^T$
6	20	−20	$15\sqrt{2}$	$\{20,\ 0,\ 10\}^T$	$\{0,\ -20,\ 10\}^T$	$\{15,\ -15,\ 10\}^T$
7	−20	20	$15\sqrt{2}$	$\{-20,\ 0,\ 10\}^T$	$\{0,\ 20,\ 10\}^T$	$\{-15,\ 15,\ 10\}^T$
8	−20	−20	$15\sqrt{2}$	$\{-20,\ 0,\ 10\}^T$	$\{0,\ -20,\ 10\}^T$	$\{-15,\ -15,\ 10\}^T$

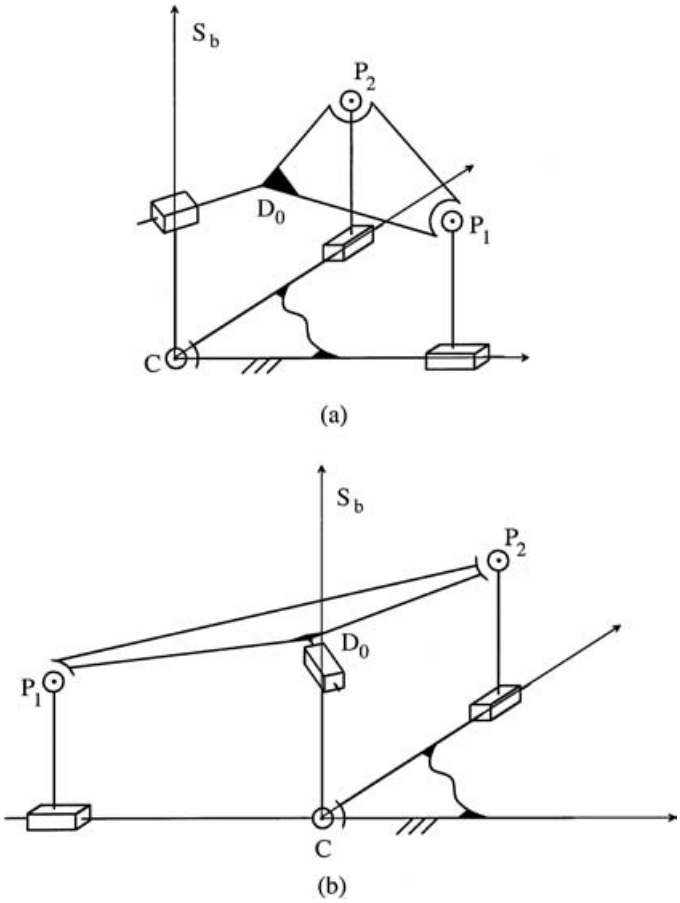


Fig. 4. Results of the numerical example: (a) assembly mode No. 1 of Table I and (b) assembly mode No. 3 of Table I.

${}^b\mathbf{C}$, ${}^b\mathbf{u}_2 \equiv \{0,\ 1,\ 0\}^T$, ${}^b\mathbf{v}_2 \equiv \{0,\ 0,\ 1\}^T$, $h_2 = 10$, ${}^p\mathbf{P}_1 \equiv \{20,\ 0,\ 0\}^T$, ${}^p\mathbf{P}_2 \equiv \{0,\ 20,\ 0\}^T$, ${}^p\mathbf{D}_0 \equiv \{5,\ 5,\ 0\}^T$, ${}^p\mathbf{s} \equiv \{1/\sqrt{2},\ 1/\sqrt{2},\ 0\}^T$, ${}^p\mathbf{t} \equiv \{0,\ 0,\ -1\}^T$, $m = 10$.

The eight assembly modes determined with the above procedure are reported in Table I. In Table I, each row corresponds to one assembly mode, the columns 2, 3 and 4 report the values of the joint variables q_1, q_2 and q_3; whereas the columns 5, 6 and 7 report the coordinates of the platform points P_1, P_2 and D_0, measured in S_b, which make it possible to easily visualize the assembly mode they refer to. Figure 4 shows the assembly modes 1 and 3 of table I as example.

V. CONCLUSIONS

The assembly modes of the parallel structures with three legs of type PS or SP correspond to the solutions of the direct position analysis of all the three-legged parallel manipulators which have, in each leg, one not actuated prismatic pair, one not actuated spherical pair and one or two one-dof actuated pairs of any type, placed along the leg in any order. There are two types of such structures: (i) 3PS structures and (ii) SP-2PS structures. The 3PS structures have been already studied in the literature.[10]

The analytical form solution of the closure equations of the SP-2PS structures has been presented in this paper. In particular, the closure equations of a generic SP-2PS structure has been written and their solution has been implemented via a Sylvester's dialytic elimination procedure which reduced it to the solution of an eight-degree univariate polynomial equation with real coefficients. Finally, the proposed procedure has been applied to a real case.

The result of this study is that the assembly modes of any SP-2PS structure are at most eight, and the end-effector poses, which solve the direct position analysis of the parallel manipulators that generate those structures, are as many.

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