Macroeconomic Dynamics, **14** (2010), 257–268. Printed in the United States of America.
doi:10.1017/S1365100509090154

OPTIMAL MONETARY POLICY UNDER PARAMETER UNCERTAINTY IN A SIMPLE MICROFOUNDED MODEL

TAKUSHI KUROZUMI
Bank of Japan

This paper examines optimal monetary policy under uncertainty about fundamental parameters of a dynamic stochastic general-equilibrium model. In contrast to previous studies, a microfoundation of the model leads this uncertainty to generate uncertainty not only about the transmission of monetary policy but also about the transmission of shocks and about a social welfare loss function. In the presence of such uncertainty, this paper finds conditions under which optimal discretionary policy responds to shocks more aggressively than in the absence of the uncertainty. These conditions depend crucially on the persistence of shocks and the magnitude of policy multipliers. To obtain the conditions, taking proper account of uncertainty about the transmission of shocks and about the welfare loss function is of crucial importance.

Keywords: Optimal Monetary Policy, Parameter Uncertainty, Microfoundation

1. INTRODUCTION

A considerable amount of research has investigated the "Brainard conservatism principle" named by Blinder (1998) for Brainard (1967), who was the first to show that uncertainty about policy multipliers causes a less aggressive policy response to shocks. In the existing literature there are two approaches for inspecting the validity of the Brainard principle. One takes a Bayesian approach in which a policymaker has a unique prior distribution of uncertain parameters.[1] Another uses a robust-control approach, which can be justified by Gilboa and Schmeidler's (1989) decision theory with multiple priors.[2] Most studies indicate

This article is part of the author's Ph.D. dissertation at Carnegie Mellon University. The author would like to thank Kosuke Aoki, the editor William Barnett, Daniele Coen-Pirani, John Duffy, Marvin Goodfriend, Christian Jensen, Takeshi Kimura, Finn Kydland, his adviser Bennett McCallum, Willem Van Zandweghe, an associate editor, and an anonymous referee for helpful comments and suggestions. Any remaining errors are the sole responsibility of the author. The views expressed herein are those of the author and should not be interpreted as those of the Bank of Japan. Address correspondence to: Takushi Kurozumi, Bank of Japan, Tokyo 103-8660, Japan; e-mail: takushi.kurozumi@boj.or.jp.

© 2010 Cambridge University Press 1365-1005/10

https://doi.org/10.1017/S1365100509090154 Published online by Cambridge University Press

that the Brainard principle is supported by the Bayesian approach but not by the robust-control one. Yet there is still no consensus about its validity.[3]

This paper adopts the Bayesian approach to examine optimal monetary policy under uncertainty about fundamental parameters of a dynamic stochastic general-equilibrium (DSGE) model of Woodford (2003), which has been a canonical model in the recent literature. The consideration of this uncertainty is in stark contrast to previous studies in that a microfoundation of the model leads such uncertainty to generate uncertainty not only about the transmission of monetary policy but also about the transmission of shocks and about a social welfare loss function. This is because the fundamental parameters determine coefficients of the shocks and weights of the welfare loss function, as well as policy multipliers. Thus, taking proper account of the relationships among these via the fundamental parameters, we can obtain precise implications of uncertainty about parameters of the microfounded model for optimal policy.[4]

In the model this paper focuses on two key fundamental parameters: the frequency of price adjustment and the intertemporal elasticity of substitution. This is because actual values of these two parameters are issues around which there is great uncertainty as shown in previous empirical studies. In the presence of uncertainty about each of the two parameters, the paper finds a necessary and sufficient condition under which optimal discretionary policy responds to shocks more aggressively than in the absence of such uncertainty. This condition depends crucially on the persistence of shocks and the magnitude of policy multipliers. In the case of uncertainty about the price adjustment frequency, the optimal discretionary policy responds more aggressively to low persistent (inefficient) supply shocks, but it does less aggressively to high persistent ones if the policy multipliers are sufficiently large.[5] By contrast, in the face of uncertainty about the intertemporal substitution elasticity, the optimal discretionary policy responds less aggressively to both low persistent supply and natural-rate-of-interest (NRI) shocks, but it does more aggressively to high persistent ones as long as the multipliers are large enough. For these results, taking proper account of uncertainty about the transmission of the shocks and about the weights of the welfare loss function is of crucial importance. As shown later, if this uncertainty is disregarded, the exactly opposite implications of uncertainty about the fundamental parameters for optimal monetary policy may be obtained.[6]

Among related studies, Levin and Williams (2003) examine how an optimal Taylor rule is affected by uncertainty about fundamental parameters of a microfounded model. Levin et al. (2006) numerically investigate effects of uncertainty about fundamental parameters of a medium-scale DSGE model à la Christiano, Eichenbaum, and Evans (2005). Kimura and Kurozumi (2007) use a microfounded model with inflation and output persistence and show that uncertainty about such structural persistence leads to a more aggressive optimal policy response to shocks.

The remainder of this paper is organized as follows. Section 2 presents a DSGE model. Section 3 illustrates how uncertainty about fundamental parameters of

https://doi.org/10.1017/S1365100509090154 Published online by Cambridge University Press

this microfounded model affects a monetary policymaker's problem. Section 4 examines effects of such uncertainty on optimal discretionary policy. Finally, Section 5 concludes.

2. THE MODEL

The model is a DSGE model developed by Woodford (2003). The economy consists of a representative household, a continuum of monopolistically competitive firms, and a central bank. The household's spending behavior is represented as

$$x_t = E_t x_{t+1} - \sigma\left(i_t - E_t \pi_{t+1} - r_t^n\right), \tag{1}$$

where x_t is the output gap, E_t is the expectation operator conditional on private agents' period-t information set Ω_t, $\sigma > 0$ measures the intertemporal elasticity of substitution in spending, i_t is the nominal interest rate, π_t is the inflation rate, and $r_t^n = \sigma^{-1}\tilde{r}_t^n$ is the NRI, where $\tilde{r}_t^n$ is a fundamental shock relevant to the household's preferences and the natural rate of output. This shock is assumed to follow an AR(1) process with a persistence coefficient $\rho_r \in (-1, 1)$ and a white noise $\varepsilon_{r,t}$, given by $\tilde{r}_t^n = \rho_r \tilde{r}_{t-1}^n + \varepsilon_{r,t}$.

The firms set prices of their differentiated products on a staggered basis as in Calvo (1983). In each period a fraction $\alpha \in (0, 1)$ of firms keeps prices unchanged and the remaining fraction $1 - \alpha$ adjusts prices. The firms' pricing behavior is represented as

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + u_t, \tag{2}$$

where $\kappa = \kappa_u(\omega + \sigma^{-1}) > 0, \kappa_u = (1-\alpha)(1-\alpha\beta)\{\alpha(1+\omega\theta)\}^{-1} > 0, \beta \in (0, 1)$ is the discount factor, $\omega > 0$ is the inverse of the labor-supply elasticity, $\theta > 1$ is the steady-state price elasticity of demand, and $u_t = \kappa_u \tilde{u}_t$ denotes the inefficient supply shock relevant to a time-varying markup factor $\tilde{u}_t$,[7] which is assumed to follow an AR(1) process with a persistence coefficient $\rho_u \in (-1, 1)$ and a white noise $\varepsilon_{u,t}$, given by $\tilde{u}_t = \rho_u \tilde{u}_{t-1} + \varepsilon_{u,t}$.

The central bank conducts monetary policy by setting the nominal interest rate so as to maximize the welfare of the representative household. As Woodford (2003) shows, a second-order approximation to this household's utility function is proportional to the loss function:

$$E_0 \sum_{t \geq 0} \beta^t \left(\lambda_\pi \pi_t^2 + \lambda_x x_t^2\right), \tag{3}$$

where $\lambda_\pi = \theta \kappa_u^{-1}$ and $\lambda_x = \omega + \sigma^{-1}$. Here, it is important to stress that when the inflation weight λ_π is uncertain, the widely used relative output-gap weight $\lambda_x \lambda_\pi^{-1}$ yields a misleading implication for optimal monetary policy. This is because in that case the ratio of the central bank's expected value of the output-gap weight to that of the inflation weight differs from the bank's expected value of the relative output-gap weight.

https://doi.org/10.1017/S1365100509090154 Published online by Cambridge University Press

3. UNCERTAINTY ABOUT FUNDAMENTAL MODEL PARAMETERS

We now consider the central bank's uncertainty about parameters of the microfounded model. Previous studies have analyzed the bank's uncertainty about policy multipliers (e.g., σ, κ in the model). By contrast, this paper examines its uncertainty about fundamental model parameters (e.g., $\alpha, \beta, \theta, \omega, \sigma$). These determine the multiplier σ and the NRI shock r_t^n in the household's spending equation (1), the multiplier κ and the supply shock u_t in firms' pricing equation (2), and the weights λ_π, λ_x in the welfare loss function (3). The bank's uncertainty about the fundamental parameters thus generates its uncertainty not only about the transmission of monetary policy but also about the transmission of shocks and about the welfare loss function. Therefore, taking proper account of the relationships via the fundamental parameters is crucial in examining optimal policy under uncertainty about parameters of the microfounded model.

This paper focuses on two key fundamental parameters: the frequency of price adjustment α and the intertemporal elasticity of substitution σ.[8] This is because actual values of these two parameters are issues around which there is great uncertainty. As Walsh (2005) indicates, the early estimates of the frequency α [Galí and Gertler (1999)] are much larger than the micro evidence [Bils and Klenow (2004)] or the recent estimates [Eichenbaum and Fisher (2004)]. Also, there are several estimates of the intertemporal elasticity σ. The estimates of McCallum and Nelson (1999) are much smaller than those of Rotemberg and Woodford (1997).

It is assumed that the central bank knows the current state of the economy, except the uncertain parameter, when determining the current nominal interest rate. This assumption implies that current values of the inflation rate, the output gap, and the fundamental shocks are included in the bank's current information set, i.e., $\pi_t, x_t, \tilde{r}_t^n, \tilde{u}_t \in \Omega_t^{\mathrm{CB}}$.[9]

3.1. Uncertainty about the Frequency of Price Adjustment

We begin with the case of uncertainty about the price adjustment frequency $\alpha \notin \Omega_t^{\mathrm{CB}}$. In the face of this uncertainty, the central bank has a probability distribution of the frequency α. The bank then confronts uncertainty not only about the policy multiplier κ and the supply shock u_t in firms' pricing equation (2) but also about the inflation weight λ_π in the welfare loss function (3), i.e., $u_t, \kappa, \lambda_\pi \notin \Omega_t^{\mathrm{CB}}$. In what follows, $\bar{\kappa}_u, \bar{\kappa}, \bar{\lambda}_\pi$ denote the bank's expectations of $\kappa_u, \kappa, \lambda_\pi$ based on its probability distribution of the uncertain fundamental parameter. (Here, it is α.)

Under parameter uncertainty the principle of certainty equivalence ceases to hold. This is because future variability of inflation and the output gap in the welfare loss function (3) is no longer independent of monetary policy. It is also because there is uncertainty about the inflation weight λ_π, which has not been considered in previous studies. Under the timing assumption (i.e., $\pi_t \in \Omega_t^{\mathrm{CB}}$), we have $E_t^{\mathrm{CB}}[\lambda_\pi \pi_t^2] = E_t^{\mathrm{CB}}[\lambda_\pi]\pi_t^2 = \bar{\lambda}_\pi \pi_t^2$. Then, taking expectations of the welfare

https://doi.org/10.1017/S1365100509090154 Published online by Cambridge University Press

loss function (3) conditional on the bank's information set Ω_0^{CB} yields

$$E_0^{\mathrm{CB}} E_0 \sum_{t\geq 0} \beta^t \left(\lambda_\pi \pi_t^2 + \lambda_x x_t^2\right) = \bar{\lambda}_\pi \pi_0^2 + \lambda_x x_0^2 + \beta\left(\bar{\lambda}_\pi V_0^{\mathrm{CB}} \pi_1 + \lambda_x V_0^{\mathrm{CB}} x_1\right)$$
$$+ E_0^{\mathrm{CB}} \sum_{t\geq 1} \beta^t \big[\bar{\lambda}_\pi \left(E_{t-1}^{\mathrm{CB}} \pi_t\right)^2 + \lambda_x \left(E_{t-1}^{\mathrm{CB}} x_t\right)^2$$
$$+ \beta\left(\bar{\lambda}_\pi V_t^{\mathrm{CB}} \pi_{t+1} + \lambda_x V_t^{\mathrm{CB}} x_{t+1}\right)\big], \tag{4}$$

where V_t^{CB} is the variance operator conditional on Ω_t^{CB}. Moreover, from (1) and (2) we have

$$V_t^{\mathrm{CB}} \pi_{t+1} = v_\kappa \beta^{-2} \{x_t + (\omega + \sigma^{-1})^{-1} \tilde{u}_t\}^2 + \text{t.i.p.}, \tag{5}$$
$$V_t^{\mathrm{CB}} x_{t+1} = \sigma^2 V_t^{\mathrm{CB}} \pi_{t+1} + \text{t.i.p.}, \tag{6}$$

where $\varepsilon_{t+1}^z = z_{t+1} - E_t z_{t+1}$, $z = \pi, x$, are private agents' expectation errors, v_κ is the variance of κ based on the bank's probability distribution of the uncertain fundamental parameter, and "t.i.p." represents terms independent of policy.[10] Substituting (5) and (6) into (4) leads to the loss function that the bank seeks to minimize under uncertainty about the frequency α:

$$\bar{\lambda}_\pi \pi_0^2 + \lambda_x x_0^2 + v_\kappa \beta^{-1} \left(\bar{\lambda}_\pi + \sigma^2 \lambda_x\right) \{x_0 + (\omega + \sigma^{-1})^{-1} \tilde{u}_0\}^2 + E_0^{\mathrm{CB}} \sum_{t\geq 1} \beta^t$$
$$\times \left[\bar{\lambda}_\pi \left(E_{t-1}^{\mathrm{CB}} \pi_t\right)^2 + \lambda_x \left(E_{t-1}^{\mathrm{CB}} x_t\right)^2 + v_\kappa \beta^{-1} (\bar{\lambda}_\pi + \sigma^2 \lambda_x) \{x_t + (\omega + \sigma^{-1})^{-1} \tilde{u}_t\}^2\right]. \tag{7}$$

To set up the central bank's problem under the parameter uncertainty, we also take expectations of private agents' equations (1) and (2) conditional on Ω_t^{CB}:

$$x_t = E_t^{\mathrm{CB}} x_{t+1} - \sigma \left(i_t - E_t^{\mathrm{CB}} \pi_{t+1} - r_t^n\right), \tag{8}$$
$$\pi_t = \beta E_t^{\mathrm{CB}} \pi_{t+1} + \bar{\kappa}\, x_t + \bar{\kappa}_u \tilde{u}_t. \tag{9}$$

Here, it is important to stress that the central bank cannot observe private agents' expected inflation and output gap (i.e., $E_t \pi_{t+1}$, $E_t x_{t+1}$) nor can the bank extract these expectations or the uncertain parameters from current information available to the bank. This is because the bank has less information by the uncertain parameters than private agents, i.e., $\Omega_t^{\mathrm{CB}} \subset \Omega_t$.

3.2. Uncertainty about the Intertemporal Elasticity of Substitution

We turn next to the case of uncertainty about the intertemporal substitution elasticity $\sigma \notin \Omega_t^{\mathrm{CB}}$. Under such uncertainty the central bank has a probability distribution of this elasticity with a mean $\bar{\sigma} > 0$ and a variance $v_\sigma > 0$. The bank then faces uncertainty about the policy multiplier σ and the NRI shock r_t^n in the household's spending equation (1), about the multiplier κ in firms' pricing

https://doi.org/10.1017/S1365100509090154 Published online by Cambridge University Press

equation (2), and about the output-gap weight λ_x in the welfare loss function (3), i.e., $r_t^n, \kappa, \lambda_x \notin \Omega_t^{\mathrm{CB}}$. From (1) and (2) we have

$$V_t^{\mathrm{CB}} x_{t+1} = v_\sigma \{i_t - \beta^{-1}(\pi_t - \kappa_u \omega x_t - u_t)\}^2 + \text{t.i.p.},$$
$$V_t^{\mathrm{CB}} \pi_{t+1} = v_\kappa \beta^{-2} x_t^2 + \text{t.i.p.}$$

Thus, it follows that under uncertainty about the intertemporal elasticity σ, the central bank's expectations of the welfare loss function (3) conditional on Ω_0^{CB} can be rewritten as

$$\lambda_\pi \pi_0^2 + \left(\bar{\lambda}_x + v_\kappa \lambda_\pi \beta^{-1}\right) x_0^2 + v_\sigma \beta \bar{\lambda}_x \{i_0 - \beta^{-1}(\pi_0 - \kappa_u \omega x_0 - u_0)\}^2$$
$$+ E_0^{\mathrm{CB}} \sum_{t \geq 1} \beta^t \Big[\lambda_\pi \left(E_{t-1}^{\mathrm{CB}} \pi_t\right)^2 + \bar{\lambda}_x \left(E_{t-1}^{\mathrm{CB}} x_t\right)^2 + v_\kappa \lambda_\pi \beta^{-1} x_t^2 + v_\sigma \beta \bar{\lambda}_x$$
$$\times \{i_t - \beta^{-1}(\pi_t - \kappa_u \omega x_t - u_t)\}^2 \Big], \tag{10}$$

where $\bar{\lambda}_x$ denotes the bank's expectations of λ_x based on its probability distribution of the elasticity σ. Taking expectations of private agents' equations (1) and (2) conditional on Ω_t^{CB} yields[11]

$$x_t = E_t^{\mathrm{CB}} x_{t+1} - \bar{\sigma}\left(i_t - E_t^{\mathrm{CB}} \pi_{t+1}\right) + \bar{\sigma}\beta^{-1}\{\bar{\kappa} - \kappa_u(\omega + \bar{\sigma}^{-1})\} x_t + \tilde{r}_t^n, \tag{11}$$

$$\pi_t = \beta E_t^{\mathrm{CB}} \pi_{t+1} + \bar{\kappa}\, x_t + u_t. \tag{12}$$

4. OPTIMAL DISCRETIONARY POLICY

This section examines optimal discretionary policy under uncertainty about each of the two fundamental parameters. In line with previous studies, it is assumed that the central bank does not revise its prior distribution of uncertain parameters in later periods. That is, the issue of the bank's learning about uncertain parameters is beyond the scope of the paper.

We first consider the case of no uncertainty about parameters of the model. In each period, optimal discretionary policy minimizes the welfare loss function (3) subject to private agents' equations (1) and (2), taking their expectations as exogenously given. As Clarida, Galí and Gertler (1999) point out, this discretionary-policy problem can be reduced to a sequence of the static minimization of the current-period welfare loss in (3), $\lambda_\pi \pi_t^2 + \lambda_x x_t^2$, subject to (2). Thus, since $\lambda_x \lambda_\pi^{-1} \kappa^{-1} = \theta^{-1}$, the optimality condition for the minimization problem is

$$\pi_t + \theta^{-1} x_t \;=\; 0. \tag{13}$$

A rational-expectations equilibrium (REE) attained by the optimal discretionary policy is a solution to the system consisting of the central bank's optimality condition (13) and private agents' equations (1) and (2). Because this system has a unique solution, we can obtain the unique REE process of the nominal interest

https://doi.org/10.1017/S1365100509090154 Published online by Cambridge University Press

rate, given by

$$i_t = r_t^n + \frac{\rho_u + \theta\sigma^{-1}(1-\rho_u)}{1-\beta\rho_u+\kappa\theta}u_t. \tag{14}$$

4.1. Uncertainty about the Frequency of Price Adjustment

In the face of uncertainty about the price adjustment frequency α, optimal discretionary policy minimizes the current-period loss in (7), $\bar{\lambda}_\pi \pi_t^2 + \lambda_x x_t^2 + \tilde{v}_\kappa \lambda_x \{x_t + (\omega + \sigma^{-1})^{-1}\tilde{u}_t\}^2$, where $\tilde{v}_\kappa = v_\kappa \beta^{-1}(\sigma^2 + \bar{\lambda}_\pi \lambda_x^{-1})$, subject to (9). The optimality condition for discretionary policy is

$$\pi_t + \left(\lambda_x \bar{\lambda}_\pi^{-1}\bar{\kappa}^{-1}\right)(1+\tilde{v}_\kappa)x_t + \tilde{v}_\kappa\left(\lambda_x\bar{\lambda}_\pi^{-1}\bar{\kappa}^{-1}\right)(\omega+\sigma^{-1})^{-1}\tilde{u}_t = 0. \tag{15}$$

Solving the system that consists of the central bank's optimality condition (15) and private agents' equations (1) and (2) leads to a unique REE attained by the optimal discretionary policy under uncertainty about the frequecy α. Consequently, the unique REE process of the nominal interest rate is given by

$$i_t = r_t^n + \frac{\rho_u + \left(\lambda_x\bar{\lambda}_\pi^{-1}\bar{\kappa}^{-1}\right)^{-1}\sigma^{-1}(1-\rho_u)\left\{1+\tilde{v}_\kappa\lambda_x\bar{\lambda}_\pi^{-1}(\kappa\bar{\kappa})^{-1}(1-\beta\rho_u)\right\}}{(1-\beta\rho_u)(1+\tilde{v}_\kappa)+\kappa\left(\lambda_x\bar{\lambda}_\pi^{-1}\bar{\kappa}^{-1}\right)^{-1}}u_t. \tag{16}$$

By comparing the REE processes (14) and (16), we have the following result.

PROPOSITION 1. *In the presence of uncertainty about the frequency of price adjustment α, the optimal discretionary policy responds to inefficient supply shocks u_t more aggressively in every period than in the absence of such uncertainty if*

$$(1-\rho_u)(1-\beta\rho_u) - \rho_u\sigma\kappa > 0; \tag{17}$$

otherwise, the policy responds to the supply shocks less aggressively. This uncertainty, however, has no effect on optimal discretionary policy responses to NRI shocks r_t^n.[12]

Condition (17) depends crucially on the persistence of inefficient supply shocks ρ_u and the magnitude of policy multipliers σ, κ. This condition illustrates that in the presence of uncertainty about the price adjustment frequency α, the optimal discretionary policy adjusts the nominal interest rate more aggressively for low persistent supply shocks in every period than it does in the absence of such uncertainty, but it adjusts the interest rate less aggressively for high persistent supply shocks if the policy multipliers are sufficiently large.

Here, it is important to stress that taking no account of uncertainty about the transmission of supply shocks and about the inflation weight of the welfare loss function yields the exactly opposite implication for the optimal discretionary policy. To see this, suppose that the central bank knows the supply-shock coefficient and the inflation weight with certainty even in the presence of uncertainty about

https://doi.org/10.1017/S1365100509090154 Published online by Cambridge University Press

the frequency α. Then, the optimality condition for discretionary policy and the unique REE process of the nominal interest rate are given by

$$\pi_t + \theta^{-1}(1+\tilde{v}_\kappa)x_t = 0, \qquad i_t = r_t^n + \frac{\rho_u(1+\tilde{v}_\kappa)+\theta\sigma^{-1}(1-\rho_u)}{(1-\beta\rho_u)(1+\tilde{v}_\kappa)+\kappa\theta}u_t.$$

Consequently, the optimal discretionary policy responds to inefficient supply shocks more aggressively in every period than under no uncertainty about parameters if

$$(1-\rho_u)(1-\beta\rho_u)-\rho_u\sigma\kappa < 0; \tag{18}$$

otherwise, the policy responds to the supply shocks less aggressively. Condition (18) is exactly opposite to (17). Therefore, the consideration of uncertainty about the transmission of supply shocks and about the inflation weight of the welfare loss function is of crucial importance for the precise policy implications of uncertainty about the frequency of price adjustment.

4.2. Uncertainty about the Intertemporal Elasticity of Substitution

In the case of uncertainty about the intertemporal substitution elasticity σ, the optimality condition for discretionary policy is given by

$$\left(1+\beta\theta\tilde{v}_\sigma^{-1}\right)\pi_t + \left\{\beta\tilde{v}_\sigma^{-1}(1+\tilde{v}_\kappa)-\kappa_u\omega\right\}x_t - \beta i_t - u_t = 0, \tag{19}$$

where $\tilde{v}_\kappa = v_\kappa\theta\beta^{-1}\bar{\kappa}^{-1}$ and $\tilde{v}_\sigma = v_\sigma\bar{\sigma}^{-1}(\beta+\kappa_u)$. Solving the system of the central bank's optimality condition (19) and private agents' equations (1) and (2) leads to the unique REE process of the nominal interest rate given by

$$i_t = \left[1+\frac{\tilde{v}_\sigma\{\rho_r\kappa-\sigma^{-1}(1-\rho_r)(1-\beta\rho_r)\}}{(1-\beta\rho_r)(1+\tilde{v}_\kappa)+\kappa\theta+\tilde{v}_\sigma\sigma^{-1}(1-\beta\rho_r)(1-\rho_r+\kappa_u\beta^{-1})}\right]r_t^n$$
$$+\frac{\rho_u(1+\tilde{v}_\kappa)+\theta\sigma^{-1}(1-\rho_u)+\tilde{v}_\sigma\rho_u\sigma^{-1}(1-\rho_u+\kappa_u\beta^{-1})}{(1-\beta\rho_u)(1+\tilde{v}_\kappa)+\kappa\theta+\tilde{v}_\sigma\sigma^{-1}(1-\beta\rho_u)(1-\rho_u+\kappa_u\beta^{-1})}u_t. \tag{20}$$

Comparing the REE processes (14) and (20) yields the following result.

PROPOSITION 2. *In the presence of uncertainty about the intertemporal elasticity of substitution σ, the optimal discretionary policy responds to inefficient supply shocks u_t more aggressively in every period than in the absence of such uncertainty if (18) holds; otherwise, the policy responds to the supply shocks less aggressively. Also, the policy responds to NRI shocks r_t^n more aggressively if*

$$(1-\rho_r)(1-\beta\rho_r)-\rho_r\sigma\kappa < 0; \tag{21}$$

otherwise, the policy responds to the NRI shocks less aggressively.

Both conditions (18) and (21) show that in the presence of uncertainty about the intertemporal substitution elasticity σ, the optimal discretionary policy adjusts the

https://doi.org/10.1017/S1365100509090154 Published online by Cambridge University Press

nominal interest rate less aggressively for low persistent supply and NRI shocks in every period than it does in the absence of such uncertainty, but it adjusts the interest rate more aggressively for high persistent supply and NRI shocks as long as the policy multipliers are sufficiently large. This policy implication is opposite to that given by (17), the condition under which optimal discretionary policy responds to supply shocks more aggressively in the face of uncertainty about the price adjustment frequency α. What causes these opposite policy implications? This is due to the fact that uncertainty about the frequency α generates uncertainty about the transmission of supply shocks and about the inflation weight of the welfare loss function as examined in the preceding section, but uncertainty about the elasticity σ does not.

Uncertainty about the intertemporal elasticity σ generates uncertainty about policy multipliers σ, κ. Clarida et al. (1999) examine optimal discretionary policy under uncertainty about policy multipliers of a model that has the same form as that of the present paper but no explicit microfoundation. Comparing an optimality condition for this policy with that for discretionary policy under no such uncertainty, Clarida et al. conclude that "parameter uncertainty may reduce the response of the policy instrument to disturbances in the economy" (Result 11, p. 1688). This conclusion holds in the present paper's analysis of uncertainty about the intertemporal elasticity σ if the persistence of shocks is sufficiently low. However, when the shock persistence is sufficiently high and the policy multipliers are sufficiently large, Proposition 2 claims a more aggressive policy response to shocks under uncertainty about σ. This difference between Clarida et al. and this paper is due to the fact that this paper takes account of the relationships among the policy multipliers, the NRI-shock coefficient, and the output-gap weight via the fundamental parameter σ, given by the microfoundation of the model.[13]

5. CONCLUDING REMARKS

This paper has examined optimal monetary policy under uncertainty about fundamental parameters of a canonical DSGE model in the recent literature. In contrast to previous studies, a microfoundation of the model leads this uncertainty to generate uncertainty not only about the transmission of monetary policy but also about the transmission of shocks and about a social welfare loss function. In the presence of such uncertainty, this paper has found conditions under which optimal discretionary policy responds to shocks more aggressively than in the absence of the uncertainty. These conditions depend crucially on the persistence of shocks and the magnitude of policy multipliers. To obtain the conditions, the paper has stressed that taking proper account of uncertainty about the transmission of shocks and about weights of the welfare loss function is of crucial importance. No consideration of this uncertainty causes a misleading implication of the parameter uncertainty for optimal monetary policy.[14]

https://doi.org/10.1017/S1365100509090154 Published online by Cambridge University Press

NOTES

1. See Chow (1975), Craine (1979), Clarida et al. (1999), Estrella and Mishkin (1999), Svensson (1999), Sack (2000), Rudebusch (2001), Söderström (2002), and Wieland (2003), among others.

2. See, e.g., Stock (1999), Tetlow and von zur Muehlen (2001), Onatski and Stock (2002), Onatski (2000), Onatski and Williams (2003), and Giannoni (2002, 2007).

3. Using the Bayesian approach, Craine (1979) and Söderström (2002) find that in backward-looking models, uncertainty about the persistence of a variable targeted by a policy maker leads to a more aggressive policy response to shocks. Kimura and Kurozumi (2007) confirm this finding in a forward-looking model with inflation and output persistence. Taking the robust-control approach, Onatski (2000) shows that the Brainard principle holds in a backward-looking model.

4. Walsh (2005) utilizes such relationships to examine the consequences of basing optimal monetary policy on misspecified fundamental parameters of a microfounded model. See also Angeloni, Coenen, and Smets (2003), Amano (2007), and Leitemo (2007).

5. Uncertainty about the frequency of price adjustment has no effect on optimal policy responses to the NRI shock, because this uncertainty generates no uncertainty about the transmission of this shock.

6. Using a model that has the same form as that of the present paper but no explicit microfoundation, Clarida et al. (1999) analyze implications of uncertainty about policy multipliers for optimal discretionary policy. As shown later, such an analysis, however, does not necessarily provide accurate implications of uncertainty about fundamental parameters of the microfounded model.

7. This factor represents time variation not only in price markup but also in a labor-market friction that pushes real wages away from their competitive-equilibrium values as in Clarida, Galí, and Gertler (2001).

8. This paper does not consider the central bank's uncertainty about the other fundamental parameters, β, θ, ω. The existing literature has a consensus on the quarterly value of the discount factor β of 0.99. As for the steady-state price elasticity of demand θ and the inverse of the labor-supply elasticity ω, the effects of the bank's uncertainty about these parameters are very similar to those of its uncertainty about the price adjustment frequency α, because uncertainty about each of these three parameters has the same features.

9. In the model the natural rate of output depends on the parameter σ. Yet this dependence unnecessarily complicates the analysis of the central bank's uncertainty about fundamental model parameters, and thus this paper assumes that x_t and $\tilde{r}_t^n$ are included in Ω_t^{CB}.

10. For each $z = \pi, x$, note that $E_t^{\mathrm{CB}}\varepsilon_{t+1}^z = 0$, $\mathrm{Cov}_t^{\mathrm{CB}}[\kappa, \varepsilon_{t+1}^z] = 0$, and ε_{t+1}^z is independent of monetary policy.

11. To obtain (11), we use the following three equations: $E_t^{\mathrm{CB}}[\sigma E_t\pi_{t+1}] = \bar{\sigma} E_t^{\mathrm{CB}}\pi_{t+1} + \mathrm{Cov}_t^{\mathrm{CB}}[\sigma, E_t\pi_{t+1}]$, $\mathrm{Cov}_t^{\mathrm{CB}}[\sigma, E_t\pi_{t+1}] = -\beta^{-1}\mathrm{Cov}_t^{\mathrm{CB}}[\sigma, \kappa]x_t$, and $\mathrm{Cov}_t^{\mathrm{CB}}[\sigma, \kappa] = \bar{\sigma}\{\kappa_u(\omega + \bar{\sigma}^{-1}) - \bar{\kappa}\}$.

12. This is because uncertainty about the price adjustment frequency α generates no uncertainty about the transmission of the NRI shock, and thus the optimal discretionary policy can offset this shock perfectly.

13. The difference may also be ascribed to the difference in the way that determines the effects of parameter uncertainty. This paper compares a unique REE process of the policy instrument under the uncertainty with that under no such uncertainty, but Clarida et al. (1999) do not.

14. A longer version of this paper analyzes optimal commitment policy as well and finds numerically that such an optimal policy responds to inefficient supply shocks more aggressively but to NRI shocks less aggressively. This longer version is available on request to the author.

REFERENCES

Amano, Robert (2007) Inflation persistence and monetary policy: A simple result. *Economics Letters* 94, 26–31.

https://doi.org/10.1017/S1365100509090154 Published online by Cambridge University Press

Angeloni, Ignazio, Günter Coenen, and Frank Smets (2003) Persistence, the transmission mechanism and robust monetary policy. *Scottish Journal of Political Economy* 50, 527–549.

Bils, Mark and Peter J. Klenow (2004) Some evidence on the importance of sticky prices. *Journal of Political Economy* 112, 947–985.

Blinder, Alan S. (1998) *Central Banking in Theory and Practice*. Cambridge, MA: MIT Press.

Brainard, William C. (1967) Uncertainty and the effectiveness of policy. *American Economic Review* 57, 411–425.

Calvo, Guillermo A. (1983) Staggered prices in a utility-maximizing framework. *Journal of Monetary Economics* 12, 383–398.

Chow, Gregory C. (1975) *Analysis and Control of Dynamic Economic Systems*. New York: Wiley.

Christiano, Lawrence J., Martin Eichenbaum, and Charles L. Evans (2005) Nominal rigidities and the dynamic effects of a shock to monetary policy. *Journal of Political Economy* 113, 1–45.

Clarida, Richard, Jordi Galí, and Mark Gertler (1999) The science of monetary policy: A new Keynesian perspective. *Journal of Economic Literature* 37, 1661–1707.

Clarida, Richard, Jordi Galí, and Mark Gertler (2001) Optimal monetary policy in open versus closed economies: An integrated approach. *American Economic Review* 91, 253–257.

Craine, Roger (1979) Optimal monetary policy with uncertainty. *Journal of Economic Dynamics and Control* 1, 59–83.

Eichenbaum, Martin and Jonas D.M. Fisher (2004) Evaluating the Calvo model of sticky prices. NBER working paper 10617.

Estrella, Arturo and Frederic S. Mishkin (1999) Rethinking the role of NAIRU in monetary policy: Implications of model formulation and uncertainty. In John B. Taylor (ed.), *Monetary Policy Rules*, pp. 405–430. Chicago: University of Chicago Press.

Galí, Jordi and Mark Gertler (1999) Inflation dynamics: A structural econometric analysis. *Journal of Monetary Economics* 44, 195–222.

Giannoni, Marc P. (2002) Does model uncertainty justify caution? Robust optimal monetary policy in a forward-looking model. *Macroeconomic Dynamics* 6, 111–144.

Giannoni, Marc P. (2007) Robust optimal policy in a forward-looking model with parameter and shock uncertainty. *Journal of Applied Econometrics* 22, 179–213.

Gilboa, Itzhak and David Schmeidler (1989) Maxmin expected utility with non-unique prior. *Journal of Mathematical Economics* 18, 141–153.

Kimura, Takeshi and Takushi Kurozumi (2007) Optimal monetary policy in a micro-founded model with parameter uncertainty. *Journal of Economic Dynamics and Control* 31, 399–431.

Leitemo, Kai (2007) The optimal perception of inflation persistence is zero. *Scandinavian Journal of Economics* 1, 107–113.

Levin, Andrew T., Alexei Onatski, Noah Williams, and John C. Williams (2006) Monetary policy under uncertainty in microfounded macroeconometric models. In Mark Gertler and Kenneth Rogoff (eds.), *NBER Macroeconomics Annual 2005*, pp. 297–346. Cambridge, MA: MIT Press.

Levin, Andrew T. and John C. Williams (2003) Parameter Uncertainty and the Central Bank's Objective Function. Mimeo.

McCallum, Bennett T. and Edward Nelson (1999) Performance of operational policy rules in an estimated semi-classical model. In John B. Taylor (ed.), *Monetary Policy Rules*, pp. 15–45. Chicago: University of Chicago Press.

Onatski, Alexei (2000) Minimax Monetary Policy: Comparison to Bayesian Approach, Worst Cases, and Exact minimax Rules. Mimeo.

Onatski, Alexei and James H. Stock (2002) Robust monetary policy under model uncertainty in a small model of the U.S. economy. *Macroeconomic Dynamics* 6, 85–110.

Onatski, Alexei and Noah Williams (2003) Modeling model uncertainty. *Journal of the European Economic Association* 1, 1087–1122.

Rotemberg, Julio J. and Michael Woodford (1997) An optimization-based econometric framework for the evaluation of monetary policy. In Ben S. Bernanke and Julio J. Rotemberg (eds.), *NBER Macroeconomics Annual 1997*, pp. 297–346. Cambridge, MA: MIT Press.

https://doi.org/10.1017/S1365100509090154 Published online by Cambridge University Press

Rudebusch, Glenn D. (2001) Is the Fed too timid? Monetary policy in an uncertain world. *Review of Economics and Statistics* 83, 203–217.
Sack, Brian (2000) Does the Fed act gradually? A VAR analysis. *Journal of Monetary Economics* 46, 229–256.
Söderström, Ulf (2002) Monetary policy with uncertain parameters. *Scandinavian Journal of Economics* 104, 125–145.
Stock, James H. (1999) Comment. In John B. Taylor (ed.), *Monetary Policy Rules*, pp. 253–259. Chicago: University of Chicago Press.
Svensson, Lars E.O. (1999) Inflation targeting: Some extensions. *Scandinavian Journal of Economics* 101, 337–361.
Tetlow, Robert J. and Peter von zur Muehlen (2001) Robust monetary policy with misspecified models: Does model uncertainty always call for attenuated policy? *Journal of Economic Dynamics and Control* 25, 911–949.
Walsh, Carl E. (2005) Endogenous objectives and the evaluation of targeting rules for monetary policy. *Journal of Monetary Economics*, 52, 889–911.
Wieland, Volker (2003) Monetary policy and uncertainty about the natural unemployment rate. CEPR discussion paper 3811.
Woodford, Michael (2003) *Interest and Prices: Foundations of a Theory of Monetary Policy*. Princeton, NJ: Princeton University Press.

https://doi.org/10.1017/S1365100509090154 Published online by Cambridge University Press