*J. Fluid Mech.* (2006), *vol.* 558, *pp.* 243–252. © 2006 Cambridge University Press 
doi:10.1017/S0022112006000280 Printed in the United Kingdom

# Intermittency and asymmetry in fully developed turbulence

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(Received 7 July 2005 and in revised form 27 January 2006)

Using experimental transverse velocity data for very high-Reynolds-number turbulence, we suggest a model describing both the formation of intermittency and asymmetry of turbulence. The model, called the 'bump model', is a modification of the ramp model suggested previously. The connection between asymmetry and intermittency makes it possible to study the latter with relatively low moments.

## 1. Introduction

One important feature of fluid turbulence is local isotropy. The turbulence is stirred at the large scales, and, according to the Kolmogorov theory, Kolmogorov (1941*b*), any high-Reynolds-number turbulent flow is supposed to restore isotropy in small scales. It was shown experimentally, however, that in a sheared turbulence the isotropy is not sufficiently restored for both the scalar and velocity fields, see Tavoularis & Corrsin (1981), Sreenivasan (1991), Shen & Warhaft (2000), Ferchichi & Tavoularis (2000), Schumacher, Sreenivasan & Yeung (2003). In numerical simulations, the failure to return to isotropy was linked to both the asymmetry of the probability distribution function (PDF) and to the vortex sheets, Pumir & Shraiman (1995). It became clear that the shear in the integral scale induces asymmetry down to the small scales, where it is manifested by structures like cliffs, etc., Staicu & van de Water (2003).

In case of isotropic turbulence the asymmetry of the PDF appears naturally; for the longitudinal velocity increments $u_r = u(x + r) - u(x)$ we have

$$B_u = \langle u_r \rangle = 0, \tag{1.1}$$

while, in the inertial range,

$$B_{uuu}(r) = \langle u_r^3 \rangle = -\tfrac{4}{5}\varepsilon r. \tag{1.2}$$

Equation (1.2) is the so-called 4/5-Kolmogorov law, Kolmogorov (1941*a*). The fact that the first moment vanishes whereas the third moment does not clearly indicates that the PDF for $u_r$ is asymmetric.

This asymmetry is traditionally linked to the direction of the energy cascade in fully developed turbulence, Monin & Yaglom (1971), rather than to the small-scale structures, or intermittency. Moreover, the Kolmogorov law (1.2) is exact in the inertial range, while other moments contain the so-called intermittency corrections. From the self-similar properties of turbulence, suggested by Kolmogorov (1941*b*), a simple scaling for $u_r$ follows, namely,

$$\langle |u_r|^p \rangle \sim r^{p/3}. \tag{1.3}$$

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It became clear, however, that there are corrections to these scalings, usually attributed to intermittency. The only exception is the third moment, $p=3$, according to (1.2). Note that the prediction (1.3) deals with moments of $|u_r|$ while in (1.2) we have a moment of $u_r$; nevertheless, this should not affect the properties of self-similarity of the turbulence.

Thus, at first sight, the isotropic turbulence asymmetry manifested in (1.2) is not related to the intermittency, unlike what happens in the sheared turbulence. However, this is not the case. A more detailed study of the third-order structure function has proved to be useful in understanding the intermittency. It is important to understand the contribution to third moment given by the tails of the PDF.

The contribution of the rare violent events is expected to be substantial for high-order moments. As for the low-order moments, it is natural to assume that the main events from the core of $P(u_r)$, the PDF, mainly contribute. This is certainly the case for the even (low)-order structure functions, or for moments like $\langle |u_r|^p \rangle$.

Presumably, the third-order structure functions can be considered as relatively low-order moments, and therefore one may assume that the main events essentially form them. However, the situation is more subtle: one should keep in mind that these moments, like other odd-order structure functions, do not vanish only due to the asymmetry. Again, one would assume that the asymmetry manifested by the Kolmogorov law (1.2) is presented by the main events, that is by the core of the PDF. However, if the core of the PDF were more or less symmetric, then the contribution of the tails would be substantial.

This asymmetry, described by a ramp model, is indeed related to intermittency (in addition to the asymmetry), as suggested by Vainshtein & Sreenivasan (1994) and Sreenivasan *et al.* (1996). Note that the ramp model was suggested much earlier: for example, local anisotropy was observed in heated and cooled turbulent boundary layers, Mestayer *et al.* (1976), as well as in temperature distributions, Antonia & Sreenivasan (1977). Returning to the connection between the anisotropy and intermittency, we note that further studies showed that the tails of $P(u_r)$, responsible for the intermittency, substantially contribute to the structure function, see Vainshtein (2000). Another way to check this connection between asymmetry and intermittency is to compare the positive and negative parts of the PDF for $u_r$ directly, and we have seen that the asymmetry of the PDF stretches far into the tails, Vainshtein (2000).

This relationship between asymmetry and intermittency makes it possible to study the latter with relatively low-order moments. It is interesting to note that the low-order structure functions (presumably reflecting the main events, rather than rare tail events) do indeed show intermittency corrections, see Sreenivasan *et al.* (1996), Cao, Chen & Sreenivasan (1996) and Chen *et al.* (2005). The transverse velocities give additional information about both asymmetry and intermittency, and this paper is devoted to their study.

## 2. Problem description

The transverse (vertical) component of the velocity increments $v_r = v(x+r)-v(x)$ is also assumed to possess asymmetry, although $\langle v_r^3 \rangle = 0$ (this moment does not vanish in sheared turbulence, Staicu & van de Water (2003)). Assuming isotropic turbulence, the only non-vanishing third-order correlation containing $v_r$ is

$$B_{uvv} = \langle u_r v_r^2 \rangle = \frac{1}{6}\frac{\mathrm{d}(B_{uuu}r)}{\mathrm{d}r}, \tag{2.1}$$

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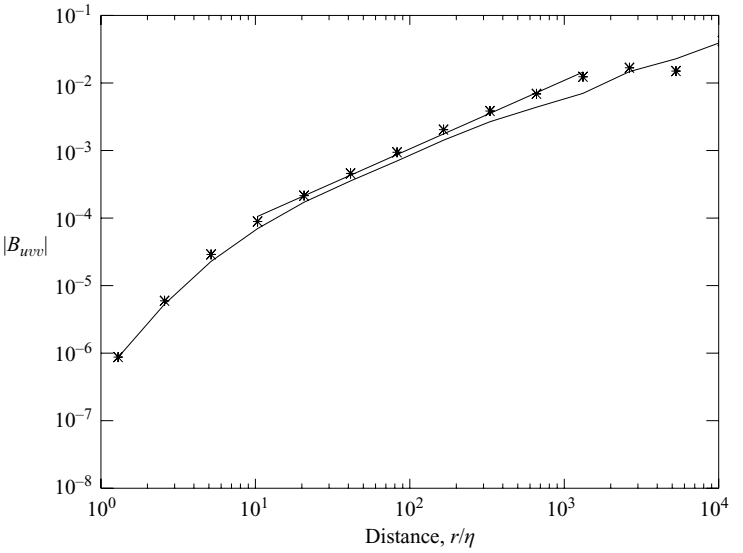



FIGURE 1. Experimental moment $\langle u_r v_r^2 \rangle$ (asterisks), the power law fit $\sim r^{1.01 \pm 0.02}$ (thick solid line) are compared to $B_{uvv}$ derived from the longitudinal moment according to (2.1), and depicted by a solid line. The distance is given in terms of the Kolmogorov microscale $\eta$.

Landau & Lifschitz (1987), Monin & Yaglom (1971). In this paper, we compare the measured moment $B_{uvv}$ with cumulative moments created in such a way that only the tail parts of the distribution (rare events) contribute to the moment. We will show that these parts (responsible for intermittency) quite satisfactorily recover the third moment.

Although this recovery using rare violent events seems to be unusual, as mentioned in the Introduction, the situation is more subtle. It is not obvious *a priori* that a 'typical' distribution would not behave in qualitatively the same way. In order to assess this recovery, we need a 'regular' distribution to compare with. For this purpose, we use Gaussian-like distributions. They are defined in such a way as to recover the third moment $B_{uvv}$, but, as is shown in § 3, the cumulative moments for these pseudo-Gaussian distributions are noticeably below the experimental cumulative moments. That is the rare events of 'regular' distributions poorly recover the moment $B_{uvv}$.

Another way to assess the deviation of the cumulative moments from the full moment $B_{uvv}$ is to consider a numerical model which supposedly contains both asymmetry and intermittency, § 4. Again, the model recovers $B_{uvv}$ by definition; however, unlike the pseudo-Gaussian distribution, it shows qualitative agreement with the experimental $B_{uvv}$, and with its cumulative parts.

We used X-wire data acquired at Brookhaven National Laboratory. The distance of probe above the ground was 35 m; the number of samples was 40960000 per component, that is, for longitudinal ($u$) and transverse ($v$) components; the sampling frequency was 10 kHz; the mean velocity was 5.15076224 m s$^{-1}$; rms $u$-velocity was 1.81617371 m s$^{-1}$; rms $v$-velocity was 1.3646025 m s$^{-1}$; Taylor Reynolds number was 10680 (courtesy of K. R. Sreenivasan). As usual, the data are interpreted using Taylor's hypothesis.

As seen from figure 1, the experimental $B_{uvv}$ is close to that obtained from (2.1), especially at small distances between the points, in agreement with earlier observations by Kurien & Sreenivasan (2001) (see their figure 2). At small scales the statistical

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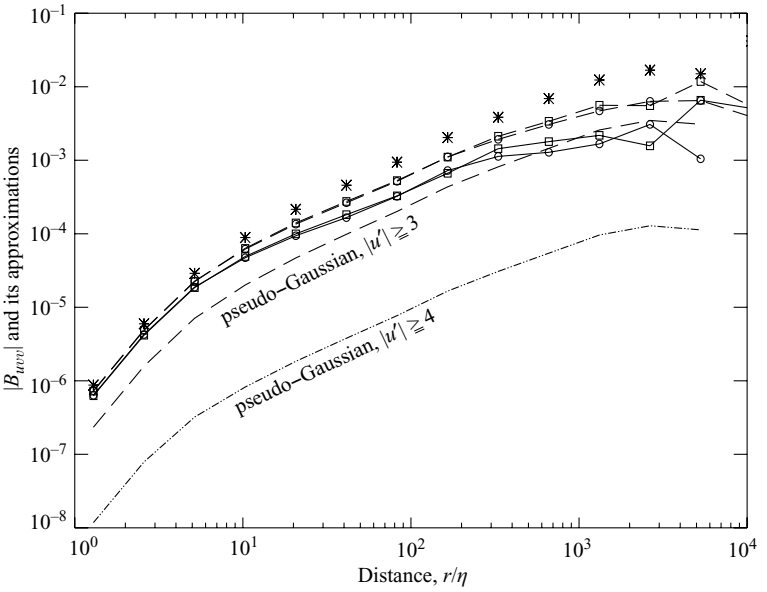



FIGURE 2. Comparing the experimental $B_{uvv}$ with cumulative moments: asterisks correspond to the measurements, open squares connected by a dashed line correspond to $|v'_r| \geqslant 3$, open squares connected by a dashed-dotted line to $|v'_r| \geqslant 4$, open circles connected by a dashed line correspond to $|u'_r| \geqslant 3$, and open circles connected by a dashed-dotted line correspond to $|u'_r| \geqslant 4$.

properties are more isotropic, in accordance with Kolmogorov's ideas about local isotropy.

Denote

$$u'_r = \frac{u_r}{\sigma_u}, \quad v'_r = \frac{v_r}{\sigma_v},$$

where

$$\sigma_u = \left\langle u_r^2 \right\rangle^{1/2} = B_{uu}^{1/2}, \quad \sigma_v = \left\langle v_r^2 \right\rangle^{1/2} = B_{vv}^{1/2}.$$

We will consider cumulative moments,

$$\left\langle u'_r v'^2_r \right\rangle\big|_{|u'_r| \geqslant t} = \left( \int_{-\infty}^{-t} + \int_t^{\infty} \right) \mathrm{d}u'_r \int_{-\infty}^{\infty} \mathrm{d}v'_r u'_r v'^2_r P(u'_r, v'_r), \tag{2.2}$$

$$\left\langle u'_r v'^2_r \right\rangle\big|_{|v'_r| \geqslant t} = \int_{-\infty}^{\infty} \mathrm{d}u'_r \left( \int_{-\infty}^{-t} + \int_t^{\infty} \right) \mathrm{d}v'_r u'_r v'^2_r P(u'_r, v'_r), \tag{2.3}$$

where $P(u'_r, v'_r)$ is the distribution function, and $t$ is a number. If $t \ll 1$, then essentially the whole distribution contributes, and the cumulative moments are expected almost to coincide with $\langle u'_r v'^2_r \rangle = k(r)$, where $k = B_{uvv}/(\sigma_u \sigma_v^2)$, analogously to skewness. For not small $t$, we are dealing with the tails of the distribution, and it is important to know what contribution they give to the moment.

## 3. Cumulative moments

Figure 2 shows these moments for $t = 3$ and 4. It can be seen that the moments thus constructed do not deviate far from the experimental $B_{uvv}(r)$.

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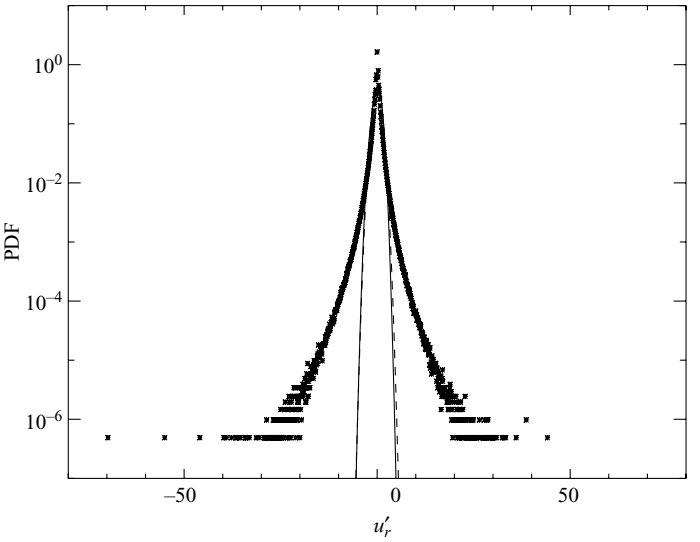



FIGURE 3. The experimental PDF for different $u'_r$, $r/\eta = 1.29$ (asterisks), compared with Gaussian (dashed line) and $I_r$ (dashed-dotted).

In order to get some idea of how strong the deviation of the cumulative moments is from experimental results, note that, for the smallest distances $r$, the values of $t = 3$ and $t = 4$ correspond to about 1% of all events; for larger distances this number being even less. Figure 2 suggest that these events alone satisfactorily recover the mixed moment $B_{uvv}$.

Still, it would be useful to have a 'regular' distribution to compare with. Indeed, the relatively rare events satisfactorily recover the moment $B_{uvv}(r)$, satisfactorily, but not completely, of course. It is not obvious that the cumulative moments for $t = 3$ and $t = 4$ for 'regular' asymmetric distributions would recover this moment to the same extent as the experimental cumulative moments do.

By a 'regular' PDF we mean a PDF that provides asymmetry through the main events (rather than through the rare tail events). A Gaussian distribution is symmetric, its third moment vanishes, and therefore it is not suitable for this purpose. We construct a series of PDFs $I_r(u')$ for different distances $r$ as a sum of two Gaussian functions in such a way that all three first moments coincide with the experimental moments. Namely, let

$$\langle u'^0 \rangle_I = \int I_r \, \mathrm{d}u' = 1, \quad \langle u' \rangle_I = \int I_r u' \, \mathrm{d}u' = 0,$$

$$\langle u'^2 \rangle_I = 1, \quad \langle u'^3 \rangle_I = k(r).$$

Then,

$$\langle u_r \rangle_I = \langle u' \rangle_I \sigma_u = 0, \quad \langle v_r \rangle_I = \langle u' \rangle_I \sigma_v = 0, \tag{3.1}$$

$$\langle u_r^2 \rangle_I = B_{uu}, \quad \langle v_r^2 \rangle_I = B_{vv}, \quad \langle u_r v_r^2 \rangle_I = B_{uu}^{1/2} B_{vv} k = B_{uvv}. \tag{3.2}$$

As seen from (3.1), (3.2), the $I_r$ distribution recovers the first, the second and the third moments by definition. Figure 3 gives an example of this PDF for $r/\eta = 1.29$. It is compared with experimental PDFs for different $u'_r$, the latter having clearly asymmetric tails (a direct comparison of positive and negative tails is given by

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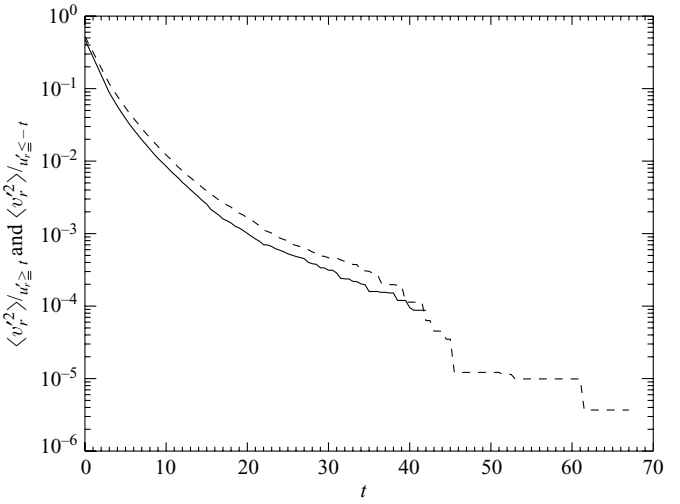



FIGURE 4. Experimental cumulative tail moments for different cut-off numbers $t$. Solid lines correspond to $u'_r \geqslant t$, dashed lines – to $u'_r \leqslant -t$.

Vainshtein (2000)). The distribution $I_r$ is very similar to the Gaussian, except the skewness of the former does not vanish, i.e. it is asymmetric. For this reason, we will call it 'pseudo-Gaussian'.

As the total moment $\langle u_r v_r^2\rangle_I$ recovers the third moment by definition (3.2), we will consider the third moment for cumulative average $\langle u_r v_r^2\rangle_I|_{|u'|\geqslant t}$, $t = 3$ or 4. Corresponding moments are depicted in figure 2. We note that even for $t = 3$, the cumulative moment constructed from $I_r$ is essentially lower than $B_{uvv}$; only for large distances does it approach the experimental cumulative moments. In the case $t = 4$, it can be seen that the pseudo-Gaussian cumulative moment is quite noticeably lower than the corresponding experimental cumulative structure function.

Thus, relatively rare events of experimental PDFs satisfactorily recover the third moment. The deviation of the cumulative moments from the total moment $B_{uvv}$ is substantially less than for the pseudo-Gaussian distribution.

As a non-vanishing $B_{uuu}$ is a result of asymmetry of the PDF for $u_r$, the correlation $B_{uvv}$, obeying (2.1), is therefore related to the asymmetry. Indeed, in order that $B_{uvv} < 0$, there should be an anti-correlation between $u_r$ and $v_r^2$: decreasing $u_r$ is accompanied by increasing $v_r^2$, and vice versa. Roughly speaking, the conditional average satisfies

$$B_{uvv}(u_r < 0) > B_{uvv}(u_r > 0).$$

If the asymmetry is indeed related to intermittency, this conditional inequality should be satisfied for $u_r > t$ versus $u_r < -t$, i.e.

$$B_{uvv}(u_r < -t) > B_{uvv}(u_r > t),$$

where $t$ is not a small number. To check this, we consider, first, the distributions for the smallest $r$ corresponding to the distance between two neighbour samples. Second, we consider cumulative moments, $\langle u_r v_r^2\rangle|_{u_r\leqslant -t}$, and $\langle u_r v_r^2\rangle|_{u_r\geqslant t}$, for different $t$.

Figure 4 presents the experimental moments. It shows, first, quite substantial tails: even when $t = 30$, or greater (in units of $\sigma_u$), the contribution to the cumulative moments is noticeable – note that the case $t = 30$ corresponds to a factor of less than $10^{-6}$ of the events. Second, we see a remarkable feature: the negative contributions

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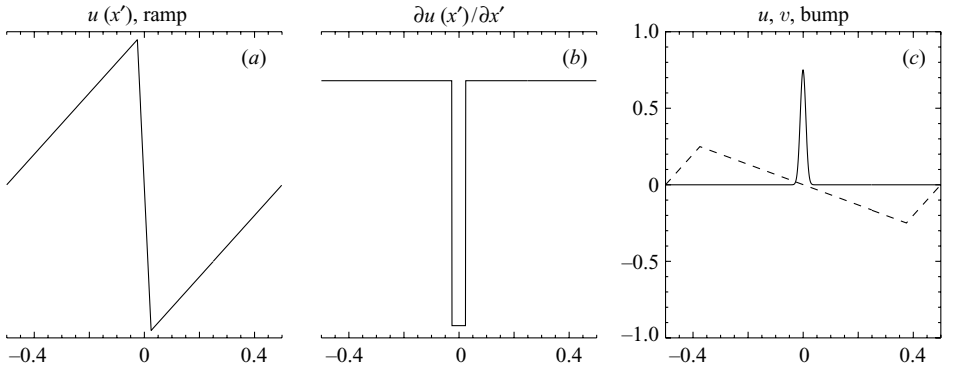



FIGURE 5. (*a*) Ramp structure. (*b*) Derivative of the ramp. (*c*) Bumpstructure. In the one-dimensional model, the velocity component $u$ is a sum of the solid line and dashed line. In the two-dimensional model, the dashed line corresponds to $v_x$, and the solid line to $v_y$, see (4.1)–(4.3).

exceed the positive not only at small $t$, corresponding to the core of the distribution, but also far into the tails.

## 4. Constructing the model

Vainshtein & Sreenivasan (1994) and Sreenivasan *et al.* (1996) suggested a model to explain how the asymmetry appears. Figure 5(*a*) shows a ramp structure. Clearly, $\langle \partial_x u(x) \rangle = 0$, while $\langle \partial_x u(x)^3 \rangle < 0$. In addition, the negative part of $\partial_x u(x)$ is certainly intermittent, see figure 5(*b*), and that is how the idea of intermittency being connected to the asymmetry came about.

This model is only heuristic, however. It was shown by Jiménez (1992), Hatakeyama & Kambe (1997), and Tanaka & Kida (1993) that a Burgers vortex, embedded in a converging motion, acquires negative skewness, this picture containing both asymmetry and intermittency. The ramp model does not exactly correspond to this. A more realistic modification of this model is the bump model by Vainshtein (2001), see figure 5(*c*), where $u(x)$ is the sum of the solid and dashed lines. Here again, $\langle \partial_x u(x) \rangle = 0$ while $\langle \partial_x u(x)^3 \rangle < 0$. This model simulates a converging motion (the dashed line in the vicinity of the solid line peak), naturally generating a vortex (solid line). It is thought that, this structure in the longitudinal velocity appears in the vicinity of a Burgers vortex.

Both the ramp model and the bump model are one-dimensional, and therefore they do not reflect any structures appearing in the transverse velocity component. Therefore, we need further modification of the model to make it two- or three-dimensional. Besides, a real vortex generated by converging motion depicted by the solid line in figure 5(*c*) would be described by a shear of the $v_y$-component of velocity (rather than by a shear of $v_x$). If that is the case, then the $u_r$–$v_r^2$ anti-correlation will appear. Indeed, when $u_r < 0$ (converging motion), the vortex is generated thus increasing $v_y^2$, that is, $v_r^2$, while for $u_r > 0$ (diverging motion), the vortex is not generated (and $v_r^2$ is smaller than average).

Consider, therefore, a two-dimensional model:

$$v_x = f_1(x'), \tag{4.1}$$

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smooth motion corresponding to the dashed line in figure 5(*c*), and

$$v_y = f_2(x'), \tag{4.2}$$

a 'bump', or vortex depicted by a solid line. Let $\alpha = f_1(x' + r_x) - f_1(x')$, and $\omega = f_2(x' + r_x) - f_2(x')$. Then,

$$u_r = [\alpha \cos\phi + \omega \sin\phi] \cos\phi, \quad v_r = [-\alpha \sin\phi + \omega \cos\phi] \cos\phi, \tag{4.3}$$

where $r_x = r\cos\phi$, $r_y = r\sin\phi$. We thus have two averages: over $x'$, and over $\phi$. As a result, $\langle u_r \rangle = \langle v_r \rangle = 0$, while

$$\langle u_r^3 \rangle = \langle \alpha^3 \rangle \phi_{60} + 3\alpha(0)\langle \omega^2 \rangle \phi_{40}, \tag{4.4}$$

$$\langle u_r v_r^2 \rangle = \langle \alpha^3 \rangle \phi_{40} + \alpha(0)\langle \omega^2 \rangle (\phi_{60} - 2\phi_{40}), \tag{4.5}$$

where

$$\phi_{60} = \langle \cos^6\phi \rangle = \tfrac{5}{16}, \quad \phi_{42} = \langle \cos^4\phi \sin^2\phi \rangle = \tfrac{1}{16},$$

and $\langle v_r^3 \rangle = 0$. Here we considered small $r$, so that $\omega$ is strongly peaked at $x' = 0$, and therefore, when combined with $\omega$, the value of $\alpha$ contributes only at $x' = 0$. Note that $\alpha(0) < 0$ (converging motion), and $|\omega| \gg |\alpha|$, and therefore both $\langle u_r^3 \rangle$ and $\langle u_r v_r^2 \rangle$ are negative.

It is useful to simulate both smooth motion (4.1) and the bump (4.2) numerically. The numerical model should be constructed in such a way that the first three moments coincide with the experimental values: expressions (4.4) and (4.5) guarantee only that the signs of the two third-order structure functions are correct. Then, we will have several free parameters such as the width of the bump (4.2), etc. It is interesting to note that by choosing them simply in a 'reasonable' way, we immediately reproduce the real experimental values for $\langle u_r^3 \rangle$, and $\langle u_r v_r^2 \rangle$ with a good accuracy. To do this better, we used computer routines to optimize these parameters so that they fit the experimental values in the best way.

On the other hand, though, the model, being a little more complicated than the previous one-dimensional model, is still much too simple to reflect all the real properties of turbulence. In particular, it does not reflect the multi-scale nature of turbulence. Nevertheless, it makes sense to simulate the model (4.1)–(4.3) numerically, constructing the third-order moments (longitudinal and mixed), and to compare them with the experimental data. We may expect some qualitative agreement with the experiment.

We now are ready to calculate the cumulative moments $\langle u_r v_r^2 \rangle|_{u_r \leqslant -t}$, and $\langle u_r v_r^2 \rangle|_{u_r \geqslant t}$, for different $t$, corresponding to this model. They are shown in figure 6. Qualitatively, the we see the same features as in experimental cumulative moments depicted in figure 4. Namely, there are substantial tails, obviously related to the presence of the vortex (or the 'bump'), and the negative part always exceeds the positive one.

Note however that the agreement is only qualitative. Quantitatively, we can see that the intermittency in the model, figure 6, is much less pronounced than in figure 4. As mentioned above, this is because the model is too simple. We would get more intermittency if these bumps were random and multi-scaled, in which case the effective Reynolds number would increase. At present, we believe that the numerical model illustrates the connection between asymmetry and intermittency.

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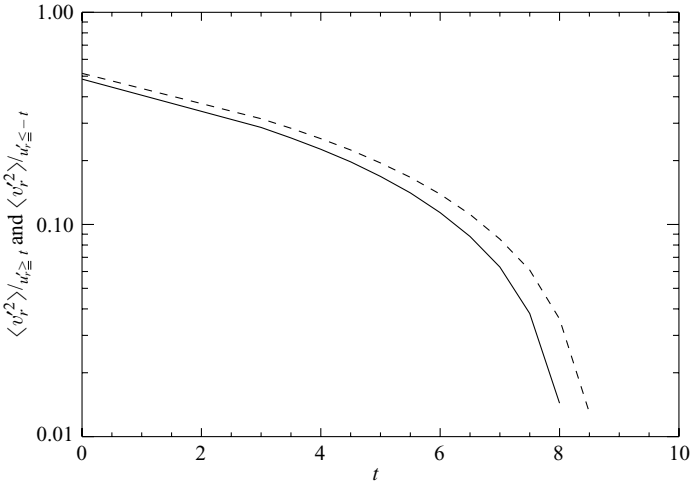



FIGURE 6. Cumulative tail moments for different cut-off numbers $t$ corresponding to the bump model, (4.1)–(4.3). Solid line corresponds to $u_r' \geqslant t$, dashed line to $u_r' \leqslant -t$.

## 5. Discussion

Thus, isotropic turbulence has something in common with a sheared turbulence. Namely, the asymmetry is related to the intermittency. Note that the cumulative moments proved to be useful in studying the tails of the distributions: we thus consider the contribution of the tails, as if the core of the distribution does not contribute at all.

We saw that the Kolmogorov law, and related third-order $u_r$–$v_r^2$ correlation, can be satisfactory reproduced by the tail events only. In contrast, the third-order moments corresponding to pseudo-Gaussian distributions are poorly reproduced by the contribution of the tails.

As these third-order moments do not vanish because of the asymmetry of the distributions, we assume that the intermittency (i.e. the substantial contribution of the tails of the distributions) is related to the asymmetry. This conjecture can be checked directly, by comparing positive and negative contributions. We see that the predicted difference between the positive and negative parts is present not only at the core of the distribution but also stretches far into the tails.

The intermittency related to the asymmetry comes out naturally from the ramp model and its two-dimensional modification – the bump model. It simply presents a vortex embedded in converging motion. An analytical representation of this model shows qualitatively the same behaviour as the experimental data. We conclude that it is consistent with the above interpretation of intermittency related to the asymmetry. As the third moment is a relatively low moment, this conjecture suggests a useful tool for studying the intermittency of turbulence.

I thank K. R. Sreenivasan, S. Kurien, and R. Rosner for sending me valuable data, and for discussions.

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https://doi.org/10.1017/S0022112006000280 Published online by Cambridge University Press