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Relativistic collisionless shocks formation in pair plasmas

ANTOINE BRET[1], A. STOCKEM[2], F. FIUZA[2], C. RUYER[3], L. GREMILLET[3], R. NARAYAN[4] and L. O. SILVA[2]

[1]ETSI Industriales, Universidad de Castilla-La Mancha, 13071 Ciudad Real, Spain
(antoineclaude.bret@uclm.es)

[2]GoLP/Instituto de Plasmas e Fusão Nuclear – Laboratório Associado, Instituto Superior Técnico, Lisboa, Portugal
[3]CEA, DAM, DIF F-91297 Arpajon, France
[4]Harvard-Smithsonian Center for Astrophysics, 60 Garden Street, MS-51 Cambridge, MA 02138, USA

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Abstract. Collisionless shocks are ubiquitous in astrophysics and in the laboratory. Recent numerical simulations and experiments have shown how these can arise from the encounter of two collisionless plasma shells. When the shells interpenetrate, the overlapping region turns unstable, triggering the shock formation. As a first step toward a microscopic understanding of the process, we here analyze in detail the initial instability phase. On the one hand, 2D relativistic PIC simulations are performed where two unmagnetized, symmetric, and initially cold pair plasmas collide. On the other hand, the instabilities at work are analyzed, as well as the field at saturation and the seed field which gets amplified. For mildly relativistic motions and onward, Weibel modes with $\omega = 0 + i\delta$ govern the linear phase. We derive an expression for the duration of the linear phase in reasonable agreement with the simulations.

1. Introduction

Colliding plasma shells are expected in a variety of physical settings. Astrophysical jets produced by black holes are expected to generate a shock when interacting with the interstellar medium (Begelman et al. 1984; Harris and Krawczynski 2006). Still in astrophysics, the Fireball scenario for Gamma-Ray-Bursts (Nakar 2007) relies on shock particle acceleration (Bell 1978; Blandford and Ostriker 1978) where the shock arises from the encounter of two ultra-relativistic plasma blobs ejected from a central engine. Non-relativistic supernova remnant shocks are also instrumental in accelerating high-energy cosmic rays (Koyama et al. 1995).

For the collisionless environments considered, Particle-In-Cell (PIC) simulations are an excellent tool to study these kinds of processes. The formation of a shock following the collision of two pair plasma shells is now numerically well documented (Silva et al. 2003; Spitkovsky 2008), and subsequent particle acceleration has been observed. Similar features have been reproduced when modeling pair/electron–ion (ei) plasma interaction (Nishikawa et al. 2005), or ei/ei interaction (Dieckmann et al. 2000). In addition, shock generation through counter-streaming plasmas has already been observed in laboratory (Joseph et al. 2011), and the corresponding particle acceleration could be a promising alternative to current plasma accelerator schemes (Haberberger et al. 2012).

Although the full shock formation process has thus been repeatedly observed, a first principle understanding of the very birth of the shock is still lacking. Such an understanding could provide an accurate timing of the shock formation time, and could constraint the conditions required to form a shock in the first place. Whether they are in the laboratory, in a computer simulation, or in the vicinity of a supernova, it should be possible to separate the scenario leading to shock into two phases. In the first phase, plasma shells make contact, then overlap, and the overlapping region turns unstable. An instability grows and saturates. At this junction, the total density in the overlapping region is roughly the sum of each plasma density. A second phase is therefore needed during which nonlinear processes pick up the system from the end of the linear phase, and build up the Rankine–Hugoniot expected density jump near the borders of the interpenetrating shells.

The present paper is concerned with the first of these two phases. The collision of two identical cold relativistic pair plasmas has been simulated in 2D with the PIC Code OSIRIS (Fonseca et al. 2002). This setup has been chosen for its simplicity, allowing for a neat comparison with theory, as the only free parameter is the initial Lorentz factor γ_0 of the shells. Periodic boundary conditions are applied in the transverse direction. The evolution of the magnetic field in the overlapping region is displayed in Fig. 1 for $\gamma_0 = 25$. The saturation time τ_s (t_1 in Fig. 1) is defined as the end of the exponential growth of the field energy integrated over the overlapping region. The field at saturation is simply $B(\tau_s)$.

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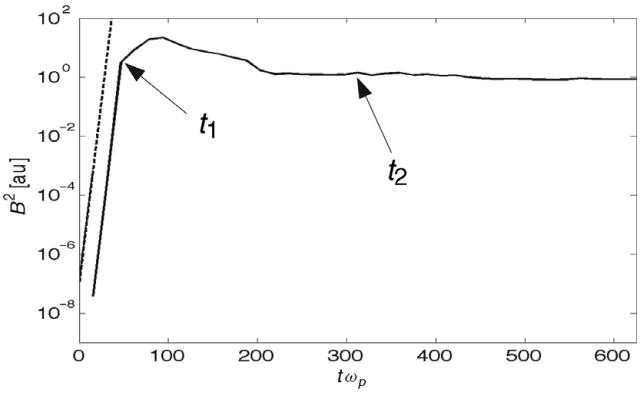


Figure 1. Growth of the magnetic energy integrated over the transverse direction in the overlapping region. The dashed line is the theoretical growth rate. The initial Lorentz factor was $\gamma_0 = 25$. All the field growth plots look qualitatively the same until $\gamma_0 = 10^4$. The saturation time τ_s is t_1. By this time the density in the overlapping region is still twice the shells' density. The density jump of $\sim$3.3 then builds up, and the shock is formed around $t = t_2$. The field at saturation is $B(\tau_s)$.

As will be checked, the instabilities at play can be interpreted in terms of the homogenous theory for such, although geometry is finite here, since our shells have one border. As it amplifies a seed field from its initial fluctuation value to saturation, the instability governs this first phase of the shock formation process for a time τ_s that we labeled as 'saturation time'. A theoretical determination of this time, in reasonable agreement with simulations, is the main result of this paper. Not only τ_s gives a lower bound for the shock formation time, it also sheds light on the amplitude of the amplified initial fluctuations.

2. Instability analysis

We here deal with the first phase of the shock formation, namely the instability of the overlapping region. Indeed, if counter-streaming collisionless plasmas were not unstable, they would simply go through each other without anything happening. Let us start ignoring the finite geometry at stake here, and analyze the system as if it were homogenous. The full unstable **k** spectrum has been analyzed long ago in the cold regime where a shell is much denser than the other (Bludman et al. 1960; Watson et al. 1960; Faĭnberg et al. 1970). These early results were recently generalized to the hot symmetric case (Bret and Deutsch 2005; Bret et al. 2010). For wave-vectors aligned with the flow, we find two-stream unstable modes. Then for wave-vectors normal to the flow, we find the filamentation, or Weibel, instability. Finally, oblique modes propagating at an arbitrary angle with the flow are also unstable (Bret et al. 2004). As the two plasmas penetrate each other, all the modes are excited. But the fastest growing one quickly overcomes the other and shapes the linear phase. For a cold symmetric and relativistic system such as the one considered here, a calculation of the growth rate for every possible **k** shows that only oblique and filamentation modes can dominate (Bret and Deutsch 2005; Bret et al. 2010). The transition from oblique to filamentation occurs for $\gamma_0 > \sqrt{3/2}$ (Bret et al. 2012). Note that although the analysis of Bret and Deutsch (2005) and Bret et al. (2010) was conducted for counter-streaming *electron* beams, counter-streaming *pair* beams are linearly equivalent because the linear regime scales like the square of the charge.

We thus find that unless $\gamma_0 < \sqrt{3/2}$, filamentation should govern the interaction with the growth rate,

$$\frac{\delta}{\omega_p} = \frac{v_0}{c}\sqrt{\frac{2}{\gamma_0}} \sim \sqrt{\frac{2}{\gamma_0}}. \tag{2.1}$$

Comparing this value with the growth of the field observed in the overlapping region results in a very satisfactory agreement, as evidenced in Fig. 1. We also checked that the Weibel/Oblique transition does occur around $\gamma_0 = \sqrt{3/2}$. Note that a shock is *also* formed for $\gamma_0 < \sqrt{3/2}$ (not shown).

How can a theory developed for a homogenous system apply to the present inhomogenous system? We think that can be explained as follows: before they come in contact, each plasma shell is stable. Once they interpenetrate, it takes a time $\sim \omega_p^{-1}$ for collective effects to set in and start the instability process. By that time shells already overlap by a distance $d \sim c/\omega_p$. But the parallel scale length relative to instabilities is precisely $\lambda_s = c/\omega_p$. Even if at the very beginning of the instability process $d \gg \lambda_s$ is not fulfilled, the strong inequality is quickly realized with time passing so that most of the instability process develops in a setting fulfilling the homogenous approximation.

Knowing the growth-rate expression (2.1) should allow for an accurate timing of the linear phase. Assuming the instability amplifies a seed field of amplitude B_i up to a saturation level B_s, we can write for the saturation time τ_s,

$$B_f = B_i e^{\delta\tau_s} \quad \Rightarrow \quad \tau_s = \frac{1}{2\delta}\ln\left(\frac{B_f^2}{B_i^2}\right), \tag{2.2}$$

where, for convenience, we consider the field energy B^2 ratio instead of the field itself. Determining the saturation time then amounts to the determining of the initial and final fields. We will first discuss the saturation field.

3. Field at saturation

One way to derive the value of the saturation field $B_f = B(\tau_s)$ consists in stating that the field grows exponentially as long as it is small enough for the system to fit the linear approximation. Since a field B_f affects particles on a time scale given by the cyclotron frequency, this implies (Medvedev and Loeb 1999),

$$\frac{qB_f}{\gamma_0 mc} = \delta \quad \Rightarrow \quad B_f = \frac{\gamma_0 m}{q}\delta c. \tag{3.1}$$

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Accounting in addition for the growth-rate expression (2.1) gives,

$$B_f^2 = 2\gamma_0 b^2, \tag{3.2}$$

where

$$b = \frac{mc\omega_p}{q} \tag{3.3}$$

is the magnetic field unit of the simulations. This expression fits very well with the values observed in the simulations. A consequence of the observed γ_0 scaling is that the field energy relative to the beam one reads as

$$\frac{B_f^2/8\pi}{\gamma_0 nmc^2} \sim 1, \tag{3.4}$$

displaying the near-equipartition already noted by various authors (Medvedev and Loeb 1999; Silva et al. 2003).

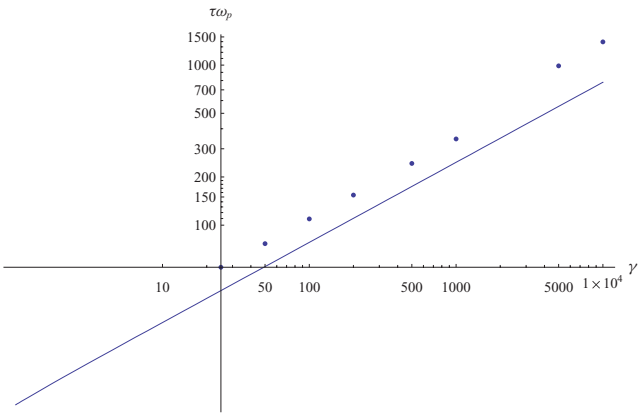


Figure 2. (Colour online) Saturation time in ω_p units measured from simulations, *circles*, vs. (4.4).

4. The initial field amplitude

We now turn to the evaluation of the initial field amplitude. The idea is that the instability mechanism picks up a seed field from the spontaneous fluctuations of the system and amplifies it. Various authors have been dealing with plasma fluctuations (Sitenko 1967; Tautz and Schlickeiser 2007; Yoon 2007; Schlickeiser and Yoon 2012), and the ability of PIC simulations to correctly render them has been checked (Dieckmann et al. 2004). But the first question to ask could be the following: Should we consider that the instability starts from the fluctuations of one single plasma, or from the fluctuations of the system formed by the union of the two plasmas? In other words, should we consider the fluctuations of the system *before* it turns unstable, or *after*?

The fluctuations of the two plasmas will wait for a time, at least ω_p^{-1}, before they start to adapt to each other. But this is also the time it takes for the instability to set in. We thus consider that the instability mechanism amplifies the fluctuations of a single shell *before* it turns unstable.

4.1. Fluctuation power spectrum

The power spectrum of the fluctuations for a Maxwellian plasma drifting at relativistic velocity has been recently calculated (Ruyer and Gremillet 2012). Calculations have been conducted considering a Maxwell–Jüttner distribution function for the plasma, allowing for any relativistic drift and temperature. For the modes of interest here with $\mathbf{k} \perp \mathbf{v}_0$, the fluctuations energy density integrated over ω read

$$\frac{B_{k_\perp}^2}{8\pi} = \frac{k_B T}{2} \frac{\frac{k_\perp^2 c^2}{\omega_p^2}\frac{k_B T}{mc^2} + 1}{\frac{k_\perp^2 c^2}{\omega_p^2}\frac{k_B T}{mc^2} + 1/\gamma_0^2}. \tag{4.1}$$

Equation (4.1) now needs integration over a $\mathbf{k}$ domain to obtain the available power in the corresponding fluctuations. Wave-vectors selected for growth form a narrow band around the normal axis, extending to infinity and width (Bret et al. 2010),

$$Z_\parallel \sim \sqrt{\frac{2}{\gamma_0}} \Longrightarrow k_{\parallel,\max} \sim \sqrt{\frac{2}{\gamma_0}}\frac{\omega_p}{c} \tag{4.2}$$

in the parallel direction (we set here $v_0 \sim c$).

Regarding the normal direction, we numerically found that the fastest growing $k_\perp$ has $k_\perp \sim \omega_p/c\sqrt{\gamma_0}$. We thus integrated in this direction from $k_{\perp,\min} = \omega_p/2c\sqrt{\gamma_0}$ to $k_{\perp,\,\max} = \omega_p/2c\sqrt{\gamma_0}$.

4.2. Saturation time

The energy density (4.1) is thus integrated the following way,

$$\frac{B_i^2}{8\pi} = \int_{k_{\perp,\min}}^{k_{\perp,\max}} 2\pi k_\perp dk_\perp \int_{-k_{\parallel,\max}}^{k_{\parallel,\max}} dk_\parallel d\omega \frac{B_{c/\omega_p}^2}{8\pi}. \tag{4.3}$$

Clearly, it is an *averaged* initial amplitude over the modes which are likely to grow the most. Inserting the resulting B_i in (2.2) for the saturation time,

$$\tau\omega_p = \frac{\sqrt{\gamma_0}}{2\sqrt{2}} \ln\left[\frac{n(c/\omega_p)^3}{\sqrt{2}\pi} \frac{\mu\gamma_0^{3/2}}{\frac{15}{4\gamma_0} + \mu \ln\left[\frac{1+4\gamma_0/\mu}{1+\gamma_0/4\mu}\right]}\right], \quad \mu = \frac{mc^2}{k_B T}, \tag{4.4}$$

where n is the plasma density.

Simulations have been run with nearly cold colliding plasma shells, with thermal velocity $v_{\rm th} = 10^{-3}c$, i.e. $k_B T/mc^2 = (v_{\rm th}/c)^2$. With eight particles per cells, and the size of a cell being $\Delta x = 0.05\sqrt{\gamma_0}c/\omega_p$, we come to

$$n\left(\frac{c}{\omega_p}\right)^3 = \frac{8}{0.05^3\gamma_0^{3/2}}. \tag{4.5}$$

The time to reach saturation observed from the simulations is compared with (4.4) in Fig. 2. Clearly, the scaling is correct while a factor 1.7 is consistently lacking. This systematic underestimate of the saturation time probably comes from the fact that the initial field amplitude is determined from the ω-integrated density instead of the density near $\omega = 0$ (filamentation has $\omega = 0$). As a result the initial field amplitude

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is overestimated, yielding an underestimated saturation time.

Still another source of discrepancy could lie in the correspondence between simulations and theory. The simulation particles do not have the mass of a real electron. Fewer heavier particles yield a noise power that is higher than that of many light particles. These issues will be the topic of future works.

5. Conclusion

The saturation time has been determined from the expression of the dominant growth rate, the field at saturation, and the seed field amplified by the instability. The dominant growth rate could be determined from the theory derived for an homogenous, infinite system in spite of the limited extension of the overlapping region. The agreement observed between this theory and the simulation is due to the fact that although finite, the center of the unstable region, where modes start growing, satisfies the homogeneity criterion.

The seed field B_i which the instability picks up for amplification is computed from the amplitude of the spontaneous fluctuations of one single relativistically streaming shell. This density integrated over the **k** domain is likely to grow.

The theory elaborated consistently underestimates the saturation time. Solutions to this problem are currently being investigated (Bret et al. 2012).

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