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Ion-dust streaming instability in a radially bounded dusty plasma

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Abstract. The effect of radial boundaries on the ion-dust streaming instability is investigated. Possible applications of our work to certain low-temperature dusty plasma experiments are discussed.

The effect of radial boundaries on dust-acoustic (DA) [1] and dust-ion-acoustic [2] waves in a collisional dusty plasma has been investigated previously [3] (see also [4, 5]). In a cylindrical geometry, the radial boundaries lead to a frequency cut-off in the DA wave dispersion, i.e. the wave can have finite frequency as the axial wavenumber $k \to 0$ (see also [6]). Recently, this behavior was reported in a dusty plasma wave experiment [7]. Since the DA wave can be driven by ions streaming relative to the dust grains [8, 9], it is of interest to investigate the conditions under which radial boundaries may affect the ion-dust streaming instability. Thus, we extend our previous analysis of the DA wave in a radially bounded collisional dusty plasma [3] accounting for the streaming of charged particles in an electric field. We note that recently, an analysis of an ion streaming instability in a radially bounded collisionless quantum dusty plasma has been reported [10].

Our dusty plasma model is weakly ionized and contains electrons, singly charged ions, and negatively charged dust grains of the charge state Z_d (i.e. the dust charge $q_d = -Z_d e$). In the equilibrium, the quasineutrality condition reads

$$n_{e0} + Z_d n_{d0} = n_{i0}, \tag{1}$$

where n_{j0} is the equilibrium density of the plasma species j, where $j = e, i, d$ stands for the electrons, ions, and dust, respectively. The plasma has a cylindrical geometry with coordinates r, θ, and z. There is an electric field $\mathbf{E}_0 = E_0\hat{\mathbf{z}}$, where $\hat{\mathbf{z}}$ is the unit vector along the longitudinal axis of the cylinder, viz. the z-direction. Each charged species acquires a drift due to this electric field, given by

$$\mathbf{V}_{j0} = \frac{q_j E_0}{m_j \nu_j}\,\hat{\mathbf{z}}. \tag{2}$$

Here q_j, m_j, and ν_j are the charge, mass, and collision frequency, respectively, of the particle species j. It is assumed that the collision rates are due primarily to collisions of charged particles with neutrals.

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The system is described using the fluid equations for each charged particle species

$$\partial_t n_j + \nabla \cdot (n_j \mathbf{v}_j) = 0, \tag{3a}$$

$$n_j m_j (\partial_t + \mathbf{v}_j \cdot \nabla \mathbf{v}_j) + T_j \nabla n_j - q_j n_j \mathbf{E} = -\nu_j n_j m_j \mathbf{v}_j , \tag{3b}$$

where n_j and $\mathbf{v}_j$ are the number density and velocity of the species j, respectively, and T_j the temperature of the species j in energy units.

We linearize (3) by letting $n_j = n_{j0}+n_{j1}$ and $\mathbf{v}_j = \mathbf{V}_{j0}+\mathbf{v}_{j1}$ and consider electrostatic waves with the perturbed electric field $\mathbf{E}_1(\mathbf{r}, t) = -\nabla\phi(\mathbf{r}, t)$, where $n_{j1} \ll n_{j0}$, $\mathbf{v}_{j1} \ll \mathbf{V}_{j0}$, and ϕ is the wave potential. We consider the frame where the dust is at rest ($V_{d0} = 0$) and assume that the wave frequency ω is $\ll$ the electron and ion collision frequencies. The perturbed quantities n_{j1}, $\mathbf{v}_{j1}$, and ϕ are assumed to vary as $n_{j1} = f_{j1}(r)\exp(ikz - i\omega t)$ and $\phi = \varphi(r)\exp(ikz - i\omega t)$. Then, for the electrons, we have from (3)

$$kV_{e0}(kV_{e0} + i\nu_e)n_{e1} = \frac{en_{e0}}{m_e}\nabla^2\phi - v_e^2\nabla^2 n_{e1}, \tag{4}$$

while for the ions we have from (3)

$$-kV_{i0}(-kV_{i0} + i\nu_i)n_{i1} = -\frac{en_{i0}}{m_i}\nabla^2\phi - v_i^2\nabla^2 n_{i1}. \tag{5}$$

Here, $v_j = (T_j/m_j)^{1/2}$ is the thermal speed of the particle species j. For the dust fluid, (3) gives

$$\omega(\omega + i\nu_d)n_{d1} = \frac{Z_d e n_{d0}}{m_d}\nabla^2\phi, \tag{6}$$

assuming that the dust grains are cold. Equations (4)–(6) are closed by Poisson's equation

$$\nabla^2\phi = 4\pi e(n_{e1} - n_{i1} + Z_d n_{d1}), \tag{7}$$

where the Laplacian operator in cylindrical co-ordinates is given by $\nabla^2 = \nabla_\perp^2 + \partial_z^2$, where $\nabla_\perp^2 = \partial_r^2 + (1/r)\partial_r + (1/r)^2\partial_\theta^2$. For simplicity, we consider the case of azimuthal symmetry with $\partial_\theta = 0$.

We now consider the kinetic regime for the electrons, with $\nu_e \ll v_e|\nabla|$ and $kV_{e0} \ll v_e|\nabla|$. Here, (4) yields

$$n_{e1} \approx n_{e0}\frac{e\phi}{T_e}. \tag{8}$$

For the ions, we assume the hydrodynamic regime $|kV_{i0}(kV_{i0} - i\nu_i)| \gg v_i^2|\nabla^2|$ so that (5) gives

$$n_{i1} \approx -\frac{n_{i0}e}{m_i k V_{i0}(kV_{i0} - i\nu_i)}\nabla^2\phi. \tag{9}$$

Combining (6)–(9), we readily obtain

$$\frac{d^2\varphi}{dr^2} + \frac{1}{r}\frac{d\varphi}{dr} + \beta\varphi \approx 0, \tag{10}$$

where

$$\beta = -k^2 - \frac{1}{\lambda_{De}^2(1 + \chi_i + \chi_d)}, \tag{11}$$

where $\lambda_{De} = (T_e/4\pi n_{e0}e^2)^{1/2}$ is the electron Debye radius, $\chi_i = -\omega_{pi}^2/kV_{i0}(kV_{i0} - i\nu_i)$ is the ion susceptibility, $\omega_{pi} = (4\pi n_{i0}e^2/m_i)^{1/2}$ is the ion plasma frequency,

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$\chi_d = -\omega_{pd}^2/\omega(\omega + iv_d)$ is the dust susceptibility, and $\omega_{pd} = (4\pi n_{d0} Z_d^2 e^2/m_d)^{1/2}$ is the dust plasma frequency.

Equation (10) is a zero-order Bessel equation whose solution that is finite at the origin is $\varphi(r) = J_0(\sqrt{\beta}r)$, where J_0 is the Bessel function of zero order. Requirement of $\varphi = 0$ at $r = R$, where R is the radius of the cylindrical dust cloud, implies that $J_0(\sqrt{\beta}R) = 0$. Thus, $\sqrt{\beta}R = \gamma_n$, where γ_n are the roots of J_0 (the first few zeroes of J_0 are $\gamma_1 = 2.4$, $\gamma_2 = 5.5$, $\gamma_3 = 8.7$, $\gamma_4 = 11.8$) [11]. Accordingly, we can rewrite (11) as

$$1 - \frac{\omega_{pi}^2}{kV_{i0}(kV_{i0} - iv_i)} - \frac{\omega_{pd}^2}{\omega(\omega + iv_d)} + \frac{1}{\left[k^2 + \left(\frac{\gamma_n}{R}\right)^2\right]\lambda_{De}^2} \approx 0. \tag{12}$$

Equation (12) has the same form as for a cold ion-dust streaming instability in an unbounded dusty plasma, as given, e.g. in [9,12], the difference being the quantization of an effective radial wavenumber γ_n/R.

We solve (12) at the resonance condition for the maximum growth rate of an ion-dust streaming instability

$$kV_{i0} \sim \frac{\omega_{pi}}{\sqrt{A}}, \tag{13}$$

with

$$A = 1 + \frac{1}{\left[k^2 + \left(\frac{\gamma_n}{R}\right)^2\right]\lambda_{De}^2}.$$

In the limits $kV_{i0} > v_i$ and $v_d \ll \omega$, the solution of (12) is [9]

$$\frac{\omega}{\omega_{pd}} \sim \frac{1+i}{\sqrt{2}}\left(\frac{\omega_{pi}}{v_i}\right)^{1/2}\frac{1}{A^{3/4}}. \tag{14}$$

Since A decreases as γ_n increases, it turns out that the lowest order γ_1 mode may have larger growth rate, if it satisfies the resonance condition (13).

We now speculate on possible applications of our work to certain dusty plasma experiments where the dust cloud appears to have radial boundaries. First, we note that the resonance condition (13) can be written in the following form, which is more useful for comparison with experimental parameters,

$$k^2\lambda_{Di}^2 \approx \frac{v_i^2}{V_{i0}^2} - \frac{k^2}{k^2 + \left(\frac{\gamma_n}{R}\right)^2}\frac{T_i}{\delta T_e}, \tag{15}$$

where $\delta = n_{i0}/n_{e0}$.

First, we consider possible application to a cryogenic dusty plasma in a high-frequency discharge reported by Fortov et al. [13]. In the latter paper, it was suggested that an ion-dust streaming instability may play a role in the formation of sharp layers observed in a thin dusty plasma cloud (see the Fig. in [13]). If we assume that the dusty plasma cloud is roughly cylindrical with a radius of about 1 mm, we have $\gamma_1/R \sim 24$ cm^{-1}. The wavelength corresponding to the layers in the cloud is approximately 300 μm [13] so that $k \sim 200$ cm^{-1}. We assume the following nominal parameters for the weakly ionized cryogenic plasma: gas (air) pressure $\sim$0.1 torr, $T_i \sim 0.007$ eV (77 K), $T_e \sim 2$ eV, $n_i \sim 10^9$ cm^{-3}, and assume an ion mass of $\sim$28. To estimate V_{i0}, we also need the ion-neutral collision cross-section, which we assume is the resonant cross section for N_2^+ collisions with N_2, and use

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the cross section $\sigma_i \sim 2 \times 10^{-14}$ cm^2 [14]. Owing to the very large ratio of T_e/T_i, the second term in the right-hand side of (15) is negligible. (This implies that modes characterized by various γ_n values may become unstable, so it would be interesting to look for radial structure in the dusty cloud). Then balancing the remaining two terms, we can obtain an estimate for the axial electric field providing the ion drift (see (2)). Using $\lambda_{Di} \sim 20$ μm, $v_i \sim \sigma_i n_n V_{i0}$ with a neutral density $n_n \sim 3 \times 10^{15}$ cm^{-3}, we find that (15) is satisfied for $V_{i0}/v_i \sim 2.5$, which implies $E_0 \sim 300$ V m^{-1}. It would be interesting to compare this value with any available data. Note, however, that the dusty plasma may be strongly coupled, and the waves may be nonlinear, as indicated in Fortov et al. [13].

Next, we consider possible application to a small dusty plasma cloud trapped in a magnetized plasma discussed in Pilch et al. [7]. The latter paper reported measurements of the dispersion of self-excited dust density waves in the cloud and showed that the dispersion shows an influence of the boundaries. An interesting question is whether the cloud boundaries affect the ion-dust streaming instability, which may generate the waves. We note that our analysis in this paper does not take into account the effect of a magnetic field, while it appears that for the experimental parameters given in Ref. [7], namely $B = 200$ G, argon pressure $P \sim 3$ Pa and $T_e \sim 3$ eV, the electrons would be magnetized since the electron gyrofrequency $\omega_{ce} \sim 4 \times 10^9$ rad s^{-1} is much larger than $\nu_e \sim 2.5 \times 10^7$ s^{-1} (using an electron-neutral collision cross section of $\sim 5 \times 10^{-16}$ cm^2). However, in the limit where $V_{e0} \ll v_e$, which may be expected since the dominant electric field component appears to be oblique to the magnetic field (see Fig. 9 in Ref. [7]), and in the limit $\omega \ll v_e \ll k v_e$ the Boltzmann response for the electrons in (8) may not be an unreasonable approximation (see e.g. [15]). In other words, the electrons that can move along the magnetic field can provide screening since the wavevector has a component in the direction of the magnetic field. The ions, on the other hand, may be considered unmagnetized since the ion gyrofrequency $\omega_{ci} \sim 4.8 \times 10^4$ rad s^{-1} is small compared with ν_i, as will be shown below. The dust is similarly unmagnetized.

We estimate the ion drift V_{i0} by using $V_{i0} = \mu_i(E_0) E_0$ [16], where

$$\mu_i(E_0) = 0.146 \frac{P_c}{P} \left[1 + \frac{C E_0}{P}\right]^{-1/2} \frac{\text{m}^2}{\text{V s}},$$

where $P_c = 133$ Pa and $C = 0.035$ (Pa m V^{-1}). By using $P \sim 3$ Pa and $E_0 \sim 1000$ V/m [7], we obtain $V_{i0} \sim 2 \times 10^5$ cm s^{-1}. Then, assuming the ion-neutral collision cross section $\sigma_i \sim 4 \times 10^{-15}$ cm^2, we obtain $\nu_i \sim \sigma_i n_n V_{i0} \sim 6 \times 10^5$ s^{-1}, which is $\gg \omega_{ci}$ estimated in the previous paragraph, so the ions may be considered unmagnetized to zeroth order. Assuming $T_i \sim 0.1$ eV [7], we have $V_{i0}/v_i \sim 4$. Further, taking $n_i \sim 10^9$ cm^{-3} [7], we have $\omega_{pi}/\nu_i \sim 10$.

We assume that the radius of the dust cloud is ~ 2.5 mm [7] so that $\gamma_1/R \sim 9.6$ cm^{-1}. The measured values of k range from about 5 to 20 cm^{-1} (see Fig. 10 in Pilch et al. [7]). Thus, for this set of parameters, γ_1/R can be comparable to k. Interestingly, using our estimated value of V_{i0}/v_i along with the above values of T_e/T_i and γ_1/R in the resonance condition (15), and taking $\delta \sim 1$, we find that (15) can be satisfied for $k \sim 25$ cm^{-1}, for a wavelength somewhat smaller than the measured values shown in Ref. [7]. At this wavelength, the growth rate from (14) for this set of parameters is roughly on the order of ω_{pd}.

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To summarize, we have considered the effect of radial boundaries on a dissipative ion-dust streaming instability in a cylindrical geometry, assuming azimuthal symmetry. The radial boundary influences the unstable dust wave spectrum when the roots of the zero-order Bessel function are on the order of kR, where k is the axial wavenumber and R is the radius of the cylinder. Possible applications of our work to ion-dust streaming instability in localized dust clouds in laboratory dusty plasmas [7, 13] were considered.

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