Freedom and Social Choice: Notes in the Margin

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I comment on Amartya Sen's study of the relations between the analysis of freedom and the theory of social choice. Two of his themes are analysed with regard to their contribution to an analytic understanding of the issues. These are: (1) the multiple interpretations of the concept of 'preferences' as a foundation for the formal conceptualizations of social choice and freedom; and (2) some issues in the formalization of freedom as a value to be compared with outcomes. Under (2), I mainly point out some difficulties in the existing analyses and mildly support a 'flexibility' interpretation of freedom that I have advanced earlier. I conclude with some observations drawn from history and literature which complicate the value *bien-pensant* thinkers are prone to place on freedom.

In the Franciscan monastery in Assisi, I was once shown a manuscript of a thirteenth-century tax code. In the middle of each page was a text; surrounding this central passage were three or four others scattered on the page. The central text was a tax provision; the surrounding passages were explanations and rulings as to the applicability of that particular tax.

This commentary is like a series of comments on the lectures, named after me, which Amartya Sen delivered at Stanford University in May 1991, under the general heading 'Freedom and Social Choice'. These lectures have seen published form.[1] The analogy is imperfect; the tax code contained brief comments on each particular tax provision, while my distinctly baggier remarks deal with underlying themes. I do not intend to set forth any very definitive doctrines but rather comment in broad and non-rigorous fashion on some of the perspectives which recur in Sen's deep and penetrating argument.

More precisely, I will take up three different themes; these are marginal comments on three distinct strands in Sen's tapestry. First, I consider Sen's emphasis on the flexibility of the concept of 'preferences'. Second, I consider in some detail the distinction between 'comprehensive' and 'culmination' outcomes; I suggest a different way of looking at this distinction, which emphasizes the idea of constitution-making. Finally, I ruminate on some ways of looking at the value of freedom, however formalized, to individuals.

[1] A. K. Sen, *Rationality and Freedom* (Cambridge, Mass., and London, 2002), chs. 20–2.

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I. THE INTERPRETATION OF PREFERENCES

Sen provides a valuable reminder that the concept of preference may have many interpretations.[2] 'Preference' is a formal concept which may have several different empirical referents. It may refer to behaviour ('revealed preference') or, more exactly, to potential behaviour (if offered the choice between x and y, I (or society) would choose x), It may refer to well-being in some more or less objective sense (I am better off objectively if I have x rather than y). This might include better functioning or higher capability in the sense that Sen has made us aware of.[3] Another direction would be to interpret preference in terms of mental states, as indeed the founders of utility theory in economics tended to argue (most explicitly, Francis Edgeworth:[4] I am happier if I have x rather than y). Still another interpretation is suggested by Sen:[5] 'a valuational ranking that can survive reasoned scrutiny'.[6]

The problem of aggregation of preferences is much the same *formally* no matter which interpretation of preferences we may consider. The conditions which I have suggested to define a 'good' aggregation or social welfare function[7] seem equally applicable (or inapplicable) under any of these interpretations. It follows that for many purposes the differing interpretations of preferences do not matter. Of course, the interpretations do matter in terms of actual value judgements, and they matter in such questions as implementability of social choice through the market, through the democratic process or through the legal system.

[2] Sen, *Rationality*, pp. 588–90.

[3] A. K. Sen, *Commodities and Capabilities* (Amsterdam, 1985; republished, Delhi, 1999).

[4] F. Y. Edgeworth, *Mathematical Psychics* (London, 1881; reprinted, New York, 1967).

[5] Sen, *Rationality*, p. 590.

[6] As a side remark, I have always been sceptical of the idea that values can be completely based on reasoning (policies and actions can be argued about in a world of imperfect information, because information is revealed in the process). This may just be the effect of the period of my early intellectual formation, the 1930s, when Freud and Marx were the idols of self-defined intellectuals; reasons given for actions were not the true causes, which were the unconscious outcomes of personal or class conflicts. But the doubts are very much older. Cicero (?) said, 'de gustibus non est disputandum', and David Hume (*A Treatise of Human Nature* (Edinburgh, 1739–40; L. A. Selby-Bigge (ed.) and P. H. Nidditch (text ed.), (Oxford, 1978), p. 415) notoriously held, 'Reason is, and ought only to be the slave of the passions, and can never pretend to any other office than to serve and obey them.' The entirety of Book II, Section III in Hume argues this point in greater detail.

[7] K. J. Arrow, *Social Choice and Individual Values* (New York, 1951; 2nd edn. New Haven, 1963), ch. 3.

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II. THE FORMALIZATION OF FREEDOM AS A VALUE

Any discussion of freedom as a value implies a trade-off between freedom and outcomes. There may indeed be other elements of process which have been regarded as having value, for example, democratic participation (which, as John Stuart Mill argued, may be quite different from freedom), deference to authority, equality before the law, conformity to certain moral principles, and protection of a private sphere of choice, possibly exemplified by property rights. Process values in short are not self-evident truths.

But I will confine myself to the value of freedom. Later, I want to meditate on the meaning and acceptability of freedom as a value, but here I touch on the alternative ways in which freedom can be incorporated into a formal theory of choice.

A. Ranking of Opportunity Sets with a Single Preference Ordering

Most of the studies on incorporating freedom as a value have really dealt with individual choice. The underlying intuition is that an individual is freer if he or she can choose from an expanded set and is therefore 'better off' in some sense even if the choice actually made is the same. However, the question was how this intuition can be generalized to comparisons between two opportunity sets neither of which includes the other. (This still leaves open the social choice aspect of freedom: how do individual freedoms affect each other?) But a second line of study is closer in spirit to social choice theory. It recognizes that in some sense even an individual may contemplate alternative preferences.

The most general formalization of the single-preference case is Sen's.[8] He argues that the objects that should be ordered are not the outcomes x but the pairs (x, A), where A is the set from which the choice is to be made ('menu' in Sen's terminology) and x belongs to A. He refers to simple outcomes as 'culmination outcomes', and the pairs of simple outcomes and menus as 'comprehensive outcomes'.

There is not really a full analysis of the ordering (partial or complete) of comprehensive outcomes. Much of the literature[9] has been devoted to showing that certain conditions on the ordering of menus with respect to freedom imply that they are ordered by the number of elements in them. (Of course, this cardinality criterion is usually restricted to finite menus, although one could imagine using something like Lebesgue measure for opportunity sets which are continua.) The argument always starts with the assumption that when the menu is a unit set

[8] Sen, *Rationality*, pp. 159–63, 664–9.
[9] Summarized in ibid., pp. 600–3, 683–90.

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(i.e. there is no choice), then there is no freedom, and therefore all unit sets are indifferent. Note that this analysis is a special case of ranking 'comprehensive outcomes': if S is a menu with more elements than T, it could happen that the pair (x, S) would be preferred to (y, T) even though y would be preferred to x within the menu T.[10]

Are there more general but still usable ways of ordering the pairs (x, S)? That is not clear from Sen's work. His point of view is clear; the sense in which one menu offers more freedom than another cannot be independent of preferences. Freedom to starve or be tortured is not significant freedom.[11] I am thoroughly in agreement with this position.

However, the uses that Sen makes of the ordering of comprehensive outcomes are rather limited. On the one hand, he returns several times to the idea that the ordering of culmination outcomes for a given menu may depend on the menu,[12] i.e. one may prefer (x, S) to (y, S) and yet prefer (y, T) to (x, T) (where it is implicitly assumed that both x and y belong to both S and T). This implication does not relate very directly to the measurement of freedom.

On the other hand, Sen discusses criteria for the ordering of menus, without imposing any conditions on the trade-off between outcomes and menus. His criteria tend to be utilitarian or at least consequentialist in nature. Suppose there is an ordering (possibly partial) of all culmination outcomes. One criterion, called Elementary Option Superiority,[13] simply says that S is at least good as T if there exists an element, x, in S such that x is at least good as y for every element y in T. This condition exemplifies strongly, perhaps extremely strongly, the idea that opportunities are not independent of preferences. If S contained just one element but that element was better (in some relevant ordering) than every outcome available in T, S would be judged better than T *as a menu or opportunity*, that is, in addition to the preference for x as a culmination outcome. (Sen's other proposed criterion is similar but differs if the ordering of culmination outcomes is not complete.) This implies a fundamental contradiction with the cardinality criterion of freedom. T may have an arbitrarily large number of members, and yet will be judged inferior to a set with only one element. Hence, there is no trade-off between cardinality and preference.

[10] For an elegant axiomatic treatment, see K. Suzumura and Y. Xu, 'Consequences, Opportunities, and Generalized Consequentialism and Non-Consequentialism', *Journal of Economic Theory*, 111 (2003).

[11] See Sen, *Rationality*, pp. 594–5, for somewhat more benign examples.

[12] For example, ibid., pp. 167–8.

[13] Ibid., p. 647.

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B. Ranking of Opportunity Sets with Multiple Preference Orderings

An alternative approach is to judge among menus on the basis of a number of preference orderings. This way of looking at the choice among menus is more like the theory of social choice. The menu becomes a 'constitution', a choice of opportunities which has to be offered without knowing the preference ordering. Of course, the orderings need not be arbitrary; they may all give preference to common human needs, physical and spiritual. Foster[14] presented this viewpoint vigorously, and Sen has given an excellent account of Foster's work and his own modification,[15] which I will not repeat. I note, however, that both versions, and especially Sen's, tend to what might be called the 'Pareto' viewpoint; one menu is better than another if it is better for each of the possible preferences.

I have already offered a different but related approach, also based on multiple preferences.[16] Before summarizing it, let me relate it to a broader stream of thought, namely, social choice theory or, to put it another way, constitution-making. It is natural to think this way when we consider the possibility of multiple preferences. We want a procedure, in this case a menu, which will be applicable to a set of preferences. In social choice theory or alternative views of constitutional formation, the contemplated situation is that of a number of different individuals with differing preferences over social states. Here, we have a single individual but one who considers a whole range of preferences as possible.

The paradoxes in social choice theory arise in good measure from the assumed incomparability of different individuals' preference intensities. If one were to drop this restriction, the utilitarian solution of adding up the individuals' utilities would satisfy all the other conditions usually suggested. With a single individual facing multiple preferences, it is at least more plausible to assume 'inter-preference' comparability.[17]

[14] J. Foster, 'Notes on Effective Freedom', mimeographed, Vanderbilt University. Presented at Stanford Workshop on Economic Inequality, 11–13 March 1993.

[15] Sen, *Rationality*, pp. 676–8.

[16] K. J. Arrow, 'A Note on Freedom and Flexibility', *Choice, Welfare, and Development*, ed. K. Basu, P. Pattanaik and K. Suzumura (Oxford and New York, 1995), pp. 7–16.

[17] There has been one interesting attempted resolution of the social choice paradox in which interpersonal ordinal comparability is assumed; see P. Hammond, 'Equity, Arrow's Condition, and Rawls' Difference Principle', *Econometrica* 44 (1976), and S. Strasnick, 'Preference Priority and the Maximization of Social Welfare', doctoral thesis (Harvard University, 1975). Let i, j represent different individuals (really, their inner states, including preferences). It is assumed that there is an ordering of the pairs (x, i). To say that (x, i) is preferred to (y, j) means that it is better to be individual i in culmination outcome x than to be individual j in outcome y. (In particular, when $i = j$, the preference ordering is that of individual i.) Then it can be shown that there are several solutions to the social choice problem. As was shown by C. D'Aspremont and L. Gevers, 'Equity

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I suppose that the menu has to be chosen before the preferences are known, in accordance with Foster's viewpoint. Let the different possible preferences be represented by von Neumann-Morgenstern utility functions, $U_k(x)$. The uncertainty about which utility function will be relevant can be represented by probabilities P_k. Given the menu, S, the individual will choose an outcome x to maximize the actual preference, U_k, and receive the payoff,

$$P_k(S) = [\max U_k(x) \mid x \varepsilon S].$$

If, however, the menu, S, has to be chosen before knowing which utility function will govern, then the preferences are determined by the function,

$$V(S) = \Sigma p_k P_k(S),$$

the expected value of the optimal choice.[18]

Sen explores two criteria for multiple preferences, called, 'single ranking action choice' (SRAC) and 'multiple ranking action choice' (MRAC). Let R_k be the kth preference ordering. Then SRAC defines menu S to be at least as good as menu T if, for every k, there is an x in S for which $x\, R_k\, y$ for all y in T. MRAC defines menu S to be at least as good as menu T if there is an x in S for which, for all k, $x\, R_k\, y$ for all y in T.[19] The two conditions differ in the order of their quantifiers. As Sen shows, MRAC is the stronger condition. Note that, clearly, ordering according to expected maximum utility is implied by (i.e. is weaker than) SRAC and therefore than MRAC. It is therefore not inconsistent with these conditions. The relation between the expected-maximum-utility criterion and MRAC, in particular, is roughly the same as that between a utilitarian criterion and a Pareto or unanimity ranking in social choice theory.

Sen is not attracted to this approach and has expressed his reservations.[20] He seems to have the view that uncertainty about one's preferences would only be relevant if they were determined externally, by a mechanism which is random to the subject. But I see no logical or other problem in the view that, at the time of choosing the underlying

and the Informational Basis of Collective Choice', *Review of Economic Studies* 44 (1977), Theorem 3, one of them is the utilitarian solution.

[18] As both Sen and I have remarked, this approach is similar to and motivated by the concept of flexibility, where there is a sequence of decisions in the face of uncertainties. The uncertainties are resolved over time, but the earlier decisions constrain the later ones. The concept seems to have been first introduced into economics by A. G. Hart, 'Risk, Uncertainty, and the Unprofitability of Compounding Probabilities', *Studies in Mathematical Economics and Econometrics*, ed. O. Lange, F. McIntyre and T. O. Yntema (Chicago, 1942), pp. 110–18.

[19] Sen, *Rationality*, pp. 677–8. The condition SRAC is the one proposed by Foster.

[20] Ibid., pp. 618–20, 681–3.

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mechanisms of choice, an individual does not feel that he or she knows his or her preferences. The individual wants to retain his or her autonomy for the future. This point has even greater validity if we think of the choice mechanism (the menu) as being chosen for a long future and perhaps also for many individuals.

III. SOME OBSERVATIONS ON THE VALUE OF FREEDOM

It has been assumed by all of us that freedom is indeed a value. In the United States Declaration of Independence, 'liberty' follows only 'life' and precedes 'the pursuit of happiness' in the list of 'inalienable rights'. John Rawls insists repeatedly on the 'priority of liberty' among his conditions for justice.[21] Yet fairly casual observation shows that this value cannot be described as universal. What follows is an unorganized series of reflections, and I do not draw any deep conclusions.

A. Can Freedom Be Compelled?

During the period between the two world wars, Austrian politics was dominated by the contest between two major parties, the Socialists and a right-wing party greatly influenced by Italian fascism. During a period in which the Socialists were strong, the Socialist leader, Otto Bauer, held that the opposition should subscribe to democratic principles. In his analogy, politics was like chess; the loser should have the right to another game. If the opposition would not accept this principle, they should be suppressed as violating the rules of democracy. The opposition leader said 'no'. If we win, we will have an authoritarian state without opposition: those are our principles. But you cannot suppress us for this position: those are your principles.

This dialogue was prophetic about Austria, which indeed drifted into authoritarianism. But much more seriously, it was prophetic about Germany. Here was a democratic country with all the freedoms to which we aspire. In a free and democratic election, the people of Germany voted in 1932 to abolish their democracy and their freedoms. This was not a *coup d'état*, a surrender to superior force. Although there are not many cases, Germany in 1932 is not the only example of a free choice to give up freedom: Perón in Argentina and Mussolini in Italy come to mind.

B. The Willingness to Die for Freedom

A few years ago, my wife and I were in Kenya, on a tour mainly devoted to seeing wildlife. The President there had been in power for over twenty years and was widely regarded as repressive. The Kenyan tour

[21] J. Rawls, *A Theory of Justice* (Cambridge, Mass., 1971).

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director was vehement in his denunciations of the President. My wife, concerned for his safety, suggested he should be careful how and in what circumstances he expressed his opinions. He replied that if one wants freedom, one must be prepared to die for it. He continues a long tradition, with its inextricable mixture of fact and fantasy, which will cite Spartacus, the nobles at Runnymede, the Peasant Rebellion of 1525 in Germany, the Great and the Glorious Rebellions of seventeenth-century England, the American and French Revolutions, the slave rebellions in Haiti, the United States and Jamaica, the Revolutions of 1830 and 1848 in Europe, and the twentieth-century anti-colonial movements in Africa and in India, where Mohandas K. Gandhi became a world-famous hero to such as me.

Does this 'willingness to pay' imply that freedom must have an extraordinary value? Unfortunately, people are willing to die for all sorts of reasons. Certainly, a much longer list can be compiled of those willing to die for their country. Worse yet, there are those who risk their lives to prevent freedom. For instance, the followers of Franco in the Spanish Civil War fought and died to suppress democracy and the Southerners in the American Civil War to retain slavery.

C. The Fear of Freedom

In an attempt to understand the German abandonment of freedom in 1932, the psychoanalyst Erich Fromm argued that people find freedom, with its accompanying responsibility, a burden.[22] I have some doubts that this hypothesis does much to explain German history, but the idea itself has powerful support in everyday experience, as expressed in literature. Byron's poem, *The Prisoner of Chillon*, tells the story of a prisoner (a victim of tyranny) condemned to solitary confinement. After some time of anger and resentment, he gradually adjusts and begins to appreciate the life around him, the rats and the spiders and other creatures. When, after twenty years, he is liberated by a revolutionary movement, he is relatively indifferent, 'Even I / regained my freedom with a sigh'.

The *locus classicus* of the dilemmas of freedom is the story of the 'The Grand Inquisitor' in Fyodor Dostoevsky's great novel, *The Brothers Karamazov*. The story is told by the sceptic, Ivan Karamazov, to his brother, Alyosha, a novice monk. Christ appears in Seville during the era of the Spanish Inquisition. Though he does not identify himself, he is immediately recognized and followed by the people. However, the Grand Inquisitor has him arrested. He then reproaches Christ for trying to make people free. It was impossible, the Grand Inquisitor tells

[22] E. Fromm, *Escape from Freedom* (New York, 1941).

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Christ, to spread his religion without authority bolstered by miracles and bread. In the end, Christ kisses the Grand Inquisitor and leaves.

The implications of the story are clearly meant to be ambiguous.

D. Clarification

None of these thoughts makes me doubt the importance of freedom. They do constitute a warning that a consensus among well-meaning academics may not be a universal.

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