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The Infinitely Iterated Labyrinth: Conceivability and Higher-Order Knowledge

ABSTRACT: *Some time ago I wrote a paper about conceivability and knowledge. An anonymous referee rejected it on the grounds that the result had already been established in a short story by Jorge Luis Borges. Intrigued, I looked for the story but found no mention of it in Louis and Ziche's extensive bibliography. I spent months consulting archives and electronic records to no avail. I had begun to doubt whether the story even existed when I had the curious good luck to encounter Sir Thomas Browne of Pembroke College, Oxford, who assured me of the piece's authenticity and introduced me to Brother Christian Rosenkreuz of Invisible College who in turn generously put me into correspondence with Borges himself. The literary defects in what follows reflect not a diminution of Borges's undoubted power as a storyteller, but merely my limited ability as an amateur translator. Whatever the translation's literary faults may be, I hope the story's philosophical interest—i.e., an argument that ideal conceivers have higher-order knowledge of necessary truths—remains. [Trans.]*

KEYWORDS: epistemology, philosophy of logic, philosophy of mind, higher-order knowledge, conceivability

Shortly after I died, a distinguished society of Latin literature invited me to deliver a reading of one of my short stories. I arrived at the appointed time and place to find my host, Fr. Francisco Suárez, standing behind a small wrought-iron gate nestled in a towering stone wall. He unlocked the gate, and as he escorted me across the cloistered courtyard, he explained that the hall belonged to an ancient but obscure college that had generously donated its use to the society.

Upon entering, I was startled to find the interior to be a sumptuous library. Ornate shelves of dark wood filled with thousands of leather-bound volumes lined the walls. Against the back, an enormous mirror, slightly darkened by soot, hung horizontally above a grand fireplace. Within the fireplace, a small fire crackled, warming the room to slight discomfort.

I wish to thank Sir Thomas Browne and Brother Christian Rosenkreuz for their help contacting Borges and, of course, Borges himself for permission to translate his story and present it to the philosophical public. I wish to thank Nathan Ballantyne and two anonymous referees for valuable editorial help in preparing the translation and appendix.

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Figure 1: The Labyrinth

By the fire, the fellows—I recognized them all immediately—were engaged in heated conversation. They fell silent as I entered. Fr. Suárez beckoned me to the dais on the near side of the room as the fellows shuffled to their chairs. Then, as if calling the session formally to order, he solemnly intoned, 'It is inquired: Whether the blessed in heaven who know God exists know they know He exists?'

I had no idea what to make of this peculiar introduction, so I fished nervously in my briefcase for my manuscript as Suárez motioned me to the lectern. Having found my papers, I braced myself to meet the gaze of my audience; looking up, I caught a glance of the mirror against the back wall. The mirror reflected the pattern of the circular rug lying before the fireplace: a labyrinth constructed of concentric circles. I was struck with déjà vu. I had encountered the same peculiar pattern many years before in a Swedish encyclopedia while researching a story.[1] Shaking off the uncanny feeling, I cleared my throat and began reading *Argumentum ornithologicum*, the story I had been asked to prepare:

> *I close my eyes and see a flock of birds. The vision lasts a second, or perhaps less; I am not sure how many birds I saw. Was the number of birds definite or indefinite? The problem involves the existence of God. If God exists, the number is definite, because God knows how many birds I saw. If God does not exist, the number is indefinite, because no one can have counted. In this case, I saw fewer than ten birds (let us say) and more than one, but I did not see nine, eight, seven, six, five, four, three, or two birds. I saw a number between ten and one, but not nine, eight, seven, six, five, etc. That integer—not-nine, not-seven, not-six, not-five, etc.—is inconceivable. Ergo, God exists.*[2]

1 [See figure 1; the original image appeared in (Linder 1885: 475).]

2 [I have simply quoted verbatim the excellent translation by Hurley in Borges (1998: 299).]

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Having finished the story, I paused. It slowly dawned on the audience that this was the *entire* story, and they applauded politely. Before the applause had died down, William of Ockham stood up. Without waiting to be acknowledged, Ockham sighed loudly and began:

'This is simply the abuse of language. The word "indeterminate" is indeterminate. Distinguish ontological from epistemological indeterminacy, and the riddle resolves itself. It is merely an *epistemic* fact that you were not able to determine the number of birds in the flock precisely. But from this epistemic fact, no ontological thesis about an inconceivable number of birds follows. You *did* see either nine birds or eight or so on; you simply cannot tell which. Hence, the fact that it is inconceivable for there to be an indeterminate integer between one and ten—which I gladly concede—shines no light on the question whether we know that we know God exists. Conceiving of a nonideal conceiver making a mistake is not ideally conceiving of an ideal conceiver ideally conceiving the denial of a necessary truth'.

I was dumbfounded. All eyes turned to me. I instinctively reached out for the cane I no longer needed. Seeing my perplexity, Anselm of Canterbury stood up and addressed me in a kindly tone: 'Perhaps a few words of clarification will shed some light upon Brother William's comment'.

I nodded for him to go on.

'In its most general form, our question is whether an ideal conceiver's knowledge of a necessary truth would entail that the conceiver would not only know the necessary truth, but know that he knows the truth and know that he knows that he knows and so on'.

'I see', I said.

'The problem arises from three claims. First, if an ideal conceiver can conceive something, that thing is possible. Upon death, God removed from us those cognitive limitations that in mortal life had darkened our intellects; therefore, we are now *ideal conceivers*. The removal of our cognitive limitations has two consequences: First, we are logically omniscient: we know the consequences of all of our knowledge. Second, we are able to imagine any consistent state of affairs.

'Thus our first premise simply says ideal conceivability entails possibility. May I now proceed?'[3]

I nodded and Anselm continued:

'The second premise says that if an ideal conceiver cannot conceive a possible world in which that sentence is true, the ideal conceiver knows the sentence to be false. This follows from the fact that we are ideal conceivers and know ourselves to be so. Therefore, for any sentence either we can conceive of a possible world in which that sentence is true or we cannot; there aren't any indeterminate cases. Now suppose there is a sentence that is true in some possible world, and yet we cannot conceive of that sentence being true. In this case, there would have to be some limitations on our ability to conceive. But since we know we are ideal conceivers,

3 [The careful reader will have noted that Borges's (or is it Anselm's?) 'ideal conceiver' is an ideal positive conceiver, in the sense described in Chalmers (2002: 148ff.), which also contains a defense of the claim at stake here, i.e., ideal positive conceivability entails metaphysical possibility.]

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we know there are no limitations on our ability, and hence there can be no such sentence. Which is to say: we know if something is possible, then it is conceivable as well. From this it follows, by *modus tollens*, that if something is not conceivable, then it is not possible either. And if something isn't possible, of course, it isn't the case. Hence, as our premise states, if an ideal conceiver cannot conceive of a possible world in which some sentence is true, the ideal conceiver knows the sentence to be false in the actual world'.

'Do you mean that you could know all truths?' I inquired.

'Yes and no', replied the archbishop coolly. 'It is a theorem that if all truths are knowable, then all truths are actually known.[4] We do know whether any *first-order* modal sentence like, "It is possible Socrates is pale" is true. But it is not in the least surprising that an ideal conceiver who *could* know whether any such sentence is true does *in fact* know whether those sentences are true. However this *first-order* omniscience does not entail *higher-order* omniscience. So, even though we know it is possible Socrates is pale, we do not automatically know it is possible it is possible Socrates is pale.'

The room had grown stuffy, and I could see the reflected flames dancing in the mirror vigorously. With trepidation, I nodded that I had understood Anselm's second premise and beckoned him to go on. Suddenly, from the back row, Thomas Aquinas lumbered to his feet and interrupted Anselm.

'Let us move the exposition along a bit more quickly, Your Grace. The final claim is that it is necessarily the case that God exists. We know this because He told us so himself, just as He also told us the first two claims and to lie would be inconsistent with His perfection.'

Aquinas returned to his chair and again the audience turned intently toward me.

'I cannot yet see what this has to do with knowing that one knows God exists', I admitted.

John Duns Scotus leapt to his feet and spoke: 'Consider: is it conceivable that it is conceivable that God does not exist? It seems not. We know from our first claim that if it is conceivable that it is conceivable that God does not exist, then it is possible that it is conceivable that God does not exist. But if it is necessary that God does exist, as our third claim says, it is necessary that it is necessary that God exists.[5] And, if it is necessary it is necessary God exists, then it is false it is possible it is false it is false it is possible it is false God exists. Which—obviously—is simply to say it is false it is possible it is possible that God exists. Now let us assume, for the sake of a *reductio,* it is conceivable it is conceivable God does not exist. By the principle that everything conceivable is possible, this would be just to say it is possible it is conceivable that God does not exist. And by the same principle again, this would be just to say it is possible it is possible that God does not exist, which is precisely what was denied above. Contradiction. Hence we know that our assumption, to wit, that it is conceivable that it is conceivable that God does not exist, is false.

4 [Cf. Kvanvig (2006).]

5 [A careful reader will again have noticed that the argument turns on the principle, $\Box p \longrightarrow \Box\Box p$, the axiom logicians call **4**.]

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'But, per the principle that anything an ideal conceiver cannot conceive is such that an ideal conceiver knows it to be false, we know it is false an ideal conceiver can conceive God not to exist. Therefore, by a second application of this principle, the ideal conceiver knows he knows God exists, which was the claim to be demonstrated.'[6]

To my great surprise, I found that I had followed what had just been said quite clearly, despite the sweltering heat in the room. Yet, I was still uncertain of one thing, so I asked the Subtle Doctor for a final clarification: What illumination did they expect from me, Borges, about this question?

Again Ockham stood up, and with an air of exasperation he said, 'Some among us think we know we know God exists; others, such as myself, deny this. We who deny we know we know God exists grant—as a matter of logic—the validity of the argument just given, provided some logical axioms be accepted. Nevertheless, we do not agree whether everything necessary is necessarily necessary. Our disagreement, then, is about which logical axioms provide the *correct* model of reality.

'Since anything necessary is necessarily necessary if and only if it is conceivable that it is conceivable that a necessary truth be false, we resolved to invite you, the writer than whom none more imaginative can be conceived', he said waving his hand with an air of mockery, 'in an attempt to test empirically that metaphysical thesis. We reckon that if you, Borges, are not able to conceive what is inconceivable, *a fortiori* no one else will be able to do so either. Hence, the argument that we know we know God exists will go through. So you can see the source of my disappointment when your story turned out to be nothing but you conceiving of someone making a mistake, for that result does nothing to help us untangle this riddle. So, I will ask you directly: can you conceive of an ideal conceiver conceiving a contradiction?'

Again, all eyes turned to me. I loosened my stifling tie to get a breath of air and steadied myself against the lectern. Like Theseus grasping for Ariadne's thread, I racked my brain for an example of someone who had conceived of an impossibility. A candidate appeared. I said, 'Alain of Lille conceived of God as an infinite circle whose center was everywhere and whose circumference was nowhere.[7] If we can conceive of him conceiving this, have we not conceived of someone conceiving the inconceivable?'

A murmur went through the crowd. 'It turns out I can only conceive of a circle whose radius gets arbitrarily large', said Alain of Lille. He added dejectedly, 'I cannot conceive of a *closed* figure that is actually infinite. Since I cannot conceive of this, it appears you cannot conceive of my conceiving it either, if you are actually an ideal conceiver.'

Perspiring heavily, I felt the piercing gaze of the fellows even more keenly. I sopped at my brow with my handkerchief trying to keep my composure and said, 'Perhaps the answer here is to be found in fiction.

'In fiction it is possible at least for a nonideal conceiver to conceive of someone conceiving a contradiction. With *At Swim-Two-Birds,* Flann O'Brien, has written

6 [See the appendix.]

7 [See Borges (1999a).]

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a book whose unnamed narrator is himself an author one of whose characters, Trellis, is also an author who accidentally impregnates one of *his* characters. It is inconceivable for a fictional character to conceive a child by her author, and yet the unnamed narrator has conceived of this, and of course, O'Brien has conceived of the narrator.[8]

'You want to know whether an ideal conceiver can conceive of an ideal conceiver conceiving a contradiction, but allow me to suggest another possibility. Perhaps we are merely the conceptions of some nonideal conceiver who is trying to conceive whether ideal conceivers would be able to conceive of an ideal conceiver's conceiving a contradiction. In other words, perhaps we ourselves are fictions, just characters in some wiseacre's story. If so, it would not be surprising if we are too frail to resolve these riddles, for we would not actually be ideal conceivers, after all.'

Having said this, I suddenly felt a pang of pity for our poor nonideal creator: for if it is conceivable that ideal conceivers would not agree about which modal axioms are correct, then it is possible ideal conceivers would not agree about which modal axioms are correct. (No one who takes conceivability as a guide to possibility could deny that.) But if it is possible ideal conceivers would not agree about which modal axioms are correct, then *a fortiori* nonideal conceivers, such as our creator, cannot know which modal axioms are correct. Finally, it is conceivable that ideal conceivers would not agree which modal axioms are true. (Our current predicament makes this clear.) Therefore, nonideal conceivers simply cannot know which modal axioms are correct.

My reverie was interrupted as the room burst into an uproar. The assembled scholars began to shout recriminations. Faint from the heat and aghast at having caused such a tumult, I leaned back and the fiery carpet reflected in the mirror again caught my gaze. As Fr. Suárez tried to call the assembly back to order, I began to trace the labyrinth with my eye. As I followed the lines, the walls of the labyrinth seemed to shrink at a constant rate, until at the end of the labyrinth I found a monster. Not the Minotaur, but another labyrinth, exactly the same as the original. Again I traced with my eye. As the walls grew smaller it took me longer and longer to follow the labyrinth as it twisted and turned.

Fr. Suárez gaveled the audience to order, and bade John of Salisbury, who had not spoken previously, to propose the question to be investigated in the next meeting. With a trembling voice, Salisbury said, 'Are we in fact the blessed in heaven or not?'

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8 [For more on Borges's relation to O'Brien, cf. Borges (1999b). I thank an anonymous reviewer for drawing my attention to this passage.]

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Appendix: Proof of the Main Result

[To aid contemporary philosophers who might be reading along, I offer the following reconstruction the long argument given above. —Trans.]

We begin with four logical axioms.

$\Box p \vdash \neg\Diamond\neg p$	*Duality*
$\Box p \vdash \Box\Box p$	**4**
$(\Diamond p \wedge (p \longrightarrow q)) \vdash \Diamond q$	$\mathfrak{A}$
$(Kp \wedge (p \longrightarrow q)) \vdash Kq$	$\mathfrak{B}$

The second axiom is valid in any normal modal logic whose accessibility is at least transitive.

$\mathfrak{A}$ is true in any normal modal logic whose accessibility relation is at least reflexive. (Suppose $\Gamma \models p$, $p \longrightarrow q$. By *modus ponens* then, $\Gamma \models q$ as well. Since in this case accessibility is reflexive, $\Gamma \models \Diamond p$. Thus, $\Gamma \models \Diamond p \wedge (p \longrightarrow q)$. But notice that in this case $\Gamma \models \Diamond q$ also, Q.E.D.)

$\mathfrak{B}$ says knowledge is closed for ideal conceivers. This is actually a quite weak principle that places no requirements on the epistemic accessibility relation. (In all normal epistemic modal logics $p \longrightarrow q \vdash Kp \longrightarrow Kq$, and we already have the antecedent of that condition in the right conjunct of $\mathfrak{B}$.) Furthermore, this assumption is justifiable because we are interested in finding out the limits of ideal epistemic agents, and so we are regarding logical omniscience as unproblematic.

From these axioms, we can demonstrate that the **KK** principle holds for at least one item of an ideal epistemic agent's knowledge, given three conditions we capture in three premises. The first two conditions specify that the epistemic agent has to be an 'ideal conceiver'. The third says there is at least one necessary truth.

$\forall p(Cp \longrightarrow \Diamond p)$	Premise	(1)
$\forall p(\neg Cp \longrightarrow K\neg p)$	Premise	(2)
$\Box g$	Premise	(3)
$CC\neg g \longrightarrow \Diamond C\neg g$	$\forall$I $C\neg g/p$, [1]	(4)
$\Box\Box g$	$\longrightarrow$ E, **4**, [3]	(5)
$\neg\Diamond\neg\neg\Diamond\neg g$	Duality, [5]	(6)

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$\neg\Diamond\Diamond\neg g$	$\neg$E, [6]	(7)
$CC\neg g$	Assumption	(8)
$\Diamond C\neg g$	$\longrightarrow$ E, [4], [8]	(9)
$C\neg g \longrightarrow \Diamond\neg g$	$\forall$I $\neg g/p$, [1]	(10)
$\Diamond C\neg g \wedge (C\neg g \longrightarrow \Diamond\neg g)$	$\wedge$I, [9], [10]	(11)
$(\Diamond C\neg g \wedge (C\neg g \longrightarrow \Diamond\neg g)) \longrightarrow \Diamond\Diamond\neg g$	$\forall$I $\mathfrak{A}$	(12)
$\Diamond\Diamond\neg g$	$\longrightarrow$ E, [11], [12]	(13)
$\Diamond\Diamond\neg g \wedge \neg\Diamond\Diamond\neg g$	$\wedge$I, [7], [13]	(14)
$\bot$		(15)
$\neg CC\neg g$	$\neg$I, [8] – [15]	(16)
$\neg CC\neg g \longrightarrow K\neg C\neg g$	$\forall$I $C\neg g/p$, [2]	(17)
$K\neg C\neg g$	$\longrightarrow$ E, [16], [17]	(18)
$\neg C\neg g \longrightarrow K\neg\neg g$	$\forall$I $C\neg g/p$, [2]	(19)
$K\neg C\neg g \wedge (\neg C\neg g \longrightarrow K\neg\neg g)$	$\wedge$I, [18], [19]	(20)
$(K\neg C\neg g \wedge (\neg C\neg g \longrightarrow K\neg\neg g)) \longrightarrow KK\neg\neg g$	$\forall$I $\mathfrak{B}$	(21)
$KK\neg\neg g$	$\longrightarrow$ E, [20], [21]	(22)
KKg	$\neg$E, [22]	(23)

(Note line 12 follows from $\mathfrak{A}$ by universal instantiation, substituting $C\neg g/p$, $\neg g/q$ and line 21 follows from $\mathfrak{B}$ by universal instantiation, substituting $\neg C\neg g/p$ and $K\neg\neg g/q$.)

The third premise is an arbitrarily chosen necessary truth. Readers of an atheistic stripe are invited to read g as the sentence '$2 + 2 = 4$' rather than as 'God exists.' The argument will go through just the same.

From these three premises and our four axioms, we have established that an ideal epistemic agent does luminously know necessary truths. Because this proof requires $\mathfrak{A}$ and **4**, it requires the *alethic* accessibility relation to be both transitive and reflexive. But note that this does not beg the question, because we have not required the *epistemic* accessibility relation to be either transitive and reflexive. Indeed, our axiom $\mathfrak{B}$ placed no restriction on epistemic accessibility at all.

What we have shown, in summary, is a condition: If our four axioms be accepted and if conceivability entails possibility and if what is inconceivable is known to be false, then ideal epistemic agents do indeed have higher-order knowledge of necessary truths, and so we should endorse the **KK** principle at least for necessary truths.

Determining whether the antecedents of that condition are fulfilled, however, is another labyrinth for nonideal conceivers like ourselves.

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