THE REVIEW OF SYMBOLIC LOGIC
Volume 5, Number 4, December 2012

INDISPENSABILITY ARGUMENTS AND INSTRUMENTAL NOMINALISM

RICHARD PETTIGREW
Department of Philosophy, University of Bristol

Abstract. In the philosophy of mathematics, indispensability arguments aim to show that we are justified in believing that mathematical objects exist on the grounds that we make indispensable reference to such objects in our best scientific theories (Quine, 1981a; Putnam, 1979a) and in our everyday reasoning (Ketland, 2005). I wish to defend a particular objection to such arguments called *instrumental nominalism*. Existing formulations of this objection are either insufficiently precise or themselves make reference to mathematical objects or possible worlds. I show how to formulate the position precisely without making any such reference. To do so, it is necessary to supplement the standard modal operators with two new operators that allow us to shift the locus of evaluation for a subformula. I motivate this move and give a semantics for the new operators.

In the philosophy of mathematics, indispensability arguments aim to show that we are justified in believing that abstract mathematical objects exist. I wish to defend a particular objection to such arguments that has become increasingly popular recently. It is called *instrumental nominalism*. I consider the recent versions of this view and conclude that it has yet to be given an adequate formulation. I provide such a formulation and show that it can be used to answer the indispensability arguments.

There are two main indispensability arguments in the literature, though one has received nearly all of the attention. They correspond to two ways in which we use mathematics in science and in everyday life. We use mathematical language to help us describe nonmathematical reality; and we use mathematical reasoning to help us perform inferences concerning nonmathematical reality using only a feasible amount of cognitive power. The former use is the starting point of the Quine–Putnam indispensability argument (Quine, 1980a, 1980c, 1981a, 1981b; Putnam, 1979a, 1979b); the latter provides the basis for Ketland's more recent argument (Ketland, 2005). I begin by considering the Quine–Putnam argument and introduce instrumental nominalism to defuse it. Then I show that Ketland's argument can be defused in a similar way.

§1. The Quine–Putnam indispensability argument. Neither Quine nor Putnam explicitly formulated the indispensability argument that bears their names. And it is not clear that they would have endorsed the particular versions that we find in the literature. Here, I will focus on a particular formulation for the sake of precision; but everything I will say will count against other versions. The present formulation is close to that given by Mark Colyvan in his survey article on such arguments (Colyvan, 2011):

Received: October 30, 2011.

© Association for Symbolic Logic, 2012
doi:10.1017/S1755020312000044

https://doi.org/10.1017/S1755020312000044 Published online by Cambridge University Press

(QP1) We know that our best scientific theory entails the existence of mathematical objects.

(QP2) We are justified in believing any proposition we know to be entailed by our best scientific theory.

(QC) Therefore, we are justified in believing in the existence of mathematical objects.

The idea is that, just as our best scientific theory includes reference to physical objects, such as quarks, electrons, and photons, so it also includes reference to mathematical objects, such as real numbers, Hilbert spaces, and differentiable manifolds. And just as the first fact justifies our belief in the physical objects, so the second fact justifies our belief in mathematical objects.

The instrumental nominalist denies the second premise, (QP2). According to her, it is not our best scientific theory that we are justified in believing, but rather its nominalistic content; that is, what it says that only concerns the concrete world. And this, she claims, entails nothing about the existence of mathematical objects. Thus, her position is analogous to van Fraassen's constructive empiricism (van Fraassen, 1980). The constructive empiricist admits that our best scientific theories entail the existence of unobservable objects. But he denies that we are justified in believing those theories. Rather, while we are justified in *accepting* our best theory, we are only justified in *believing* its empirical content; that is, what it says that only concerns the observable world. The instrumental nominalist takes a similar line: the constructive empiricist believes that we are only justified in believing that our best theories are *empirically adequate*; similarly, in Rosen's terminology, the instrumental nominalist believes that we are only justified in believing that our best theories are *nominalistically adequate* (Rosen, 2001).

In this paper, I am concerned only to formulate the instrumental nominalist's thesis precisely in such a way that she is not vulnerable to various objections, and to show that she can resist the two different versions of the indispensability argument: the Quine–Putnam version and the Ketland version. I am not concerned to argue for her position. That is a separate task. Its difficulty lies in saying what it is that leads us to retreat from believing that our best theory is true to believing that it is nominalistically adequate, but does not lead us to retreat from believing that our best theory is nominalistically adequate to believing only that it is empirically adequate. Any answer to this must rebut the increasingly popular claim that what justifies us in going beyond the mere nominalistic adequacy of our theory to believe in its truth is the explanatory power that its truth provides.[1] I leave that topic for another time.

To formulate instrumental nominalism precisely, we must be able to say precisely which propositions it is that she takes us to be justified in believing; that is, we must be able to identify exactly which propositions constitute the nominalistic content of her best theory. That is the task of this paper. In the coming sections, I consider a number of different ways in which we might make the position precise, and I will propose my own alternative.

In proposing my own alternative, I will make some assumptions. I collect them together here for ease of reference. I assume that the distinction between concrete and nonconcrete objects and relations is determinate and fixed. I will not inquire into how this distinction is drawn or how we come to know it.[2] Also, I will assume that quantifying into modal contexts created by the standard primitive modal operators—in particular, necessity,

[1] See, for instance, Baker (2005) and Colyvan (2010).

[2] See Resnik (1983) and Pincock (2007) for scepticism concerning this assumption.

https://doi.org/10.1017/S1755020312000044 Published online by Cambridge University Press

possibility, and actuality operators—is acceptable to the nominalist. Note that Quine would not accept this (Quine, 1980b). In fact, I will go on to supplement these operators with two new modal operators; their purpose is to shift the locus of evaluation of the subformulae to which they are applied. I argue for their legitimacy by noting that they are required in order to paraphrase certain natural sentences of everyday English. Again, Quine would not accept this way of arguing. For him, we can elicit the ontological commitment of a theory by regimenting the language, but we cannot elicit its ideological commitment; the ideological commitment is always the framework of first-order logic (Quine, 1980a).

§2. A naive formulation of instrumental nominalism. In this section, I draw on an insight due to Joseph Melia to show that a certain naive formulation of instrumental nominalism will not work (Melia, 1995). Above, I characterized the instrumental nominalist as follows: We are not justified in believing that our best theory is true; rather, we are justified in believing that what it says about the concrete world is true. One might be tempted to make this precise by saying that the propositions we are justified in believing are those that are consequences of our best theory and which are stated using only nominalistically-acceptable language; that is, using only predicates that express relations between the concrete objects, constants that name concrete objects, and quantifiers that are restricted so that they range over only concrete objects. However, as Melia notes, this will not do. Of course, each of these propositions will indeed be justified. But there will be others that the nominalist wishes to say are justified that are not included in this characterization.

Here is Melia's example (Melia, 1995, p. 226):

(1) The average star has 2.4 planets.

The problem is that it seems that this proposition can only be made precise using mathematical language. We can say:

(2) There are natural numbers m and n such that m is the number of the planets and n is the number of the stars and $m = 2.4n$.

But (2) is not stated entirely in nominalistically-acceptable language. The closest we can get to a nominalistically-acceptable statement of (1) is this:

There are 12 planets and 5 stars $\vee$
There are 24 planets and 10 stars $\vee$ ...

But this is not nominalistically-acceptable, since any way of stating precisely how to 'fill in' the dots of ellipsis will have recourse to the natural numbers. Nonetheless, we surely wish to say that this sentence somehow speaks only about the concrete world. Its use of mathematics may be indispensable; but the content we wish to convey when we assert it concerns only the concreta and the concrete relations in which they stand. Melia's point is that the instrumental nominalist must think that we are justified in believing (1); but (1) will not be among the propositions that follow from our best theory and are stated only using nominalistically-acceptable language.

The upshot of the foregoing considerations is that the instrumental nominalist cannot say simply that the propositions we are justified in believing are those that are entailed by our best theory and which are stated in nominalistically-acceptable language. The nominalistically-acceptable language is simply too weak to say everything we wish to say about the nominalistically-acceptable world.

https://doi.org/10.1017/S1755020312000044 Published online by Cambridge University Press

§3. The way of the weasel. Melia, himself an instrumental nominalist, proposes an alternative to the naive formulation of the position. It runs as follows. Usually, when we make one assertion and then another, we intend our second assertion to *add* to the claim about the world that we intend to convey by our first assertion—that is, we intend to convey that both are true. However, sometimes, Melia says, we do not intend this. Rather, sometimes, we intend our second assertion to *retract* some, but not all, of what we conveyed by our first assertion. He gives (something close to) the following example (Melia, 2000, p. 13):

(3) Everyone is happy. Except Harry.

We do not intend to convey by this pair of assertions the following contradictory proposition:

(4) Everyone is happy and Harry is not happy.

Rather, we intend to convey:

(5) Everyone who isn't Harry is happy.

That is, when we make the second assertion, we intend to retract something that is asserted in the first assertion, namely, that Harry is happy. But we wish to retain everything else, namely, that everyone else is happy.

Melia claims that there is a general linguistic device that we employ in (3), and which can be employed by the instrumental nominalist to state her thesis precisely. Thus, for Melia, if our best scientific theory entails (1), then what we are justified in believing is the proposition expressed by the following pair of assertions:

(6) There are natural numbers m and n such that m is the number of the planets and n is the number of the stars and $m = 2.4n$. But there are no natural numbers.

According to Melia, by making the second assertion, we intend to retract something that was asserted by the first, namely, that there are natural numbers. But we wish to retain everything else. In this case, what is left after the retraction is supposed to be what the first assertion says about the stars using the mathematical language. Melia calls this *the way of the weasel*.

I agree with Melia that there is such a linguistic mechanism. But without a precise account of how it functions, we cannot tell whether the mechanism that is clearly at work in (3) is able to do the work required of it in (6). After all, in the case of (3), there is a paraphrase available, namely, (5). Thus, it might be that the linguistic mechanism at work in (3) functions by generating paraphrases in some systematic way. If this is the case, then it cannot help in the case of (6), since, as Melia himself points out, there seems to be no adequate paraphrase available in these cases.[3] What's more, there are examples that superficially resemble (6), but which are such that we certainly would not wish to say that they convey a meaningful proposition. For instance:

(7) There is a smile that is the expression on the Cheshire Cat's face. But the Cheshire Cat has no face.

Or:

(8) There is a fist that is the posture of my right hand. But I have no right hand.

[3] For a similar criticism, see Colyvan (2010, pp. 295–296).

https://doi.org/10.1017/S1755020312000044 Published online by Cambridge University Press

To answer this worry, we must say precisely how the way of the weasel works. How can we use nominalistically-unacceptable language to say something that does not entail the existence of nominalistically-unacceptable objects? One might read the remainder of this paper as an attempt to answer that question.

§4. Concrete cores and exact intrinsic duplicates. In this section, I consider Gideon Rosen's account of instrumental nominalism (Rosen, 2001) and I offer an improvement. The instrumental nominalist holds that we are not justified in believing that our best theories are true; instead, we are justified in believing that what they say of the concrete world is true. For Rosen, this amounts to the following: we are justified in believing that our best theories are *nominalistically adequate*, where he spells out this notion as follows:

> Let's say the concrete core of a world w is the largest wholly concrete part of w: the aggregate of all of the concrete objects that exist in w. [. . .] [Then] S is nominalistically adequate iff the concrete core of the actual world is an exact intrinsic duplicate of the concrete core of some world at which S is true—that is, just in case things are in all concrete respects as if S were true. (Rosen, 2001, p. 75)

Thus, for instance, we are not justified in believing that (2) is true, but we are justified in believing that (2) is nominalistically adequate, where that amounts to the following:

(9) There is a possible world w such that:

(i) At w, there are natural numbers m and n such that m is the number of the planets and n is the number of the stars and $m = 2.4n$. That is, at w, (2) relativized to w is true.

(ii) The concrete core of w is an exact intrinsic duplicate of the concrete core of the actual world.

The idea is that a sentence that talks about numbers or other nominalistically-unacceptable sorts of object is nominalistically adequate if there is a world at which all these unacceptable objects exist and at which the sentence is true and whose concrete part is exactly like the concrete part of the actual world.

There are two problems with Rosen's proposal. First, it leaves the notion of an 'exact intrinsic duplicate' unexplained. This leaves open the possibility that, when correctly analyzed, this crucial notion will reveal nominalistically unacceptable commitments. Just as the hermeneutic or revolutionary nominalist must take pains to reveal all ontological commitments when she renders our best scientific theory in nominalistically acceptable language, so must our instrumental nominalist make explicit her commitments when she states the nominalistic content of that theory. That is, while I reject Quine's antinominalist conclusion, I accept his methodology: the ontological commitments of a theory are revealed by giving its logical form and identifying the range of its quantifiers and the referents of its constants.

The second problem with Rosen's account is that it involves quantification over possible worlds in a way that cannot obviously be eliminated. In this section, I improve on Rosen's account by analyzing the notion of an 'exact intrinsic duplicate'; in the next section, I eliminate quantification over possible worlds.

The most obvious analysis of the notion of 'exact intrinsic duplicate' runs as follows: a is an exact intrinsic duplicate of b iff every intrinsic property of a is an intrinsic

https://doi.org/10.1017/S1755020312000044 Published online by Cambridge University Press

property of b, and vice versa. Now, even if it is possible to make the distinction between intrinsic and extrinsic properties tolerably clear, the problem with this analysis is that it involves quantification over properties, which the nominalist will disavow. To avoid this unacceptable commitment, we borrow some ideas from Ketland (2011).

Suppose we formalize our best scientific theory in a language $\mathcal{L}$ with the predicate P, which expresses the property of being concrete.[4] And suppose the predicates of our language are divided into three kinds: the c-predicates, which express relations amongst the concrete objects; the a-predicates, which express relations amongst the abstract, nonconcrete objects; and the m-predicates, which express the mixed relations that hold between nonconcrete and concrete objects. Let $P_1, \ldots, P_n$ be the concrete predicates. Then our theory includes axioms saying that only objects with the property expressed by P can be related by the relations expressed by the P_is. Then, given a model $\mathcal{M}$ for such a theory, the *concrete core* of $\mathcal{M}$ is the structure whose domain is the subset of the domain of $\mathcal{M}$ that interprets the predicate P—call this $P^{\mathcal{M}}$—and whose relations are the sets of n-tuples of the elements of $P^{\mathcal{M}}$ that interpret the c-predicates—call these $P_1^{\mathcal{M}}, \ldots, P_n^{\mathcal{M}}$. Thus, the concrete core of $\mathcal{M}$ is the relational structure $\langle P^{\mathcal{M}}, P_1^{\mathcal{M}}, \ldots, P_n^{\mathcal{M}} \rangle$. Using this framework, Ketland defines nominalistic adequacy as follows. Suppose S is a sentence of $\mathcal{L}$. Then, writing α for the actual world, we have:

> *S is nominalistically adequate relative to $\mathcal{L}$* iff there is a model $\mathcal{M}$ such that
>
> (i) $\mathcal{M} \models S$
>
> (ii) The concrete core of $\mathcal{M}$ is isomorphic to the concrete core of α. That is,
>
> $$(\exists f)\left(f : \langle P^{\mathcal{M}}, P_1^{\mathcal{M}}, \ldots, P_n^{\mathcal{M}} \rangle \cong \langle P^{\alpha}, P_1^{\alpha}, \ldots, P_m^{\alpha} \rangle\right)$$
>
> where P^{α} is the set of all actual concrete objects, and P_i^{α} is the intended interpretation of the concrete predicate P_i at the actual world.

Thus, Ketland's account replaces Rosen's talk of worlds by talk of models, and Rosen's talk of 'exact intrinsic duplicates' by talk of isomorphisms. Of course, quantification over models will not appeal to the nominalist, but we can use Ketland's idea to improve Rosen's account in the obvious way:

> *S is nominalistically adequate relative to $\mathcal{L}$* iff there is a possible world w such that
>
> (i) S is true at w
>
> (ii) The concrete core of w is isomorphic to the concrete core of α. That is,
>
> $$(\exists f)\left(f : \langle P^{w}, P_1^{w}, \ldots, P_n^{w} \rangle \cong \langle P^{\alpha}, P_1^{\alpha}, \ldots, P_n^{\alpha} \rangle\right)$$
>
> where P^w and P_i^w are the intended interpretations of P and P_i at w.

[4] In fact, Ketland uses a two-sorted theory in which the variables of the first sort range over the concrete objects and the variables of the second sort range over the abstract, nonconcrete objects. The effect is the same.

https://doi.org/10.1017/S1755020312000044 Published online by Cambridge University Press

Making this move, we avoid Rosen's quantification over intrinsic properties and we make his account more precise and fully explicit. However, these advantages come at a price. First, our account of nominalistic adequacy is relativized to a language $\mathcal{L}$, which supplies the predicates $P_1, \ldots, P_n$. In fact, this is not a serious problem, since our primary goal is to say when our best scientific theory is nominalistically adequate. Thus, when we come to apply our definition, there is a natural choice of language, namely, the nominalistically-acceptable part of that theory's language. So, we might say: A scientific theory T is nominalistically adequate *tout court* iff T is nominalistically adequate relative to the nominalistically-acceptable predicates in the language of T.

The second price we must pay is considerably higher: to avoid Rosen's quantification over intrinsic properties, we have made heavy use of the machinery of set-theoretic relational structures, as well as existential quantification over isomorphisms. Thus, made precise in this way, the notion of 'exact intrinsic duplicate' involves mathematical objects. And this seems to have dire consequences for the instrumental nominalist who wishes to use this account. After all, she says that what we are justified in believing is not that our best scientific theory is true, but that it is nominalistically adequate. Thus, given the above definition of nominalistic adequacy, we are justified in believing that there is a possible world at which our best scientific theory is true and such that there is an isomorphism between the concrete core of that world and the concrete core of the actual world. Thus, we are justified in believing that there are set-theoretic structures and isomorphisms. Thus, we are justified in believing that mathematical objects exist. But of course this is the very conclusion the instrumental nominalist wishes to resist.

Furthermore, the instrumental nominalist cannot avoid the commitment to mathematical objects revealed in her account of nominalistic adequacy by appealing to the same technique she uses to circumvent commitment to mathematical objects in her best scientific theory. That is, she cannot avoid the conclusion that we are justified in believing that there are mathematical objects by saying that what we are really justified in believing is not that our best scientific theory is nominalistically adequate, but rather that the claim that it is nominalistically adequate is itself nominalistically adequate. That move will not save her, since it still entails that we are justified in believing a claim of nominalistic adequacy, and this is enough to entail that we are justified in believing that there are mathematical objects.

Thus, if we are to use Rosen's account, we must remove its mathematical commitments while retaining the precision and explicitness obtained by using Ketland's appeal to set-theoretic structures and isomorphisms. Remaining within the possible worlds framework, whether or not this is possible will depend on whether we take worlds to overlap or not.

First, suppose that possible worlds do not overlap. In this case, we render our quantified modal claims using a counterpart relation. Let's follow Hazen (1976) in presenting Lewis' counterpart theory as a two-sorted theory: one sort of variables $v, w, \ldots$ ranges over the worlds; the other $x, y, \ldots$ ranges over the nonworld individuals. Then there is a counterpart relation Cxy that holds between nonworld individuals; there is a parthood relation Ixw that holds between nonworld individuals and worlds; and there is an accessibility relation Rwv that holds between worlds. Given a formula in $\mathcal{L}$ and a world w, let φ^w be the relativization of φ to w. Then we have:

> *S is nominalistically adequate relative to* $\mathcal{L}$ iff there is a world w such that:

(i) $R\alpha w$
(ii) S^w

https://doi.org/10.1017/S1755020312000044 Published online by Cambridge University Press

(iii) The counterpart relation induces an isomorphism between the concrete core of α and the concrete core of w.
That is,

(a) Every individual in w that is concrete at w has a unique counterpart at α, and this unique counterpart is concrete at α.
That is,

$$\forall x \big(Ixw \wedge Px \rightarrow (\exists! y(Iy\alpha \wedge Cyx) \wedge \forall z(Iz\alpha \wedge Czx \rightarrow Pz)) \big)$$

(b) Every individual in α that is concrete at α has a unique counterpart at w, and this unique counterpart is concrete at w.
That is,

$$\forall x \big(Ix\alpha \wedge Px \rightarrow (\exists! y(Iyw \wedge Cyx) \wedge \forall z(Izw \wedge Czx \rightarrow Pz)) \big)$$

(c) For each i, if P_i is a k_i-ary predicate, then

$$\forall x_1, \ldots, x_{k_i} \forall y_1 \cdots y_{k_i} \Big(\bigwedge_{j=1}^{k_i} Ix_j w \wedge \bigwedge_{j=1}^{k_i} Iy_j \alpha \wedge \bigwedge_{j=1}^{k_i} Cx_j y_j \rightarrow \big(P_i x_1 \ldots x_{k_i} \leftrightarrow P_i y_1 \ldots y_{k_i} \big) \Big)$$

The first point to note here is that we have successfully eliminated quantification over mathematical objects, such as functions and set-theoretic structures. The second point is that this definition fails to capture what we want. The problem is that the actual world might be such that there is no possible world w such that the counterpart relation induces a *bijection* between the concrete individuals at α and the concrete individuals at w.[5] Suppose, for instance, that the actual world contains two concrete individuals o_1 and o_2 that are utterly indiscernible from one another—that is, they share the same properties, both intrinsic and extrinsic, and they stand in exactly the same relations to exactly the same objects.[6] Now take a world exactly like the actual world in all concrete respects. Then it will contain two concrete individuals o_1' and o_2', which are again utterly indiscernible. Then it seems that there is no unique counterpart of o_1 and no unique counterpart of o_2. o_1' and o_2' are both counterparts of o_1 and they are both counterparts of o_2. In this situation, the counterpart relation doesn't even induce a function, let alone a bijective function. And so there is no world of the sort required for the definition of nominalistic adequacy. Of course, the problem might be solved if we were allowed to require not that the counterpart relation induces a bijection, but only that there is a subrelation of the counterpart relation that induces a bijection. But this requires quantifying over relations. And this is forbidden by the nominalist. I conclude that the counterpart theorist cannot give a nominalistically-acceptable account of nominalistic adequacy.

[5] Thanks to Joseph Melia for this point.

[6] Entangled bosons are thought to be like this. See Saunders (2006); for a dissenting view, see Muller & Seevinck (2009).

https://doi.org/10.1017/S1755020312000044 Published online by Cambridge University Press

Fortunately, no such problems arise if we suppose that possible worlds overlap. Then, given a formula φ of $\mathcal{L}$, let φ^w be the relativization of φ to w. Thus, $P^w x$ says that x exists at w and is concrete at w. Then we have:

> *S is nominalistically adequate relative to* $\mathcal{L}$ iff there is a world w such that:

(i) $R\alpha w$

(ii) S^w

(iii) The transworld identity relation induces an isomorphism between the concrete core of α and the concrete core of w.
That is,

(a) Every individual in w that is concrete at w exists at α and is concrete at α.
That is,

$$\forall x \left(P^w x \rightarrow P^\alpha x \right)$$

(b) Every individual in α that is concrete at α exists at w and is concrete at w.
That is,

$$\forall x \left(P^\alpha x \rightarrow P^w x \right)$$

(c) For each i, if P_i is a k_i-ary predicate, then

$$\forall x_1, \ldots, x_{k_i} \left(P_i^w x_1 \ldots x_{k_i} \leftrightarrow P_i^\alpha x_1 \ldots x_{k_i} \right)$$

In sum, in the framework of possible worlds, it is possible to define nominalistic adequacy in a precise and fully explicit manner without appeal to mathematical objects, providing we assume that worlds overlap. We combine the modal framework of Rosen's account with the precision of Ketland's account; but we paraphrase away the quantification over set-theoretic structures and isomorphisms.

§5. Primitive modality and instrumental nominalism. Nonetheless, certain nominalists will not yet be satisfied. After all, the definition of nominalistic adequacy at which we arrived in the previous section involves existential quantification over possible worlds; and many nominalists disavow our knowledge of these just as they disavow our knowledge of mathematical objects. This brings us to the central question of the paper: Can we give an account of nominalistic adequacy that does not appeal to possible worlds? Can we give an account that appeals only to primitive modality? In this section, I show that we can; but it is not straightforward.

It will be useful to begin by presenting the only other attempt to give such an account of which I am aware. It is due to Joel Friedman, who calls the position *modal platonism* (Friedman, 2005). Friedman takes the view that a proposition can be actually true even if some of the objects with which the proposition is concerned do not actually exist. Thus, for instance, he takes the following proposition to be true even if, actually, there are no sets:

(10) It is possible that there be a set containing all and only the actual moons of Earth, and it be actually true that the set contain exactly one element.

https://doi.org/10.1017/S1755020312000044 Published online by Cambridge University Press

More precisely:

$$(11)\ \Diamond\exists S\left[@(\forall x(x \in S \leftrightarrow Mx) \wedge \exists x(x \in S) \wedge \forall x, y(x, y \in S \rightarrow x = y))\right]$$

where '@' is the actuality operator.

For Friedman, then, what we are justified in believing is not our best scientific theory; rather, we are justified in believing that it is possible that there be the mathematical objects with which that theory is concerned and that they *actually* bear the relations to the *actual* concrete objects that are required to make that theory *actually* true.

This seems odd in two ways: first, it is odd for a set at one world to contain only elements from a different world; second, it is odd for a proposition involving a set that doesn't exist at the actual world to be true at the actual world. And the same goes for other sorts of mathematical objects. Nonetheless, Friedman answers:

> [W]e may ask: why isn't it logically possible that the actual Earth moon is actually a member of a non-actual set? As before, isn't there enough ambiguity in our use of "possibility" and "actuality" to make this work? Suppose you just don't think this is possible, as you understand modal/actuality statements and possible worlds semantics. Then here is my reply:
>
> SO MUCH THE WORSE FOR POSSIBLE WORLDS SEMANTICS AS THE EXCLUSIVE INTERPRETATION OF MODAL/ACTUALITY STATEMENTS. (Friedman, 2005, p. 239)

Perhaps. But it would certainly be preferable if we were able to give an account of the use of mathematics in science and everyday life that would not commit us to the truth of the following modal sentence: There are no actual winged horses, but it is possible that I'm actually riding one. It is this that I hope to provide in this section.

At first sight, it might seem obvious how to give such an account. After all, at the end of the previous section, we gave an account in terms of possible worlds; and it might seem straightforward to translate this into a language with primitive modal operators, possibly including an actuality operator. However, this is a case that witnesses the well-known fact that there is not always an adequate translation of possible worlds discourse into standard primitive modal discourse with the operators $\Box$, $\Diamond$, and @.

To render the definition of nominalistic adequacy, we need to render the clauses (ii) and (iii) from the overlapping worlds definition of nominalistic adequacy given above. Once this is done, we can conjoin these renderings and apply the possibility operator. (ii) is straightforward, for it is simply rendered as S. But (iii) presents serious difficulties. We can use the actuality operator to say, within the scope of a possibility operator, that every concrete thing at the world introduced by the possibility operator exists at the actual world and is concrete at the actual world—that is, we can render (iii)(a):

$$(12)\ \forall x(Px \rightarrow @\exists y(y = x \wedge Py))$$

But we cannot say that every concrete thing at the actual world exists at the world introduced by the possibility operator and is concrete at that world—that is, we cannot render (iii)(b). For instance,

$$(13)\ @\forall x(Px \rightarrow \exists y(y = x \wedge Py))$$

will not do, since the existential quantifier is still bound by the actuality operator, and thus ranges over the objects at the actual world, not the objects at the possible world introduced

https://doi.org/10.1017/S1755020312000044 Published online by Cambridge University Press

by the initial possibility operator. Indeed, the attempted formulation just given in (13) is a logical truth![7]

A couple of remarks about the semantics for first-order modal logic. First, we have assumed here that the range of the first-order quantifiers in our modal language is the domain of the possible world introduced by the modal operator in whose scope the quantifier falls. That is, we are considering an *actualist* or *variable domain* semantics for first-order modal logic. This contrasts with a *possibilist* or *fixed domain* semantics in which all quantifiers range over all possibilia. It is clear that the actualist semantics is most appealing to the nominalist, since it avoids any quantification over nonactual possibilia outside the scope of a modal operator. Second remark: On our semantics, we have the following logical validity: $\Box[\Phi(x) \rightarrow \exists y(y = x)]$ for any formula Φ. That is, a property can be predicated of something at a world only if that thing exists at a world. This is what Friedman denies in the quotation above. One consequence of this is that $\Diamond\forall x(Px \rightarrow @\exists y(y = x \wedge Py))$ is equivalent to $\Diamond\forall x(Px \rightarrow @Px)$.

Let us return to the inexpressibility of (iii)(b). I favor a solution to this problem that supplements $\Box$, $\Diamond$, and $@$ with two new primitive modality operators. I will call these *secondary modal operators* to distinguish them from $\Box$, $\Diamond$, and $@$, which are hereby dubbed *primary modal operators*.[8] The problem with (13) is the range of the existential quantifier. We need it to range over the objects at the possible world introduced by the possibility operator in whose scope it ultimately falls; but, since the modal operator in whose scope it most immediately falls is the actuality operator, it ranges over the objects at the actual world instead. Our inability to overcome this obstacle highlights a general expressive limitation in modal logic that only really becomes apparent in the first-order version of the theory. The problem is this: The possibility and necessity operators allow us to introduce *new* possible worlds and talk about what holds at those. But once a world has been introduced, the truth of the formula that follows the operator that introduces it is evaluated entirely in that world, or in some new world that is introduced by another possibility or necessity operator that lies in the scope of the original operator. Thus, for instance, there is no way to speak within the scope of one modal operator of what is true at a world introduced by a *previous* modal operator. The exception is the actual world. We can talk within the scope of any modal operator of what is true at the actual world. The solution that I propose is that we introduce two new modal operators. The first allows us to change the world w at which the subsequent formula is evaluated to the world w' introduced by the *previous primary* modal operator $\Box$, $\Diamond$, or $@$. I will write this operator $\triangleleft$. But, of course, having introduced $\triangleleft$, we need a second modal operator that 'undoes' the work done by $\triangleleft$. That is, once we have shifted the locus of evaluation from w to w', and said something

[7] For the technical details and proof strategies for inexpressibility results like this one, see Hazen (1976) and Hodes (1984a, 1984b). Roughly speaking, the idea is this: Suppose T is a set of sentences in our modal language and suppose that $(\mathcal{M}, w_0)$ is a Kripke model with distinguished world w_0 such that $\mathcal{M}, w_0 \models T$ and $(\mathcal{M}, w_0)$ has the property we wish to capture. Then we use Henkin-style constructions to produce a further model $(\mathcal{M}', w_0')$ such that $\mathcal{M}', w_0' \models T$ and $(\mathcal{M}', w_0')$ does not have the property we wish to capture.

[8] Alternative solutions to similar problems are given in Forbes (1989), Peacocke (1978), and Correia (2007). For an overview of how these various solutions relate to one another, see Appendix 2. I am very grateful to Graham Leach-Krouse for extremely useful correspondence about these new operators.

https://doi.org/10.1017/S1755020312000044 Published online by Cambridge University Press

about that world, we may wish to shift back again to w in order to say something else. Thus, we introduce $\triangleright$, which does exactly this.[9]

The semantics for quantified modal logic supplemented by $\triangleleft$ and $\triangleright$ is given in Appendix 1. But to understand how it works, it is best to see it in action. It is often said that, in order to render the following proposition, we need an actuality operator:

(14) It might have been that everyone who is in fact rich was poor.

Those who argue in this way usually proceed to render (14) as follows:

(15) $\Diamond\forall x(@Rx \rightarrow Px)$

However, as Michael Fara and Timothy Williamson observe, if our quantifiers are actualist, this is not sufficient: (15) does not capture (14) (Fara & Williamson, 2005, p. 4). (15) is made true by the existence of a possible world at which none of the people who are rich at the actual world exist—for instance, a world inhabited entirely by the actual poor. But, intuitively, (14) is not made true by the existence of such a world. Rather, it is made true by the existence of a world at which all of the actual rich exist, but at which they are poor. Thus, we need to add to (15) the claim that, at the possible world introduced by the possibility operator, all the people who are actually rich exist. But the same problem arises here as arose when we tried to say that all actual concrete things exist and are concrete at the possible world introduced by the initial possibility operator in the definition of nominalistic adequacy. Quantified modal logic equipped with possibility and necessity operators and an actuality operator cannot help. But our new operator can. We can now render (14) as follows:

(16) $\Diamond@\forall x(Rx \rightarrow \triangleleft Px)$

This asserts that, at some possible world, everything that is rich at the actual world is poor. I submit that this is an adequate rendering of (14). But it is only possible if we use our new modal operator $\triangleleft$ (or some equally expressive extension of modal logic, such as those surveyed in Appendix 2).[10]

Next, let's consider an example that requires both of our new modal operators. Consider the following conversation:

(17) A: It might have been that everyone who is in fact rich was poor.
B : That's necessarily so.

There may be a number of ways to parse what B asserts, but one natural rendering is this: It is not a quirk of how things actually are that the rich might all have been poor; it is true however things are. More precisely: For any world w, there is a world w' at which all the rich *of* w exist and are poor. However, without $\triangleright$, this proposition cannot be rendered.[11] With $\triangleright$, it is rendered as follows:

---

[9] Harold Hodes considers the operator $\triangleleft$ and gives the following useful gloss: "['$\triangleleft$'] is to '@' as on a typewriter "back-space" is to "carriage-return"." (Hodes, 1984a, p. 426). Oddly, he does not consider $\triangleright$ as well. We require $\triangleright$ for our purposes.

[10] Again, the relevant results follow from those in Hazen (1976) and Hodes (1984b).

[11] The problem is that we need a universally quantified formula with the predicate R in the antecedent and P in the consequent. But the quantification and the antecedent must be evaluated at the world introduced by $\Box$, while the consequent must be evaluated at the world introduced by $\Diamond$. But, since $\Diamond$ must occur after $\Box$, this is impossible without using $\triangleright$.

https://doi.org/10.1017/S1755020312000044 Published online by Cambridge University Press

(18) $\Box\Diamond(\triangleleft\forall x(Rx \rightarrow \triangleright Px))$

That is: For any world w, there is a world w' such that, everything that is rich *at* w is poor *at* w'.

These examples serve two purposes. The first is to illustrate how our new modal operators work; both examples help with this. The second is to show that they are necessary in order to render certain ordinary sentences in English, and thus that their introduction is not *ad hoc*. In this respect, the second example is particularly important. In Appendix 2, I consider a number of different ways in which we might enrich first-order modal logic in order to extend its expressive power. On all of these ways, we are able to express (14); but on only a handful are we able to express (17). Of that handful, I will argue that $\triangleleft$ and $\triangleright$ are the most acceptable to the nominalist.

So much for our examples. Now we are ready to state nominalistic adequacy using the primitive modal operators $\Diamond$, @, and $\triangleleft$. We will need $\Box$ and $\triangleright$ when we come to consider the instrumental nominalist's response to Ketland's indispensability argument below.

S is nominalistically adequate relative to language $\mathcal{L}$ iff

$$\begin{aligned}
&\Diamond[S\wedge\\
&\quad\forall x(Px \rightarrow @Px)\wedge\\
&\quad @\forall x(Px \rightarrow \triangleleft Px)\wedge\\
&\qquad\forall x_1,\ldots,x_{k_1}(\textstyle\bigwedge_{j=1}^{k_1} Px_j \rightarrow (P_1x_1\ldots x_{k_1} \leftrightarrow @P_1x_1\ldots x_{k_1}))\wedge\\
&\qquad\ldots\wedge\\
&\qquad\quad\forall x_1,\ldots,x_{k_n}(\textstyle\bigwedge_{j=1}^{k_n} Px_j \rightarrow (P_nx_1\ldots x_{k_n} \leftrightarrow @P_nx_1\ldots x_{k_n}))]
\end{aligned}$$

where k_i is the arity of P_i.

The first conjunct after the initial possibility operator renders (ii). The second conjunct renders (iii)(a). The third renders (iii)(b). And the remaining conjuncts render (iii)(c).

Thus, for any theory, there is a proposition that does not quantify over mathematical objects or over possible worlds, but which gives the nominalistic content of that theory. According to the instrumental nominalist, it is this proposition that we are justified in believing; not the proposition expressed by the theory. In this way, they avoid the Quine–Putnam indispensability argument.

§6. Ketland's indispensability argument. The Quine–Putnam indispensability argument starts from our use of mathematics in our best scientific theories to describe the concrete world. But of course we do not use mathematics only for this purpose; we also use it to reason about the concrete world. And not only in our best scientific theories; we also use it to reason about everyday facts about the concrete world. Ketland's new indispensability argument begins in this use of mathematics (Ketland, 2005). Ketland argues that there is a vast array of inferences concerning the concrete world that we know to be valid, yet whose validity cannot be established in a feasible amount of time without appealing to mathematics. To obtain such knowledge of validity, Ketland claims, knowledge of mathematical objects is practically indispensable. Therefore, we must have it.

I will argue that, in fact, we do not need knowledge of mathematical objects in order to know these validity facts. Rather, the instrumental nominalist can point to nominalistically-acceptable knowledge that will suffice instead. Moreover, I will argue that knowledge of mathematical objects does not even suffice.

https://doi.org/10.1017/S1755020312000044 Published online by Cambridge University Press

To make Ketland's argument and our response more precise, let's consider a particular inference of the sort to which Ketland appeals:

(i) The number of people in the room is 100.
(ii) The number of houses in the street is 99.
(iii) Each person in the room lives in exactly one house in the street.

Therefore,

(vi) At least two people in the room share the same house.

Call this inference I. The relevant features of I are these: First, its premises and conclusion can be rendered in first-order logic with identity using the standard recursive definitions of finite cardinality quantifiers $\exists_k$. Thus, they are nominalistically-acceptable statements. Second, once rendered in first-order logic with identity, the conclusion can be derived from the premises in any standard proof system for that logic. Third, the shortest proof of the conclusion from the premises in any standard proof system for first-order logic with identity is unfeasibly long when the length is measured by counting the number of symbols in the proof. Fourth, there is a proof of feasible length that employs mathematics. As it is sometimes put in the terminology of proof theory, a proof system that allows certain mathematical reasoning provides *speed up* over a proof system that does not. For more detailed discussion of these features, I refer the reader to Ketland's paper; here, I simply assume that they are true. On the basis of these features, Ketland mounts his indispensability argument:

> [T]he 'speed-up' phenomenon under discussion suggests a modified version of the indispensability argument, based now on unfeasibility considerations. Presumably the nominalist does not wish to deny the validity of [I]. But there is no feasible direct verification for [I], and the short mathematical derivations involve practically indispensable assumptions about numbers, sets and functions. So, how might a nominalist account for our knowledge that such inferences are valid? After all, the anecdotal evidence is that even non-mathematicians find [I] 'obvious'. [. . .] [T]he nominalist cannot give a nominalistically acceptable reason for thinking that [I is] indeed valid. (Ketland, 2005, p. 23)

We might restate the argument as follows:

(K1) We are justified in believing that I is valid.
(K2) We are justified in believing that I is valid only if we are justified in believing that mathematical objects exist.

Therefore,

(K3) We are justified in believing that mathematical objects exist.

Thus, Ketland's argument is a sort of transcendental argument. It attempts to draws out the epistemic conditions for justified belief in certain validity facts. It does not explain what it is that justifies our belief that mathematical objects exist. Instead, it claims to show that, only if we have such justification can we be justified in believing certain things that we ordinarily take ourselves to be justified in believing.

I will argue that the instrumental nominalist can and should deny (K2). But first we must make the argument precise by saying what it means to say that I is valid. This is crucial

https://doi.org/10.1017/S1755020312000044 Published online by Cambridge University Press

since it is the proposition that *I* is valid that Ketland takes us to be justified in believing, and justified belief in which he takes to require justified belief in mathematical objects.

Here are four interpretations of the claim that *I* is valid:

- (*Model-theoretic*) If we formalize (i)–(iv) as (i')–(iv') respectively, then every model of (i')–(iii') is a model of (iv').
- (*Proof-theoretic*) If we formalize (i)–(iv) as (i')–(iv') respectively, then there is a derivation of (iv') from (i')–(iii') in a sound proof system.
- (*Epistemic*) If we are justified in believing (i)–(iii), then we are justified in believing (iv).
- (*Modal*) Necessarily, if (i)–(iii), then (iv).

I submit that the modal interpretation is primary. The proof-theoretic interpretation simply provides evidence for the model-theoretic interpretation, since we assume that the proof system is sound. And the model-theoretic interpretation provides evidence for the modal interpretation, since we assume that the variety of possible situations is adequately captured by the variety of models. Finally, the epistemic interpretation seems to follow from the modal interpretation together with certain assumptions about the aims of belief. In any case, I will take the modal interpretation to be primary. And I will take it to provide the proposition Ketland intends in his argument. Thus, we are justified in believing the following:

(19) $\Box((i) \wedge (ii) \wedge (iii) \rightarrow (iv))$.

It is this proposition that Ketland claims we can be justified in believing only if we are justified in believing that there are mathematical objects.

Both nominalist and antinominalist can agree that we can only justify our belief in (19) by using reasoning that employs mathematics. For instance, we may use arithmetic reasoning, which employs postulates concerning the natural numbers—call the conjunction of these (NN)—and bridge principles, such as abstraction principles, which relate the concrete world to the postulated numbers—call the conjunction of these (BNN). Or, we may use set-theoretic reasoning, which employs postulates concerning finite sets—call the conjunction of these (ST)—and bridge principles, such as comprehension principles, relating the concrete world to the postulated sets—call the conjunction of these (BST). Thus, what our mathematical reasoning establishes initially is this:

(20) $\Box((i) \wedge (ii) \wedge (iii) \wedge (NN) \wedge (BNN) \rightarrow (iv))$

Or:

(21) $\Box((i) \wedge (ii) \wedge (iii) \wedge (ST) \wedge (BST) \rightarrow (iv))$

That is, the mathematical reasoning that we take to establish the validity of *I* in fact shows that (iv) follows of necessity from (i)–(iii) *together with our assumptions about the mathematical world and its relation to the concrete world*. If it is to show further that (iv) follows of necessity from (i)–(iii) *alone*, we must show how to move from (20) or (21) to (19). How are we to do this?

One standard move for the antinominalist is this. Add to the antinominalist claim that numbers and sets actually exist and that the bridge principles that relate them to concrete world actually hold, the further claim that they exist necessarily and that the bridge principles hold necessarily. That is:

(22) $\Box((NN) \wedge (BNN))$

https://doi.org/10.1017/S1755020312000044 Published online by Cambridge University Press

or

(23) $\Box((ST) \wedge (BST))$

Then it is easy to see that (19) follows from (20) and (22) or from (21) and (23) by uncontroversial rules of modal logic. Thus, if the antinominalist can argue that we are justified in believing (22) or (23), then she can use this to argue that we are justified in believing (19) on the basis of our mathematical reasoning. But such a move will certainly not appeal to all antinominalists. In particular, it will not appeal to many whose antinominalism is motivated by the Quine–Putnam indispensability argument. After all, while they might hold that the use of mathematics in science gives us reason to believe that there are mathematical objects, it is a significant extra step to hold that this use gives us reason to believe that these mathematical objects exist necessarily; and it is not clear what could motivate this step. However, if she cannot, she is in the same position as the nominalist. Thus, it is clear that knowledge of $(NN) \wedge (BNN)$ is not sufficient for knowledge that I is valid. If we are to follow the antinominalist route, we must know further that this is necessary.

So let's see what we can do if we do not assume either (22) or (23). How else might we move from (20) or (21) to (19)? One natural suggestion is this: Suppose we were justified in believing that, for any world w at which (i)–(iii) are true, there is a world w' whose concrete core is exactly like the concrete core of w, but at which there are numbers that satisfy (NN), say, and at which the bridge principles (BNN) hold. If we were justified in believing this, then we could infer (19) from (20) (and similarly for (21)). For, by (20), we would know that (iv) is true at w'. And, since the concrete core of w' is exactly like the concrete core of w, and since the truth or falsity of (iv) depends only on the way the concrete core is, we would know that (iv) is true at w, as required.

In fact, we might know something more general than that. We might know that, for *any* world w, there is a world w' whose concrete core is exactly like the concrete core of w, but at which there are numbers that satisfy (NN) and at which the bridge principles (BNN) hold. In this case, we would be able to know that any use of (NN) and (BNN) in our reasoning about the concrete world would give rise to an inference whose validity does not depend on the truth of (NN) or (BNN). In this case, we may say that (NN) $\wedge$ (BNN) is *nominalistically conservative*.

Now, so far, we have not been completely precise: we have spoken of concrete cores at different worlds being exactly alike. But, as we know from previous sections, such talk can be made precise without reference to mathematical objects, and even without reference to possible worlds. First, here is the precise definition that appeals to overlapping possible worlds; a definition involving nonoverlapping worlds faces the same problems as before. Suppose T is a theory of abstracta with bridge principles relating the abstracta and the concreta:

> T *is nominalistically conservative* iff for all possible worlds w such that $R\alpha w$, there is a possible world w' such that

(i) Rww'

(ii) $T^{w'}$

(iii) The transworld identity relation induces an isomorphism between the concrete core of w' and the concrete core of w. That is,

  (a) Every individual in w' that is concrete at w' exists at w and is concrete at w.

https://doi.org/10.1017/S1755020312000044 Published online by Cambridge University Press

That is,

$$\forall x \left(P^{w'} x \rightarrow P^{w} x \right)$$

(b) Every individual in w that is concrete at w exists at w' and is concrete at w'.
That is,

$$\forall x \left(P^{w} x \rightarrow P^{w'} x \right)$$

(c) For each i, if P_i is a k_i-ary predicate, then

$$\forall x_1, \ldots, x_{k_i} \left(P_i^{w'} x_1 \ldots x_{k_i} \leftrightarrow P_i^{w} x_1 \ldots x_{k_i} \right)$$

Next, here is the definition that uses the primitive modal operators $\Box$, $\Diamond$, $\triangleleft$, and $\triangleright$:

T is nominalistically conservative iff
$\Box\Diamond[T \wedge$
$\quad \forall x(Px \rightarrow \triangleleft Px) \wedge$
$\quad \triangleleft \forall x(Px \rightarrow \triangleright Px) \wedge$
$\quad\quad \forall x_1, \ldots, x_{k_1} (\bigwedge_{j=1}^{k_1} Px_j \rightarrow (P_1 x_1 \ldots x_{k_1} \leftrightarrow \triangleleft P_1 x_1 \ldots x_{k_1})) \wedge$
$\quad\quad \ldots \wedge$
$\quad\quad \forall x_1, \ldots, x_{k_n} (\bigwedge_{j=1}^{k_n} Px_j \rightarrow (P_n x_1 \ldots x_{k_n} \leftrightarrow \triangleleft P_n x_1 \ldots x_{k_n}))]$

where k_i is the arity of P_i.

Thus, in order to move from (20) to (19), we needn't go via (22). Instead, we can go via the claim that (NN) $\wedge$ (BNN) is nominalistically conservative. From that claim and (20), we can deduce (19).

Therefore, in order to answer Ketland's argument by denying (K2), we must argue that we are justified in believing this latter claim. That is, we must argue that we are justified in believing that, to any possible world, it is possible to add objects that make (NN) and (BNN) true without changing anything about the concrete relations between concrete objects. This is not trivial. For one thing, we must be justified in believing that (NN) is consistent; for another, we must be justified in believing that, once the numbers are added to a world, the bridge principles (BNN) that relate the concrete objects to the numbers will turn out true. We know from Frege's Basic Law V that there are plausible-looking theories of mathematical objects that are inconsistent. And we know from Boolos' ingenious abstraction principles that there are bridge principles relating concreta and abstracta such that, though the theory of the abstracta is consistent, these abstracta cannot always be added to a world in such a way as to make the bridge principles true. For instance, Boolos' Parity Principle will only turn out true when the abstracta are added to worlds that contain only finitely many objects (Boolos, 1990).[12]

It is not obvious how we might justify the belief that it is always possible to add objects to a world in order to make (NN) and (BNN) come out true. The epistemology of consistency is a woefully neglected topic in the philosophy of mathematics, and I have little to add to it here. However, I wish to note that this is not a problem that the nominalist faces alone.

[12] Though note that, in each of these cases, the background theory to which the abstraction principle is added is second-order logic rather than first-order logic.

https://doi.org/10.1017/S1755020312000044 Published online by Cambridge University Press

It is equally pressing for the antinominalist. The antinominalist has two options. She must either say that we are justified in believing (22), and then justify the move from (20) to (19) by appealing to that claim; or she must say that we are justified in believing that (NN) $\wedge$ (BNN) is nominalistically conservative, and justify the move from (20) to (19) in that way. In the second case, it is clear that she faces the same justificatory demands that the instrumental nominalist faces. But in the first case, she also faces such demands, since she claims that (NN) and (BNN) are true at *all* worlds. Thus, she must certainly be justified in believing that (NN) is consistent, and that (BNN) places no restrictions on how the concrete world is—that is, it is not like Boolos' parity principle.

I conclude that the instrumental nominalist can answer Ketland's argument. Although we must use mathematical reasoning to justify our belief that I is valid, this does not entail that we must be justified in believing that mathematical objects exist. Rather, it entails only that we are justified in believing that our mathematical reasoning is nominalistically conservative. But this does not entail that we are justified in believing that there are mathematical objects. And, besides, the antinominalist must also justify our belief in the nominalistic conservativeness of our mathematical reasoning prior to carrying out any such reasoning.

§7. Conclusion. In sum: Neither the Quine–Putnam indispensability argument nor Ketland's new indispensability argument defeats the nominalist. In the former case, she simply denies that we are justified in believing our best scientific theory. Instead, we are justified in believing that our best scientific theory is nominalistically adequate. That is, she appeals to instrumental nominalism. Crucially, the definition of nominalistic adequacy is given without appeal to mathematical objects, and without even appeal to possible worlds. What is required instead is the introduction of two new primitive modal operators. But, as we saw, these are required anyway in order to render certain natural assertions in English. So their introduction is not *ad hoc*.

In the case of Ketland's argument, on the other hand, we are justified in believing that inference I is valid. But this does not entail that we are justified in believing that mathematical objects exist. Rather, we are justified in believing that our mathematical reasoning is nominalistically conservative, where again this is defined without appeal to mathematical objects, and without appeal to possible worlds. And this suffices to justify our belief that I is valid.

In short: a theory need only be nominalistically adequate in order for us to use it to describe the concrete world; and it need only be nominalistically conservative in order to for us to use it to reason about the concrete world. And neither of these entail that there are mathematical objects.

§8. Appendix 1: The semantics for $\triangleleft$ and $\triangleright$. The crucial difference between the semantics for modal logic with $\Box$, $\Diamond$, @, $\triangleleft$, and $\triangleright$ and the semantics for modal logic with only $\Box$, $\Diamond$, and @ is this: in the latter, a formula φ is evaluated at a *single world* w; but in the former, it is evaluated at a *finite sequence of worlds* $s(1), \ldots, s(n)$ with a distinguished term $s(i)$. The idea is that $s(i)$ is the world at which the formula is to be evaluated; while the sequence $s(1), \ldots, s(n)$ keeps track of the worlds that have been introduced by previous primary modal operators.

- A model for our expanded language is a standard Kripke model for variable domain quantified modal logic with an actuality operator: that is, $\mathcal{M} = (W, R, I, D, \alpha)$ where W is thought of as the set of possible worlds; $R \subseteq W \times W$ is the accessibility

https://doi.org/10.1017/S1755020312000044 Published online by Cambridge University Press

relation between worlds; D is the domain function, which assigns to each world $w \in W$ a domain; I is the interpretation function, which assigns to each m-ary predicate P and each world $w \in W$ a set of m-tuples $I_w(P) \subseteq D(w)^m$; and α is the distinguished world that is the actual world for this model.

- A variable assignment is a function τ from the free variables $a_1, a_2, \ldots$ of the language to the union of all domains, $\bigcup_w D(w)$.
- Given a model $\mathcal{M} = (W, R, I, D, \alpha)$, a sequence $s(1), \ldots, s(n)$ of worlds, a number $1 \leq i \leq n$, and a variable assignment τ, we define $\mathcal{M}, s, i, \tau \models \varphi$ recursively as follows:

(i) $\mathcal{M}, s, i, \tau \models Pa_1 \ldots a_m \Leftrightarrow (\tau(a_1), \ldots, \tau(a_m)) \in I_{s(i)}(P)$, where P is an m-ary predicate.

(ii) $\mathcal{M}, s, i, \tau \models \varphi \wedge \psi \Leftrightarrow \mathcal{M}, s, i, \tau \models \varphi$ and $\mathcal{M}, s, i, \tau \models \psi$.

(iii) $\mathcal{M}, s, i, \tau \models \neg\varphi \Leftrightarrow M, s, i, \tau \not\models \varphi$.

(iv) $\mathcal{M}, s, i, \tau \models \forall x\varphi[x/a_j] \Leftrightarrow$ for all $d \in D(s(i))$, we have

$$\mathcal{M}, s, i, \tau\,[a_j \mapsto d] \models \varphi,$$

where $\tau[a_j \mapsto d]$ is obtained from τ by mapping the free variable a_j to the element d of $D(s(i))$ and leaving all other values unchanged.

(v) $\mathcal{M}, s, i, \tau \models \Box\varphi \Leftrightarrow$ for all $w \in W$ such that $Rs(i)w$, we have

$$\mathcal{M}, s\,[i+1 \mapsto w], i+1, \tau \models \varphi,$$

where

$$s\,[i+1 \mapsto w]\,(k) = \begin{cases} s(k) & \text{if} \quad 1 \leq k \leq i \\ w & \text{if} \quad k = i+1 \\ s(k-1) & \text{if} \quad k > i+1 \end{cases}$$

In other words, the sequence $s[i+1 \mapsto w]$ is obtained from the sequence s by 'slotting in' w between $s(i)$ and $s(i+1)$.

(vi) $\mathcal{M}, s, i, \tau \models \Diamond\varphi \Leftrightarrow$ for some $w \in W$ such that $Rs(i)w$, we have

$$\mathcal{M}, s\,[i+1 \mapsto w], i+1, \tau \models \varphi.$$

(vii) $\mathcal{M}, s, i, \tau \models @\varphi \Leftrightarrow$ we have

$$\mathcal{M}, s\,[i+1 \mapsto \alpha], i+1, \tau \models \varphi.$$

(viii) $\mathcal{M}, s, i, \tau \models \triangleleft\varphi \Leftrightarrow$ we have

$$\mathcal{M}, s, i \overset{1}{-} 1, \tau \models \varphi,$$

where

$$i \overset{1}{-} 1 = \begin{cases} i-1 & \text{if} \quad i > 1; \\ 1 & \text{if} \quad i = 1. \end{cases}$$

That is, $\overset{1}{-}$ is cut-off subtraction with cut-off at 1.

(ix) $\mathcal{M}, s, i, \tau \models \triangleright\varphi \Leftrightarrow$ we have

$$\mathcal{M}, s, i \overset{n}{+} 1, \tau \models \varphi,$$

https://doi.org/10.1017/S1755020312000044 Published online by Cambridge University Press

where

$$i \overset{n}{+} 1 = \begin{cases} i+1 & \text{if } i < n; \\ n & \text{if } i \geq n. \end{cases}$$

That is, $\overset{n}{+}$ is cut-off addition with cut-off at n.

- If φ is a sentence, we write $\mathcal{M} \models \varphi$ iff $\mathcal{M}, s = (\alpha), 1, \tau \models \varphi$, for any variable assignment τ.

§9. Appendix 2: Alternative extensions of first-order modal logic. In order to be able to express the properties of *nominalistic adequacy* and *nominalistic conservativity*, we must be able to express the following claims:

(24) There is a possible world w such that everything that is concrete at α is concrete at w and everything that is concrete at w is concrete at α.

(25) For every possible world w there is a possible world w' such that everything that is concrete at w is concrete at w' and everything that is concrete at w' is concrete at w.

In Section §5, I suggested that we should enrich our first-order modal language by adding the operators $\triangleleft$ and $\triangleright$ in order to be able to express these. In Appendix 1, I gave the semantics for these operators. In this appendix, I wish to survey the alternative ways in which we might enrich modal language to obtain this expressive power. I conclude that $\triangleleft$ and $\triangleright$ are preferable for the nominalist.

There are three ways in which we might enrich our modal language in order to be able to express (24) and (25): we might replace the actualist semantics for the first-order quantifiers with a possibilist semantics; we might add second-order or plural quantifiers; we might add new operators, such as Vlach operators or $\triangleleft$ and $\triangleright$.

Possibilist first-order quantifiers In the actualist semantics for first-order modal logic (without $\triangleleft$ and $\triangleright$), the clause for quantification is this:

(iii′) $\mathcal{M}, w, \tau \models \forall x \varphi[x/a_j] \Leftrightarrow$ for all $d \in D(w)$, we have $\mathcal{M}, w, \tau[a_j \mapsto d] \models \varphi$.

In the possibilist semantics, it is this:

(iii″) $\mathcal{M}, w, \tau \models \forall x \varphi[x/a_j] \Leftrightarrow$ for all $d \in \bigcup_w D(w)$, we have $\mathcal{M}, w, \tau[a_j \mapsto d] \models \varphi$.

With this in hand, we can express (24) as follows:

$(24)_p$ $\Diamond \forall x (@Px \leftrightarrow Px)$

But we still cannot express (25). The problem is that, in order to introduce the worlds w and w' we need $\Box$ followed by $\Diamond$. And we need universally quantified conditionals with P in the antecedent and P in the consequent. In one of these conditionals, we need the antecedent to be evaluated at the world introduced by $\Box$ and the consequent to be evaluated at the world introduced by $\Diamond$. And in the other conditional it must be the other way round. In the case of (24), we achieved this using the actuality operator to shift the locus of evaluation whilst inside the scope of $\Diamond$. But here we have no way of doing this.

Second-order (or plural) quantifiers We introduce a new (actualist or variable domain) second-order quantifier $\forall X$ and extend the standard semantics as follows: a variable assignment τ must now take values at the free second-order variables A, B, ... as well

https://doi.org/10.1017/S1755020312000044 Published online by Cambridge University Press

as the free first-order variables a, b, ... For each such variable A, $\tau(A) \subseteq \bigcup_w D(w)$. Then we add new clauses to the semantics:

(iiib) $\mathcal{M}, w, \tau \models a \in A \Leftrightarrow \tau(a) \in \tau(A) \subseteq D(w)$.

(iiic) $\mathcal{M}, w, \tau \models \forall X\varphi[X/A] \Leftrightarrow$ for all $D \subseteq D(w)$, we have $\mathcal{M}, w, \tau[A \mapsto D] \models \varphi$.

Using this new expressive power, we can express (24) and (25) as follows:

$(24)_s$ $\exists X[\forall x(x \in X \leftrightarrow Px) \wedge \Diamond\forall x(x \in X \leftrightarrow Px)]$

$(25)_s$ $\Box\exists X[\forall x(x \in X \leftrightarrow Px) \wedge \Diamond\forall x(x \in X \leftrightarrow Px)]$.

New operators Graeme Forbes suggests that we should add a pair of *Vlach operators* to our modal logic in order to overcome the inexpressibility of sentences like (14) (Forbes, 1989). These are written $\uparrow$ and $\downarrow$. Grammatically, $\uparrow$ must occur immediately after a modal operator, and $\downarrow$ must occur within the scope of $\uparrow$. The idea is that $\uparrow$ 'stores' the world introduced by the modal operator after which it occurs and $\downarrow$ 'retrieves' this world and makes it the locus of evaluation. With these operators, we can express (24):

$(24)_v$ $\Diamond \uparrow [@\forall x(Px \rightarrow \downarrow Px) \wedge \forall x(Px \rightarrow @Px)]$.

But we cannot express (25). The problem is that we need to store the world introduced by $\Box$ and the world introduced by $\Diamond$ and retrieve them again within one another's scope (Correia, 2007). Thus, we need at least two pairs of Vlach operators. Correia suggests that we introduce infinitely many pairs of Vlach operators $\uparrow_i$ and $\downarrow_i$. (A similar suggestion by Peacocke, 1978, turns out to be expressively equivalent.) With these in hand, we can express (25) as follows:

$(25)_{v*}$ $\Box \uparrow_1 \Diamond \uparrow_2 [\downarrow_1 \forall x(Px \rightarrow \downarrow_2 Px) \wedge \downarrow_2 \forall x(Px \rightarrow \downarrow_1 Px)]$

Alternatively, using the operators introduced in Section §5, we can render (24) and (25) as follows:

$(24)_b$ $\Diamond[\forall x(Px \rightarrow @Px) \wedge @\forall x(Px \rightarrow \triangleleft Px)]$

$(25)_b$ $\Box\Diamond[\forall x(Px \rightarrow \triangleleft Px) \wedge \triangleleft\forall x(Px \rightarrow \triangleright Px)]$

The upshot of all this is that there are (at least) three ways in which we might enrich our first-order modal language in order to express (24) and (25). We might add second-order quantifiers; we might add infinitely many Vlach operators; or we might add $\triangleleft$ and $\triangleright$. I claim that the latter is to be preferred. The ontological innocence of second-order quantifiers and even the corresponding plural versions is dubious (Resnik, 1988; Linnebo, 2003) and the nominalist should surely prefer a system with two new operators over one with infinitely many, especially when we note that it is straightforward to give translation schema that witness the expressive equivalence of adding infinitely many Vlach operators and adding $\triangleleft$ and $\triangleright$. In sum: the nominalist should add $\triangleleft$ and $\triangleright$ to her first-order modal language.

§10. Acknowledgments. I am very grateful to the extremely useful discussion at Mic Detlefsen's Intersem 2011 seminar; in particular, I thank Kevin Kelly, Timothy McCarthy, and Mic Detlefsen. I owe a great debt of gratitude to Cat St Croix and Graham Leach-Krouse for giving their time generously to think through the modal logic with me. Their comments greatly improved the paper; though any mistakes are, of course, still mine. My greatest debt is to Jeff Ketland for many very helpful discussions of this material.

https://doi.org/10.1017/S1755020312000044 Published online by Cambridge University Press

BIBLIOGRAPHY

Baker, A. (2005). Are there genuine mathematical explanations of physical phenomena? *Mind*, **114**, 223–238.

Boolos, G. (1990). The standard of equality of numbers. In Boolos, G., editor. *Meaning and Method: Essays in Honor of Hilary Putnam*. Cambridge, UK: Cambridge University Press, pp. 261–277.

Colyvan, M. (2010). There is no easy road to nominalism. *Mind*, **119**(474), 285–306.

Colyvan, M. (2011). Indispensability arguments in the philosophy of mathematics. In Zalta, E. N., editor. *Stanford Encyclopedia of Philosophy*. Stanford, CA: CSLI.

Correia, F. (2007). Modality, quantification, and many Vlach-operators. *Journal of Philosophical Logic*, **36**(4), 473–488.

Fara, M., & Williamson, T. (2005). Counterparts and actuality. *Mind*, **114**(453), 1–30.

Forbes, G. (1989). *Languages of Possibility*. Oxford, UK: Blackwell.

Friedman, J. (2005). Modal platonism: An easy way to avoid ontological commitment to abstract entities. *Journal of Philosophical Logic*, **34**(3), 227–273.

Hazen, A. (1976). Expressive completeness in modal language. *Journal of Philosophical Logic*, **5**, 25–46.

Hodes, H. T. (1984a). On modal logics which enrich first-order S5. *Journal of Philosophical Logic*, **13**(4), 423–454.

Hodes, H. T. (1984b). Some theorems on the expressive limitations of modal languages. *Journal of Philosophical Logic*, **13**, 13–26.

Ketland, J. (2005). More curious inferences. *Analysis*, **65**(1), 18–24.

Ketland, J. (2011). Nominalistic adequacy. *Proceedings of the Aristotelian Society*, **111**(2), 201–217.

Linnebo, Ø. (2003). Plural quantification exposed. *Noûs*, **37**(1), 71–92.

Melia, J. (1995). On what there's not. *Analysis*, **55**(4), 223–229.

Melia, J. (2000). Weaseling away the indispensability argument. *Mind*, **109**(435), 455–479.

Muller, F. A., & Seevinck, M. P. (2009). Discerning elementary particles. *Philosophy of Science*, **76**, 179–200.

Peacocke, C. (1978). Necessity and truth theories. *Journal of Philosophical Logic*, **7**, 473–500.

Pincock, C. (2007). A role for mathematics in the physical sciences. *Noûs*, **41**(2), 253–275.

Putnam, H. (1979a). Philosophy of logic. In *Mathematics, Matter, and Method: Philosophical Papers* (second edition), Vol. 1. Cambridge, UK: Cambridge University Press, pp. 323–357.

Putnam, H. (1979b). What is mathematical truth. In *Mathematics, Matter, and Method: Philosophical Papers* (second edition), Vol. 1. Cambridge, UK: Cambridge University Press, pp. 60–78.

Quine, W. V. O. (1980a). On what there is. In *From a Logical Point of View* (second edition). Cambridge, MA: Harvard University Press, pp. 1–19.

Quine, W. V. O. (1980b). Reference and modality. In *From a Logical Point of View* (second edition). Cambridge, MA: Harvard University Press, pp. 139–159.

Quine, W. V. O. (1980c). Two dogmas of empiricism. In *From a Logical Point of View*. Cambridge, MA: Harvard University Press, pp. 20–46.

Quine, W. V. O. (1981a). Success and limits of mathematization. In *Theories and Things*. Cambridge, MA: Harvard University Press, pp. 148–155.

Quine, W. V. O. (1981b). Things and their place in theories. In *Theories and Things*. Cambridge, MA: Harvard University Press, pp. 1–23.

https://doi.org/10.1017/S1755020312000044 Published online by Cambridge University Press

Resnik, M. (1983). Review of *Science without Numbers* by Hartry Field. *Noûs*, **17**(3), 514–519.
Resnik, M. (1988). Second-order logic still wild. *Journal of Philosophy*, **85**, 75–87.
Rosen, G. (2001). Nominalism, naturalism, and epistemic relativism. *Philosophical Perspectives*, **15**, 69–91.
Saunders, S. (2006). Are quantum particles objects? *Analysis*, **66**, 52–63.
van Fraassen, B. (1980). *The Scientific Image*. Oxford, UK: Clarendon Press.

DEPARTMENT OF PHILOSOPHY
UNIVERSITY OF BRISTOL
43 WOODLAND ROAD, BRISTOL BS8 1TB, UK
E-mail: richard.pettigrew@bristol.ac.uk

https://doi.org/10.1017/S1755020312000044 Published online by Cambridge University Press