DYNAMIC STRESS ANALYSIS AROUND A CIRCULAR CAVITY IN TWO-DIMENSIONAL INHOMOGENEOUS MEDIUM WITH DENSITY VARIATION

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ABSTRACT

Based on the complex function theory and the homogenization principle, an universal approach of solving the dynamic stress concentration around a circular cavity in two-dimensional (2D) inhomogeneous medium is developed. The Helmholtz equation with variable coefficient is converted to the standard Helmholtz equation by means of the general conformal transformation method (GCTM) analytically. As an example, the inhomogeneous medium with density varying as a function of two spatial coordinates and the constant elastic modulus is studied. The dynamic stress concentration factors (DSCF) are calculated numerically. It shows that medium inhomogeneous parameters and wave numbers have significant influence on the dynamic stress concentration by the circular cavity in two-dimensional inhomogeneous medium.

Keywords: Complex function theory, 2D inhomogeneous medium, Variable coefficient Helmholtz equation, Dynamic stress concentration.

1. INTRODUCTION

Dynamic stress concentration around the discontinuities as a very important and fundamental reference standard in the design and security of structures is being researched in the field of elastodynamics. The scattering of elastic waves by obstacles has received considerable attention due to its direct relationship with dynamic stress concentration [1]. Since 1960s, many researchers have investigated the scattering of elastic waves by cylindrical cavities in homogeneous medium with different boundaries, such as full space [2-5], semi-infinite space [7,8], right-angle space [9] and infinite wedge [10]. Some studies have been used in engineering applications, *e.g.* pipelines or tunnels under earth's surface and plates containing cylindrical cavities [11-13].

In order to reach a higher precision, the case of inhomogeneous medium with various physical phenomena of the incidence, reflection and refraction of elastic/acoustic waves is to be studied [14-19]. This becomes particularly important with the rapid development of functionally graded materials. In recent years, stress analysis around a circular cavity in inhomogeneous infinite medium receives great interest. With regard to the Young's modulus and Poisson's ratio varying exponentially along the radial direction only, Yang *et al*. [20] investigated the two-dimensional stress distribution around a circular hole under arbitrary loads by using the method of piece-wise homogeneous layers. Zhang *et al*. [21] provided an exact thermal stress solution for a functionally graded plate that has a circular hole, with the material properties and applied temperature varying arbitrarily in the radial direction exponentially. An analytical solution was obtained by Sburlati [22] to reduce the stress concentration factor around a circular hole surrounding by a functionally graded material annular ring in an isotropic homogeneous plate. For both Young's modulus and Poisson's ratio being limited to smooth monotonic functions of the radius and attaining limiting values far from the hole, Mohammadi *et al*. [23] provided an analytical calculation for the stress concentration factor around a circular hole in an infinite inhomogeneous plate. In view of the difficulty resulting from the inhomogeity, the numerical solution techniques, such as the finite element and boundary

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element methods, have been extended to the investigation of inhomogeneous medium. Kubair and Bhanu-Chandar [24] employed the multiple isoparametric finite element formulation to simulate the elastostatic boundary value problem for the stress concentration factor due to a circular hole in functionally graded panels with material properties varying along the radial direction according to the exponential or power law. Afsar and Go [25] studied thermoelastic field in a thin rotating functionally graded material disk with a concentric hole by using finite element method, which is assumed to have exponential variation of material properties in radial direction only. In contrast to radial variation of inhomogeneous medium, Ashrafi *et al*. [26] constructed a full three-dimensional graded boundary element model to investigated elastostatic response of functionally graded plates with circular cutouts, where the constituent material varies continuously in the transverse /longitudinal direction to an exponential law.

However, much less analytical investigations on dynamic stress concentration in inhomogeneous medium have been studied because of the complexity. Fang *et al*. [27] investigated multiple scattering and strain energy density in a semi-infinite functionally graded material with a circular cavity by means of wave functions expansion method. Martin [28] studied the far-field and near-field behaviors of antiplane shear waves in an exponentially graded half-space with a circular cavity. He then investigated the incident and scattering waves by a defect in an inhomogeneous layer sandwiched between homogeneous half spaces and discussed the misuse of the image method [29]. Recently, Yang and Gao [30] presented a complex function method for calculating the dynamic stress concentration around a circular hole in a functionally graded material plate in which the material properties are graded in the radial direction. The literatures mentioned above all focus on medium varying along one direction. As far as we known, dynamic stress concentration around a circular cavity in an inhomogeneous medium varying by a general manner has not been studied so far.

In light of this, the present paper presents an universal method to deal with dynamic stress concentration around a circular cavity in variable density two-dimensional (2D) inhomogeneous medium by using the general conformal transformation method (GCTM). Based on the complex function theory and the homogenization principle, Helmholtz equation with variable coefficient is reduced into the standard Helmholtz equation. An inhomogeneous medium has been considered as an example, where the medium density varies as a function of two spatial coordinates and the elastic modulus is a constant. The dynamic stress concentration factors (DSCF) are calculated numerically. Finally, the influence of incident wave number and inhomogeneous parameters on DSCF by a circular cavity are discussed in detail.

2. GOVERNING EQUATION

The governing equation for the propagation of elastic waves in inhomogeneous medium, in absence of the body force, is give by

$$\left[\nabla^2 + k^2(\boldsymbol{x})\right]\varphi(\boldsymbol{x}) = 0 \tag{1}$$

where $\boldsymbol{x} = (x,\ y)$ is the position vector, ∇^2 is the Laplacian, $k(\boldsymbol{x})$ is the wave number of inhomogeneous medium, and $\varphi(\boldsymbol{x})$ is the potential function.

Basing on the theory of complex functions, we introduce complex-conjugate variables as $z = x + iy$ and $\bar{z} = x - iy$ and represent $k(\boldsymbol{x})$ by the product of a reference wave number k_0 of elastic waves and a function $n(z, \bar{z})$ describing the inhomogeneity. Then Eq. (1) can be rewritten as

$$\frac{\partial^2 \varphi}{\partial z \partial \bar{z}} + \frac{1}{4}k_0^2 n^2(z, \bar{z})\,\varphi = 0 \tag{2}$$

where k_0 is the reference wave number of elastic waves and $n(z, \bar{z})$ indicates the inhomogeneity of medium.

By introducing the general conformal transformation

$$\zeta = w(z) \tag{3}$$

the inhomogeneity of medium can be expressed as [31]

$$n^2(z, \bar{z}) = \left|d\zeta / dz\right|^2 = \left|J(z, \bar{z})\right| \tag{4}$$

where J denotes the Jacobian matrix.

Substituting of Eqs. (3) and (4) into Eq. (2) yields

$$\frac{\partial^2 \varphi}{\partial \zeta \partial \bar{\zeta}} + \frac{1}{4}k_0^2 \varphi = 0 \tag{5}$$

The above equation corresponds to the standard Helmholtz equation in the mapping plane.

3. FIELDS OF DISPLACEMENTS AND STRESSES

An infinite inhomogeneous medium with a circular cavity subjected to time-harmonic elastic waves with arbitrary incident angle is shown in Fig. 1. The medium density varies as a function of the spatial positions coordinates and the elastic modulus is a constant.

In the complex plane $(\zeta, \bar{\zeta})$, the expression of scattering waves stratifying Eq. (5) can be expressed as

$$\varphi^{(s)}(\zeta, \bar{\zeta}) = \sum_{n=-\infty}^{\infty} A_n H_n^{(1)}\left(k_0 \left|\zeta\right|\right)\left\{\frac{\zeta}{\left|\zeta\right|}\right\}^n \tag{6}$$

where $H_n^{(1)}(\cdot)$ is the nth Hankel function of the first kind and A_n are coefficients of scattered waves.

In the complex plane $(z, \cdot\bar{z})$, the above equation becomes

$$\varphi^{(s)}(z, \bar{z}) = \sum_{n=-\infty}^{\infty} A_n H_n^{(1)}\left(k_0 \left|w(z)\right|\right)\left\{\frac{w(z)}{\left|w(z)\right|}\right\}^n \tag{7}$$

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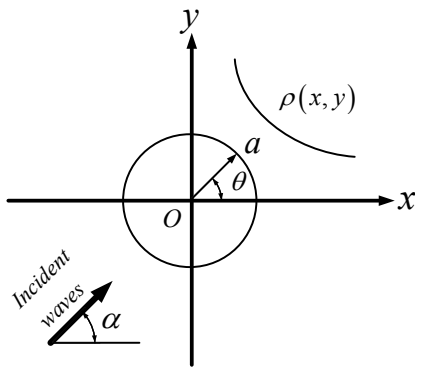


Fig. 1 Model of scattering due to a circular cavity in inhomogeneous infinite medium.

We consider the incidence of elastic waves propagating with arbitrary angle in 2D inhomogeneous medium with a circular cavity. The general form of incident waves can be written as

$$\varphi^{(i)} = \varphi_0 \exp\left[\frac{ik_0}{2}(\zeta e^{-i\alpha} + \overline{\zeta} e^{i\alpha})\right] e^{-i\omega t} \tag{8}$$

where φ_0 is the amplitude of incident waves.

Manolis and Shaw [32] compared numerical calculations of the real and imaginary components of the displacement for both reference homogeneous and inhomogeneous medium using the above Eq. (8) with the incident angle of $\alpha = 0$.

The total wave field can be expressed by the superposition of incident and scattering waves

$$\varphi = \varphi^{(i)} + \varphi^{(s)} \tag{9}$$

The components of the radial and hoop shearing stresses are given by

$$\tau_{rz} = \mu\left(\frac{\partial\varphi}{\partial z} e^{i\theta} + \frac{\partial\varphi}{\partial\overline{z}} e^{-i\theta}\right) \tag{10}$$

$$\tau_{\theta z} = i\mu\left(\frac{\partial\varphi}{\partial z} e^{i\theta} - \frac{\partial\varphi}{\partial\overline{z}} e^{-i\theta}\right) \tag{11}$$

Substituting the transformed function (3) into Eqs. (10) and (11), the components of stress becomes

$$\tau_{rz} = \mu\left(\frac{\partial\varphi}{\partial\zeta} w'(z)\, e^{i\theta} + \frac{\partial\varphi}{\partial\overline{\zeta}} \overline{w'(z)}\, e^{-i\theta}\right) \tag{12}$$

$$\tau_{\theta z} = i\mu\left(\frac{\partial\varphi}{\partial\zeta} w'(z)\, e^{i\theta} - \frac{\partial\varphi}{\partial\overline{\zeta}} \overline{w'(z)}\, e^{-i\theta}\right) \tag{13}$$

For the incident waves, the stress components are

$$\tau_{rz}^{(i)} = \frac{i}{2}\mu k_0 \varphi_0 \left[w'(z)\, e^{i(\theta-\alpha)} + \overline{w'(z)}\, e^{i(\alpha-\theta)}\right] \cdot \exp\left\{ik_0\left[w(z)\, e^{-i\alpha} + \overline{w(z)}\, e^{i\alpha}\right]/2\right\} \tag{14}$$

$$\tau_{\theta z}^{(i)} = -\frac{1}{2}\mu\varphi_0 k_0 \left[w'(z)\, e^{i(\theta-\alpha)} - \overline{w'(z)}\, e^{i(\alpha-\theta)}\right] \cdot \exp\left\{ik_0\left[w(z)\, e^{-i\alpha} + \overline{w(z)}\, e^{i\alpha}\right]/2\right\} \tag{15}$$

Analogously, for scattering waves, the stress components are

$$\tau_{rz}^{(s)} = \frac{k_0\mu}{2}\sum_{n=-\infty}^{\infty} A_n \left\{H_{n-1}^{(1)}\left(k_0|w(z)|\right)\left[\frac{w(z)}{|w(z)|}\right]^{n-1} \cdot w'(z)\, e^{i\theta} - H_{n+1}^{(1)}\left(k_0|w(z)|\right)\left[\frac{w(z)}{|w(z)|}\right]^{n+1} \cdot \overline{w'(z)}\, e^{-i\theta}\right\} \tag{16}$$

$$\tau_{\theta z}^{(s)} = \frac{k_0\mu i}{2}\sum_{n=-\infty}^{\infty} A_n \left\{H_{n-1}^{(1)}\left(k_0|w(z)|\right)\left[\frac{w(z)}{|w(z)|}\right]^{n-1} \cdot w'(z)\, e^{i\theta} + H_{n+1}^{(1)}\left(k_0|w(z)|\right)\left[\frac{w(z)}{|w(z)|}\right]^{n+1} \cdot \overline{w'(z)}\, e^{-i\theta}\right\} \tag{17}$$

4. BOUNDARY CONDITION AND DYNAMIC STRESS CONCENTRATION FACTOR (DSCF)

Without loss of generality, the case that the circular cavity is free of traction is investigated. The boundary condition is that the radial shear stress is equal to zero, *i.e.*

$$\tau_{rz} = \tau_{rz}^{(i)} + \tau_{rz}^{(s)} = 0\,;\ |z| = a \tag{18}$$

Submitting Eqs. (14) and (16) into Eq. (18), we obtain

$$\sum_{n=-\infty}^{\infty} A_n \Theta_n = \Theta \tag{19}$$

where

$$\Theta_n = H_{n-1}^{(1)}\left(k_0|w(z)|\right)\left[\frac{w(z)}{|w(z)|}\right]^{n-1} \cdot w'(z)\, e^{i\theta} - H_{n+1}^{(1)}\left(k_0|w(z)|\right)\left[\frac{w(z)}{|w(z)|}\right]^{n+1} \cdot \overline{w'(z)}\, e^{-i\theta} \tag{20}$$

$$\Theta = i\varphi_0\left[w'(z)\, e^{i(\theta-\alpha)} + \overline{w'(z)}\, e^{i(\alpha-\theta)}\right]\exp\left[ik_0(\zeta e^{-i\alpha} + \overline{\zeta} e^{i\alpha})/2\right] \tag{21}$$

Multiplying both sides of Eq. (19) with $e^{-im\theta}$ and integrating on the interval $(-\pi, \pi)$, we find A_n defined infinity algebraic equation set as follows

$$\sum_{n=-\infty}^{\infty} A_n \Theta_{mn} = \Theta_m \quad m = n = 0\,,\ \pm 1\,,\ \pm 2\,,\ldots \tag{22}$$

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where

$$\Theta_{mn}=\frac{1}{2\pi}\int_{-\pi}^{\pi}\Theta_n e^{-im\theta}d\theta\ ,\ \ \Theta_m=\frac{1}{2\pi}\int_{-\pi}^{\pi}\Theta e^{-im\theta}d\theta \quad (23)$$

The present paper focuses on the dynamic stress concentration in the vicinity of the cavity, since the maximum occurs in this region depending on the cavity. The DSCF is defined as the ratio of the stress $\tau_{\theta z}$ to the stress τ_0. For the harmonic elastic waves, the DSCF is given by

$$\tau_{\theta z}^{*}=\left|\tau_{\theta z}/\tau_0\right| \quad (24)$$

where

$$\tau_0=\mu k_0\varphi_0/2 \quad (25)$$

Substituting Eqs. (15) and (17) into Eq. (24), the final expression is obtained as follows

$$\begin{aligned}\tau_{\theta z}^{*}=&-\left[w'(z)\,e^{i(\theta-\alpha)}-\overline{w'(z)}\,e^{i(\alpha-\theta)}\right]\\&\cdot\exp\left\{ik_0\left[w(z)\,e^{-i\alpha}+\overline{w(z)}\,e^{i\alpha}\right]/2\right\}\\&+\frac{i}{\varphi_0}\sum_{n=-\infty}^{\infty}A_n\left\{H_{n-1}^{(1)}\left(k_0\left|w(z)\right|\right)\left[\frac{w(z)}{|w(z)|}\right]^{n-1}\cdot w'(z)\,e^{i\theta}\right.\\&\left.+H_{n+1}^{(1)}\left(k_0\left|w(z)\right|\right)\left[\frac{w(z)}{|w(z)|}\right]^{n+1}\cdot\overline{w'(z)}\,e^{-i\theta}\right\}\end{aligned} \quad (26)$$

5. STATEMENT OF THE INHOMOGENEITY OF MEDIUM

For purpose of verifying the accurate of the present approach, a special-type inhomogoneity with combines exponential with polynomial terms is introduced [32]

$$\zeta=w(z)=z+\beta_1 e^{\beta_2 z} \quad (27)$$

where β_1 and β_2 are the inhomogeneous parameters.

Substituting Eq. (27) into (4), the corresponding relation can be obtained as follows

$$n^2(z,\bar{z})=\left|d\zeta/dz\right|^2=\left|J(z,\bar{z})\right|=\left|1+\beta_1\beta_2 e^{\beta_2 z}\right|^2 \quad (28)$$

The corresponding wave number k in the actual inhomogeneous medium is given by

$$k^2(x,y)=k_0^2(1+\beta_1^2\beta_2^2e^{2\beta_2 x}+2\beta_1\beta_2e^{\beta_2 x}\cos\beta_2 y) \quad (29)$$

Here, we focus on shear waves. $k(x,y)=\omega/c(x,y)$, $c(x,y)=\sqrt{\mu/\rho(x,y)}$ is the velocity of shear wave. The corresponding variable density is written as

$$\rho(x,y)=\rho_0(1+\beta_1^2\beta_2^2e^{2\beta_2 x}+2\beta_1\beta_2e^{\beta_2 x}\cos\beta_2 y) \quad (30)$$

where ρ_0 is the reference density. In [32] the solution for incident wave can be obtained by an integral over the linear combination of the separated solution. The three-dimensional displacement graphs were plotted by using the boundary elements method.

6. COMPUTATION PROCEDURE

In this section, the DSCF can be produced by substituting Eq. (27) into (26)

$$\begin{aligned}\tau_{\theta z}^{*}=&-\left[(1+\beta_1\beta_2e^{\beta_2 z})\,e^{i(\theta-\alpha)}-(1+\beta_1\beta_2e^{\beta_2\bar{z}})\,e^{i(\alpha-\theta)}\right]\\&\cdot\exp\left\{ik_0\left[(z+\beta_1e^{\beta_2 z})\,e^{-i\alpha}+(\bar{z}+\beta_1e^{\beta_2\bar{z}})\,e^{i\alpha}\right]/2\right\}\\&+\frac{i}{\varphi_0}\sum_{n=-\infty}^{\infty}A_n\left\{H_{n-1}^{(1)}\left(k_0|z+\beta_1e^{\beta_2 z}|\right)\left[\frac{z+\beta_1e^{\beta_2 z}}{|z+\beta_1e^{\beta_2 z}|}\right]^{n-1}\cdot(1+\beta_1\beta_2e^{\beta_2 z})\,e^{i\theta}\right.\\&\left.+H_{n+1}^{(1)}\left(k_0|z+\beta_1e^{\beta_2 z}|\right)\left[\frac{z+\beta_1e^{\beta_2 z}}{|z+\beta_1e^{\beta_2 z}|}\right]^{n+1}\cdot(1+\beta_1\beta_2e^{\beta_2\bar{z}})\,e^{-i\theta}\right\}\end{aligned} \quad (31)$$

To analyze the dynamic stress concentration factors (DSCF) around the circular cavity in inhomogeneous medium, numerical calculations were carried out by adopting the following dimensionless variables: The reference wave number is k_0a, the inhomogeneous parameters are β_1 and β_2a. The results are provided by truncating the set of infinite algebraic equations. Obviously, the accuracy of the numerical results is dependent on the choice of n. Here, we truncate the infinite matrix to $n=8$, where the accuracy can arrived at 10^{-12}.

To validate the present approach, the suggested approach and numerical scheme are checked by comparing our solution with the elastodynamic counterpart reported by Pao and Mow [1]. Figure 2 shows that the distribution of DSCF around the circular cavity under the degeneration case of inhomogeneous medium with inhomogeneous parameter $\beta_1=0$ under the horizontal incidence, which corresponds to the homogeneous medium. From the graphics we can see that the results in this paper can well coincide with the reference above.

For $\beta_1=0.1$, $\beta_2a=-1.5$ and $\alpha=0$, Fig. 3(A) displays the DSCF along the boundary of a circular cavity of $a=1.0$ with different dimensionless wave numbers, the solid line representing $k_0a=0.1$, the dashed line representing $k_0a=1.0$, and the dotted line representing $k_0a=2.0$. It can be seen that the maximum of the DSCF increases along the y-axis. As inhomogeneous parameter β_2a changes to 1.5, Fig. 3(B) shows that the DSCF increase along the positive direction of x-axis significantly, which is the increasing direction of the density. Meanwhile, as we seen form Fig. 3, the distributions of dynamic stress concentration are symmetric with respect to the x-axis. As the incident angle α changes to $\pi/2$, Fig. 4 indicates the influence of wave number k_0a on DSCF in the medium around the circular cavity for the case of $\beta_1=0.1$ (**A.** $\beta_2a=-1.5$, **B.** $\beta_2a=1.5$). It reveals that the distributions of DSCF have a trend of shifting towards the increasing direction of the density.

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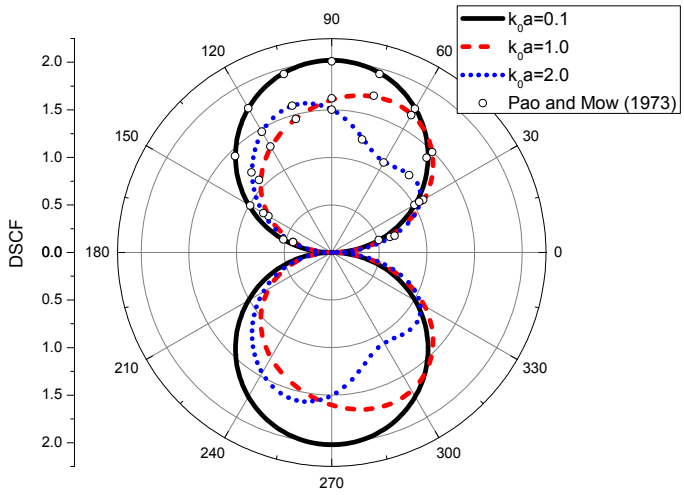
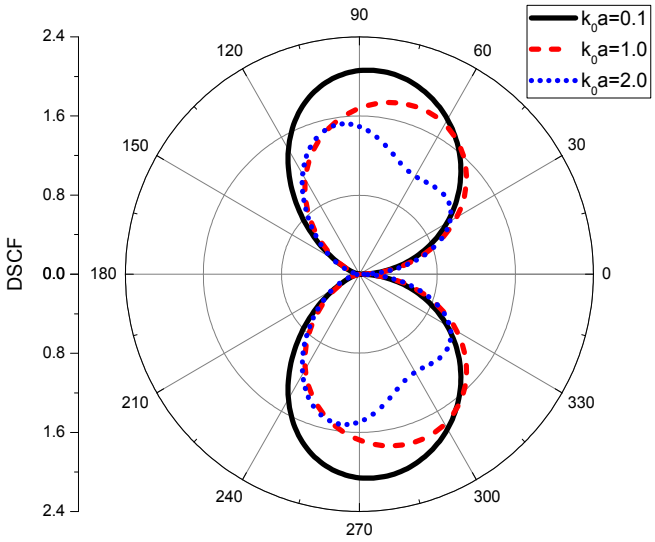
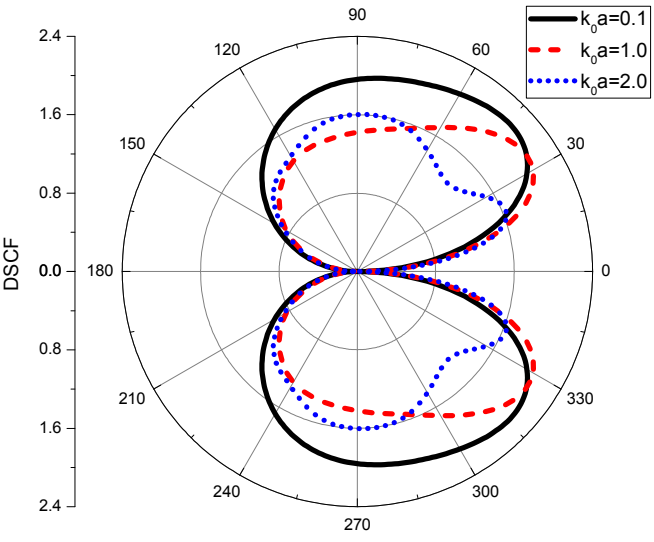


Fig. 2 Verification of DSCF by the degeneration procedure ($\beta_1 = 0$, $\alpha = 0$).

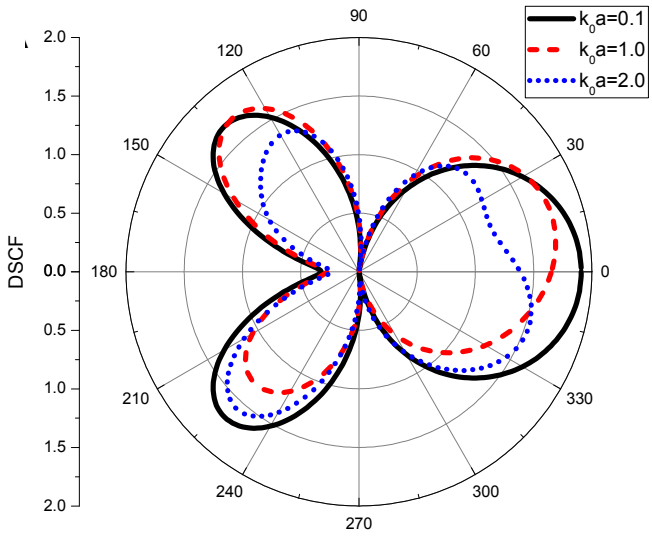


(A)

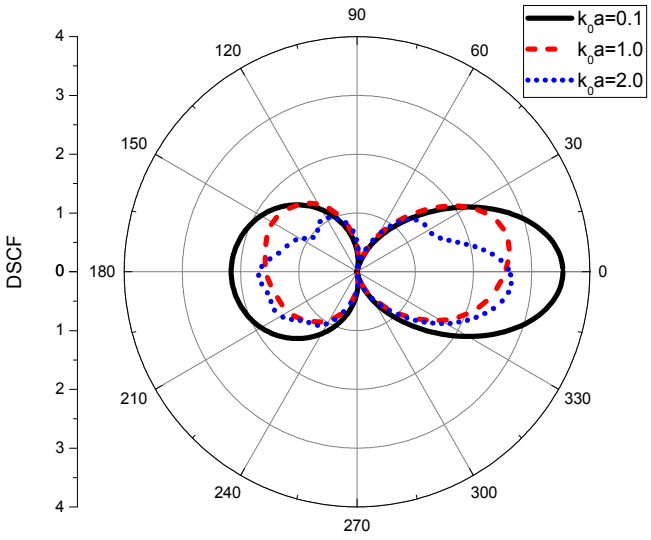


(B)

Fig. 3 The influence of k_0a on DSCF in the medium around the circular cavity with $\beta_1 = 0.1$, $\alpha = 0$. (**A.** $\beta_2 a = -1.5$, **B.** $\beta_2 a = 1.5$).



(A)



(B)

Fig. 4 The influence of k_0a on DSCF in the medium around the circular cavity with $\beta_1 = 0.1$, $\alpha = \pi/2$. (**A.** $\beta_2 a = -1.5$, **B.** $\beta_2 a = 1.5$).

As shown in Figs. 5 ~ 6, the DSCF at the position of $\theta = \pi/2$ with inhomogeneous parameters: $\beta_1 = 0.4$ and $\beta_2 a = 1.0$ under the incident angle with $\alpha = 0$ and $\alpha = \pi/2$ are illustrated, respectively. The variation of dimensionless wave number k_0a has a great effect on the DSCF with inhomogeneous parameters β_1 or $\beta_2 a$ increasing. It indicates that the resulting DSCF appears to be very sensitive upon the inhomogeneous parameter β_1 and $\beta_2 a$. It is interesting to note that the fluctuation of the DSCF appears successively with k_0a increasing for different $\beta_2 a$. Note that the greater the inhomogeneous parameter is, the more evident the fluctuation is.

Figure 7 displays the influence of $\beta_2 a$ on DSCF in the medium around the circular cavity with $ka = 0.1$ and $\beta_1 = 0.1$ (**A.** $\alpha = 0$, **B.** $\alpha = \pi/2$). It can be seen that the maximum of DSCF varies significantly and the location trends to the increasing direction of the density along the x-axis. Obviously, the presence of inhomogeneous

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parameter $\beta_2 a$ generates a pronounced effect in the resulting variation of the DSCF along the increasing direction of the density. Figure 8 depicts the influence of $\beta_2 a$ on DSCF in inhomogeneous medium at $\theta = \pi/2$ with $\beta_1 a = 0.1$ (**A.** $\alpha = 0$, **B.** $\alpha = \pi/2$). The results show that a weak effect is acted on the DSCF with the wave number $k_0 a$ increasing for the incident angle with $\alpha = 0$. While the incident waves propagates vertically, the DSCF increase as inhomogeneous parameter $\beta_2 a$ increasing. Meanwhile, the greater the inhomogeneous parameter is, the more evident the fluctuation is.

The influence of $\beta_2 a$ on DSCF in the medium around the circular cavity with $k_0 a = 2.0$ and $\beta_2 a = 0.5$ (**A.** $\alpha = 0$, **B.** $\alpha = \pi/2$) is shown in Fig. 9 for different inhomogeneous parameter, the solid line representing $\beta_1 = 0$, the dashed line representing $\beta_1 = 0.2$, and the dotted line representing $\beta_1 = -0.2$. As illustrated in Figs. (A) and (B), the maximum of the DSCF occurs in the illuminated region. Moreover, an increase for the maximum of the DSCF is accompanied by a significant increase with inhomogeneous parameter β_1 increasing from the negative to positive value slightly. It reveals that the inhomogeneous parameter has great effects on the distribution of DSCF.

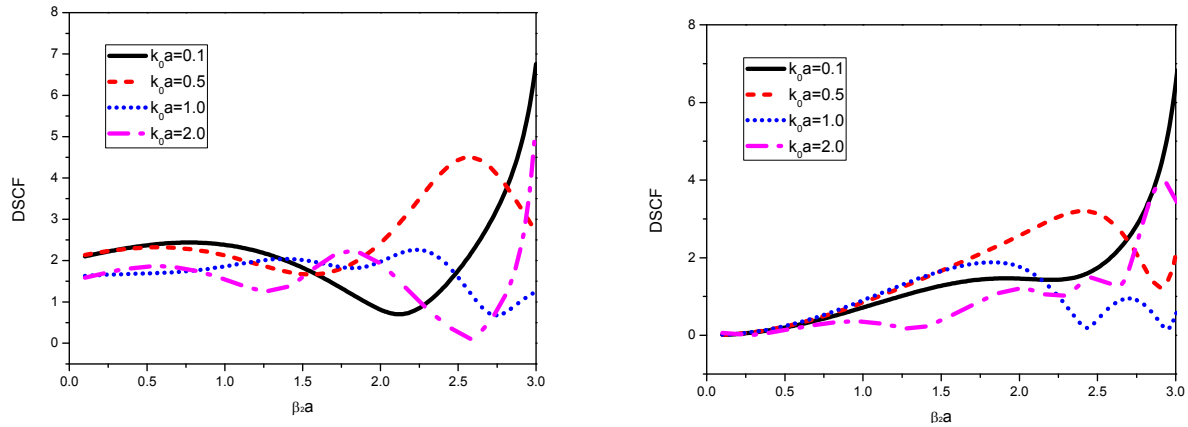


Fig. 5 The influence of $k_0 a$ on DSCF in inhomogeneous medium at $\theta = \pi/2$ with $\beta_1 = 0.4$ (**A.** $\alpha = 0$, **B.** $\alpha = \pi/2$).

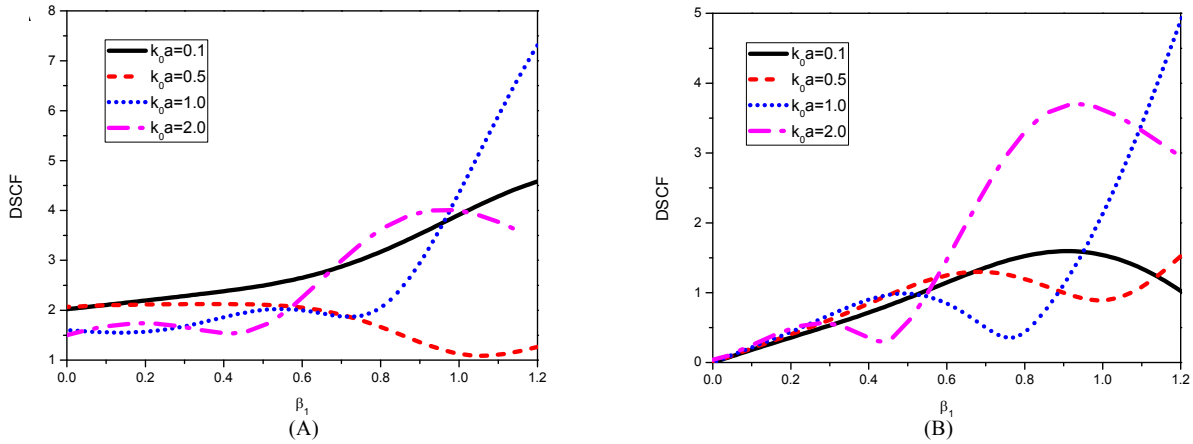


Fig. 6 The influence of $k_0 a$ on DSCF in inhomogeneous medium at $\theta = \pi/2$ with $\beta_2 a = 1.0$ (**A.** $\alpha = 0$, **B.** $\alpha = \pi/2$).

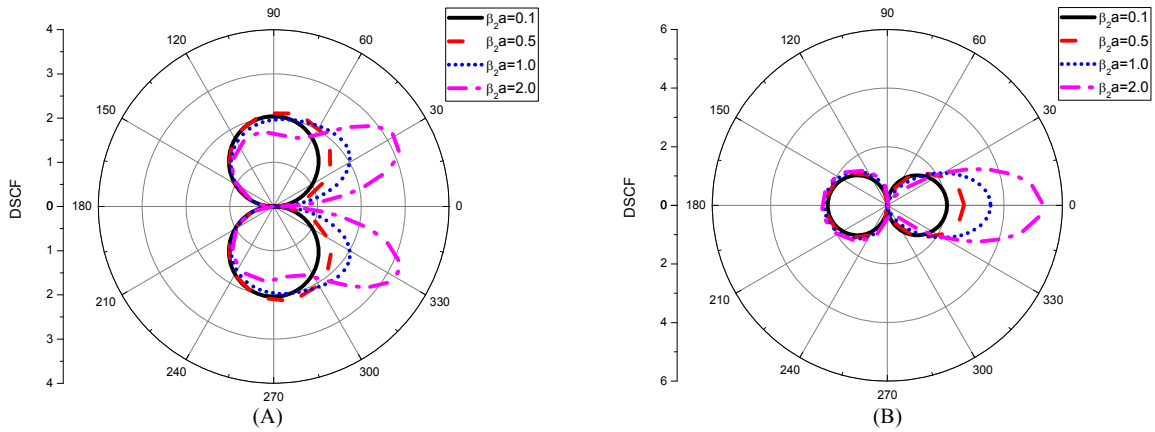


Fig. 7 The influence of $\beta_2 a$ on DSCF in the medium around the circular cavity with $k_0 a = 0.1$, $\beta_1 = 0.1$ (**A.** $\alpha = 0$, **B.** $\alpha = \pi/2$).

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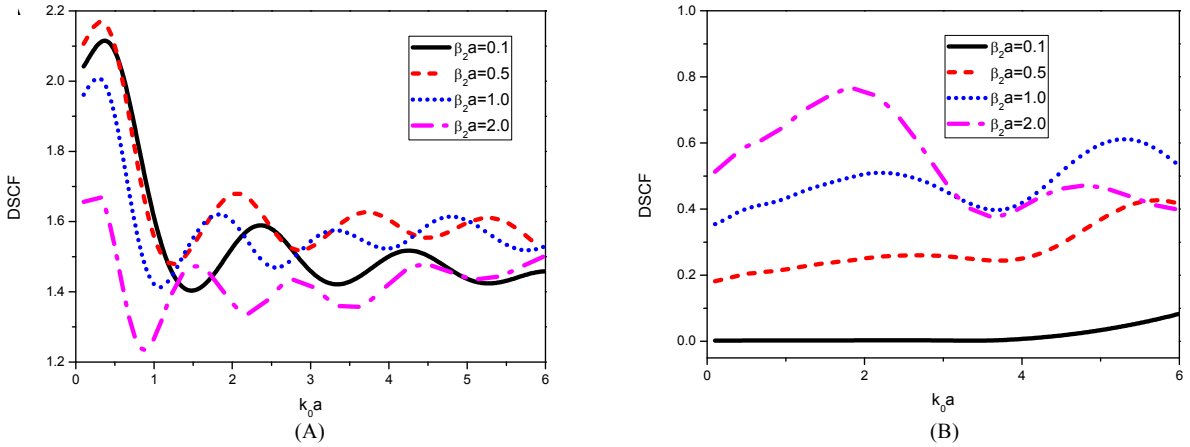


Fig. 8 The influence of $\beta_2 a$ on DSCF in inhomogeneous medium at $\theta = \pi/2$ with $\beta_1 = 0.1$ (**A.** $\alpha = 0$, **B.** $\alpha = \pi/2$).

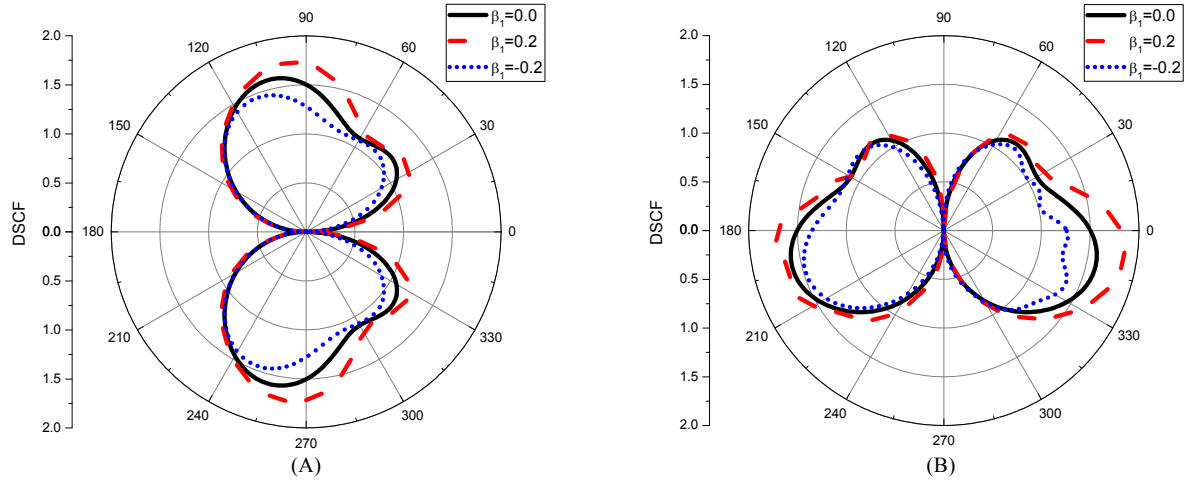


Fig. 9 The influence of β_1 on DSCF in the medium around the circular cavity with $k_0 a = 2.0$, $\beta_2 a = 0.5$ (**A.** $\alpha = 0$, **B.** $\alpha = \pi/2$).

7. CONCLUSIONS

Dynamic stress analysis around a circular cavity embedded in two-dimensional (2D) inhomogeneous medium is investigated by using the complex function method in this paper. Based on the homogenization principle, the variable coefficient wave equation is converted to the standard Helmholtz equation by the general conformal transformation method. An inhomogeneous medium with density variation has been considered as an example to simulate the distribution of the dynamic stress concentration in the neighborhood of the circular cavity. Numerical results show shows that the wave number, as well as different inhomogeneous parameters have considerable effects on the dynamic stress concentration factor (DSCF), and the extent to which this occurs depends considerably on the physical-mechanical characteristics of the medium. Meanwhile, the results can provide a reference for investigating stress concentration around the arbitrary cavity (inclusion) in a functionally graded palate. It is noteworthy in nondestructive inspection of damaged material and earthquake resistance of engineering structure under the ground.

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REFERENCES

1. Pao, Y. H. and Mow, C. C., *Diffraction of Elastic Waves and Dynamic Stress Concentrations*, Crane and Russak, New York (1973).
2. Baron, M. L. and Matthews, A. T., "Diffraction of a Pressure Wave by a Cylindrical Cavity in an Elastic Medium," *ASME Journal of Applied Mechanics*, **28**, pp. 347–354 (1961).
3. Baron, M. L. and Matthews, A. T., "Diffraction of a Shear Wave by a Cylindrical Cavity in an Elastic Medium," *ASME Journal of Applied Mechanics*, **29**,

https://doi.org/10.1017/jmech.2016.7 Published online by Cambridge University Press

pp. 205–207 (1962).
4. Mow, C. C. and Mente, L. J., "Dynamic Stresses and Displacements Around Cylindrical Discontinuities Due to Plane Harmonic Shear Waves," *ASME Journal of Applied Mechanics*, **30**, pp. 598–604 (1962).
5. Moodie, T. B. and Barclay, D. W., "Wave Propagation from a Cylindrical Cavity," *Acta Mechanica*, **27**, pp. 103–120 (1977).
6. Luco, J. E. and De Barros, F. C. P., "Dynamic Displacements and Stresses in the Vicinity of a Cylindrical Cavity Embedded in a Half-space," *Earthquake Engineering and Structural Dynamics*, **23**, pp. 321–340 (1994).
7. Coşkun, İ., Engin, H. and Özmutlu, A., "Dynamic Stress and Displacement in an Elastic Half-space with a Cylindrical Cavity," *Shock and Vibration*, **18**, pp. 827–838 (2011).
8. Liao, W. I. and Teng, T. J., "Dynamic Stress Concentration of a Cylindrical Cavity in Half-Plane Excited by Standing Goodier-Bishop Stress Wave," *Journal of Mechanics*, **28**, pp. 269–277 (2012).
9. Shi, W. P., Liu, D. K., Song, Y. T., Chu, J. L. and Hu, A. Q., "Scattering of Circular Cavity in Right-angle Planar Space to Steady SH-wave," *Applied Mathematics and Mechanics (English Edition)*, **27**, pp. 1619–1626 (2006).
10. Liu, G., Ji, B. H., Chen, H. T. and Liu, D. K., "Antiplane Harmonic Elastodynamic Stress Analysis of an Infinite Wedge with a Circular Cavity," *ASME Journal of Applied Mechanics*, **76**, pp. 061008.1–061008.9 (2009).
11. Pao, Y. H., "Dynamical Stress Concentrations in an Elastic Plate," *ASME Journal of Applied Mechanics*, **29**, pp. 299–305 (1962).
12. Mccoy, J. J., "On Dynamic Stress Concentrations in Elastic Plates," *International Journal of Solids and Structures*, **3**, pp. 399–411 (1967).
13. Gao, S. W., Meng, Q. Y., Wang, B. L. and Ma, X. R., "BEM for Diffraction of Elastic Wave and Dynamic Stress Concentrations in Thin Plate with a Circular Hole," *Journal of Harbin Institute of Technology*, **6**, pp. 40–46 (1999).
14. Ewing, W. M., Jardetzky W. S. and Press, F., *Elastic Waves in Layered Media*, McGraw-Hill Book Co., New York (1957).
15. Brekhovskikh, L. M., *Waves in Layered Media*, Academic Press, New York (1960).
16. Brekhovskikh, L. M. and Godin, O. A., *Acoustics of Layered Meida 1. Plane and Quasi-Plane Waves*, Spring-Verlag Berlin Heidelberg, New York (1998).
17. Zhang, H. L., *Theoretical Acoustics (Revised Edition)*, Higher Education Press, Beijing (2012).
18. Butrak, I. O., Kil'nitskaya, T. I. and Mikhas'kiv, V. V., "The Scattering of a Harmonic Elastic Wave by a Volume Inclusion with a Thin Interlayer," *Journal of Applied Mathematics and Mechanics*, **76**, pp. 342–347 (2012).
19. Cieszko, M., Drelich R. and Pakula, M., "Acoustic Wave Propagation in Equivalent Fluid Macroscopically Inhomogeneous Materials," *The Journal of the Acoustical Society of America*, **132**, pp. 2970–2977 (2012).
20. Yang, Q., Gao C. F. and Chen, W., "Stress Analysis of a Functional Graded Material Plate with a Circular Hole," *Archive of Applied Mechanics*, **80**, pp. 895–907 (2010).
21. Zhang, X. Z., Kitipornchai, S., Liew, K. M., Lim, C. W. and Peng, L. X., "Thermal Stresses around a Circular Hole in a Functionally Graded Plate," *Journal of Thermal Stresses*, **26**, pp. 379–390 (2003).
22. Sburlati, R., "Stress Concentration Factor due to a Functionally Graded Ring around a Hole in an Isotropic Plate," *International Journal of Solids and Structures*, **50**, pp. 3649–3658 (2013).
23. Mohammadi, M., Dryden, J. R. and Jiang, L., "Stress Concentration around a Hole in a Radially Inhomogeneous Plate," *International Journal of Solids and Structures*, **48**, pp. 483–491 (2011).
24. Kubair, D. V. and Bhanu-Chandar, B., "Stress Concentration Factor due to a Circular Hole in Functionally Graded Panels under Uniaxial Tension," *International Journal of Mechanical Sciences*, **50**, pp. 732–742 (2008).
25. Afsar, A. M. and Go, J., "Finite Element Analysis of Thermoelastic Field in a Rotating FGM Circular Disk," *Applied Mathematical Modelling*, **34**, pp. 3309–3320 (2010).
26. Ashrafi, H., Asemi, K. and Shariyat, M., "A Three-dimensional Boundary Element Stress and Bending Analysis of Transversely/Longitudinally Graded Plates with Circular Cutouts under Biaxial Loading," *European Journal of Mechanics—A/Solids*, **42**, pp. 344–357 (2013).
27. Fang, X. Q., Hu, C. and Du, S. Y., "Strain Energy Density of a Circular Cavity Buried in Semi-infinite Functionally Graded Materials Subjected to Shear Waves," *Theoretical and Applied Fracture Mechanics*, **46**, pp. 166–174 (2006).
28. Martin, P. A., "Scattering by a Cavity in an Exponentially Graded Half-space," *Journal of Applied Mechanics*, **76**, pp. 31009-1–31009-4 (2009).
29. Martin, P. A., "Scattering by Eefects in an Exponentially Graded Layer and Misuse of the Method of Images," *International Journal of Solids and Structures*, **48**, pp. 2164–2166. (2011).
30. Yang, Q. and Gao, C. F., "Dynamic Stress Analysis of a Functionally Graded Material Plate with a Circular Hole," *Meccanica*, **48**, pp. 91–101 (2013).
31. Ramo, S., Whinnery, J. R. and Duzer, T. V., *Field and Waves in Communication Electronics*, Third Edition, Wiley, New York (2007).
32. Manolis, G. D. and Shaw, R. P., "Fundamental Solutions for Variable Density Two-dimensional Elastodynamic Problems," *Engineering Analysis with Boundary Elements*, **24**, pp. 739–750 (2000).

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