NUMERICAL SIMULATION OF PARTICLE-GAS FLOW THROUGH A FIXED PIPE, USING ONE-WAY AND TWO-WAY COUPLING METHODS

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ABSTRACT

A numerical simulation of the particle-gas flow in a vertical turbulent pipe flow was conducted. The main objective of the present article is to investigate the effects of dispersed phase (particles) on continuous phase (gas). In so doing, two general forms of Eulerian-Lagrangian approaches namely, one-way (when the fluid flow is not affected by the presence of the particles) and two-way (when the particles exert a feedback force on the fluid) couplings were used to describe the equations of motion of the two-phase flow. Gas-phase velocities which are within the order of magnitude as that of particles, volume fraction, and particle Stokes number were calculated and the results were subsequently compared with the available experimental data. The simulated results show that when the particles are added, the fluid velocity is attenuated. With an increase in particle volume fraction, particle mass loading and Stokes number, velocity attenuation also increases. Moreover, the results indicate that an increase in particle Stokes number reduces the special limited particle volume fraction, according to which one-way coupling method yields plausible results. The results have also indicated that the significance of particle fluid interaction is not merely a function of volume fraction and particle Stokes number.

Keywords: One-way and two-way coupling, Eulerian-Lagrangian approaches, Volume fraction, Particle Stokes number.

1. INTRODUCTION

Suspension of particles in a turbulent fluid has been frequently observed in many important natural and technological situations such as circulating fluidized bed reactors, aerosol transport and deposition, pneumatic conveyance of solid materials, pulverized coal and spray fuel combustion, and dispersion of pollutants. In most of these applications, the turbulent behavior of the carrier phase plays a dominant role in transporting or mixing the particles. To improve the efficiency of these processes and the quality of the final products, it is necessary both to discover and to predict the mechanisms that influence the flow field. The interactions between the carrier phase (continuous phase) and the particle phase (dispersed phase) comprise a complex issue by themselves. No surprise that turbulence flow analysis has yet remained a challenging topic for the classical mechanics

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since Osborne Reynolds originally formulated the well known closure problem in the late 1800 s. The ambiguity of which, however, is expected to increase following the injection of particles to a turbulent flow [1]. Turbulence intensifies the dispersion of particles, and the particles may in return affect the properties of the carrier phase flow. The degree of coupling between the particulates and the gaseous carrier phase depends on the amount of existing particulates [2]. For dilute flows (where the effect of particle/particle collision is negligible), and for particle volume fractions less than 10^{-6}, the coupling between phases is one-way, that is, the particulate motion is driven by gas phase but the presence of the particulates does not influence the carrier phase flow. As the volume fraction of the particulate phase increases, the fluid phase velocities and turbulence characteristics alter as a result of interacting with the particles. This situation has been termed as two-way coupling by Crowe [3]. Several reviews on the particle- laden turbulent flow have been published during the past decade. [See Hetsroni [4], Elghobashi [2], Crowe *et al*. [5], Mashayek and Pandya [6], Nasr and Ahmadi [7], Doisneau [8], Namazian *et al* [9] for instance].

Gore and Crowe [10] demonstrated that in most cases, small particles have a tendency to minimize disturbance whereas larger particles increase turbulence levels. In the study of Kulick *et al*. [11] in developed turbulent flow in the channel, particles smaller than the micro scale were considered. They showed that the fluid turbulence decreases with increasing number of particles. Portela and Oliemans [12] reported that the presence of particles leads to a decrease in turbulent eddy intensity without significant change in their size and shape.

Strömgren *et al*. [13] investigated the difference between one-and two-way coupling for different particle volume fractions and particle diameters. It was found that particles with a much higher density than the fluid substantially affect the gas-phase in turbulent channel flow for particle volume fractions as low as 10^{-4}. They showed that two-way coupling and the turbophoretic effect must be taken into account in models even at relatively low particle volume fractions. Also in the other study, Mousavian *et al*. [14] showed that using of two way coupling model predict more accurate separation performance in the hydrocyclone due to high solid concentration in outlet.

The obtained experimental data for Particle-laden turbulent flows, reported adding the particles with increases and decreases turbulence. Tsuji and Morikawa [15, 16] showed that small particles with a diameter of about 200-500 micrometers reduce turbulence intensity and velocity of the fluid phase. Breuer and Alletto [17] investigated the simulation of particle-laden two-phase flows based on the Euler–Lagrange approach. They used two-phase flows, the fluid–particle interaction (two-way coupling) as well as particle–particle collisions (four-way coupling). Also in order to tackle complex turbulent flows eddy-resolving schemes (large-eddy) is used. They found good agreement between computational results and experimental measurements.

Some researchers study the appropriateness of modeling method of solid fluid interaction. Benra *et al*. [18] investigated the different problems with one-way and two-way coupled methods and showed under what conditions a one-way coupling solution gives plausible results.

The present study intends to make a comparison between two general forms of Euler-Lagrange approaches, namely one-way and two-way couplings. This analogy has been tested by the use of two well-known parameters, namely particle Stokes number and volume fraction. To serve this purpose, first for each particle Stokes number, volume fraction is increased and then velocity in a specific point of the gas flow-field is determined both by one-way coupling and two-way coupling methods. In one-way coupling, due to negligible effects of particles on the gas flow, flow-field is not affected by the volume fraction, whereas in two-way coupling, flow-field changes with different volume fraction. The intriguing point observed in this study is the deviation of the results of one-way coupling from those of two-way coupling which is due to the increasing effects of particles on continuous phase when the volume fraction is raised.

Typically the use of one-way coupling method can be justified as a result of its low computational cost; hence, particular limited volume fractions have been specified for several particle stokes numbers for which, the use of one-way coupling is plausible without any significant error. Finally, agreement of existing experimental data [16] and predictions obtained by the model in question is completely observable.

2. ANALYSIS AND MODELLING

A number of parameters which are of importance to turbulent flow suspension as well as those used in this review are defined below. The volume fraction of the dispersed phase (i.e. particles) is:

$$\alpha_p = \frac{\Delta V_p}{\Delta V} \tag{1}$$

Where ΔV_p is the volume of the dispersed phase in the volume ΔV of the mixture. An additional parameter that may have a significant influence on the flow is the particulate loading ratio. It may be written mathematically as:

$$\beta = \frac{\dot{m}_p}{\dot{m}_g} \tag{2}$$

Where $\dot{m}_p$ and $\dot{m}_g$ are mass flow rate of particles and gas respectively. For systems with intermediate particulate loadings, estimating the value of the particle Stokes number may contribute to the selection of the most appropriate model. The particle Stokes number can be defined as a relation between the particle response time $\left(\tau_p = \frac{\rho_p d_p^2}{18\mu_g}\right)$ and that of the system $\left(\tau_s = \frac{L_s}{V_s}\right)$ as follows

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$$St = \frac{\tau_p}{\tau_s} \tag{3}$$

Whereas $St \to 0$, the particle behaves as a fluid tracer and as $St \to \infty$, the particle is unresponsive to the fluctuations of the flow. Obviously, the most physically interesting situation occurs when it approaches the unity: particles follow the large scale fluid motion. Finally, the flow regime in the vicinity the particles can be characterized by means of the particle Reynolds number as follows:

$$\mathrm{Re}_p = \frac{|u - u_p| d_p}{\upsilon} \tag{4}$$

Where u_p is particle velocity. At small values of Re_p viscous effects are dominant and the streamlines are essentially symmetrical around the particle center, whereas at larger Re_p numbers it can be dispersed and a wake may develop behind the particle path. When Re_p raises enough, the wakes become turbulent. It has been postulated by Hetsroni [4] that Re_p can be used to determine whether particles attenuate ($\mathrm{Re}_p < 100$) or enhance ($\mathrm{Re}_p > 400$) the fluid turbulence level.

2.1 Equation of Particle's Motion

In the present work we have concentrated on spherical un-deformable particles of density ρ_p and diameter d_p smaller than all the flow length scales, and low values of the particle Reynolds number. The Lagrangian particle equation of motion proposed by Basset [19], Boussinesq [20], and Oseen [21] and later expanded by Tchen [22] is most frequently used. That is,

$$\begin{aligned} \rho_p V_p \frac{du_{pi}}{dt} &= (\rho_p - \rho_g) V_p g + \rho_g V_p \frac{Du_i}{Dt} \\ &- C_D \frac{1}{2} \frac{\pi d_p^2}{4} \rho_g |u_{pi} - u_i| (u_{pi} - u_i) \\ &- \frac{3}{2} d_p^2 \sqrt{\pi \upsilon} \int_{t_0}^{t} \frac{\frac{d}{dt}(u_{pi} - u_i) d\tau}{\sqrt{t - \tau}} \\ &+ \rho_g V_p C_M \left(\frac{Du_i}{Dt} - \frac{du_{pi}}{dt} \right) - C_L \rho_g V_p (u_{pi} - u_i) \times \omega \end{aligned} \tag{5}$$

Where V_p and ρ_g are volume of particle and gas density respectively. The six terms on the right hand-side of Eq. (5) represent, from left to right, the buoyancy force, pressure gradient force (excluding the gravity term that we included in the buoyancy force), the Stokes drag force, the basset history force, added mass or virtual mass force, and the lift force, respectively. In most practical dilute flow applications, the fluid-to-particle density ratios are in the order of 10^{-3}. Under such a circumstance, the added mass, the basset history force, and pressure gradient force can be neglected [23, 24]. The lift force can also be neglected, since it does not affect the particle trajectory except within the viscous sub layer [6]. According to the assumption above, drag force is the most influential force on the moving of particles and Eq. (5) will be reduced to

$$\frac{du_{pi}}{dt} = \frac{C_D \mathrm{Re}_p}{24 \tau_p} (u_{pi} - u_i) + g_i \tag{6}$$

and

$$\frac{dx_{pi}}{dt} = u_{pi} \tag{7}$$

Where C_D is the drag coefficient and for the solid particles is expressed as follows:

$$C_D = \begin{cases} \dfrac{24}{\mathrm{Re}_p} & \mathrm{Re}_p < 1 \\ \dfrac{24}{\mathrm{Re}_p} \left(1 + 0.15 \mathrm{Re}_p^{0.678}\right) & 1 < \mathrm{Re}_p < 1000 \\ 0.44 & \mathrm{Re}_p > 1000 \end{cases} \tag{8}$$

2.2 Fluid-phase Flow Equations

For incompressible steady flow governing equations are Navier-Stocks and continuity equations. The corresponding governing equations for fluid phase are:

Continuity

$$\frac{\partial \overline{u}_i}{\partial x_i} = 0 \tag{9}$$

Momentum equation

$$\overline{u}_j \frac{\partial \overline{u}_i}{\partial x_j} = -\frac{1}{\rho_g} \frac{\partial \overline{p}}{\partial x_i} + \frac{\partial}{\partial x_j} \left(\upsilon \frac{\partial \overline{u}_i}{\partial x_j} + \upsilon_T (\overline{u_{i,j}} + \overline{u_{j,i}}) - \frac{2}{3} k \delta_{ij} \right) + S_{pi} \tag{10}$$

For turbulence modeling standard k-ε Turbulence model is used.

Turbulence kinetic energy equation

$$\begin{aligned} \overline{u}_j \frac{\partial k}{\partial x_j} &= \frac{\partial}{\partial x_j} \left[\left(\upsilon + \frac{\upsilon_T}{\sigma_k} \right) \frac{\partial k}{\partial x_j} \right] \\ &- \left(\frac{2}{3} k \delta_{ij} - \upsilon_T (\overline{u_{i,j}} + \overline{u_{j,i}}) \right) \frac{\partial \overline{u}_i}{\partial x_j} - \varepsilon + S_{kp} \end{aligned} \tag{11}$$

Turbulence dissipation energy equation

$$\begin{aligned} \overline{u}_j \frac{\partial \varepsilon}{\partial x_j} &= \frac{\partial}{\partial x_j} \left[\left(\upsilon + \frac{\upsilon_T}{\sigma_\varepsilon} \right) \frac{\partial \varepsilon}{\partial x_j} \right] - C_{\varepsilon 1} \frac{\varepsilon}{k} \left(\frac{2}{3} k \delta_{ij} - \upsilon_T (\overline{u_{i,j}} + \overline{u_{j,i}}) \right) \frac{\partial \overline{u}_i}{\partial x_j} \\ &- C_{\varepsilon 2} \frac{\varepsilon^2}{k} + S_{\varepsilon p} \end{aligned} \tag{12}$$

S_{pi}, S_{kp} and $S_{\varepsilon p}$ are the effect of particles on momentum equation, turbulence kinetic energy and turbulence dissipation energy respectively.

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3. DESCRIPTION OF THE FLOW

In Tsuji *et al.'s* experiment [16], particles were injected into the flow by the use of an electromagnetic feeder and hence passed around a horizontal-vertical bend to enter the vertical test section as a two phase flow, as shown in Fig. 1. The pipe diameter changed abruptly at the bend outlet from 40 mm to 30.5 mm. It made the particles slide on the bend outer wall and become dispersed. The distance between the bend outlet and the test section was 5.11 m. At the end of the test, they made measurements using Laser Doppler Velocimetry (LDV) in the fully developed region of the glass pipe. There were five kinds of polystyrene particles (four types of spherical and one cylindrical shapes) utilized in their experiment. A wide range of particle sizes (from 200 μm ~ 3000 μm in diameter) with densities ranging from 1020 kg/m^3 to 1030 kg/m^3 and loading ratios ranging from 0.5 to 5.0 were employed for spherical particles. The 200 μm and 500 μm particles were coated with a charge-preventing substance to avoid the electrostatic charging caused by the friction between the particles and the wall. The mean air velocities were ranged from 8 to 20 *m/s*.

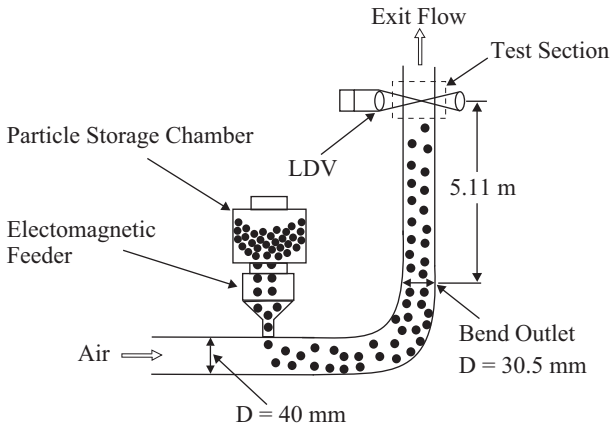


Fig. 1 Experimental set up of Tsuji *et al*. [16].

4. NUMERICAL SIMULATION

In this study, the system of equations has been numerically solved by the application of a 2D- ax symmetric double precision model using CFD code based on finite volume procedure. The equations are discredited using Presto's scheme, which is similar to the staggered-grid scheme with a first-order upwind scheme for the convective terms. The Simple algorithm is used to solve the coupling between continuity and momentum through pressure. The convergence criterion in each case was $(\phi^{(i+1)} - \phi^{(i)})/\phi^{(i)} < 10^{-8}$, where i denotes the iteration number and ϕ could stand for any of the dependent variables. When applying one-way coupling simulations, the continuous-phase flow field is solved and then the particle trajectory for discrete phase is determined. On the other hand, when we employ two-way coupling simulations, the continuous phase flow pattern is impacted by the dispersed phase (and vice versa); thus, iterative calculations of continuous phase and dispersed phase are performed until a converged coupled solution is obtained.

The particle phase equations are solved using the implicit and the trapezoidal schemes. In all the simulations, the phases are assumed to be incompressible, and density and viscosity are constant. The walls were adiabatic and no-slip wall conditions were used for none of the phases. A velocity inlet boundary condition was applied on the inlet, while on the outlet pressure outlet boundary condition was applied. The quadrilateral mesh was used to simulate this axi-symmetric vertical pipe flow (Fig. 2). Tests were conducted earlier to ensure grid-independent results and in the end, a non- uniform grid of 300 × 20 was found to suffice. Aimed to verify the independence of initial conditions from turbulence, different turbulent kinetic energy and turbulent dissipated energy at the inlet of pipe are considered, then independence of results at the end of pipe are studied.

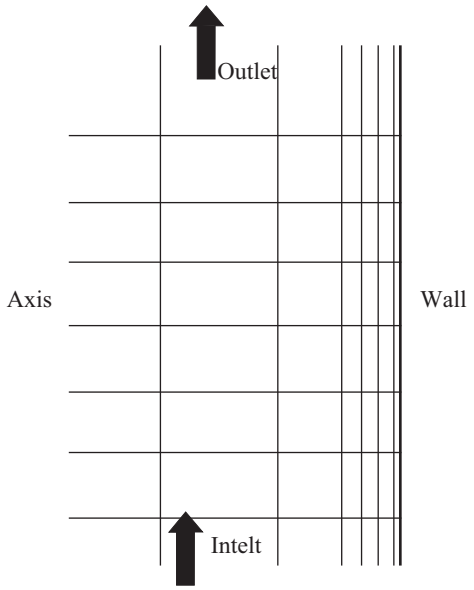


Fig. 2 Mesh scheme used in the present work.

Physical domain was long enough to obtain fully developed condition. The initial velocities of the gas-phase were not provided in the experiment of Tsuji *et al*. [16], therefore in order to obtain an average velocity to be used as the inlet velocity, integration was performed on the available velocity profiles for each test case [1]. The inlet velocity was considered to be constant across the inlet plane. The particles were injected with a constant mass flow rate across the inlet section. Actually, using the following equation the inlet velocity was calculated based on a given loading ratio (β)

$$\dot{m}_p = \beta \rho_g \bar{u} A \tag{13}$$

The test cases in the particle laden vertical pipe flow by Tsuji *et al*. [16] are summarized in Table 1.

The particle volume fraction was calculated as [1]

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Table 1 A summary of the test cases in the particle laden vertical pipe flow by Tsuji *et al*. [16]

Quantity	Case I	Case II	Case II
$dp(\mu m)$	200	500	1000
$\rho_p(kg/m^3)$	1020	1020	1030
β	3.2	3.4	3.0
$\bar{u}_g(m/s)$	9.62	9.66	11.4
$\dot{m}_p(kg/s)$	2.64×10^{-2}	2.83×10^{-2}	2.95×10^{-2}
Re_p	9.17	65.95	251.56
α_p	3.68×10^{-3}	3.91×10^{-3}	3.42×10^{-3}

$$\alpha_p = \frac{\beta}{\beta + \rho_p / \rho_g} \tag{14}$$

The average particle Reynolds number was calculated by solving the non-linear equation as follows [1]:

$$\mathrm{Re}_p \, \mu\left(1 + 0.15\,\mathrm{Re}_p^{0.687}\right) - \rho_g \tau_p g d_p = 0 \tag{15}$$

This equation was obtained based on the definition of the particle Reynolds number. The initial velocity of the particles was set equal to those of the gas phase.

5. RESULTS AND DISCUSSION

5.1 Model Validation

In order to validate the reliability of the proposed model, obtained numerical results were compared with several existing experiments. Therefore, for radial distribution of axial velocity at the pipe outlet, we made a comparison between numerical (One-way and Two-way coupling) and experimental results for cases I, II, and III, as illustrated in Fig. 3. As expected, compared to the simulation results of the one-way coupling, those of the two-way coupling are in a better agreement with the experimental results for the axial velocities. Moreover, it is observed that increasing the particle size leads to a decrease in the velocity attenuation (see case I and case III).

5.2 Comparing One-way and Two-way Couplings

In this section, the simulation results for fluid velocities as well as the discrepancy between two-way and one-way couplings for different particle volume fractions and Stokes numbers are presented. The simulations were performed for 2D-axisymmetric upward fully developed pipe with the diameter and length of 30.5 mm and 5.11 m long, respectively. We assumed the values of T, μ and ρ for air to be as: $T = 298.15\ K$, $\mu = 1.789 \times 10^{-5}\ Ns/m^2$ and $\rho = 1.773\ kg/m^3$, respectively.

5.2.1 Variable Particle Stokes Number

Figure 4 illustrates the percentage of the deviation of

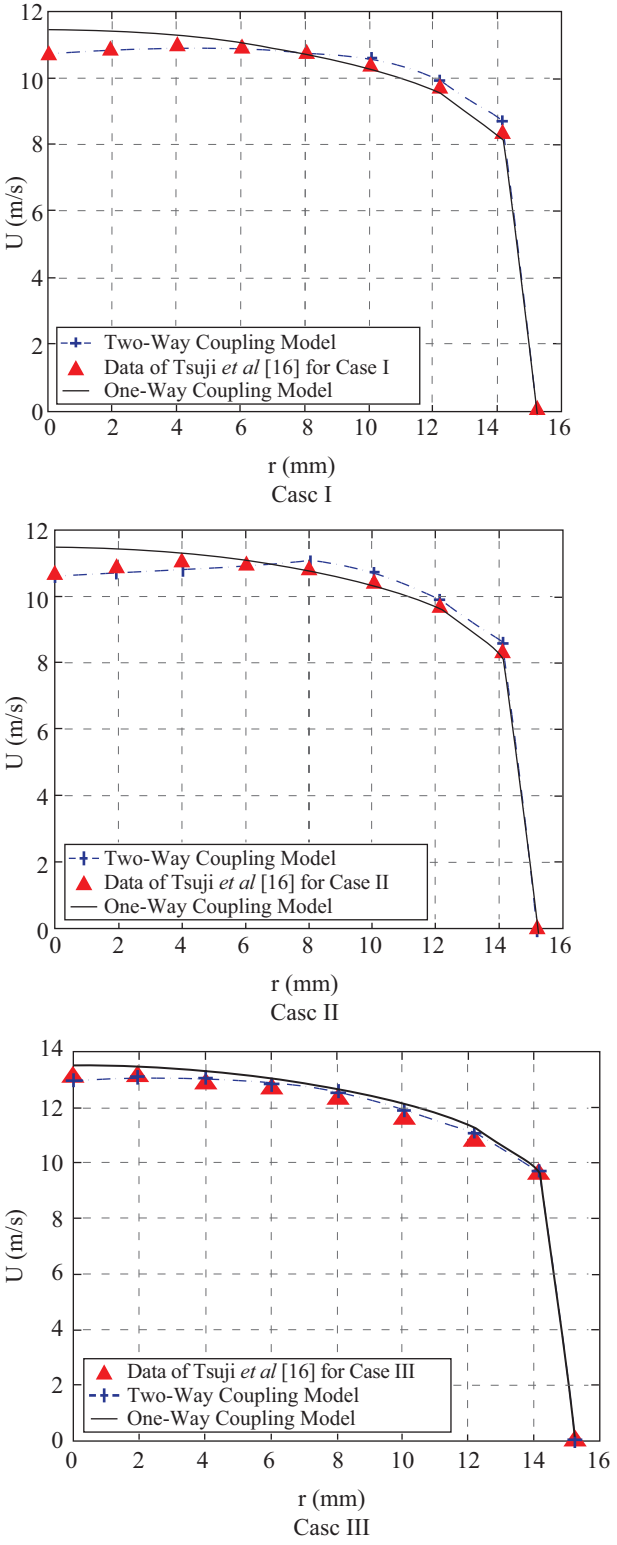


Fig. 3 Comparisons of predicted radial distribution of axial mean velocity w.r.t the experimental data of Tsuji *et al*. [16], $\mathrm{Re} \approx 23000$ Cases I, II, and III.

axial mean gas velocities at the end of centerline of pipe section predicted by one-way and two-way couplings. The simulations were performed at 10 *m*/s velocity of inlet air and 400 μm particles size in different particle volume fractions and Stokes numbers. As illustrated in Fig. 4, velocity predicted deviation is highly dependent

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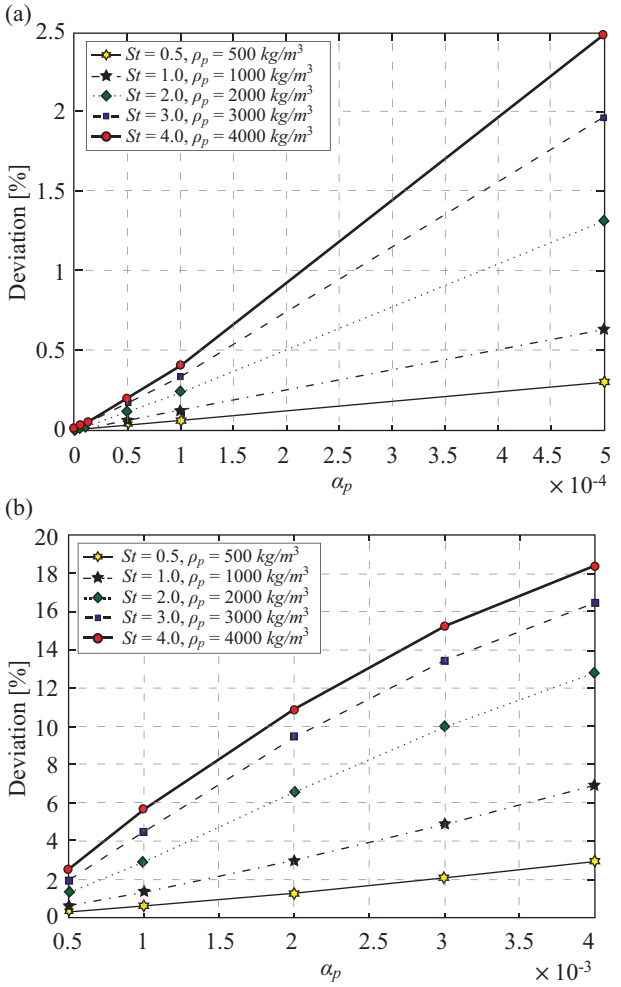


Fig. 4 Percentage of deviation of axial mean gas velocities at the end of the centerline pipe section obtained by one-way and two-way couplings for 400 μm particles; (a) $\alpha_p < 5\times10^{-4}$, (b) $\alpha_p > 5\times10^{-4}$.

Table 2 Special limited particle volume fraction for different particle Stokes numbers for 1% deviation

Stokes number	0.5	1	2	3	4
volume fraction × 10^{-4}	15	7.6	3.9	2.6	2

on the particle Stokes number and volume fraction. With a simple observation, it could be perceived that the velocity obtained deviation increases with an increase in the volume fraction, which is due to increasing the number of particles, at constant Stocks number. In addition, it can be seen at a specific volume fraction, the obtained velocity deviation grows with increasing the particle Stokes number. This may be due to the fact that increasing the stock number at constant particle diameter is equal to an increase in the particle density, i.e. heavy particles with more influence on continuous phase.

Because of low computational cost of one-way coupling, this method is commonly preferred. Therefore, specifying special conditions in which particle effects are negligible are generally accentuated. In this study, the limitations of volume fraction were indicated in several Stoke numbers where the one-way coupling method can be used without any significant error. For example, Table 2 has been obtained for 1% deviation.

5.2.2 Constant Particle Stokes Number

Figure 5 illustrates the percentage of axial mean gas velocity deviation at the end of the pipe section centerline. The simulations were performed by one- way and two-way couplings at 10 *m/s* velocity of inlet air and the particle Stokes number equal to unity ($St \approx 1$) in different particle volume fractions, and particle sizes. This has demonstrated that the particle fluid interaction does not depend merely on the volume fraction and particle Stokes number. In other words, at a certain constant particle volume fraction and particle Stokes number, the particle's influences are not identical but they are a function of particle's density and particle's diameter. The result shows that denser and smaller particles have more effects on gas flow at specific constant particle Stokes number and particle's volume fractions. When the particle's diameter is decreased, in order to have a fixed

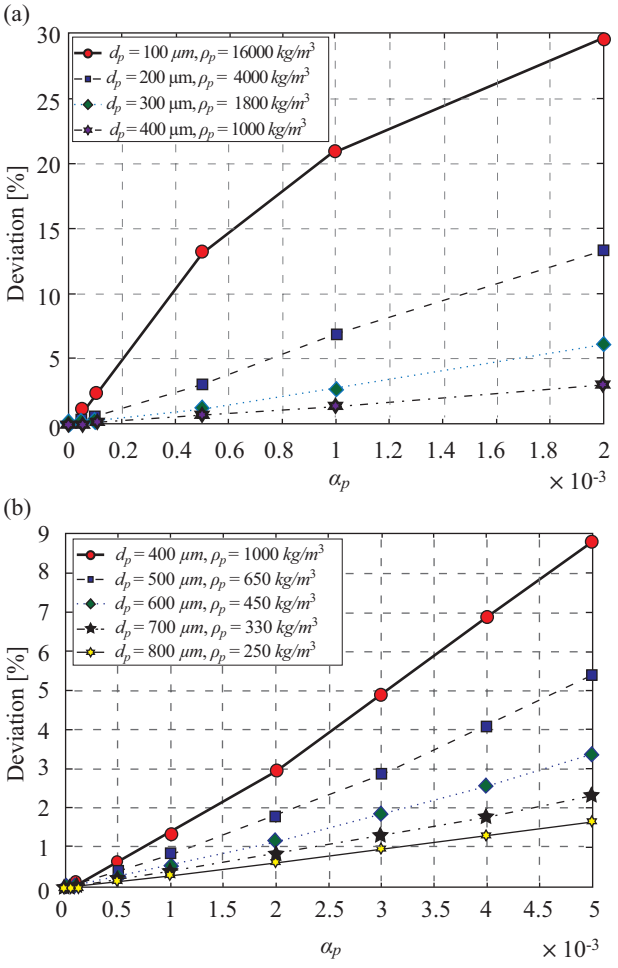


Fig. 5 Percentage of deviation of axial mean gas velocities at the end of the centerline pipe section obtained by one-way and two-way couplings for $St \approx 1$.

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volume fraction, the number of particles should be increased; thus, at a specific particle Stokes number, the particle density should be increased. In a fixed particle Stokes number and volume fraction, it has been shown that with a decrease in particles' size the effects of particles increase, which is a notable growth of the number of particles.

6. CONCLUSIONS

In the present study, we performed CFD analysis on internal turbulent particle-gas flow. The Lagrangian model has been used to track the particles. In order to validate the numerical model, comparisons between experimental and numerical results have been made. Thus, in order to specify the importance of particle fluid interaction, one-way and two-way coupling methods have been compared. The results have shown when the particle volume fraction and particle Stokes number are increased, the two-way coupling method leads to more accurate prediction of axial mean gas velocity compared to one way coupling method in which the particles-gas interaction is neglected. In other words, considerations of gas-particle interaction appear to be more significant at higher volume fractions, particle Stokes numbers and particle mass loading. The results also indicate that particle Stokes number enhancement reduces the volume fraction limitation. Moreover, it has been observed that the importance of particle fluid interaction does not only depend on volume fraction and particle Stokes number. The results have shown in a fixed particle Stokes number and volume fraction, with a decrease in particles size, which leads to a growth both in particles' density and number of particles, the effects of particles become more significant.

NOMENCLATURE

A	cross-sectional area [m^2]
$C_{\varepsilon 1}$, $C_{\varepsilon 2}$, C_μ	k-ε model constant
C_D	drag coefficient [-]
C_L	lift coefficient [-]
C_M	added mass [-]
d_p	particle diameter [m]
g	acceleration of gravity [m/s^2]
i, j	grid coordinate [-]
k	turbulent kinetic energy [m^2/s^2]
$\dot{m}$	mass flow rate [kg/s]
p	pressure [N/m^2]
Re	Reynolds number [-]
S_{pi}	time-averaged particle source [m/s^2]
S_{kp}	particle source to k equation [m^2/s^3]
$S_{\varepsilon p}$	particle source to ε equation [m^2/s^4]
St	particle Stokes number [-]
t	time [s]
u	instantaneous velocity [m/s]
$\bar{u}$	time-average velocity [m/s]
V	volume [m^3]
x_i	dimensional component [m]
	Greek symbols
α_p	particle volume fraction [-]
β	particulate loading ratio [-]
ϕ	dependent variable [-]
σ_k, σ_ε	constants in k-εmodel [-]
ε	turbulent dissipated energy [m^2/s^3]
ω	fluid vorticity [1/s]
ρ	particle density [kg/m^3]
τ	response time [s]
υ	kinematic viscosity [m^2/s]
υ_T	turbulence kinematic viscosity [m^2/s]
	Subscript
G	gas phase
P	Particle
	Superscripts
$(\bar{\ })$	time averaged property

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