Fuel burn prediction algorithm for cruise, constant speed and level flight segments

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ABSTRACT

This paper presents a new algorithm that predicts the quantity of fuel burned by an aircraft flying at a constant speed and altitude. It considers the continuous fuel burn rate variation with time caused by the gross weight (and centre of gravity position) modification due to the fuel burn process itself. The algorithm was developed for use by the Flight Management System (FMS) and employs the same aircraft performance data as the existing FMS fuel burn prediction algorithms. The new fuel burn method was developed for aircraft models that use the centre of gravity position as well as for models that do not consider the centre of gravity position. This algorithm was developed for normal flight conditions. Algorithm performances were evaluated for two aircraft models: one for models that use an aircraft's centre of gravity position – a more complex and computing intensive method, and one for those that do not use the centre of gravity position. The validation data were generated based on the information produced on a CMC Electronics – Esterline FMS platform that used identical aircraft models and performance data for identical flight conditions.

Paper No. 3825. Manuscript received 7 March 2012, revised version received 18 August 2012, accepted 7 September 2013.

https://doi.org/10.1017/S0001924000008149 Published online by Cambridge University Press

NOMENCLATURE

air_temperature	air temperature
altitude	aircraft altitude
ALT_LIMIT	the maximal flying altitude under any conditions
cg	centre of gravity position
CGREFDIST	aircraft centre of gravity reference point's position, as a distance from the aircraft's reference point
CG_AT_GW	centre of gravity position as a function of the total weight
CG_SLOPE	centre of gravity position variation coefficient
crossover_altitude	the altitude at which the true air speeds corresponding to each of the *IAS* and Mach values defined by the *speed_schedule* are equal
datum	aircraft's longitudinal reference point
dcg	centre of gravity position variation
dgw	gross weight variation
fbr	fuel burn rate
fcr	fuel correction factor
ff	fuel flow
flight_time	the duration of the flight for which the quantity of fuel burned is computed
fuel	fuel weight
fuel_burn	the quantity of fuel burned
gw	gross weight
IAS	indicated airspeed
LEMAC	the leading-edge, mean aerodynamic chord position with respect to the datum
M_a	aircraft moment
M_f	fuel moment
M_{fgw}	fuel moment as a function of the total weight
MIN_ALTITUDE	minimal cruise altitude
M_t	total aircraft moment
MAC	Mean Aerodynamic Chord
%MAC	percentage of MAC
Mach	Mach index
speed	an *IAS* or Mach defined cruise speed value
speed_schedule	a pair of *IAS* and Mach values
TAS	true air speed
T_{flight}	cruise segment flight time
zfgw	zero fuel gross weight
zfwcg	zero fuel weight centre of gravity

1.0 INTRODUCTION

The Flight Management System is a key element in observing the aircraft navigation, operational and safety requirements. As such, its performance improvement and capabilities development are subject to a special and continuous interest of the aviation industry. This is determined by two factors: first, the introduction of new aviation standards and requirements; second, the ongoing increase in computing power and the development of new hardware and algorithms. An analysis

https://doi.org/10.1017/S0001924000008149 Published online by Cambridge University Press

of Liden[1] provides a comprehensive description of the development and evolution of the FMS, at Honeywell, since its initial design, in 1982. In a more recent work, Herndon *et al*[2] describe some of the current key FMS concepts and directions of development, including Area Navigation (RNAV), Required Navigation Performance (RNP), Optimal Profile Descents (OPDs), and Continuous Descent Arrivals (CDA). A comparative analysis of the capabilities and performances of several modern FMS equipment is also provided. Other directions of research and development relate to new navigation standards and concepts such as ADS-B and NextGen Navigation, adaptive trajectory planning (Alonso-Portillo *et al*[3]) or improved weather forecast prediction tools (Wu *et al*[4]) and algorithms (Knapp *et al*[5]) for trajectory optimisation.

Key to any current and future FMS device is its performance prediction module which implements the set of functions used in predicting various aircraft parameters along the flight path and for trajectory optimisation (Liden[1], Liden[6]). Fuel burn prediction is a central element of the performance prediction module as it affects the quality and validity of many other aircraft parameters' prediction (such as gross weight, centre of gravity position, maximal altitude, flight distance etc.), trajectory computations and optimisations (altitudes, flight times, costs). The equations describing the instant fuel burn consumption are complex and dependent to the particular aircraft/engine configuration (Ardema *et al*[7]). Therefore, due to its limited computing power, the FMS uses a simplified fuel burn performance description model based on linear interpolation tables (Liden[6]).

This paper presents the development of an algorithm for a Flight Management System (FMS) that determines the quantity of fuel burned (*fuel_burn*) by an aircraft flying at constant speed and altitude, for a given period of time. A number of limitations were imposed in the development of this algorithm. It must be deterministic and compatible with the real-time nature of the FMS application. It must use the aircraft performance and capabilities description model, based on linear interpolation tables, used by FMS platforms. It must also be applicable to aircraft cruise performance description models that are dependent of the centre of gravity position (*cg*) and those that are not. The algorithm is used only for normal cruise operation conditions, on cruise segments defined by their altitude and *flight_time*. The altitude values considered by the algorithm are multiples of 1,000ft and are situated between a minimum altitude, provided as an algorithm input parameter, and an aircraft's maximum attainable altitude. Aircraft speed is described by the *speed_schedule*, defined as a pairing of Indicated Air Speed (*IAS*) and Mach number (Mach) values. Their use is a function of the *crossover_altitude*, defined as the altitude at which the true air speed (TAS) computed using the *IAS* equals the TAS computed using the Mach number. Aircraft speed corresponds to the *IAS* value below *crossover_altitude*, and Mach value at or above *crossover_altitude*.

The aircraft's fuel burn rate (*fbr*) description model uses one or more linear interpolation aircraft performance tables that characterise its dependency on an aircraft's state parameters and flight conditions, such as *gw*, *cg*, *IAS*/Mach, *air_temperature* and *altitude*. These tables thus describe the value of the fuel burn rate at particular instants of time corresponding to the set of values of an aircraft's state and the flying condition parameters. Computing the value of the fuel burned on flying a cruise segment, however, requires performing the continuous summation of the fuel burned at each instant of time of the cruise segment's flight. Considering a *time-dependent* formulation of the fuel burn rate, *fbr*(*t*), the *fuel_burn* can be computed using the equation below:

$$fuel_burn = \int_{0}^{T_{flight}} fbr(t) * \mathrm{d}t \qquad \ldots (1)$$

Existing implementations compute an approximation of the *fuel_burn* value by decomposing the given cruise segment into sub-segments, usually of a maximum length of 50Nm, within which the

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fuel burn rates are considered constant and equal to those determined from the interpolation tables for the particular conditions at the start of the corresponding sub-segment[6]. The present article proposes a new method of computing the *fuel_burn* that considers the continuous variation of the fuel burn rate along the cruise segment by using linear interpolation aircraft performance tables.

2.0 FUEL BURN COMPUTATION

Fuel burn computation is dependent upon an aircraft's cruise fuel burn model (the actual definition of the aircraft performance tables used for the cruise flight *fuel_burn* computations). Two aircraft models are presented here: one that considers the aircraft's centre of gravity position and one that does not. Both models compute the *fuel burn rate* (*fbr*) as a product of two values, the *fuel flow* (*ff*) and the fuel correction factor (*fcr*). The fuel flow is described as a linear interpolation performance table compiled as a function of the aircraft' speed (*IAS* or Mach), *gross weight*, *air_temperature* and *altitude* – *ff*(*speed*, *gw*, *air_temperature*, *altitude*) – and is common to both models. The *fcr* allows for a more flexible and accurate description of the fuel burn rate's dependency on aircraft and flight parameters. The *fcr* is described as a constant value or as a linear interpolation performance table, depending on the aircraft model. In this paper the *fcr* is considered as a constant value for the aircraft model that does not consider the *cg* position, and as a linear interpolation table that is a function of the aircraft's *speed*, *gw*, *cg* and *altitude* – *fcr*(*speed, gw, cg, altitude*) – for the aircraft model that considers the cg position. Two sets of *ff* and *fcr* tables characterise the aircraft's performances with respect to *IAS* and Mach operations – and are identified by a corresponding I or M index.

For the aircraft model that considers the *cg* position, additional aircraft design and constructive data are used for computing the *cg* position: the mean aerodynamic chord length (MAC), the leading edge mean aerodynamic chord position (LEMAC), the aircraft centre of gravity reference point position (CGREFDIST), the aircraft moment ($\mathbf{M_a}$) – a function of the aircraft's zero fuel weight (*zfgw*) and the zero fuel weight centre of gravity (*zfwcg*) position – M_a(*zfgw, zfwcg*), and the fuel moment ($\mathbf{M_f}$) – a function of the fuel weight (*fuel*) – M_f(*fuel*).

The set of algorithm input parameters can be regrouped in three categories:

- as parameters that have a constant value for the duration of the flight – *zfgw*, *zfwcg*, MIN_ ALTITUDE, as well as the aircraft performance and design data;
- as variables whose values may change, but at longer time intervals, such as the *speed_schedule* (*IAS, Mach*), and *air_temperature* – and thus are considered to be constant on the segment for which the optimal altitude is computed; and
- as variables whose values are changing constantly under the influence or as a consequence of the fuel burn – i.e. the *fuel*, and thus the *gw* and the *cg*.

As noted, the parameters considered to having a constant value for the entire duration of the flight belong to one of the three types:

- aircraft performance and design data, described by the aircraft's performance description database. Their value can change upon aircraft maintenance which entails a performance database update.
- zero-fuel weight and balance settings (*zfgw, zfwcg*) which are determined by the aircraft`s structural and payload weights and distribution. For general aviation aircrafts these values are set before take-off and remain constant for the entire duration of the flight.
- algorithm configuration (*MIN_ALTITUDE*), selected as an aircraft, route or company policy,

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determining the range of cruise altitudes for which the fuel burn characterisation is produced.

The constants described above have an impact on algorithm accuracy in two main areas:

- modeling – the extent at which the aircraft performance description model and the corresponding data (the interpolation tables) are accurate.
- availability of the fuel burn characterisation data – as configured by MIN_ALTITUDE parameter.

In order to minimise the algorithm's response time and taking advantage of the observation regarding the different rate of variation of the input parameters the fuel burn algorithm presented in this paper is composed of three modules as follows:

- An initialisation module, executed before take-off after the aircraft's *zfgw*, (*zfwcg*) and fuel values are set. A series of auxiliary tables are built and subsequently used by the next two modules;
- The intermediary module, which builds the *IAS* and Mach-based fuel burn look-up tables that describe the relationship between the initial gw, *IAS*/Mach, *air_temperature*, cruise altitude and *flight_time*, and the final gw; thus, the *fuel_burn*. The module is executed each time the *speed_schedule* or *air_temperature* parameters change, or as required by the update strategy. The tables are generated for altitudes situated between MIN_ALTITUDE and ALT_LIMIT – the aircraft's maximal operational altitude; and
- The fuel burn module, which extracts the *fuel_burn* quantity as a function of the *gw* at the initial point of the cruise segment, the evaluated cruise *altitude*, and the corresponding *flight_time*.

3.0 THE INITIALISATION MODULE

The main goal of this module is to determine the set of gross weight values, $\{gw_i\}$, for which every interval $(gw_i, gw_i + 1)$ maps the linearity domains in each performance interpolation table used in the fuel burn rate computation. For the *cg*-independent model, the module only identifies and assembles the gw values defined in the $\mathbf{ff_I}$, and $\mathbf{ff_M}$ performance tables. For the cg-dependent model however, the gw values explicitly or implicitly defined by the $\mathbf{fcr_I}$, $\mathbf{fcr_M}$, and $\mathbf{M_f}$ performance tables are also considered. The initialisation module first determines the set of $\{gw_j\}$ values from the $\mathbf{fcr_I}$ and $\mathbf{fcr_M}$ tables. A new fuel moment table ($\mathbf{M_{fgw}}$) is then generated as a function of *gw* and $M_f(gw)$. The input variable, fuel, is replaced by the corresponding gross weight:

$$gw_k = \text{zfgw} + fuel_k \quad \ldots (2)$$

The set of *gw* values, $\{gw_p\}$, that corresponds to the *cg* values defined in the *fcr* tables can then be determined by the following succession of operations. Since $M_{fgw}(gw)$ is defined as a linear interpolation table, the rate of variation of the *fuel moment* is constant on each domain (gw_k, gw_{k+1}). Its value is computed using the equation:

$$\frac{\partial M_{fgw}}{\partial gw}\Big|_{gw_k} = \frac{\mathbf{M_{fgw}}(gw_{k+1}) - \mathbf{M_{fgw}}(gw_k)}{gw_{k+1} - gw_k} \quad \ldots (3)$$

Consequently, the value of the *fuel moment* corresponding to a gross weight in the domain (gw_k, gw_{k+1}) is computed using the equation:

$$M_{fgw}(gw) = \mathbf{M_{fgw}}(gw_{k+1}) - \frac{\partial M_{fgw}}{\partial gw}\Big|_{gw_k} * gw_{k+1} + \frac{\partial M_{fgw}}{\partial gw}\Big|_{gw_k} * gw \quad \ldots (4)$$

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As indicated by the Federal Aviation Administration[8], the position of the centre of gravity expressed as a percentage of the mean aerodynamic chord length, %MAC, is computed using the equation:

$$cg(gw) = \frac{\left(\frac{M_a(\text{zfgw},\text{zfwcg}) + M_{fgw}(gw)}{gw} + \text{CGREFDIST} - \text{LEMAC}\right) * 100}{\text{MAC}} \quad \ldots (5)$$

Consequently, on a gross weight domain (gw_k, gw_{k+1}), the gross weight value corresponding to a given *cg* position is computed using the following equation (derived from Equations (4) and (5)):

$$gw(cg) = \frac{\left(M_a(\text{zfgw},\text{zfwcg}) + \mathbf{M_{fgw}}(gw_{k+1}) - \frac{\partial M_{fgw}}{\partial gw}|_{gw_k} * gw_{k+1}\right)}{\left(\frac{cg * \text{MAC}}{100} - \text{CGREFDIST} + \text{LEMAC} - \frac{\partial M_{fgw}}{\partial gw}|_{gw_k}\right)} \quad \ldots (6)$$

The set of gross weight values, $\{gw_p\}$, is then computed using equation for all (gw_k, gw_{k+1}) gross weight domains by evaluating each *cg* value defined by the fcr tables. A gw_p thus computed is considered valid, and retained, if its value lies within the gross weight domain on which it was computed. A gw_p which lies outside the gross weight domain on which it was computed is discarded as the values of the parameters used in its computation are valid only inside the said domain.

Finally, the $\{gwf_k\}$, $\{gw_j\}$ and $\{gw_p\}$ sets are added to the initial set of $\{gw_i\}$ values:

$$\{gw_i\} = \{gw_i\} \cup \{gw_j\} \cup \{gwf_k\} \cup \{gw_p\} \quad \ldots (7)$$

This set of gross weight values ensures that each domain (gw_i, gw_{i+1}) corresponds to *gw* and *cg* linearity domains in all the performance interpolation tables used for computing the fuel burn rate. Next, the **Mfgw** table is rebuilt according to the new set of $\{gw_i\}$ values. Two more auxiliary tables, used by the intermediary module, are built for the same set of $\{gw_i\}$ values. First, the **CG_AT_GW**(gw_i) table that stores the *cg* position corresponding to each gw in the $\{gw_i\}$ set is computed using Equation (5). The second table, called **CG_SLOPE**(gw_i), stores a coefficient that is used to determine the cg variation (*dcg*) function of the gross weight variation (dg_w) for each domain (gw_i, gw_{i+1}). Both of these coefficients are referenced to their corresponding value at gw_{i+1}. To determine the equation used to compute the elements of the table we start with the following equation:

$$dcg(gw) = cg(gw_{i+1}) - cg(gw) \quad \ldots (8)$$

Replacing the *cg* values in Equation 8 with the values given by Equation (5), and subsequently replacing $M_{fgw}(gw)$ with its equivalent in Equation (4) we obtain:

$$dcg(gw) = \left(\frac{100 * \left(M_a(\text{zfgw},\text{zfwcg}) + \mathbf{M_{fgw}}(gw_{i+1}) - \frac{\partial M_{fgw}}{\partial gw}|_{gw_i}\ gw_{i+1}\right)}{\text{MAC}}\right) * \left(\frac{1}{gw_{i+1}} - \frac{1}{gw}\right) \quad \ldots (9)$$

Next, denoting;

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$$dgw = gw_{i+1} - gw \quad \ldots (10)$$

and

$$\mathbf{CG_SLOPE}\left(gw_i\right) = \left(\frac{100 * \left(M_a\left(\text{zfgw}, \text{zfwcg}\right) + \mathbf{M_{fgw}}\left(gw_{i+1}\right) - \frac{\partial M_{fgw}}{\partial gw}\Big|_{gw_i} gw_{i+1} \right)}{\text{MAC} * gw_{i+1}} \right) \quad \ldots (11)$$

Equation can be written as a function of *dgw*:

$$dcg\left(dgw\right) = \mathbf{CG_SLOPE}(gw_i) * \left(\frac{dgw}{gw_{i+1} - dgw} \right) \quad \ldots (12)$$

4.0 THE INTERMEDIARY MODULE

This module constructs, for a given *speed_schedule* and *air_temperature*, a structure describing the relationship between the aircraft's initial *gw*, the cruise altitude, the *flight_time*, and the final *gw*. The data are assembled for a number of *altitudes*, multiples of 1,000ft, situated between the MIN_ALTITUDE and the ALT_LIMIT. The *IAS* or Mach performances are characterised according to their corresponding altitude positions relative to the *crossover_altitude*. At each altitude, the fuel burn data are generated for a number of gross weight domains; a number that depends upon a structure's update strategy. This data should, at a minimum, cover the *fuel_burn* that can occur from flying the cruise segment under any conditions.

The present paragraph describes the computations performed on cruise segments flown at speeds specified by a Mach index, at a given altitude on a gross weight domain (gw_i, gw_{i+1}), for *cg*-dependent and *cg*-independent aircraft models. At *IAS* speeds, the computations are identical and use the *IAS* performance interpolation tables ($\mathbf{ff_1}$, and $\mathbf{fcr_1}$). First, the fuel burn rate (*fbr*) equation, a function of the dgw, is developed using the linear interpolation equation. This equation is then rewritten to describe the fbr variation as a function of time. Finally, the time-dependent fbr function is integrated using the Runge-Kutta 4 (RK4) algorithm[9].

The fuel burn rate (*fbr*) is computed as the product between the fuel flow (*ff*) and the fuel correction factor (*fcr*). The steps required for its calculation are shown next. Denoting;

$$ff_1 = ff_M\left(Mach, gw_{i+1}, air_temperature, altitude\right) \quad \ldots (13)$$

and

$$ff_2 = ff_M\left(Mach, gw_i, air_temperature, altitude\right) \quad \ldots (14)$$

the value of the fuel flow, at a gross weight value described by its variation from gw_{i+1}, *dgw*, is computed by linear interpolation using the equation:

$$ff\left(dgw\right) = ff_1 - \frac{ff_1 - ff_2}{gw_{i+1} - gw_i} dgw \quad \ldots (15)$$

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Denoting;

$$A_0 = ff_1 \quad \ldots (16)$$

and;

$$A_1 = \frac{ff_1 - ff_2}{gw_{i+1} - gw_i} \quad \ldots (17)$$

Equation (15) becomes:

$$ff\left(dgw\right) = A_0 - A_1 * dgw \quad \ldots (18)$$

For the *cg*-independent aircraft model and Mach index operations, the *fuel correction* factor, fcr_M, is a constant value. Therefore, denoting $C_0 = fcr_M$, the equation of the fuel burn rate as a function of the *dgw* is:

$$fbr\left(dgw\right) = C_0\left(A_0 - A_1 * dgw\right) \quad \ldots (19)$$

Observing that the *dgw* changes with time, as the fuel is being burned, the *fbr* variation can be described as a function of time:

$$fbr\left(t\right) = C_0\left(A_0 - A_1 * dgw\left(t\right)\right) \quad \ldots (20)$$

There is also a direct relationship between $dgw(t)$ and $fbr(t)$, since the gross weight variation is produced as a result of burning the fuel at a rate described by the function $fbr(t)$. Consequently, the equation that connects *dgw* and *fbr* is:

$$dgw\left(t\right) = \int fbr\left(t\right)dt \quad \ldots (21)$$

As mentioned in the section describing the initialisation module, on each gross weight domain (gw_i, gw_{i+1}) – where $gw_{i+1} > gw_i$, all computations are performed relative to parameters' value at gw_{i+1}. Therefore, starting at gw_{i+1} and considering positive fuel-burn rate for a time period *dt* leads to a final gross weight value $gw < gw_{i+1}$. Consequently, as indicated by Equation (10), a positive fuel burn rate produces a positive gross weight variation (*dgw*). Denoting $dgw(t) = I$ implies that $fbr(t) = \mathrm{d}I/\mathrm{d}t$. Therefore, equation becomes:

$$\frac{\mathrm{d}I}{\mathrm{d}t} = C_0\left(A_0 - A_1 * I\right) \quad \ldots (22)$$

This differential equation describes the gross weight variation (*dgw*) as a function of *flight_time* on a gross weight domain of (gw_i, gw_{i+1}) for the *cg*-independent aircraft model.

For the *cg*-dependent aircraft model, the fuel correction factor (*fcr*) for Mach speeds is obtained from the $\mathbf{fcr_M}$ interpolation table. We denote:

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$$cr_{11} = fcr_M\left(\mathbf{CG_AT_GW}\left(gw_{i+1}\right), Mach, gw_{i+1}, altitude\right)$$

$$cr_{12} = fcr_M\left(\mathbf{CG_AT_GW}\left(gw_i\right), Mach, gw_{i+1}, altitude\right)$$

$$cr_{21} = fcr_M\left(\mathbf{CG_AT_GW}\left(gw_{i+1}\right), Mach, gw_i, altitude\right) \quad \ldots (23)$$

$$cr_{22} = fcr_M\left(\mathbf{CG_AT_GW}\left(gw_i\right), Mach, gw_i, altitude\right)$$

Interpolating along the *gw* input parameter of the fuel correction factor performance table leads to the equations:

$$fcr_1 = cr_{11} - \frac{cr_{11} - cr_{21}}{gw_{i+1} - gw_i} dgw$$

$$\ldots (24)$$

$$fcr_2 = cr_{12} - \frac{cr_{12} - cr_{22}}{gw_{i+1} - gw_i} dgw$$

Subsequently, interpolating between the values of fcr_1 and fcr_2 to account for the variation of the cg with respect to $cg_{i+1} = cg(gw_{i+1})$ produces:

$$fcr = fcr_1 - \frac{fcr_1 - fcr_2}{cg\left(gw_{i+1}\right) - cg\left(gw_i\right)} dcg \quad \ldots (25)$$

Replacing the terms of Equation (25) with their definitions given in Equations (23), (24) and (25) and denoting;

$$B_0 = cr_{11} gw_{i+1}$$

$$B_1 = cr_{11} + \frac{cr_{11} - cr_{21}}{gw_{i+1} - gw_i} gw_{i+1} + \frac{\left(cr_{11} - cr_{12}\right)\mathbf{CG_SLOPE}\left(gw_i\right)}{\mathbf{CG_AT_GW}(gw_{i+1}) - \mathbf{CG_AT_GW}(gw_i)}$$

$$B_2 = \frac{1}{gw_{i+1} - gw_i}\left(cr_{11} - cr_{21} + \frac{\left(cr_{11} - cr_{12} - cr_{21} + cr_{22}\right)\mathbf{CG_SLOPE}\left(gw_i\right)}{\mathbf{CG_AT_GW}(gw_{i+1}) - \mathbf{CG_AT_GW}(gw_i)}\right)$$

$$C_0 = gw_{i+1} \quad \ldots (26)$$

the equation describing $fcr(dgw)$ becomes:

*

$$fcr\left(dgw\right) = \frac{B_0 - B_1 * dgw + B_2 * dgw^2}{C_0 - dgw} \quad \ldots (27)$$

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Consequently, the equation describing *fbr*(*dgw*) for the *cg*-dependent aircraft model is:

$$fbr\left(dgw\right)=\frac{\left(A_0-A_1*dgw\right)\left(B_0-B_1*dgw+B_2*dgw^2\right)}{C_0-dgw} \quad \ldots (28)$$

As with the *cg*-independent model, denoting $dgw(t) = I$ implies that $fbr(t) = dI/dt$. Consequently, Equation (28) leads to the time-dependent differential equation:

$$\frac{\mathrm{d}I}{\mathrm{d}t}=\frac{\left(A_0-A_1*I\right)\left(B_0-B_1*I+B_2*I^2\right)}{C_0-I} \quad \ldots (29)$$

For each *gross weight* domain and each altitude, a look-up table describing the gross weight variation with time is produced by the integration of Equation (22) or of Equation (29) (depending on the aircraft modeling) using the RK4 numerical integration algorithm. The default value of the integration time step is chosen to balance the precision and memory/processing time constraints imposed on the fuel burn computing algorithm. On any gross weight domain (gw_k, gw_{i+1}), the actual value of the integration time step is equal to the minimum of the default value and the time required to burn the entire quantity of fuel corresponding to the gross weight domain. The gross weight information stored in the look-up table corresponds to time instances that are multiples of the integration time step value, referenced to the beginning of the gross weight domain (gw_{i+1}). Therefore, searches in the look-up table can be performed either as a function of the gross weight or of the *flight_time*.

5.0 THE FUEL BURN MODULE

The fuel burn module computes *fuel_burn* values, at any given altitude, as the difference between the value of the gross weight at the beginning of the considered cruise segment and the value of the gross weight after flying for a period equal to the cruise segment's *flight_time*. The gross weight domain, containing the aircraft's gross weight value at the beginning of the cruise segment, is identified. Subsequently, the corresponding time offset is determined from the domain's initial gross weight. Subtracting the time offset from the gross weight domain's total time results in the *flight_time* required to reach the domain's final gross weight. A series of iterations are then performed on successive gross weight domains, starting with the initial domain. The *flight_time* corresponding to the selected domain and the cruise *flight_time* are compared at each iteration. If the domain's *flight_time* is smaller, its value is subtracted from the cruise *flight_time*, and the processing advances to the next iteration, corresponding to the next gross weight domain. If the domain's *flight_time* is larger than the cruise *flight_time*, the final gross weight is computed as the value corresponding to the time offset, from the beginning of the domain, equal to the value of the cruise *flight_time*. The *fuel_burn* is then computed as the difference between the initial and the final gross weights.

6.0 RESULTS

The algorithm was implemented in MATLAB and executed on a Microsoft Windows XP, 2·79GHz AMD Phenom(tm) II PC- platform. It considered the generation of the *fuel_burn* data for the entire set of gross weight domains, starting with the domain containing the initial aircraft's *gw* value, and continuing to the domain in which *gw* corresponded to *fuel* = 0. This allowed for a

https://doi.org/10.1017/S0001924000008149 Published online by Cambridge University Press

broader characterisation of the intermediary module's response time performance, on a wide set of *gross weight* ranges.

The objective of the validation was to evaluate the performances of the fuel burn computing algorithm. It was performed using the (*cg*-dependent) performance model data of an Airbus A310 aircraft, a model that determines a more complex and computation-intensive algorithm, and the (*cg*-independent) performance model data of a Sukhoi RRJ. The algorithm-computed *fuel_burn* values were compared with the corresponding values produced, for identical flight conditions, on a CMA9000 PTT platform provided by our research partner CMC Electronics – Esterline. For a better analysis of the fuel burn computing algorithm`s performances the flight times used for each test scenario and set of altitudes were the same T_{flight} values as computed by the CMA9000 PTT. Consequently, the *fuel_burn* differences are produced exclusively by the fuel burn computing algorithm.

The algorithm performances presented in this paper were investigated on 181 test scenarios for the A310 model, and 43 test scenarios for the RRJ model. Each test scenario's *fuel_burn* values and execution times were evaluated for a *flight_time* corresponding to a cruise segment distance of 500Nm and a set of altitudes situated between 20,000ft. and the maximal altitude accepted by the PTT for that configuration. The fuel burn performances are computed as the percent difference between the values obtained from the PTT data and those obtained from the algorithm-computed data (a negative value indicates an algorithm-computed *fuel_burn* value that is smaller than that computed by the PTT). The results, describing the minimal, maximal and average difference values – at each altitude – are presented in the Table 1 below:

The minimal, maximal and average execution times (in seconds) for each algorithm module and for each aircraft model are presented in Table 2, below, and they correspond to the values recorded during the generation of look-up tables for 22 and 21 altitudes, respectively, and fuel quantities up to each aircraft's fuel capacity:

CONCLUSIONS

The fuel burn process and by consequence the function describing the instant fuel burn rate as a function of aircraft parameters and flying conditions is not linear. This is caused by the non-linearity associated with engine thrust characteristics and aircraft aerodynamic forces (lift and drag) and moments. The FMS uses a performance model, based on linear interpolation tables, that approximates the actual fuel burn rate function. The linearisation may consider only a sub-set of the actual parameters that influence the *fbr*, and on a selected set of domains for each of the parameters. The accuracy of such model is highly dependent on the set of variables and linearity domains selected in the process of modeling – a trade-off between accuracy and memory and processing time requirements. Another limitation of the FMS fuel burn computation algorithm's accuracy, currently implemented on FMS platforms, is the assumption that the fuel burn rate is constant on cruse segments of less than a set distance (usually 50Nm or 100Nm.). This assumption, introduced for simplifying the fuel burn computations, determines an approximation error relative to the linearised model which itself is an approximation of the actual fuel burn function.

The fuel burn algorithm presented in this paper, while maintaining the accuracy limitations due to the aircraft's performance modeling, provides two important features: Firstly, the value of the *fuel_burn* follows the continuous variation of the fuel burn rate with time, based on the variation of the gross weight, and thus it is a more accurate approach than what is currently available. Secondly, this algorithm eliminates the need to decompose the cruise segments into

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Table 1
fuel burn prediction differences (ALG – FMS), relative to the FMS value, function of the cruise altitude

Altitude (ft)	A310 (cg - dependent)			RRJ (cg - independent)		
	Min (%)	Max (%)	Average (%)	Min (%)	Max (%)	Average (%)
20000	-0,504	0,062	-0,328	-0,447	0,475	-0,164
21000	-0,413	0,197	-0,178	-0,380	0,457	-0,144
22000	-0,420	0,213	-0,171	-0,453	0,332	-0,215
23000	-0,451	0,109	-0,322	-0,351	0,512	-0,133
24000	-0,499	0,324	-0,087	-0,392	0,434	-0,145
25000	-0,182	0,260	-0,001	-0,440	0,238	-0,197
26000	-0,491	0,380	-0,034	-0,429	0,457	-0,155
27000	-0,444	0,180	0,021	-0,446	0,237	-0,289
28000	-0,315	0,036	-0,163	-0,436	0,417	-0,188
29000	-3,545	0,317	-0,094	-0,428	0,595	-0,112
30000	-0,363	0,362	-0,036	-0,447	0,294	-0,293
31000	-3,524	0,123	-0,294	-0,383	0,727	-0,114
32000	-0,454	0,221	-0,019	-0,246	0,851	-0,104
33000	-3,929	0,084	-0,319	-0,384	0,881	-0,158
34000	-0,500	0,178	-0,104	-0,412	1,012	-0,156
35000	-0,483	0,179	-0,238	-0,333	1,048	-0,167
36000	-0,657	-0,151	-0,396	-0,681	0,879	-0,491
37000	-1,275	-0,774	-0,881	-1,299	0,397	-1,082
38000	-1,847	-1,349	-1,440	-1,906	-0,077	-1,648
39000	-2,067	-1,992	-2,023	-2,505	-0,276	-2,178
40000	-2,643	-0,706	-2,261	-3,252	-0,474	-2,857
41000	-3,160	-3,114	-3,134			

smaller sub-segments. This, in turn, opens up the possibility of computing *fuel_burn* values for cruise flights spanning long distances or multiple segments simultaneously, using the corresponding total *flight_time*s.

The results show that for both aircraft performance models the difference between the *fuel_burn* values computed by the PTT and those computed by the algorithm, depending on the aircraft configuration and flying conditions, can be significant. As presented in Table 1, they were situated between –3·9% and +0·38% for the A310 and between –3·2% and +1·04% for the RRJ. These results are consistent with the modeling differences. For instance, an analysis of the A310 model indicated that, especially for a number of configurations and certain gross

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Table 2
Algorithms' modules' execution times

Module	A310 (cg - dependent)			RRJ (cg - independent)		
	Min time (s)	**Max time** (s)	**Average time** (s)	**Min time** (s)	**Max time** (s)	**Average time** (s)
Initial	0.005277	0.022264	0.010064	0.000084	0.000457	0.000096
Intermediary	12.4464	148.5208	35.4625	3.6148	49.7099	11.955
Fuel Burn	0.000083	0.001540	0.000226	0.00008	0.000419	0.000206

weight domains, the *cg* variation with the gross weight was significant, determining important variations of the fuel burn rate with the decrease of the *gw*. These variations can represent either a steep increase or steep decrease of the *fbr* with the reduction of the *gw*. This variation accentuates the differences already introduced by the constant fuel burn rate approximation.

The analysis of the algorithm's modules' execution time statistics shows that, as expected, the times required for the execution of the initial and intermediary modules corresponding to the *cg*-dependent model are significantly larger than those of the *cg*-independent model. The time corresponding to the initialisation module is not critical as it is executed only once, upon FMS' aircraft parameters configuration. The execution time of the intermediary module, however, corresponds to delays in the availability of *fuel_burn* look-up tables (upon a change of the *speed_schedule* or *air_temperature* values) for computing the *fuel_burn* predictions. These delays, although large, do not occur frequently and may be regarded as an algorithm limitation. Furthermore, suitable advanced-generation and update strategies can lead to significant reductions of the delay. The analysis also suggests that once the *fuel_burn* tables constructed, the time required for computing the *fuel_burn* prediction is very small (less than 0·5ms for the RRJ and less than 1·6ms for the A310, with an average of 0·2ms for both aircrafts).

Given the results obtained to date, future work will look to expand the number of aircraft models and test conditions for which the algorithm's performances are evaluated. As well, algorithm implementation, integration and testing on the real-time FMS target platform, using the platform's development and test environment, would provide a better insight regarding the algorithm's execution times and fuel burn data availability/ latency times.

ACKNOWLEDGEMENTS

The authors would like to thank CMC Electronics – Esterline and the Green Aviation Research and Development Network (GARDN) for their support in conducting this research. We also wish to express our sincere appreciation to Ms N. Dumondel and Messrs J. Dupont, R. Glumineau, J.Hemmerle, T. Klotz, B. Langlet, F. Millet, T. Salamah, S. Souleymane, and L. Tevoedjre for their collaboration and important contribution to producing the PTT data used for algorithm validation.

https://doi.org/10.1017/S0001924000008149 Published online by Cambridge University Press

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