For a ship at latitude 60°, with velocity components $n = w = 20$ knots, equation (4) shows that $\alpha = 2° 40'$. Hence if in (15) $\sec\alpha$ is replaced by unity the error in the calculations for λ is about 0·1 %. For ships at sea (15) can be replaced, in general, by

$$\tan\lambda = \frac{M_2}{M_1}. \tag{16}$$

When $n = 0$ this formula is accurate.

Again, for most ships not only is α small but w is negligible in comparison with $\Omega a \cos\lambda$. A useful approximation to (9) is then

$$\cos\lambda = \frac{M_1}{Cs\Omega} \tag{17}$$

and the value of λ can be found by using one gyroscope only.

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NOTE ON THE ITERATIVE METHOD IN NUCLEAR PROBLEMS

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1. With the assumption of two-body forces it has been found possible to estimate approximately the binding energies of the simplest atomic nuclei (H^2, H^3, He^4) for a number of potential functions. The iterative technique developed by Svartholm is eminently suited to this purpose for well-known reasons; in what follows we shall therefore adopt his approach(1).

A fairly adequate treatment of the two-, three- and four-body problems is possible with the assumption of Gaussian interactions. Of the other potentials at present in use that due to Yukawa is in many ways the most important, but at the same time its adoption raises great computational difficulties. In this note we shall be concerned with the improvement of the iterative treatment by simple modification of the initial function. Since, however, this first approximation to the wave function is in essence a product function of appropriate symmetry, based on the deuteron function, the most suitable analytical form will be suggested by a preliminary examination of the two-body case.

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2. With (effectively) the potential $V = -Ce^{-\kappa r}/\kappa r$, where κ defines the range and C is a constant depending on the assumed admixture of exchange forces and on the spin states, the deuteron wave equation in momentum space reduces to

$$(p_0^2+p^2)\langle p|\rangle = \lambda\frac{\kappa}{2\pi^2}\int\frac{d\mathbf{p}'}{\kappa^2+|\mathbf{p}-\mathbf{p}'|^2}\langle \mathbf{p}'|\rangle \quad (\lambda = CM/\kappa^2,\ p_0^2 = -ME), \tag{1}$$

where E is the energy and a unit of action $\hbar = 1$ is employed. λ is regarded as the unknown eigenvalue parameter, and for any particular choice of $p_0./\kappa$ compatible values of κ and C may be obtained by computing λ^0, the least eigenvalue. With an 'initial function' $\langle p|\rangle^0$ and the definitions

$$\langle p|\rangle^{j+1} = \frac{1}{(p_0^2+p^2)}\frac{\kappa}{2\pi^2}\int\frac{d\mathbf{p}'}{\kappa^2+|\mathbf{p}-\mathbf{p}'|^2}\langle p'|\rangle^j, \quad T_j = \int{}^j\langle|p\rangle(p_0^2+p^2)\langle p|\rangle^j d\mathbf{p},$$

$$\Phi_j = \frac{\kappa}{2\pi^2}\iint{}^j\langle|p\rangle\frac{d\mathbf{p}\,d\mathbf{p}'}{\kappa^2+|\mathbf{p}-\mathbf{p}'|^2}\langle p'|\rangle^j, \quad T_j/\Phi_j = \lambda_j, \quad \Phi_j/T_{j+1} = \lambda_{j+\frac{1}{2}}$$
$$(j = 0, 1, 2, \ldots),$$

the sequence $$\lambda_0 > \lambda_{\frac{1}{2}} > \lambda_1 > \ldots$$

converges monotonically to λ^0. In the present case, for example, with $p_0/\kappa = 0{\cdot}25$ Svartholm(1) takes the function

$$\langle p|\rangle^0 = \frac{1}{(p^2+a^2)^2}. \tag{2}$$

The results of iteration are quoted below:

λ_0	$\lambda_{\frac{1}{2}}$	λ_1	$\lambda_{\frac{3}{2}}$	H
2·31	2·240	2·2346	2·2335	2·2331

The rather accurate value in the last column is due to Hulthén(2).

Unfortunately, initial functions based on (2) do not lend themselves to the many-body problem, and one is compelled to substitute, say, a Gaussian function in order to make progress possible. Use of such a function in the present case yields a very poor value of λ_0 (the variational approximation) and suggests that this type of function is not very satisfactory with Yukawa forces. Since in the many-body case progress is limited to the computation of $\lambda_{\frac{1}{2}}$, this constitutes a serious initial handicap.

We shall now show that a natural generalization of (2) proves extremely efficient in the case of the deuteron, and, moreover, may be fairly well represented as a sum of two Gaussian functions; this latter result presumably enables one to write down rather better initial functions for the three- and four-body problems, whilst preserving an analytical form admitting iteration. With the function

$$\langle p|\rangle^0 = \frac{1}{(p^2+a^2)(p^2+b^2)}, \tag{3}$$

the corresponding (variational) value λ_0 is easily computed. In the case already quoted ($p_0/\kappa = 0{\cdot}25$) we find $\lambda_0 = 2{\cdot}2333$, in very good agreement with the accurate value. The function (3) appears to be fairly generally useful in momentum space as

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a 'single particle' function. Its coordinate space transform, however, is most unwieldy, being of the form

$$\langle r\,|\rangle = \frac{\sinh Ar}{r} e^{-Br}. \tag{4}$$

The approximate representation of (3) by a sum of Gaussian terms is suggested by the identity

$$\frac{1}{(p^2+a^2)(p^2+b^2)} = \int_0^\infty \int_0^\infty \exp\{-(ta^2+sb^2)\} \exp\{-p^2(s+t)\}\, ds\, dt. \tag{5}$$

The function $\langle p\,|\rangle = \sum_r \mu_r \exp\{-rSp^2\}$ may thus be made to represent (3) as closely as desired by taking a sufficient number of terms; in practice the two-term function

$$\langle p\,|\rangle = \exp\{-Sp^2\} + \mu \exp\{-rSp^2\} \tag{6}$$

provides the best compromise between efficiency and simplicity. In the table below λ_0 is computed by using the single-term Gaussian function, the proposed variant (6), and the functions (2) and (3). This example refers to the virtual state $p_0/\kappa = 0$, for which the failure of the single-term function is particularly marked.

Function	$\exp\{-Sp^2\}$	(6)	(2)	(3)	Hulthén
λ_0	2·71	2·03	2·00	1·6804	1·6799

3. The initial function for the many-body problem may be extended in an essentially similar manner. A significant improvement of the initial function is highly desirable on account of the completely false nature of the simple Gaussian type employed by Svartholm; the important point is that this improvement is best secured by adding members of the non-orthogonal set

$$\exp\{-Sp^2\}, \quad \exp\{-2Sp^2\}, \quad \ldots, \quad \exp\{-rSp^2\}, \quad \ldots,$$

rather than by adding symmetrized oscillator functions. It will be remembered that this latter method is very slowly convergent. The reason for this lies in the false asymptotic behaviour of the Gaussian function; this cannot be corrected substantially by adding members of the same function system.

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