J. Plasma Physics (2007), *vol.* 73, *part* 6, *pp.* 831–838. © 2006 Cambridge University Press
doi:10.1017/S0022377806006295 First published online 19 December 2006 Printed in the United Kingdom

Gravitational instability of partially ionized molecular clouds

A. C. BORAH and A. K. SEN

Department of Physics, Assam University, Silchar 788011, India
(asokesen@sancharnet.in)

(Received 24 October 2006)

Abstract. Stars are formed as a result of the gravitational (Jeans) collapse of dense clumps in interstellar clouds. These clouds are partially ionized by nearby ionizing sources. We investigate the gravitational instability in such molecular clouds considering the non-Boltzmannian distribution for electrons and ions, which is more realistic than the Boltzmannian distribution. Assuming the perturbation (fluctuation) response in a radial direction as a mathematical analogue of the x-direction in the plane geometry approximation in the form $f \sim \exp(ikx - i\omega t)$, the equations of motion for different species of the multi-fluid plasma are linearized. Jeans' swindle is used as a local approximation for the equilibrium and the dispersion relation is derived by usual normal mode analysis. Then, an analytical solution to the dispersion equation with an explanation of the effects on star formation is given.

1. Introduction

Spectroscopic observations have shown that the major part of the interstellar material is concentrated in giant molecular clouds, i.e. clouds with a total mass that exceeds $10^5 M_\odot$ (Balsara 1996). In the densest parts of such clouds, substantial self-gravitational forces among the cloud particles are present (Bliokh et al. 1995; Verheest 2000). Within the domain of uniform mass distribution, the equilibrium force of gravity could be considered as unimportant with regards to the equilibrium dynamics in a molecular cloud. Sometimes, because of nearby ionizing sources, gases in a molecular cloud are also ionized (Goertz and Ip 1948; Shu et al. 1990; Hazelton and Yadlowsky 1994; Walch et al. 1995). The gravitational instability in such a mass distribution has been studied by many authors (Jeans 1920; Kolb and Turner 1990; Pandey et al. 1994; Nakano et al. 1995; Jacobs and Shukla 2005, etc.) and the condition of Jeans' swindle (Binney and Tremaine 1987) has been utilized to study the gravitational collapse of such dust mass. If the symmetry of uniformity is broken, either owing to some mass fluctuations, as in the case of a purely neutral dust mass, or owing to the charge fluctuations, as in the case of the charged dust mass, gravitational collapse will take place. Considering the Boltzmannian distribution for electrons ($n_\mathrm{e} = n_{\mathrm{e}0} \exp\{e\varphi/T_\mathrm{e}\}$) and ions ($n_\mathrm{i} = n_{\mathrm{i}0} \exp\{-e\varphi/T_\mathrm{i}\}$), and the one-dimensional (1D) fluctuation response, Dwivedi et al. (1999) have shown that there is a possibility of a pulsational mode of gravitational collapse (PMGC) of the molecular cloud. The particle distribution is important when collisions between different species are considered. When such collisions are not considered the distribution is Boltzmannian (Dwivedi et al. 1999). When collision

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terms are included we call this distribution non-Boltzamannian. The frictional forces (drag) in the neutral and charged dust dynamics play an important role and it may be noted that the distribution for ions and electrons may not always be Boltzmannian. We recalculate the distribution of particles considering the collision terms.

2. Calculation of distributions

To calculate the distribution functions for different species in the system of a partially charged dusty plasma with neutral drag force, we consider a multi-fluid system with charged dust grains, neutral dust grains, electrons, ions, and neutral gases. The number density, size, mass, charge, and temperature of charged dust grains are assumed, respectively, to be $n_{\rm dc}, a, m_{\rm d}, q_{\rm d}, T_{\rm d}$ and the corresponding quantities for neutral dust grains are, respectively $n_{\rm dn}, a, m_{\rm d}, 0, T_{\rm d}$. Furthermore, for an electron and an ion, we assume their density, charge, mass, and temperature to be $n_{\rm e}, e, m_{\rm e}, T_{\rm e}$ and $n_{\rm i}, e, m_{\rm i}, T_{\rm i}$, respectively. The quantities which vary with the perturbation of the form $f \sim \exp(ikx - i\omega t)$ can be written, for example, as $n_{\rm e} = n_{\rm e0} + \tilde{n}_{\rm e}$, $n_{\rm i} = n_{\rm i0} + \tilde{n}_{\rm i}$. It is also noted that 'dust', 'dust grain', and 'grain' are used to mean one and the same. Assuming a complete uniformity in the gravito-electrostatic fluid distribution, the equation of continuity for electrons can be written as

$$\frac{\partial n_{\rm e}}{\partial t} + \nabla \cdot (n_{\rm e}\mathbf{v}_{\rm e}) = 0. \tag{2.1}$$

Here $\mathbf{v}_{\rm e} = \mathbf{v}_{\rm e0} + \tilde{\mathbf{v}}_{\rm e}$ where the first term is the equilibrium static part and the second term is the result of the perturbation. Since, before the fluctuation, the system was in static equilibrium, $\mathbf{v}_{\rm e0} = 0$, and since the fluctuation is of the form $f \sim \exp(ikx - i\omega t)$, the operators in the above equations can be substituted as $\partial/\partial t = -i\omega$ and $\nabla = ik$. Thus, linearizing the above equation one obtains

$$-i\omega\tilde{n}_{\rm e} + ikn_{\rm e0}v_{\rm e} = 0. \tag{2.2}$$

The equation of motion for the electrons is

$$\frac{e}{m_{\rm e}}\nabla\varphi - c_{\rm e}^2\frac{\nabla\tilde{n}_{\rm e}}{n_{\rm e0}} - \nu_{\rm en}\mathbf{v}_{\rm e} = 0. \tag{2.3}$$

Here $\nu_{\rm en}$ is the binary collision rate of momentum transfer from electrons to the neutral particles and is given by $\nu_{\rm en} = \sigma c_{\rm e} n_{\rm n}$ where σ is the cross-section of the collision of the electron with the neutral particle, $c_{\rm e}$ is the thermal velocity of electrons, $n_{\rm n}$ is the number density of the neutral components, and ϕ is the electrostatic potential. Linearizing (2.3) in the same way as for (2.1) and substituting $v_{\rm e}$ from (2.2) one obtains

$$i\frac{e}{m_{\rm e}}k\tilde{\varphi} - ic_{\rm e}^2 k\frac{\tilde{n}_{\rm e}}{n_{\rm e0}} - \nu_{\rm en}\frac{\omega}{k}\frac{\tilde{n}_{\rm e}}{n_{\rm e0}} = 0. \tag{2.4}$$

Thus, the expression for the perturbed number density of electrons is

$$\tilde{n}_{\rm e} = n_{\rm e0}\frac{i(e/m_{\rm e})(1/\nu_{\rm en})k^2}{\omega + i(c_{\rm e}^2k^2/\nu_{\rm en})}\tilde{\varphi}. \tag{2.5}$$

Similarly, for ions,

$$\tilde{n}_{\rm i} = -n_{\rm i0}\frac{i(e/m_{\rm i})(1/\nu_{\rm in})k^2}{\omega + i(c_{\rm i}^2k^2/\nu_{\rm in})}\tilde{\varphi}. \tag{2.6}$$

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Analogous to the case where we are dealing with electrons, $\nu_{\rm in}$ is the binary rate of momentum transfer from ions to neutral particles and is given by $\nu_{\rm in} = \sigma c_{\rm i} n_{\rm n}$ where $c_{\rm i}$ is the thermal velocity of ions. The rate of change of the unperturbed parts of the quantities is considered to be zero. These two expressions (2.5) and (2.6) give the perturbed densities of electrons and ions, respectively, which clearly depend on the collision rate of these species with the neutral component.

3. Equations of motion

The equation of continuity and equation of motion for charged dust grains can be written as

$$\frac{\partial n_{\rm dc}}{\partial t} + \nabla \cdot (\mathbf{v}_{\rm dc} n_{\rm dc}) = 0 \tag{3.1}$$

and

$$\frac{\partial \mathbf{v}_{\rm dc}}{\partial t} = -\frac{q_{\rm d}}{m_{\rm d}} \nabla \tilde{\varphi} - \nabla \tilde{\psi} - \nu_{\rm cn} (\tilde{v}_{\rm dc} - \tilde{v}_{\rm dn}) \tag{3.2}$$

where ψ is the gravitational potential; $\nu_{\rm cn}$ is the binary collisional rate of momentum transfer from charged dust grains to neutral dust grains and can be given by $\nu_{\rm cn} \sim \pi a^2 n_{\rm dn0} c_{\rm td}$, and the opposite quantity, i.e. the collisional rate of momentum transfer from neutral dust grains to charged dust grains, can be written as $\nu_{\rm nc} \sim \pi a^2 n_{\rm dc0} c_{\rm td}$ where $c_{\rm td}$ corresponds to the thermal velocity of dust grains and πa^2 corresponds to the geometrical cross-section. The Coulomb scattering of the plasma charge particles by the charged dust grains is ignored. From (3.1) using the conditions $\partial n_{\rm dc0}/\partial t = 0$, $\mathbf{v}_{\rm dc0} = 0$ and $n_{\rm dc0} \gg \tilde{n}_{\rm dc}$, we obtain

$$\tilde{v}_{\rm dc} = \frac{\omega}{k} \frac{\tilde{n}_{\rm dc}}{n_{\rm dc0}}. \tag{3.3}$$

Similarly, from (3.2)

$$\tilde{v}_{\rm dc} (\nu_{\rm cn} - i\omega) = -\frac{q_{\rm d}}{m_{\rm d}} ik\tilde{\varphi} - ik\tilde{\psi} + \nu_{\rm cn} \tilde{v}_{\rm dn}. \tag{3.4}$$

For neutral dust grains these equations are, respectively,

$$\frac{\partial n_{\rm dn}}{\partial t} + \nabla \cdot (\mathbf{v}_{\rm dn} n_{\rm dn}) = 0 \tag{3.5}$$

and

$$\frac{\partial \mathbf{v}_{\rm dn}}{\partial t} = -\nabla \tilde{\psi} - (\mathbf{v}_{\rm dn} \cdot \nabla) \mathbf{v}_{\rm dn}. \tag{3.6}$$

Using $\partial n_{\rm dn0}/\partial t = 0$, $\mathbf{v}_{\rm dn0} = 0$, and $n_{\rm dn0} \gg \tilde{n}_{\rm dn}$, these equations yield

$$\tilde{v}_{\rm dn} = \frac{\omega}{k} \frac{\tilde{n}_{\rm dn}}{n_{\rm dn0}} \tag{3.7}$$

and,

$$\tilde{v}_{\rm dn} = \frac{k}{\omega} \tilde{\psi} \tag{3.8}$$

where the second-order term $(\tilde{\mathbf{v}}_{\rm dn} \cdot \nabla) \tilde{\mathbf{v}}_{\rm dn}$ is neglected for its small value. Thus, from (3.7) and (3.8) we obtain

$$\tilde{n}_{\rm dn} = \frac{k^2}{\omega^2} n_{\rm dn0} \tilde{\psi}. \tag{3.9}$$

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From (3.3), (3.8), and (3.4) we obtain

$$\tilde{n}_{\rm dc} = n_{\rm dc0}\left[\tilde{\psi} + \frac{q_{\rm d}}{m_{\rm d}}\left(1 + i\frac{\nu_{\rm cn}}{\omega}\right)^{-1}\tilde{\varphi}\right]\Big/\left(\frac{\omega^2}{k^2}\right). \tag{3.10}$$

It is interesting to note that the perturbed number density of charged dust grains is not only a function of the electric potential (φ) but also a function of the gravitational potential (ψ). Starting with a uniform density of charged dust grains, which has been created owing to the adsorption of charges by dust grains from the plasma environment, the density increases with an increase in the gravitational field potential. In the case that $\nu_{\rm cn} \to 0$, this implies a general growth of the perturbed number density of charged dust with an increase in the perturbed electric field potential, which is an obvious effect of electrostatics. However, with collisions of charged dust grains with neutral dust grains, the perturbed density of charged dust grains decreases. A change of charge state (due to charge exchange between dust particles) in all these collisions cannot be unexpected, which in turn leads to a loss of charge from charged dust in some collisions. Therefore, a decrease of $n_{\rm dc}$ with an increase of $\nu_{\rm cn}$ is also possible.

4. The Poisson equation and quasi-neutrality

The perturbed gravitational Poisson equation is

$$\nabla^2\tilde{\psi} = 4\pi G m_{\rm d}(\tilde{n}_{\rm dc} + \tilde{n}_{\rm dn}).$$

Using (3.9) and (3.10) and $\omega_{\rm J}^2 = 4\pi G m_{\rm d}(n_{\rm de0} + n_{\rm dn0})$, this equation yields

$$\tilde{\psi} = -\frac{4\pi G q_{\rm d}}{\omega^2}\left(1 + i\frac{\nu_{\rm cn}}{\omega}\right)^{-1}\left(1 + \frac{\omega_{\rm J}^2}{\omega^2}\right)^{-1}\tilde{\varphi}. \tag{4.1}$$

This equation relates the perturbed electric and gravitational potentials. The quasi-neutrality of the partially ionized dusty plasma demands

$$\tilde{n}_{\rm e} - \tilde{n}_{\rm i} - \frac{q_{\rm d}}{e}\tilde{n}_{\rm dc} = 0.$$

Substituting the values of $\tilde{n}_{\rm e}$, $\tilde{n}_{\rm i}$, and $\tilde{n}_{\rm dc}$ from (2.5), (2.6), and (3.10) we obtain

$$\left[\left(\frac{n_{\rm e0} m_{\rm i}\nu_{\rm in}}{n_{\rm i0} m_{\rm e}\nu_{\rm en}}\right)\frac{1}{\omega + i(c_{\rm e}^2 k^2/\nu_{\rm en})} + \frac{1}{\omega + i(c_{\rm i}^2 k^2/\nu_{\rm in})} + \frac{i m_{\rm i}\nu_{\rm in}}{T_{\rm i}}\frac{C_{\rm SCAM}^2}{\omega^2}\left(1 + i\frac{\nu_{\rm cn}}{\omega}\right)^{-1}\right]\tilde{\varphi} = -i\frac{m_{\rm i}\nu_{\rm in} q_{\rm d} n_{\rm dc0}}{e^2 n_{\rm i0}\omega^2}\tilde{\psi} \tag{4.2}$$

where $C_{\rm SCAM} = (q_{\rm d}/e)^2(n_{\rm dc0}/n_{\rm i0})(T_{\rm i}/m_{\rm d})$ corresponds to the speed of the low-frequency characteristic acoustic mode of a dusty-like multi-species plasma, namely the so-called acoustic mode (SCAM) (Dwivedi et al. 1989; Dwivedi 1997) in the limit $\lambda_{\rm De}^2/\lambda_{\rm Di}^2 \gg 1$.

5. The dispersion relation

The dispersion relation can now be derived by eliminating $\tilde{\psi}$ and $\tilde{\varphi}$ from (4.1) and (4.2):

$$\left(1 + \frac{\omega_{\rm J}^2}{\omega^2}\right)\left[\frac{\lambda_{\rm Di}^2}{\lambda_{\rm De}^2}\frac{1}{1 - i(\nu_{\rm en}\omega/k^2 c_{\rm e}^2)} + \frac{1}{1 - i(\nu_{\rm in}\omega/k^2 c_{\rm i}^2)}\right] - \frac{k^2 C_{\rm SCAM}^2}{\omega^2}\frac{1}{1 + i(\nu_{\rm cn}/\omega)}$$
$$- \frac{k^2 C_{\rm SCAM}^2}{\omega^2}\frac{1}{1 + i(\nu_{\rm cn}/\omega)}\left(\frac{\omega_{\rm J}^2 - \omega_{\rm Jd}^2}{\omega^2}\right) = 0 \tag{5.1}$$

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where $\omega_{\rm Jd}^2 = 4\pi G m_{\rm d} n_{\rm dc0}$ and $\omega_{\rm Jdn}^2 = 4\pi G m_{\rm d} n_{\rm dn0}$. Here

$$\lambda_{\rm Di}^2/\lambda_{\rm De}^2 = n_{\rm e0} m_{\rm i} c_{\rm i}^2 / n_{\rm i0} m_{\rm e} c_{\rm e}^2$$

is the ratio of squares of Debye lengths of ions to electrons.

By separating out the real and imaginary parts, (5.1) can be expressed in the form

$$c_4 \times x^4 + c_3 \times x^3 + c_2 \times x^2 + c_1 \times x + c_0 = 0 \tag{5.2}$$

where

$$x = \left(\frac{\omega}{\omega_{\rm J}}\right)^2$$

$$c_4 = \alpha^2 AB$$

$$c_3 = (1+\alpha^2)(A+B) + \alpha^2 AB(1+D) - ABC$$

$$c_2 = (2+\alpha^2) + (1+\alpha^2)(A+B)(1+D) + \alpha^2 ABD - (A+B)C - ABC\frac{\omega_{\rm Jdn}^2}{\omega_{\rm J}^2}$$

$$c_1 = (2+\alpha^2)(1+D) + (1+\alpha^2)(A+B)D - C - (A+B)C\frac{\omega_{\rm Jdn}^2}{\omega_{\rm J}^2}$$

$$c_0 = (2+\alpha^2)D - \frac{\omega_{\rm Jdn}^2}{\omega_{\rm J}^2}C.$$

Furthermore,

$$\alpha = \frac{\lambda_{\rm Di}}{\lambda_{\rm De}}$$

$$A = \left(\frac{\nu_{\rm en}\omega_{\rm J}}{c_{\rm e}^2 k^2}\right)^2, \quad B = \left(\frac{\nu_{\rm in}\omega_{\rm J}}{c_{\rm i}^2 k^2}\right)^2$$

$$C = \frac{k^2 C_{\rm SCAM}^2}{\omega_{\rm J}^2}, \quad D = \left(\frac{\nu_{\rm cn}}{\omega_{\rm J}}\right)^2.$$

Equation (5.2) is a fourth-degree equation in $x = (\omega/\omega_{\rm J})^2$, which can be solved exactly and later values of ω can be calculated for various values of k. In the plot of the dispersion relation we have chosen the wave number range from 10^{-17} to 10^{-16}, keeping in mind the large molecular clouds (1–100 pc), following the paper by Jacobs and Shukla (2005). In addition, we have also followed Jacobs and Shukla (2005) in the use of the following values of constants: $n_{\rm e0} = n_{\rm i0} = 4.5 \times 10^4$ m^{-3}, $n_{\rm dn0} = 10^8$ m^{-3}, $T_{\rm e} = T_{\rm i} = 100$ K, $T_{\rm d} = 10$ K. We have also chosen the size of the dust $a = 0.5$ μm, with mass density 2 gm cc^{-1} which should be close to that of the silicates or graphite grains found in the interstellar medium. We have taken the charged to neutral dust ratio $\eta(= n_{\rm dc0}/n_{\rm dn0}) = 10^{-8}$, which seems to be a realistic number for star-forming clouds (Dwivedi et al. 1999).

The solution of (5.2) gives four roots in which two are real (in many cases negative) and two are complex conjugates for the above values of the various parameters and the range of k values. However, the four values of ω (ω_1, ω_2, ω_3, and ω_4) calculated from these x values are mostly complex. The imaginary parts of these frequency (ω) values represent the growing Jeans mode. We have plotted the absolute value

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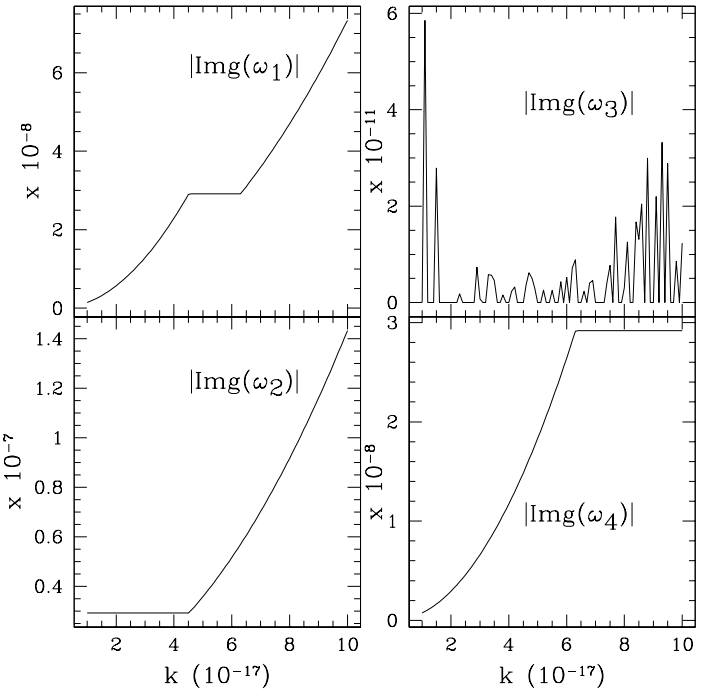


Figure 1. The dispersion relation for Jeans' cloud with charged dust grains.

of the imaginary part $|\mathrm{Im}(\omega)|$ against the k values for the four roots in Fig. 1. From this figure we also see some critical behaviour near the values of $k = 4 \times 10^{-17}$ to 6×10^{-17}. For one of the roots ω_3 we see much unstable behaviour.

In the following section we present an analysis of (5.1) under various conditions that occur in molecular clouds.

6. Discussion

6.1. Boltzmannian approximation

In the ideal situation of frictionless coupling of the neutral gas particles within the electron–ion plasma component, one obtains $\nu_{\mathrm{en}}, \nu_{\mathrm{in}} \to 0$. This approximation leads to a Boltzmannian distribution of electrons and ions, and (5.1) reduces to a fourth-degree polynomial:

$$\left[1 + \frac{\omega_{\mathrm{J}}^2}{\omega^2}\right] - \frac{(k^2 C_{\mathrm{SCAM}}^2/\omega^2)}{(1 + i(\nu_{\mathrm{cn}}/\omega))} - \frac{(k^2 C_{\mathrm{SCAM}}^2/\omega^2)}{(1 + i(\nu_{\mathrm{cn}}/\omega))}\left[\frac{\omega_{\mathrm{J}}^2 - \omega_{\mathrm{Jd}}^2}{\omega^2}\right] = 0 \qquad (6.1)$$

which is the same equation as derived by Dwivedi et al. (1999). Furthermore, if the frictional force between charged and neutral dust particles is neglected ($\nu_{\mathrm{cn}} \to 0$) the above equation reduces to the case of the usual Jeans mode (Pandey et al. 1994). On the other hand, in the other limiting case of only neutral dust particles ($n_{\mathrm{dc0}} \to 0$) the equation again reduces to the usual Jeans mode (Kolb and Turner 1990).

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6.2. Stability behaviour

The stability behaviour of the partially ionized dusty plasma system in the molecular cloud can be discussed in two extreme limiting cases. In the first case, for a system with a frictionless dust mass distribution, i.e. $\nu_{\rm cn} \to 0$, one finds that $n_{\rm dn0} \to 0$ and hence $\omega_{\rm J} \to \omega_{\rm Jd}$. Thus near the levitational equilibrium point where the critical Jeans length is $\lambda_{\rm J} \sim 2\pi C_{\rm SCAM}/\omega_{\rm J}$, one can simplify (5.1) to

$$\left(1 + \frac{\omega_{\rm J}^2}{\omega^2}\right)\left(\frac{\lambda_{\rm Di}^2}{\lambda_{\rm De}^2} + 1\right) - \frac{\omega_{\rm J}^2}{\omega^2} = 0. \tag{6.2}$$

These derivations are valid under the approximation $k^2 C_{\rm SCAM}^2 \to \omega_{\rm SCAM}^2$. The solutions are $\omega_{\rm r} \sim 0$ and $\omega_{\rm i} \sim \omega_{\rm J}(1+(\lambda_{\rm De}^2/\lambda_{\rm Di}^2))$. These solutions correspond to a gravito-electrostatic Jeans-like purely growing mode on the equilibrium levitational scale length of the total dust mass. In the other limiting case $n_{\rm dn0} \neq 0$, i.e. the dust system is not fully charged, one obtains $\omega_{\rm J} - \omega_{\rm Jd} \neq 0$. Thus (5.1) becomes

$$\left(\frac{\omega}{\omega_{\rm J}}\right)^4 - \left(\frac{\omega}{\omega_{\rm J}}\right)^2 \alpha^2 - (1+\eta)^{-1} = 0,$$

or

$$\left(\frac{\omega}{\omega_{\rm J}}\right)^2 = \alpha^2 \pm \sqrt{\alpha^4 + 4(1+\eta)^{-1}}$$

where $\alpha = \lambda_{\rm Di}/\lambda_{\rm De}$, $((\omega_{\rm J}^2 - \omega_{\rm Jd}^2)/\omega_{\rm J}^2) = 1/(1+\eta)$. Here, α^4 can be neglected with respect to $4(1+\eta)^{-1}$ and finally the roots are $\omega_{1\rm r} = \omega_{\rm J}(1+\eta)^{-1/4}\sqrt{2}$ with $\omega_{1\rm i} = 0$ and $\omega_{2\rm r} = 0$ with $\omega_{2\rm i} = \omega_{\rm J}(1+\eta)^{-1/4}\sqrt{2}$ where $\omega_{1\rm r}, \omega_{2\rm r}$ are the real roots and $\omega_{1\rm i}, \omega_{2\rm i}$ are the imaginary roots. The derivations are obtained, again under the approximation $k^2 C_{\rm SCAM}^2 \to \omega_{\rm SCAM}^2$. These two roots correspond, respectively, to the hybrid-type oscillatory mode, which can be referred to as the gravito-electrostatic mode, and the hybrid-type Jeans-like growing mode. The characteristic time scale of the oscillatory mode and that of the purely Jeans-like growing mode are equal. Furthermore, both of these modes exist on the equilibrium levitational scale length of the total dust mass. This confirms the PMGC. This mode arises due to the partial charging of the dust grain population, which leads to the survival of finite linear gravito-electrostatic coupling. The oscillation takes place because of electrostatic slippage of the charged dust fluid over the neutral dust mass around the critical Jeans length of the total dust mass distribution. For the oscillation to occur, the restoring force is supplied by the gravitational field. The charged mass oscillates around the collapsing hydrostatic equilibrium to maintain a quasi-hydrostatic equilibrium on the critical Jeans length corresponding to the total dust mass by the collapsing gravito-electrostatic fluid.

6.3. Non-Boltzmannian distribution

In the limit of frictional coupling the collisional frequencies are high enough to change the particle distribution from Boltzmannian to non-Boltzmannian and the situation can be studied in the limit $\nu_{\rm cn}/\omega \to \infty$, $\nu_{\rm en}/\omega \to \infty$, and $\nu_{\rm in}/\omega \to \infty$. In this case, the dispersion equation (5.1) is reduced to

$$\left(1 + \frac{\omega_{\rm J}^2}{\omega^2}\right)\left(\frac{\alpha^2 k^2 c_{\rm e}^2}{\nu_{\rm en}\omega} + \frac{k^2 c_{\rm i}^2}{\nu_{\rm in}\omega}\right) + \frac{k^2 C_{\rm SCAM}^2}{\nu_{\rm cn}\omega} + \frac{k^2 C_{\rm SCAM}^2}{\nu_{\rm cn}\omega}\frac{\omega_{\rm Jdn}^2}{\omega^2} = 0 \tag{6.3}$$

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or

$$\omega = \pm i\omega_{\mathrm{J}}\left[H + \frac{k^2 C_{\mathrm{SCAM}}^2}{\nu_{\mathrm{en}}}\frac{1}{(1+\eta)}\right]^{1/2}\left[H + \frac{k^2 C_{\mathrm{SCAM}}^2}{\nu_{\mathrm{en}}}\right]^{-1/2} \tag{6.4}$$

where $H = (\alpha^2 k^2 c_{\mathrm{e}}^2/\nu_{\mathrm{en}}) + (k^2 c_{\mathrm{i}}^2/\nu_{\mathrm{in}})$. This shows that there exists only one Jeans-like purely growing mode. One may interpret this result as meaning that the frictional damping of the acoustic mode, which itself acts as a dissipative agent to produce acoustic resistance for the Jeans collapse, further enhances the gravitational collapse. Thus it is seen that, in the high collisional limit, the pulsational mode is suppressed and the hybrid-type Jeans-like purely growing mode dominates.

Acknowledgements

We wish to thank Professor Ranjan Gupta of the Inter University Center of Astronomy and Astrophysics (IUCAA), Pune, India, for help with the numerical calculations in this paper and for the facilities provided by the IUCAA, where the second author is a visiting associate. We are also thankful to the anonymous referee for their valuable comments which we believe have helped to improve the quality of the paper.

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