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The ISLAND module for computing magnetic island widths in tokamaks

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Abstract. The ISLAND module in the National Transport Code Collaboration (NTCC) Module Library computes the saturated widths of multiple magnetic islands due to neoclassical tearing modes by implementing a quasi-linear model that includes the effects of arbitrary aspect ratio, cross-sectional shape, and plasma beta. This paper presents a significantly improved version of the module available at the NTCC Library. The model was updated to include the distortion of islands caused by the radial variation of the perturbed magnetic field near the mode rational surface. The computer algorithm implemented by the module was redesigned to increase its speed, reliability and accuracy. The new module has been successfully interfaced with the BALDUR integrated modeling code, where it is used to compute the saturated widths of multiple islands in tokamak discharges.

1. Introduction

A new version of the ISLAND module [1, 2] to compute the saturated widths of magnetic islands is described in this work. In Sec. 2, an extension of the model is derived to compute a distortion in the shape of the magnetic island. Section 3 details an improved computer algorithm, that utilizes the discontinuity in the radial perturbed magnetic field at the mode rational surface to obtain Δ' as a function of island width for each helical harmonic. An iteration based on a simplified form of the Rutherford equation for each harmonic is then used to vary the island widths towards convergence, while avoiding undesired higher-order eigenfunctions of the perturbed magnetic field. The new module is efficient and robust enough to be used within a predictive integrated modelling code. To demonstrate this capability, the results of simulations of magnetic islands using the BALDUR code are examined in Sec. 4. The effect of the bootstrap current and the influence of magnetic islands interacting with each other are also illustrated using prescribed plasma equilibrium and profiles.

2. Extension of the model to compute a distortion of the magnetic islands

Consider a magnetic flux stream function ψ that is uniform along magnetic field lines such that $\mathbf{B}\cdot\nabla\psi = 0$. If ψ is a smooth function near the mode rational surface,

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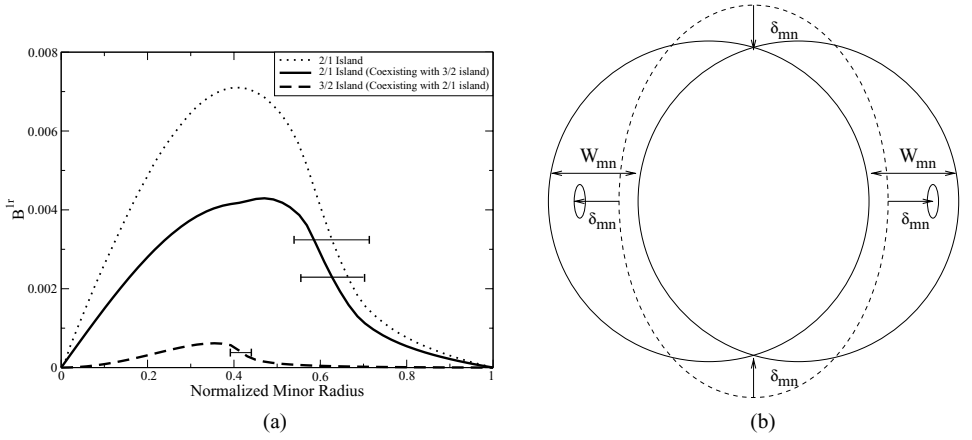


Figure 1. (a) Profile of the perturbed contra-variant radial magnetic field B^{1x} for a 2/1 island without a coexisting 3/2 island (dotted curve), B^{1x} for the 2/1 island (solid curve) and 3/2 island (dashed curve) when the 2/1 and 3/2 islands coexist. The perturbed fields are obtained using the same plasma profiles. The horizontal bars indicate the widths of the islands. The width of the 2/1 island without a coexisting 3/2 island (indicated by the upper horizontal bar) is larger than the width of the 2/1 island when it coexists with the 3/2 island (middle horizontal bar). The lower bar (on the dashed curve for the 3/2 island) indicates the width of the 3/2 island. (b) Distortion of the magnetic island due to the variation of the radial magnetic perturbation.

its Taylor series expansion can be used to determine the shape of the magnetic island. The island width is obtained from the radial magnetic perturbation B^{1x}_{mn} at the mode rational surface, x_{mn} where $q = m/n$ for poloidal mode number m and toroidal mode number n. The distortion of the magnetic island can be obtained by considering a higher-order term of the Taylor series that accounts for the radial variation of B^{1x}_{mn} across the island, which is illustrated in Fig. 1(a). It is found that the X-point is displaced from the mode rational surface by the quantity

$$\delta_{mn} = \left[\frac{dB^{1x}_{mn}}{dx} \Big/ \left(nB^{0\theta} \frac{dq}{dx} \right) \right]_{x_{mn}}. \tag{2.1}$$

Similarly, the O-point is shifted from the mode rational surface by the same distance, although in the opposite direction. The slope of the radial magnetic perturbation near the island, dB^{1x}_{mn}/dx, is usually negative, which implies that the X-point is displaced towards the magnetic axis, while the O-point is displaced towards the edge of the plasma. This is illustrated in Fig. 1(b). The value of ψ at the X-point can be equated with the value of ψ at the widest part of the island to compute the normalized radial locations of the outer and inner edges of the island.

$$\left.\begin{matrix} x_{\text{outer}} \\ x_{\text{inner}} \end{matrix}\right\} = x_{mn} \pm 2 \sqrt{\left. \frac{B^{1x}_{mn}}{nB^{0\theta}(dq/dx)} \right|_{x_{mn}}} - \delta_{mn}. \tag{2.2}$$

In this expression, the boundaries of widest part of the magnetic island are shifted away from the magnetic axis by $|\delta_{mn}|$. The resulting displacement of the local flattening of the pressure and current density profiles enhances the effect of the inboard gradients that drive the magnetic islands, which generally causes the saturated width of the islands to increase as a result of the distortion.

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3. Improved computer algorithm implemented in the ISLAND module

The computer algorithm of the ISLAND module has been modified relative to the previous work [2]. In the new version of the module, the pair of ordinary differential equations for each individual harmonic is normally integrated from the magnetic axis outwards to the mode rational surface, and from the outer boundary inwards to the mode rational surface. As a result of this change the ISLAND module is more accurate and it runs faster.

3.1. Pseudo-Rutherford equation

For each helical harmonic, the revised ISLAND module can be applied to compute the value of Δ', which is proportional to the discontinuity of the derivative of the perturbed radial magnetic field at the mode rational surface

$$\Delta'_{mn} = \lim_{\epsilon \to 0} \left[\frac{1}{B^{1x}_{mn}} \frac{dB^{1x}_{mn}}{dx} \right]^{x=x_{mn}+\epsilon}_{x=x_{mn}-\epsilon}. \tag{3.1}$$

The widths of the magnetic islands are then varied until all of the magnetic field perturbation harmonics and their first derivatives are continuous throughout the plasma, which implies that $\Delta'_{mn} = 0$. For the dominant form of the Rutherford equation

$$\frac{dW_{mn}}{dt} = \frac{x^2_{mn}}{\tau_{\mathrm{R}}} \Delta'_{mn}. \tag{3.2}$$

$\Delta'_{mn} = 0$ corresponds to the saturation condition $dW_{mn}/dt = 0$. When the ISLAND module is used within an integrated modeling code, (3.1) and (3.2) can be used to adjust the width of the magnetic islands incrementally each time the ISLAND module is called. The resistive timescale τ_{R}, the time interval, and the number of pseudo-time iterations can be specified at runtime. The algorithm implemented in the new ISLAND code has been designed to avoid higher-order eigenfunctions in which the radial magnetic field perturbation, B^{1x}_{mn}, has one or more zero crossings within the plasma.

4. Results

The effects of the bootstrap current and the interaction between magnetic islands are examined in Sec. 4.1 using a simple concentric elliptical equilibrium. An example of results from the ISLAND module in the BALDUR code, including the effects of sawtooth oscillations, is shown in Sec. 4.2.

4.1. Effect of the bootstrap current and the interaction between multiple islands

The suppression of the bootstrap current due to the flat spot in the pressure profile within each magnetic island has the effect of enhancing the island width. This phenomenon constitutes the dominant influence in what is referred to as neoclassical tearing modes. The effect of the bootstrap current for coexisting islands is illustrated in Fig. 2. The regions of the plasma covered by the islands are shown as vertical bars, while the horizontal axis corresponds to the ratio of bootstrap current density to the total current density at $r/a = 0.5$. As the relative bootstrap current density is increased, the width of the 2/1 magnetic island nearly doubles while the width of the 3/2 magnetic island more than triples.

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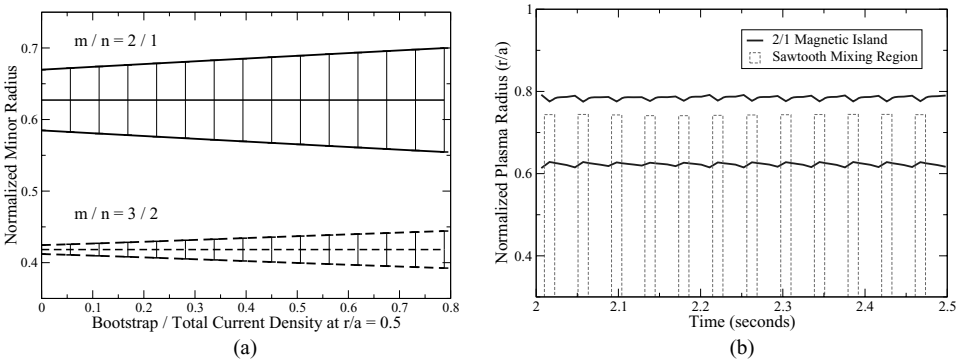


Figure 2. (a) Normalized radius at the inner edge, center and outer edge of the 2/1 and 3/2 magnetic islands as a function of bootstrap current density divided by total current density evaluated at $r/a = 0.5$. (b) The inner and outer edges of a 2/1 magnetic island are shown as a function of time in MAST. The sawtooth mixing region is indicated by hollow bars.

Magnetic islands are sensitive to the gradient of the current density near their inboard edges. Consequently, flat spots caused by an island in the current density can affect the saturated width of other islands. In general large islands that are closer to the magnetic axis tend to decrease the width of islands that are further away from the axis. In Fig. 1, for instance, the width of a 2/1 island is reduced by the presence of a 3/2 island. The effect of the flat spot in the current density produced by the 3/2 island can be seen as an undulation in the radial magnetic field perturbation curve for the 2/1 island near $r/a = 0.4$. The region covered by each island is shown as a horizontal bar on each corresponding curve.

4.2. Saturated island widths in tokamaks

The ISLAND module is used in the BALDUR code to compute the widths of saturated magnetic islands in tokamak discharges. The pressure and current density profiles are computed using transport equations, while the unperturbed axisymmetric flux surface shapes are computed using equilibrium force balance. Starting with a seed island, an initial converged island width is obtained. The converged island widths for a given time step are subsequently used as a starting point for the next time step in BALDUR. In these simulations, the ISLAND code internally computes the flattening of the pressure and current density profiles that allows a saturated island solution, but the plasma profiles used by the integrated modeling code remain unaltered.

The inner and outer edges of a saturated 2/1 island as a function of time in a MAST discharge, along with the sawtooth mixing region, are shown in Fig. 2. In this particular case, the sawtooth mixing region extends deep into the island. The effect of the sawtooth crashes upon the background profiles causes the island width to oscillate in time. The island width increases before the sawtooth crash, and then decreases immediately after it.

5. Conclusions

An improved version of the ISLAND module that includes the distortion of the magnetic islands is developed. The bootstrap current is shown to enhance the

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widths of the saturated magnetic islands. In the case of tokamaks where a high fraction of the current density is provided by the bootstrap current, this could lead to particularly wide islands. The magnetic islands are also shown to interact with one another (see, for example, Fig. 1) through the local flattening of the pressure and current density profiles. The ISLAND module can be used to compute multiple island widths within predictive integrated modeling codes, where different physical phenomena can interact with one another. The enhanced transport due to the magnetic islands is expected to cause additional effects when the ISLAND module is allowed to alter the background profiles. Consequently, future work will compute the saturated widths of magnetic islands in self-consistent discharges of DIIID and JET, and then extend these simulations to future reactors. Preliminary simulations of ITER, for example, have shown significant saturated island widths for the 2/1 and 3/2 harmonics that agree with estimates presented at the *2002 Snowmass Fusion Forum*.

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