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EXISTENTIAL IMPORT IN AVICENNA'S MODAL LOGIC

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Abstract. In this paper, I raise the following problem: what propositions have an import in Avicenna's modal logic? Which ones do not? Starting from the assumption that the singular and quantified propositions have an import if they require the existence of their subject's referent(s) to be true, I first discuss the import of the absolute propositions then I analyze the import of the modal propositions by considering Avicenna's definitions and the relations between these propositions. This leads to the following results: Avicenna's general opinion is that the affirmatives, be they assertoric or modal, have an import while the negatives do not. The possible affirmative propositions are given an import both in the externalist and the internalist post-Avicennan readings, provided that the subject is not impossible. However, the theory is not always clear, for the propositions containing 'sometimes not' are given an import, together with the negative necessaries containing 'as long as it is P', despite their negative character; the necessary affirmative propositions containing 'as long as it is P' are given an import, although they do not require it. In addition, Avicenna's analysis of the special assertorics **E** and **O** (containing the *internal* conditions 'at some times but not always') and their contradictories is erroneous, which does not help determine their import. But when correctly analyzed, these special **E** and **O** do not have an import, while their contradictories – **I** and **A** special assertorics respectively – have an import.

Résumé. Dans cet article, je pose le problème suivant: quelles propositions ont un import dans la logique modale d'Avicenne? Lesquelles n'en ont pas? Partant de l'assomption que les propositions singulières et quantifiées ont un import si elles requièrent l'existence de leur sujet pour être vraies, j'analyse d'abord l'import des propositions absolues, ensuite celui des propositions modales en tenant compte des définitions d'Avicenne et des relations entre ces propositions. Cette analyse conduit aux résultats suivants: Avicenne défend l'opinion générale selon laquelle les affirmatives, qu'elles soient modales ou assertoriques, ont un import alors que les négatives n'en ont pas. Il attribue un import aux propositions possibles affirmatives aussi bien dans l'interprétation externaliste que dans l'interprétation internaliste des logiciens post-Avicenniens, pourvu que le sujet ne soit pas impossible. Toutefois, la théorie n'est pas toujours claire, car Avicenne attribue un import aux propositions contenant 'parfois non' et aux nécessaires négatives contenant 'tant qu'il est P' malgré leur caractère négatif; les propositions nécessaires affirmatives contenant 'tant qu'il est P' sont considérées comme ayant un import alors qu'elles ne l'exigent pas. De plus, l'analyse qu'il donne des assertoriques spéciales **E** et **O** (contenant les conditions *internes* 'parfois mais pas toujours') et de leurs contradictoires est erronée, ce qui

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ne permet pas de déterminer clairement leur import. Mais quand elles sont correctement analysées, ces propositions **E** et **O** n'ont pas d'import, alors que leurs contradictoires – les assertoriques spéciales **I** et **A** respectivement – ont un import.

I. INTRODUCTION

The problem of existential import has been raised by the modern logicians as well as the medieval ones. It can be expressed as follows: What propositions require the existence of the subject to be true? Can we say that all the categorical propositions require this existence (*i.e.* have an import)? Or are there some propositions with import and some without import? Many traditional and medieval logicians, such as Apuleius, Boethius, Abelard and Buridan attributed the import to the affirmatives and denied it from the negative propositions as mentioned by Laurence Horn.[1] Buridan has even expressed the lack of import of **O** by using the following disjunction: "either nothing is F or some Fs are not Gs".[2] This opinion is a generalization of Aristotle's claim that the negative *singular* propositions do not have an import, while the affirmative ones have an import.[3] We find this same claim in Avicenna's theory, for he talks explicitly about empty terms and says that the singular propositions whose subject is empty are false when they are affirmative and true when they are negative. However, Avicenna's theory is more complex than Aristotle's for in his view, the assertoric propositions may contain various additional conditions, such as "as long as it exists", "at some times", "as long as it is S" or "as long as it is P" and all may be modalized. So, although Avicenna was more interested by the different structures of the categorical propositions and their truth conditions than by the problem of existential import strictly speaking,[4] one can nevertheless ask about the import of the propositions in his theory, because this question is one particular aspect of the general problem of the propositional structures. The questions are thus the following: which propositions, whether categorical or modal, have an import in Avicenna's theory? Which ones do not? Does the definition of possibility make any difference with regard to the import?

To answer these questions, I will first define the existential import, second analyze the import of the categorical propositions, then the import of the modal propositions by taking into account Avicenna's definitions and the truth conditions of the various kinds of propositions;

[1] Laurence Horn, *A Natural History of Negation*, The David Hume Series, CSLI Stanford (Chicago, 2001), p. 28.
[2] Horn formalizes it as follows: "$\sim\exists xFx \vee \exists x(Fx \wedge \sim Gx)$" (*A Natural History of Negation*, p. 27).
[3] *Ibid*., p. 24.
[4] I am indebted to one reviewer who drew my attention to that particular point.

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finally I will evaluate the theory by comparing it with those of the post-Avicennan logicians as well as the modern ones.

II. THE EXISTENTIAL IMPORT OF ASSERTORIC PROPOSITIONS

Avicenna presents different analyses of the assertoric propositions in his works. In *al-ʿIbāra* (the correspondent of *De interpretatione*), he presents all the traditional oppositions and defines them by considering the truth conditions of the propositions. In his view, the couples **A** / **O** and **E** / **I** are contradictories, **A** and **E** are contraries, **I** and **O** are subcontraries, **I** is the subaltern of **A**, whereas **O** is the subaltern of **E**. Subalternation is called *tadākhul* and is defined by means of the truth values of the propositions concerned. These truth values, which depend on the matters of the propositions, lead to a square of oppositions, for they make all the relations of the square valid.[5] They are presented both in *al-ʿIbāra*[6] and *al-Ishārāt wa-al-tanbīhāt*.[7]

Now what about the existential import of all these propositions? Since Avicenna defines the propositions and their relations by considering their truth conditions, we propose the following definition to existential import:

Definition 1: A categorical proposition of the form Subject-Predicate, whether singular, indefinite or quantified, has an existential import if and only if it requires the existence of its subject's referent(s) to be true.

Corollary: If such a proposition has an existential import, it implies the existence of the objects satisfying the subject term. If it can be true when the subject is empty, it does not have an import.

Given this definition, what is the import of the different propositions? In *al-Maqūlāt*, Avicenna evokes explicitly non-existent objects and claims that the singular propositions whose subject does not have an existent referent are false when they are affirmative and true when they are negative. The examples he gives are the following: "Zayd who does not exist [*al-maʿdūm*] is seeing" and "Zayd who does not exist is blind"; these sentences are both false, but the first one

[5] See Saloua Chatti, "Logical oppositions in Arabic logic: Avicenna and Averroes", in Jean-Yves Béziau, Dale Jacquette (eds.), *Around and Beyond the Square of Opposition* (Basel, 2012), pp. 21–40. The square is not drawn by Avicenna, however, for we do not find any figure in his texts.

[6] Avicenna, *al-Shifāʾ*, *al-Manṭiq* 3: *al-ʿIbāra*, ed. Mahmud El Khodeiri, rev. and intr. by Ibrahim Madkour (Cairo, 1970), p. 47.

[7] Avicenna, *al-Ishārāt wa-al-tanbīhāt*, with the commentary of Naṣīr al-Dīn al-Ṭūsī, intr. by Sulayman Dunya, Part 1, 3rd edn (Cairo, 1971), p. 305.

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contradicts "Zayd who does not exist is not seeing",[8] which is true. Thus, in case the subject's referent is non-existent, both affirmatives are false while the negative sentence is true. So the negative sentence does not have an import while the affirmative one has an import because it cannot be true when the subject's referent does not exist.

We can also note that Avicenna compares between the sentences whose subject's referent is non-existent and those whose matter is impossible, for he gives the following example: "Stones (or Zayd who does not exist) are seeing" and "Stones (or Zayd who does not exist) are not seeing"[9] and says that these sentences are contradictory, which means that "Stones are seeing" is false while "Stones are not seeing" is true. Elsewhere,[10] he also says that both "Stones are sick" and "Stones are healthy" are false, while "Stones are not healthy" is true. Given these examples and the comparison between both kinds of sentences, we can see that the impossible sentences behave like those whose subject's referent is non-existent with regard to their truth values. This shows that what holds for the singular propositions holds also for the indefinite ones. Since the subject of the indefinites is general, they can be quantified, which means that what holds for them, also holds for the quantified propositions. So the quantified sentences have an import when they are affirmative and do not have an import when they are negative.

This same opinion is hold in other treatises, such as *al-Najāt*, where Avicenna says the following: "… For the negation is true of a non-existent subject, while the affirmation, whether indefinite (= metathetic) or definite, is true only of an existent subject, so it is true to say that 'The griffin is not seeing' [= 'It is not the case that the griffin is seeing'], while it is not true to say that 'the griffin is not-seeing'".[11] So, in *al-Najāt* too, he defends the same opinion. Note that the subject 'griffin' is *not* a proper name, so this opinion does *not only* hold for the *singular* sentences; it is more general.

This opinion is even set up as a principle by Wilfrid Hodges, who says in a recent article:

On (a) he provides a new distinction between affirmative and negative sentences, namely the following two principles:

[8] Avicenna, *al-Shifā'*, *al-Manṭiq* 2: *al-Maqūlāt*, ed. George Anawati, Mahmud El Khodeiri, Fu'ad El-Ahwani, Sa'id Zayed, rev. and intr. by Ibrahim Madkour (Cairo, 1959), p. 259, 12–13.

[9] Avicenna, *al-Maqūlāt*, p. 259, 11–12.

[10] Avicenna, *al-Maqūlāt*, p. 258, 17.

[11] "[…] *li-anna al-salba yaṣuḥḥu ʿan mawḍūʿin maʿdūmin wa-al-ījābu, kāna maʿdūlan aw muḥaṣṣalan, fa-lā yaṣuḥḥu illā ʿalā mawḍūʿin mawjūdin; fa-yaṣuḥḥu an taqūla inna al-ʿanqā laysa huwa baṣīran wa-lā yaṣuḥḥu an taqūla inna al-ʿanqā huwa ghayru baṣirun*" (Avicenna, *al-Najāt*, edited by Muḥyī al-Dīn Sabrī al-Kurdī, 2nd edn [Cairo, 1938], p. 16, 3–6).

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Affirmative Principle. In every true affirmative predicative sentence the subject term is satisfied (*i.e.* non-empty) (ʿ*Ibāra*, 79. 13).
Negative Principle. A negative predicative sentence is true when its subject term is not satisfied (ʿ*Ibāra*, 81. 3f).[12]

The import is then associated with the quality of the proposition, as it is in Apuleius', Boethius', Abelard's and Buridan's theories, although no formal expression of the lack of import, similar to that of Buridan, is provided by Avicenna.

However, Avicenna's text is not always clear, for he sometimes expresses **O** as follows: "Not all men are writers, but rather some of them",[13] which gives it an existential import, because of the affirmative side of the sentence, and corresponds more to what Blanché calls the **Y** vertex (that is, '$\mathbf{I} \wedge \mathbf{O}$') than to the standard **O**.

Furthermore, Avicenna gives other interpretations of the assertoric propositions in *al-Qiyās* (the correspondent of *Prior Analytics*), for he analyzes these propositions by introducing temporal considerations and other conditions. These analyses show that these various propositions have different truth conditions and that their behavior with regard to the import is also different.

In *al-Qiyās* and *al-Ishārāt*, Avicenna says first that some assertorics may be expressed thus: "Every S is P as long as it exists", or thus: "Every man is laughing (at some times)". The latter are called "General absolutes". Other absolutes may contain "at some times but not always"; they are called "Special absolutes" or "existential".[14] Other sentences have the following structures: 'S is P as long as it is S', or 'S is P as long as it is P'.

Avicenna also gives precisions on the subject, when he says the following:

> 'Every B is A' means that each [thing] that is described and supposed as B in effect, always or not always, is also described as A, without considering when that was, and whatever classification holds.[15]

This reading is supposed to be the most general one, that is, to include the most cases, whatever additional condition they contain. It gives an import to **A**, for its subject is presupposed to be existent.

[12] Wilfrid Hodges, "Affirmative and negative in Ibn Sīnā", in Catarina Dutilh Novaes & Ole Hjortland Thomassen (eds.), *Insolubles and Consequences, Essays in Honour of Stephen Read* (Milton Keynes, UK, 2012), pp. 119–34, at p. 120.
[13] Avicenna, *al-ʿIbāra*, p. 54, 5.
[14] Avicenna, *al-Ishārāt wa-al-tanbīhāt*, p. 309, 2.
[15] "*kull [B] [A] maʿnāhu kull wāḥid wāḥid mimmā yūṣafu wa-yufraḍu annahu bi-al-fiʿli [B], dāʾiman aw ghayru dāʾimin, fa-innahu mawṣūfun ayḍan bi-annahu [A] min ghayri an yaltafita ilā matā dhālika, wa-min ayyi al-aqsāmi kāna*" (Avicenna, *al-Shifāʾ*, *al-Manṭiq* 4: *al-Qiyās*, ed. Saʿid Zayed, rev. and intr. by Ibrahim Madkour [Cairo, 1964], p. 26, 18–p. 27, 2).

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But when the subject is a space-temporal being, such as men or animals, it does not exist *eternally*, this is why Avicenna adds "always or *not always*" and the condition "as long as it exists"[16] which specifies the time of its existence. The existence is not only space-temporal too, for the object could be abstract, as is the case with the icosahedron given as an example in *al-Qiyās* (p. 21) and elsewhere.[17] Its existence is said to be "in the mind", that is, the object is considered as existent by the human mind, although it is not perceived, but only conceived. This precision is also given in *al-Ishārāt*, where Avicenna analyses the **A** proposition as follows:

> Every thing that is described as J, be it described as J in a mental assumption or in external existence, and be it so qualified always or not always, *i.e.* in just any manner, this thing is thus described as B without adding that it is so described in that [particular] time or situation or always.[18]

In his commentary of this quotation, Paul Thom notes that Avicenna "rejects the idea that the subject-term of an absolute or modal proposition applies just to what actually exists".[19] So the existence is not just space-temporal, it applies also to the mathematical and other abstract objects. Thom formalizes the "mental assumption" by means of what he calls the "simple *de re* reading of absolute and modal propositions, according to which the subject-term is ampliated to cover whatever *can* fall under that term",[20] which is rendered symbolically in the following way: "J_M",[21] where 'M' means possible, and the whole expression means 'the possible J'. A possible subject might be seen as a conceivable or a possibly existent subject, which means that the proposition has an existential import, whenever its subject is possible. It lacks import only when the subject is *not possibly* existent, as is the case with the griffin.

About this kind of existence (in the mind), Avicenna says what follows:

> Every subject of a [true] affirmative proposition is satisfied either in the world or in the mind. If one says: (34) Every icosahedron is an X. what is

[16] Avicenna, *al-Qiyās*, p. 32, 3.
[17] For instance, *al-'Ibāra*, pp. 79–80; see also Hodges "Affirmative and negative in Ibn Sīnā", p. 127.
[18] "*kull wāḥid wāḥid mimmā yūṣafu bi-[J]- fī al-faraḍi al-dhihnī aw fī al-wujūdi al-khārijī, wa-kāna mawṣūfan bi-dhālika dā'iman aw ghayru dā'imin, bal kayfa ittafaqa; fa-dhālika al-shay'u mawṣūfun bi-annahu [B] min ghayri ziyādati annahu mawṣūfun bihi fī waqti kadhā aw ḥāli kadhā aw dā'iman*" (Avicenna, *al-Ishārāt wa-al-tanbīhāt*, pp. 280–2; part of the translation reported by Paul Thom, "Logic and metaphysics in Avicenna's modal syllogistic", in Shahid Rahman, Tony Street and Hassan Tahiri [eds.], *The Unity of Science in the Arabic Tradition* [Dordrecht, 2008], pp. 361–376, at p. 362).
[19] *Ibid*., p. 362.
[20] *Ibid*.
[21] *Ibid*.

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meant by this is that every icosahedron, regardless of where it is found (*yūjadu kayfa kāna*), is an X. That does not mean that every non-existent icosahedron is a non-existent X [. . .]. But [the fact is that] the mind judges something to hold affirmatively of certain ideas, either on the basis that they themselves in their state of being satisfied satisfy the [relevant] predicate, or else on the basis that they satisfy the predicate in the mind, *not just as mental entities*, but rather on the basis that *when they are satisfied*, they satisfy the predicate[22] [my emphasis].

This means that the object must be considered as possibly existent in reality, even if it is conceived by the mind at the moment, for what the mind conceives is not *just* a mental entity, *i.e.* something that could exist *only* in the mind; rather it is an entity that could be real or that is possibly satisfied in the real world. So, what the affirmative proposition says is the following: the object existing in the mind satisfies P, not because it is mental but because it would satisfy P if it were existent in the real world. Only in that case, the affirmative proposition could be true of it. Thus the affirmation *assumes it* as existent, because the inexistence of the objects makes it impossible for them to possess properties. This is perhaps the difference between the icosahedron taken as an example and the griffin mentioned later. The icosahedron is conceived not just as existing in the mind, *i.e.* as a psychological entity; rather it is a mathematical entity that the mind conceives as such, and it is possibly existent in the real world, while the griffin could never exist in the real world, so that no affirmative proposition having the griffin as a subject could be true as Avicenna says in the following quotation:

[. . .] but it is not truthfully said that (41) The griffin is a thing that can't see. This is because 'the griffin' is a name that signifies a meaning in the imagination and there is nothing satisfying it in the real world.[23]

To make sense of the difference between the mathematical entities and the imaginary ones, we could say that the griffin could never be other than imaginary, while the mathematical objects could not be seen as pure illusions. However, the mathematical (and imaginary) objects are more clearly analyzed if one considers the distinction between the "externalist" and the "essentialist"[24] readings of the

[22] Avicenna, *al-ʿIbāra*, p. 79, 13–15, p. 80, 1, 6–10; translation Hodges in "Affirmative and negative in Ibn Sīnā", p. 132.

[23] Avicenna, *al-ʿIbāra*, p. 82; translation Hodges in "Affirmative and negative in Ibn Sīnā", p. 134.

[24] This distinction is reported and analyzed by Tony Street in the following articles: Tony Street, "Appendix: Readings of the subject term", *Arabic Sciences and Philosophy*, 20 (2010), pp. 119–24; *id*.,"Afḍal al-Dīn al-Khunājī (d. 1248) on the conversion of modal propositions", *ORIENS*, 42 (2014): 454–513; *id*.,"Kātibī, Taḥtānī and the *Shamsiyya*", forthcoming, and by Paul Thom in "Abharī on the logic of conjunctive terms", *Arabic Sciences and Philosophy*, 20 (2010): 105–17.

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subject term made by the post-Avicennan logicians of the 12th and 13th century. We will return to this analysis in the last section.

In *Manṭiq al-mashriqiyyīn*, however, Avicenna seems to express things in a different way, for he writes:

> If you say *B is J*, this means that what is described as B and is supposed B, whether it is existent or *non-existent*, possibly existent or *impossibly existent*, and it is really described as B, without adding that it is always B or not always B, this thing is described as J. And the same goes with the negative[25] [my emphasis].

This quotation is puzzling, because of the words 'impossibly existent' and 'non-existent' which seem to suggest that the subject of an affirmative sentence could be non-existent. But when one scrutinizes it more carefully, one finds that it does not really change Avicenna's position about the import because he does *not* say that the affirmative sentence, in the cases given, is *true*. And this is important because the import of the affirmative sentence is defined by the fact that the sentence is not *true* when its subject *does not exist*. So we cannot say that Avicenna changed his mind with regard to the import of the affirmative sentences.

What about the general absolute universal propositions? These propositions are said to be not contrary by Avicenna, for they may be true together. For instance, "Every man is laughing (at some times)" and "Every man is not laughing (at some times)" are both true. The latter is explained as follows by Avicenna: "the predicate 'is laughing' can be negated from every man, at some times".[26] Such a reading gives it an existential import, for 'at some times' means 'at some particular time in his life (*wujūdihi*)'.[27] Hence the sentence cannot be true if men are not existent.

Likewise, **O** general absolute and **A** general absolute, that is, "Some men do not laugh, at some times" and "Every man is laughing at some times" as well as **I** general absolute, *e.g.* "Some men are laughing at some times" and **E** general absolute, *e.g.* "Every man is not laughing at some times" seem to have an existential import, for a non-existent thing cannot be so and so at some times, given its non existence in space and time. Their contradictories, according to Avicenna, are the perpetual sentences, which are sentences containing either

[25] "*idhā qulta [B] [J] fa-maʿnāhu anna mā yūsafu bi-annahu [B] wa-yufraḍu annahu [B] sawāʾan kāna mawjūdan aw laysa bi-mawjūdin, mumkina al-wujūdi aw mumtanaʿa al-wujūdi, baʿda an yujʿala mawṣūfan bi-al-fiʿli annahu [B] min ghayri ziyādati kawnihi dāʾiman [B] aw ghayra dāʾimin fa-dhālika al-shayʾu mawṣufun bi-annahu [J]. Wa-ʿalā qiyāsihi fī al-salbi*" (Avicenna, *Manṭiq al-mashriqiyyīn*, ed. Muḥyī al-Dīn al-Khatīb and ʿAbdelfattāḥ al-Qatlane [Cairo, 1910], p. 64, 1–4).

[26] Avicenna, *al-Qiyās*, p. 82, 4.

[27] Avicenna, *al-Qiyās*, p. 82, 2.

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'always' or 'never'. In Avicenna's text, the couples of contradictories are stated as follows:

"Some Bs are A (at some times) / No B is ever A [*"fa-naqīḍu qawlinā: baʿḍu [B] ['A], laysa shayʾun min [B] ['A], al-battata"*].

Not every B is A (at some times) / Every B is always A" [*"wa-naqīḍu qawlinā: laysa kull* [*B*] ['*A*], *anna kull* [*B*] ['*A*] *dāʾiman*"].[28]

He also states the following two propositions and their contradictories:

Every B is A (at some times) / Some Bs are never A [*"fa-naqūlu al-āna: inna al-muṭlaqa bi-al-maʿnā al-ʿāmm al-mūjiba al-kulliyya, ka-qawlinā ʿkull [B] [A]' yakhruju ʿanhā shayʾāni: aḥaduhumā ʿbi-al-ḍarūrati baʿḍu [B] laysa [A]', wa-al-thānī ittifāqan ʿbaʿḍu [B] laysa [A] al-battata"*].[29]

Some Bs are always B / Every B is not A (at some times) [*"wa-in kānat al-kulliyyatu sāliba muṭlaqa ʿāmma fa-muqābiluhā ayḍan juzʾiyya mūjiba dāʾimatu al-ījābi fī al-baʿḍi"*].[30]

Among these propositions, those that contain 'at some times' have an import, even when they are negative, given the way they are analyzed by Avicenna; consequently, their contradictories should not have an import, for "Contradiction is valid only if both propositions have a different import".[31] As a matter of fact, the propositions containing 'never' do not have an import, since they are negative. However, the import of those containing 'always' is not clear in Avicenna's theory, for they contain a temporal operator, which means that what the proposition says holds in a space-temporal context. Because of this space-temporal character, it is hard to imagine that the proposition is true if its subject's referent is non-existent. But they are nevertheless the contradictories of propositions which *are said to have* an import, so according to the condition above, they should be without import, or if they have an import, their contradictories, that is, the propositions containing 'sometimes not' should be without import. But Avicenna seems to give an import both to the propositions containing 'always' and to those containing 'sometimes not'; this shows that there is something wrong or at least unclear in the theory.

As to the special assertoric propositions and their contradictories, they are stated as follows in *al-Ishārāt*, for instance:

[28] Avicenna, *al-Qiyās*, p. 49, 2–4. These propositions are stated with a different symbolism in Tony Street, "Arabic logic", in Dov Gabbay & John Woods (eds.), *Handbook of the History of Logic*, vol. 1 (Elsevier BV, 2004), pp. 523–96, p. 590.
[29] Avicenna, *al-Qiyās*, p. 47, 2–4.
[30] Avicenna, *al-Qiyās*, p. 48, 4–5.
[31] Saloua Chatti & Fabien Schang, "The cube, the square and the problem of existential import", *History and Philosophy of Logic*, 34.2 (2013): 101–32.

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A: Every J is sometimes B and sometimes not B / ∼**A**: Some Js are always B or some Js are never B[32]

I: Some Js are sometimes B and sometimes not B/ ∼**I**: Either every J is always B or no J is ever B[33]

E: No J is B sometimes but not always / ∼**E**: Some Js are always B or some Js are never B[34]

O: Some Js are not B sometimes but not always / ∼**O**: Every J is either always B or is never B.[35]

The two special assertorics **A** and **I** propositions have an import because of the affirmative part of each conjunction, but their contradictories do not have import, given that the disjunctions are true if at least one disjunct is true, and we find in each of these disjunctions a negative part which does not have an import.

However, although **A** special assertoric's contradictory is expressed in a right way, **I** special assertoric's contradictory is not rightly stated, for it should rather be the following: 'Every S is either always P or never P'. This proposition is *not* equivalent to 'Either every S is always P or no S is ever P' (stated by Avicenna) because in modern logic (x)[Ax ⊃ (Bx ∨ ∼Cx)] *differs* from (x)(Ax ⊃ Bx) ∨ (x)(Ax ⊃ ∼Cx). Note that this error is not related to the Arabic formulation; it is rather a *logical* error, for Avicenna's formulation separates explicitly the two parts of the disjunction, but *this* separation is wrong.

Now what could we say about the two special **E** and **O** assertoric propositions and their contradictories? Are these propositions and their contradictories rightly stated? It seems that they are not, for given that the special assertorics **E** and **O** are themselves negative, and given that their negation is internal and puts on the whole combination "sometimes but not always" in both cases, they can be

[32] "*(4) fa-idhā qulnā fīhā: 'Kull [J] [B]' ay ʿalā al-wajhi alladhī dhakarnāhu, kāna naqīḍuhu 'laysa innama bi-al-wujūdi Kull [J] [B], ay immā bi-al-ḍarūrati dā'iman baʿḍu [J] [B] aw [B] maslūbun ʿanhā kadhālika*' " (Avicenna, *al-Ishārāt*, p. 310, 1–5). See also *al-Qiyās*, p. 46, 11, where Avicenna says: "*Kull [B] [A] ay waqtan wa-ḥālan lā dā'iman*" and "*fa-naqūlu Laysa Kull [B] [A], waqtan bi-'aynihi, lā dā'iman, bal immā baʿḍuhu dā'iman aw baʿḍuhu lā al-battata*" (p. 47, 1–2).

[33] "(6) *wa-naqīḍu qawlinā: baʿḍu [J] [B] bi-hādhā al-wajhi, lā shay'a min [J] innamā huwa bi-al-wujūdi [B], bal immā Kull [J] [B] dā'iman, aw lā shay'a min [J] [B] dā'iman*" (Avicenna, *al-Ishārāt*, p. 311, 3–5).

[34] "*(5) wa-idhā qulnā fīhā: laysa wa-lā shay'a min [J] [B] ay ʿalā al-wajhi alladhī dhakarnāhu, kāna al-naqīḍu al-muqābilu lahu mā yufhamu min qawlinā: baʿḍu [J] dā'iman lahu ījābu [B] aw salbuhu ʿanhu; li-annahu idhā sabaqa al-ḥukmu anna Kull [J] yunfā ʿanhu [B] waqtan mā lā dā'iman, fa-innamā yuqābiluhu an yakūna nafyun dā'iman aw ithbātun dā'iman*" (Avicenna, *al-Ishārāt*, p. 310, 6–11).

[35] "*wa-naqīḍu qawlinā: laysa baʿḍu [J] [B] ay laysiya bi-hādhā al-maʿnā, huwa qawlunā: Kull [J] immā dā'iman [B] wa-immā dā'iman laysa bi-[B]*" (Avicenna, *al-Ishārāt*, pp. 311, 8–11). In *al-Qiyās*, Avicenna does not provide formulas as clear as the ones cited above, although he evokes the special absolutes and their opposites in section 5, part 1, pp. 38–50.

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formalized in the following way: **E** special assertoric: No S is P (sometimes but not always) = (x)[Sx ⊃ ∼(x is sometimes P ∧ x is sometimes ∼P)] = (x)[Sx ⊃ [∼(x is sometimes P) ∨ ∼(x is sometimes ∼P)] (according to the De Morgan law: "∼(p ∧ q) ≡ (∼p ∨ ∼q)"), which in turn leads to the following formula: (x)[Sx ⊃ (x is never P ∨ x is always P)], given that: 'never' = 'not sometimes', and 'always' = 'not sometimes not'.

Consequently, it is **E** itself that contains a disjunction, while its right contradictory should have the following structure: ∼(x)[Sx ⊃ (x is never P ∨ x is always P)], which is equivalent (by the definition of the quantifiers) to: (∃x)[Sx ∧ ∼(x is never P ∨ x is always P)], itself equivalent to: (∃x)[Sx ∧ [∼(x is never P) ∧ ∼(x is always P)] (by the other De Morgan law, that is, "∼(p ∨ q) ≡ (∼p ∧ ∼q)"). This leads to the following formula: (∃x)[Sx ∧ (x is sometimes P ∧ x is sometimes ∼P)] (which is the formal counterpart of **I** special assertoric above) given that 'not never' = 'sometimes', and 'not always' = 'sometimes not'. In ordinary language, ∼**E** special assertoric says simply the following: Some Ss are sometimes P and sometimes not P. Here, ∼**E** = **I**.

The same could be said about **O** special assertoric, which is stated as follows: "Some Ss are not P, sometimes but not always". If we formalize it, we obtain the following: (∃x)[Sx ∧ ∼(x is sometimes P ∧ x is sometimes ∼P)], which is equivalent to: (∃x)[Sx ∧ [∼(x is sometimes P) ∨ ∼(x is sometimes ∼P)], itself equivalent to: (∃x)[Sx ∧ (x is never P ∨ x is always P)], that is, "Some Ss are either never P or always P".

Its contradictory is thus the following: ∼(∃x)[Sx ∧ (x is never P ∨ x is always P)], that is, (x)[Sx ⊃ ∼(x is never P ∨ x is always P)], itself equivalent to: (x)[Sx ⊃ ∼(x is never P) ∧ ∼(x is always P)], thus in turn to: (x)[Sx ⊃ (x is sometimes P ∧ x is sometimes not P)]. In ordinary language, this is expressed as follows: "Every S is sometimes P and sometimes not P", which corresponds to **A** special assertoric above, and seems very natural since ∼**O** = **A**.

Consequently, **E** and **O** special assertorics, when they are rightly formulated, do not have an import, given that they are expressed by disjunctions containing at least one negative element. By contrast, their contradictories have an import, for they are expressed by conjunctions where at least one element is affirmative. Therefore, unlike what Avicenna says, **E** and **O** special assertorics are just the contradictories of **I** and **A** special assertorics respectively.

Now, what does Avicenna say about these special assertorics in *al-Qiyās*? In page 49, he evokes the particular (**I** and **O**) special assertorics as follows:

> If they are absolute in the special sense, their opposites are not necessarily one single thing; rather the necessary of the same quality and the perpetual

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that differs in quality, and in quantity as well, are included in their contradictories.[36]

This is not clearly expressed because Avicenna does not say precisely what are the logical relations between the components of the contradictory propositions (are they conjunctions or disjunctions?). As we noted above, **I** special assertoric's contradictory (*i.e.* **E** special assertoric) is a disjunction, while **O** special assertoric's contradictory (*i.e.* **A** special assertoric) is a conjunction. But this difference is not specified by Avicenna himself, which makes his account insufficient, and does not help determine the import of these propositions in his theory.

What about the propositions containing 'as long as it is S', called "descriptional" par some commentators,[37] that is, **A**w, **E**w, **I**w and **O**w?

The affirmatives may be true, but they would be false if the subject does not refer to any existing thing. They seem, then, to have an import. This import is suggested explicitly in *al-ʿIbāra* where Avicenna claims: "there is no affirmation at all unless the subject is [taken to be] as we said".[38] This opinion, however, can be challenged when the descriptional sentences are concerned, for instance in the modern account of the universals.

As to the negatives, they would have an import if they were false and would be without import if they were true, whenever the subject's referent is non-existent. Consequently, they should not have an import if the negation is external. For instance, a sentence like "The griffin is not a cat as long as it is a griffin" is true despite the non existence of griffins[39] when the negation is external, *i.e.* when the whole sentence is interpreted as "It is not the case that the griffin is a cat as long as it is a griffin".

Those which say: "S is P as long as it is P" are true since they could be interpreted as "Every S is P iff it is P" and "Some Ss are Ps iff they are Ps", in which case their consequent is always true. However, the negative propositions 'No S is P as long as it is P' and 'Some Ss are not P as long as they are P' are both false if their antecedents are true, given that their consequents are both always false.

[36] "*fa-in kānatā muṭlaqatayni bi-al-maʿnā al-khāṣṣ, lam yajib an yakūna muqābiluhumā shayʾan bi-ʿaynihi, bal kāna al-ḍarūrī al-muwāfiq fī al-kayf wa-al-dāʾim al-mukhālif fī al-kayf, baʿda an yukhālifa fī al-kamm, dākhilīna fī naqīḍihi*" (*al-Qiyās*, p. 49, 4–6).

[37] Street, "Arabic logic", p. 551. We use "w" because "descriptional" is the translation of "waṣfī".

[38] Avicenna, *al-ʿIbāra*, p. 80; translation Hodges in "Affirmative and negative in Ibn Sīnā", p. 132.

[39] Avicenna says that "The griffin is not a thing that can see" is true while "The griffin is a thing that cannot see" is false (*al-ʿIbāra*, p. 82; translation Hodges in "Affirmative and negative in Ibn Sīnā", p. 134).

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The two affirmatives should not have an import, given that they are always true, but we do not find in Avicenna's text anything that suggests that they are without import. On the contrary, he seems to give them an import, by saying "All what is described as B, is necessarily and always described as A, as long as its essence exists, as long as it is A, and A is not the predicate but rather a part of the predicate".[40]

As to the negatives, they would have an import if they are false, and would be without import if they are true. As a matter of fact, Avicenna holds them false for he says: "[As to] 'Not every B is A as long as it is A', this negative [sentence] is never true".[41] The word 'never' suggests that they are even *always* false; therefore they have an import in his theory. Let us now turn to the import of the modal propositions in Avicenna's theory. In order to clarify it, we will first present the definitions of the modalities provided by Avicenna.

III. AVICENNA'S DEFINITIONS OF NECESSITY AND POSSIBILITY

Avicenna talks about several kinds of possibility, which are the unilateral possible, that is "possibly α", the bilateral possible, *i.e.* "possibly α and possibly ∼α", and what he calls the "narrowest-possible" (translation Hodges 2010[42]), *i.e.* what is neither actual nor necessary, nor impossible, which he explains as follows: "the third meaning is that which is not actual and is not necessary (*ḍarūrī*) in the future".[43]

Necessity may be absolute, as in (1) "God is necessarily existent", or conditional when a condition is added. The conditions are the following: (2) "as long as S exists", (3) "as long as it is S", (4) "when it is P", (5) "In some determined times", or (6) "at regular times" not determined precisely, *e.g.* in "Every man is breathing". These conditions are stated in *al-Qiyās* as well as *al-Najāt*.[44] In *Manṭiq al-mashriqiyyīn*, they are slightly different.

The sentences containing (2), (5) and (6) have an import, for they contain the existential condition or temporal considerations. As to (3) and (4), if we consider Avicenna's examples, we would say that they have an import too, although some passages of *al-Qiyās* and *al-Ishārāt*[45] seem to suggest that they may be without import. We will return to these passages in section 4.

[40] "*Kull mā yūṣafu bi-annahu [B], fa-innahu bi-al-ḍarūrati wa-dā'iman mā dāma dhātuhu mawjūdatan yūṣafu bi-annahu [A], mā dāma alifan, wa-yakūnu al-alifu laysa huwa al-maḥmūlu bal juz'an min al-maḥmūlī*" (Avicenna, *al Qiyās*, p. 42, 3–5).

[41] "*wa-kadhālika naqīḍuhu, wa-huwa annahu laysa kull [B] [A] fī al-waqti alladhī huwa [A], fa-inna hādhā al-sāliba lā yaṣduqu al-battata*" (Avicenna, *al-Qiyās*, p. 41, 16–p. 42, 1).

[42] Wilfrid Hodges, "Ibn Sīnā on modes", *'Ibāra* ii.4, http://wilfridhodges.co.uk/arabic07.pdf, p. 11.

[43] Avicenna, *al-'Ibāra*, p. 118, 6–7.

[44] Avicenna, *al-Najāt*, pp. 20–1.

[45] For instance *al-Qiyās*, p. 32, 11–13, *al-Ishārāt*, p. 265, 8–10.

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In addition, Avicenna distinguishes in *al-ʿIbāra* between two interpretations of possible propositions, which are the following:

(1) "Every man is possibly a writer"
(2) "It is possible that every man is a writer"[46]

We will call them *internal* when the modality is in front of the predicate as in (1) and *external* when the modality is in front of the whole sentence as in (2). This distinction is not equivalent to the medieval *de re* / *de dicto* distinction, which is more complex; it simply involves the scope of the modal operator.

The contradictions between modal quantified propositions are stated very clearly in *al-Qiyās* (pp. 49–50), when the modality is external.[47] The simple (*de dicto*) ones are the following: □**A** / ◊**O**; □**E** / ◊**I**; □**I** / ◊**E**; □**O** / ◊**A**.[48]

IV. THE EXISTENTIAL IMPORT OF MODAL PROPOSITIONS

Now, what is the import of these modal propositions? For the singular proposition "Zayd is a writer" (formalized here by α), Avicenna gives the following truth values: "Possibly α" and "Possibly ∼α" are both true, while "Not possibly α" is false; "Necessarily α" and "Necessarily not α" are both false while "Not necessarily α" is true.[49] These values correspond to the case where Zayd *exists*, for in that case, the two possible propositions are both true, while the two necessary ones are both false. Avicenna does not seem to consider the case where Zayd does not exist in his evaluations, for it is when Zayd exists that either α is true or ∼α is true and consequently, ◊α and ◊∼α are both true, while □α and □∼α are both false.

But what happens if Zayd does not exist?

In this case, only ∼α is true, hence ◊∼α is true and □α is false, being contradictories. Thus ◊∼α does not have an import, while □α has an import.

What about ◊α and □∼α? Given that α is false, ◊α is indeterminate: it could be true or false. If it is true, it does not have an import, but if it is false, it has an import. The same may be said about □∼α.

Thus, absolutely speaking, we may have the following situations:

1. ◊α : with import and □∼α: without import

or

2. ◊α : without import and □∼α: with import

[46] Avicenna, *al-ʿIbāra*, p. 113, 4–5, 7–8.
[47] They are all stated in Saloua Chatti, "Avicenna on possibility and necessity", *History and Philosophy of Logic*, 35.4 (2014): 332–53, pp. 342–3.
[48] ◊: possibly and □: necessarily.
[49] Avicenna, *al-ʿIbāra*, p. 114, 5–7.

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Both cases validate the modal square, but which one is hold by Avicenna? To answer this question, we have to take into account the following facts:

1. Avicenna compares between 'stones are men', *i.e.* the affirmative sentences whose matter is impossible, and 'Zayd who does not exist is seeing', *i.e.* the affirmative sentences where the subject is empty; since the sentences whose matter is impossible are always false when they are affirmative, we would say that the affirmative sentences whose subject is empty are always false too; consequently α is always false, and ◊α is false too when Zayd is non-existent; if so, ◊α has an import, since it could not be true when the subject is empty.

2. 'never' and '□∼' are very close, even if they are not equivalent; since 'never' does not have an import, '□∼' should not have an import too. In addition □∼ = ∼◊, so if "◊α" has an import, its contradictory "∼◊α" does not have an import by the above theorem (section 2).

The above interpretation is corroborated by the following passage:

> If the icosahedron doesn't exist then it doesn't satisfy any description, because being non-existent, it can't satisfy any description. If it doesn't exist, how could it be the case that it satisfies something?[50]

Given this treatment of non-existent objects, we may say that the possible affirmative propositions have an import, while the negative necessary ones do not have an import, only because they are negative. This opinion may be generalized to the quantified propositions, for the possible **A**-proposition (1) 'Every man is possibly a writer' (**A**◊) is hold true, because the subject "man" is not empty, while the possibility **A**-proposition (2) 'Possibly Every man is a writer' (◊**A**) is said to be "doubtful" ["*qad yushakku fīhi*"][51] only because the possibility operator puts on the quantifier, that is, only because it is not very likely that every *existent* man is a writer as appears in what follows: "According to some people, it is impossible that *all* [existent] men are writers".[52] This means that (2) presupposes the existence of men. Thus the second proposition has an import and the first one too, because it is affirmative and because the possibility affects only the predicate.

The import of the substantive necessity **A** *i.e.* "Necessarily every horse is an animal (as long as it exists)" (= □**A**) and the necessary **A** "Every horse is necessarily an animal (as long as it exists)" (= **A**□) is even more obvious for they include explicitly the clause of existence.

[50] Avicenna, *al-ʿIbāra*, p. 80, 1–3; translation Hodges in "Affirmative and negative in Ibn Sīnā", p. 132.
[51] Avicenna, *al-ʿIbāra*, p. 115, 9.
[52] "*fa-inna min al-nāsi man yaqūlu: muḥālun an yakūna kullu annāsi kātibin ay muḥālun an yūjada anna kull insānun huwa kātibun*" (Avicenna, *al-ʿIbāra*, p. 115, 9–10).

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However, in *al-Qiyās*, Avicenna raises a question close to the problem of existential import by saying:

> If we say: "Every person is possibly a writer", does this judgement exclude the past persons? [...] Or does it apply to those who exist and those who will exist? Or is it exclusively related to those who will exist in the future?[53]

His answer is:

> We do not need that [...], for our expression "every person" means everyone who is a person. Thus if we say: Every person, at any time, is possibly a writer, and we mean that it is not necessary for each of them to write or not to write in the future, but it is true that he writes and that he does not write, as long as it is in the future, this discourse is true of everyone who existed or exists, its truth is not exclusively related to those who *will exist* in the future.[54] (my emphasis)

If so, **A**◊ applies *also* to future [*i.e.* non-existent yet] persons. Does this mean that this proposition does not have an import, after all, *i.e.* may be true even when its subject is non-existent? This raises a problem in the theory, because it would contradict what is said in *al-ʿIbāra* to the effect that the affirmative propositions cannot be true in case the subject's referent is not existent.

Furthermore, in *al-Qiyās*, Avicenna says that "Possibly every animal is a person" is true,

> because it is possible at some time that 'every B is C' [...], at that time 'every animal is a person' will be true.[55]

The example given in that passage suggests the narrowest-possible, for it may be read as follows: ∼(every animal is a person) ∧ ◊(every animal is a person) ∧ ◊∼(every animal is person), and formalized as: **O** ∧ ◊**A** ∧ ◊**O**. We may think that this proposition does not have an import, but if we take into account the formula which expresses it, it would have an import for if ◊**A** has an import, both "◊**A** ∧ ◊**O**" and "**O** ∧ ◊**A** ∧ ◊**O**" have an import too, because if ◊**A** is false in case the subject's referents are non-existent, the two conjunctions will be false too; they would also require the existence of the subject's referents to be true, and this is exactly what it means for a proposition to have an import.

This analysis refutes what Allan Bäck says about the import of the possible propositions in Avicenna's theory, for according to this author, the general possible has an import while the narrow-possible does not have an import. This opinion is hold in the following quotation:

[53] Avicenna, *al-Qiyās*, p. 172, 7.
[54] Avicenna, *al-Qiyās*, p. 172, 12–p. 173, 2.
[55] Avicenna, *al-Qiyās*, p. 141, 12–15.

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So the first sense of the contingent requires existential import of the subject term at some time, the second sense does not. In the first sense, it is contingent that S is P, then either S is P or S is not P. But in the second sense since S may not at all exist in the future, 'S is P' and 'S is not P' will both be false, if S never exists. Indeed, in the latter sense, something contingent need not exist at all.[56]

By saying that, A. Bäck refers to *al-Qiyās* (pp. 34–6), where Avicenna analyzes the example "Zayd is sitting". However, what Avicenna says is: "This sitting is not perpetually existent nor perpetually not existent as long as the essence of the subject exists" and he adds "The thing can never sit at all" (*al-Qiyās*, pp. 34–5); so unlike what A. Bäck says, it is the sitting that need not exist at all, not Zayd; consequently the analysis Avicenna provides in this text does not prove that the narrow-possible does not have import; on the contrary, Avicenna insists on the fact that the subject (Zayd) exists and says that what may be not existent at all is the sitting. So the narrow-possible has an import as we noted above, unlike what A. Bäck claims. In addition, when S's referent does not exist, 'S is not P' is *true*; it is *not* false, as A. Bäck says.

In addition, the fact that the possibility is *de dicto* does not necessarily mean that the subject is not satisfied. For it is clear that what is meant by Avicenna's example is that the hypothesis that in some (future) world, 'all animals would be human beings' is conceivable, in case all the other animals disappear in that particular time. But this does not mean that the extension of the subject "animal" is empty. Rather it illustrates a particular situation where the extension of the word "animal" is drastically reduced so that it becomes equivalent to that of "man".

In the passage of Avicenna's text cited above, Avicenna does not evoke necessity at all. But he does evoke necessity elsewhere in *al-Qiyās*, while talking about the same kind of situation, when he says:

> This is as if the sentence 'every animal or every white is a man' is a possible proposition with regard to the future [. . .]. These propositions, with regard to their quantification, are possibly true or false in the future, although their matter is necessary.[57]

This passage suggests a "combined *de dicto* / *de re* sense"[58] as Paul Thom calls it. For the *de dicto* meaning is related to the quantifier,

[56] Allan Bäck, "Avicenna's conception of the modalities", *Vivarium* XXX, 2 (1992): 217–55, p. 228.

[57] "*fa-yakūnu kamā anna qawlunā: 'kull ḥayawānun aw kull abyaḍun insānun' bi-ḥasbi al-mustaqbali, huwa qaḍīya mumkina* [. . .]. *fa-takūnu hādhihi al-qaḍāyā bi-ḥasbi i'tibāri ḥaṣrihā mumkina an taṣduqa aw takdhiba fī al-mustaqbali, wa-hiya fī māddatihā ḍarūriyya* [. . .]" (Avicenna, *al-Qiyās*, p. 173, 12–15).

[58] Thom, "Logic and metaphysics in Avicenna's modal syllogistic", p. 291.

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while the *de re* meaning is related to the matter of the proposition, *i.e.* the relation between the subject (animal) and the predicate (man), which is essential. This *de dicto* possibility could be interpreted as being 'truth is some possible circumstance (or world)' other that the actual one. But this 'world' should not be interpreted as some 'possible alternative' of the actual world; rather it is some 'future' possible world, which means that the *de dicto* modality is *temporal* rather than *alethic*, unlike the *de re* necessity, which seems to be *alethic*, its *temporal* character being simply a consequence of its necessity.

As to the possible affirmative particulars, they have an import, for all the reasons given above and because $\Diamond\mathbf{I}$ is equivalent to $\sim\Box\mathbf{E}$; consequently if $\Box\mathbf{E}$ does not have an import (because it is negative), then its contradictory has an import. The same may be said about $\mathbf{I}\Diamond$, *i.e.* $\sim(\mathbf{E}\Box)$.

Furthermore, if $\Box\mathbf{E}$ and $\mathbf{E}\Box$ do not have an import, then $\Diamond\mathbf{E}$ and $\mathbf{E}\Diamond$ should not have an import too, given that the possible propositions are implied by the necessary ones in Avicenna's frame, so if the necessary propositions are true when the subject's referents do not exist, then the possible propositions will be true too, and would not have an import for that reason. The same may be said about $\Diamond\mathbf{O}$ and $\mathbf{O}\Diamond$, which are the contradictories of $\Box\mathbf{A}$ and $\mathbf{A}\Box$ respectively and are implied by **O**; so given that **O** may be true when the subject is empty, $\Diamond\mathbf{O}$ and $\mathbf{O}\Diamond$ may also be true; hence they do not have an import for that reason and because they contradict the necessary universal affirmatives which do have an import.

As shown in "Avicenna on possibility and necessity",[59] the narrow-possibility propositions, are expressed as follows:

Narrow-possibility **A** = Possibly every S is P and Possibly not every S is P

which is equivalent to

Narrow-Possibility **O** = Possibly not every S is P and Possibly not (not every S is P)

And both are formalized by: $\Diamond\mathbf{A} \wedge \Diamond\mathbf{O}$ $(= \Diamond\mathbf{O} \wedge \Diamond\mathbf{A})$.

The same may be said about the universal negative, that is:

Narrow-possibility **E** = Possibly no S is P and Possibly not (no S is P)

which is equivalent to:

Narrow-possibility **I** = Possibly some Ss are P and Possibly not (some Ss are P).

Both are formalized as: $\Diamond\mathbf{E} \wedge \Diamond\mathbf{I}$ $(= \Diamond\mathbf{I} \wedge \Diamond\mathbf{E})$.

[59] Chatti, "Avicenna on possibility and necessity", p. 342.

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These propositions have an import because one of their conjuncts has an import. Their contradictories are respectively: □**O** ∨ □**A** and □**E** ∨ □**I.**

Both disjunctions do not have an import because □**O** and □**E** do not have an import.

As to **A** narrow-possible, that is: "Every S is ◊P ∧ ◊∼P = Every S is ◊P ∧ Every S is ◊∼P = **A**◊ ∧ **E**◊",[60] it has an import because **A**◊ has an import. Its contradictory, *i.e.* ∼(**A**◊ ∧ **E**◊) = ∼**A**◊ ∨ ∼**E**◊ = **O**□ ∨ **I**□ does not have an import because **O**□ does not have an import. In the same way, **I** narrow-possible is formalized as: Some Ss are ◊P and ◊∼P. Its contradictory is expressed by: Every S is □∼P or □P.[61]

If the affirmatives have an import, their contradictories should not have an import because they contain a negative predicate and a disjunction; therefore they will be true even in case the Ss do not exist, given that when □∼P is true, the disjunction, that is, the consequent of the conditional, is true.

The narrowest-possibility propositions, that is:

"Narrowest-possibility **I** = **E** ∧ ◊**I** ∧ ◊**E**"
Narrowest-possibility **E** = **I** ∧ ◊**I** ∧ ◊**E**
Narrowest-possibility **A** = **O** ∧ ◊**A** ∧ ◊**O**
Narrowest-possibility **O** = **A** ∧ ◊**O** ∧ ◊**A**"[62]

seem to have an import, for in each conjunction there is at least one conjunct which has an import, namely the affirmative possible proposition in each conjunction. Their negations do not have an import, because in each disjunction, there is at least one disjunct without import, namely the negative one(s). These negations are respectively the following: "**I** ∨ □**E** ∨ □**I**","**E** ∨ □**E** ∨ □**I**", "**A** ∨ □**A** ∨ □**O**" and "**O** ∨ □**A** ∨ □**O**".[63]

As to the necessity "descriptional" propositions, that is, □**A**w, □**E**w, □**I**w and □**O**w and their correspondent possibility ones, that is, ◊**A**w, ◊**E**w, ◊**I**w, ◊**O**w, we have to examine them closely to determine their import. The logical structure of the universals involve a conditional, for 'Every S is P as long as it is S' says what follows: 'Every S is P if it is S' and may be formalized as '(x)[Sx ⊃ (Sx ⊃ Px)]' which is equivalent to '(x)(Sx ⊃ Px)'. One can add the modalities afterwards.

If S's referents do not exist, can they be true? According to H. Lagerlund, the condition "as long as it is S" is added to the existential condition, for he says:

[60] Chatti, "Avicenna on possibility and necessity", p. 343.
[61] *Ibid*., p. 342.
[62] *Ibid*., pp. 342–3.
[63] *Ibid*.

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The substantial is necessary if what the subject term stands for is assumed to exist. As we go down the list the conditions becomes more elaborate. For the descriptional we need to *assume the same thing about the subject term*, but we need also to *add* that the substance of the moving thing is moving' (265) [my emphasis].[64]

However, the text considered is far from clear, for Avicenna says the following:

As to the perpetuity of the subject's being described in the way it is, as in: 'Every moving thing is changing', this does *not* mean absolutely, *nor does it mean* as long as its essence is existent, rather [it means] as long as the moving essence is moving [my emphasis].[65]

What he uses in front of "as long as its essence is existent" is a real negation expressed by "*nor* does it mean"; so, it seems that unless an "essence" as essence cannot be non-existent, the import is not required, for the text itself does not *say* that the import is required for this kind of propositions. But we may interpret the negation as denying the fact that the subject *always* exists. In that case, the proposition has an import, for the subject S is presupposed existent in the *time where it is S*.

In *al-Qiyās*, Avicenna claims:

We say: Every moving thing is necessarily changing [meaning] *not* that it is perpetual, has not ceased and will not cease, *nor* that it is permanent as long as that which is said to be moving is existent, *but* as long as it is moving [*bal mā dāma mutaḥarrikan*].[66]

Here too, as in *al-Ishārāt*, the durable existence of the subject's referents is not explicitly required. But the 'moving' and the 'changing' are not *just* concomitant, for the first implies the second: *by definition* a moving thing necessarily changes. If so, the logical relation between the subject and the predicate is an implication, not a conjunction, and if one considers the modern interpretation of the implication (which is material and philonian), one would say that the import is not required, because this implication is true when its antecedent is false. But the philonian implication is not hold by Avicenna. On the contrary, the syllogisms he holds such as Darapti, for instance, require the conditional to have a true antecedent, because the particular conclusion in that mood could not be true if the antecedents of its

[64] Henrik Lagerlund, "Avicenna and Ṭūsī on modal logic", *History and Philosophy of Logic*, 30.3 (2009): 227–39, p. 233.

[65] "*wa-ammā dawāmu kawni al-mawḍūʿu mawṣūfan bi-mā wuḍiʿa maʿahu, mithla qawlinā 'kull mutaḥarrikun mutaghayirun', fa-laysa maʿnāhu ʿalā al-iṭlaqi, wa-lā mā dāma mawjūda al-dhāti, bal mā dāma dhātu al-mutaḥarriki mutaḥarrikan*" (Avicenna, *al-Ishārāt*, p. 265, 8–10; part of the translation in Street "Arabic logic").

[66] Avicenna, *al-Qiyās*, p. 32, 11–13.

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two universal premises are false. This being so, the proposition should have an import, even when interpreted as an implication.

As to the propositions where the condition is related to P, they are expressed thus:

– Every B is □A as long as it is A
– Some Bs are □As as long as they are As

These should be true, whatever the truth value of the antecedent is, because 'As long as it is A' may be formalized as 'Ax ≡ Ax', which is always true.

But the negative sentences of this kind, which are expressed as follows:

– No B is □A as long as it is A
– Some Bs are □ not As as long as they are As

contain '∼Ax ≡ Ax' as a consequent; since this formula is always false, the whole proposition would be false if Bx is true, *i.e.* if there are Bs and it would be true if there are no Bs, that is if Bx is false.

Given that the affirmatives are always true, they should not have an import. But Avicenna says:

> All what is described as B is necessarily and always described as A, as long as its essence is existent [and] as long as it is A.[67]

Hence, he gives them an import quite explicitly.

As to the negatives, Avicenna considers the assertoric propositions of that kind as false and even *always* false as we saw above (section 2). As a consequence, the necessary ones are false too. But saying that these propositions are never true means that they have an import, for if they are always false, their antecedent is always true, hence it is always satisfied, and consequently never empty.

As to the possible propositions of that kind, they are expressed as follows:

– Every B is ◊A as long as it is A
– Some Bs are ◊As as long as they are As
– Every B is ◊ not A as long as it is A
– Some Bs are ◊ not As as long as they are As.

The affirmatives are true; so they should not have an import. But if the condition of existence is required in this kind of sentences too, they would have an import. Note that if possibility is interpreted in the narrow meaning, we would have in all cases a negative part in the consequent, which would make it false, because if we say "Every B is ◊A

[67] Avicenna, *al Qiyās*, p. 42, 3–4.

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and ◊~A as long as it is A", the second conjunct is contradictory, in which case the whole proposition would be false if the antecedent is true, and true if the antecedent is false. The negative assertorics are hold false by Avicenna, so the possible ones are false too; consequently the possible universals and particulars seem to have an import.

V. EVALUATION OF THE THEORY

Avicenna's position is not always clear although, generally speaking, he defends the opinion that the affirmatives have an import, while the negatives do not.

We may try to clarify Avicenna's position with regard to the subject-term, and consequently the import of the propositions in the light of the distinction between the essentialist and the externalist readings of the subject-term made by the post-Avicennan logicians.

This distinction is first made by Fakhr al-Dīn al-Rāzī who expresses it as follows (as reported by Tony Street):

Here is how Rāzī first made the distinction, for the a-proposition ("Every J is B"):

> ... we don't mean by it what is described as J externally, but rather something more general, which is: were it to exist externally (*law wujida fī al-khārij*) it would be true of it that it is J, whether it exists externally or not. For we can say "every triangle is a figure" even if there are no triangles existent externally. The meaning is rather that everything which would be a triangle were it to exist would be – in so far as it existed – a figure...
>
> By the second reading, we mean by "every J" every single thing which exists externally among the individual Js... On this hypothesis, were there no septagons existent externally, it would'nt be correct to say "every septagon is a figure"; if the only figures existent externally were triangles, it would be correct to say "every figure is a triangle". On the first reading, [that is the essentialist reading] both of these would be false."[68]

The first reading is the essentialist, while the second is the externalist. The essentialist reading gives an import to all the true affirmative propositions whose subject is a conceptual (for instance, a mathematical) entity, such as the icosahedron (Avicenna's example), and the triangle given here as an example. The proposition presupposes the existence of these subjects, (for he says "were it to exist") even if this existence is not external or actual. The second reading – the externalist one – does not give the same truth conditions to the proposition. For the truth of the proposition depends on the external existence of the things that satisfy the subject-term. This is why Rāzī

[68] Rāzī, *Mulakhkhaṣ*, pp. 141, 6–10, 142, 13–143, 1, cited and translated by Street, "Afḍal al-Dīn al-Khunājī on the conversion of modal propositions", p. 461.

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considers the proposition "every figure is a triangle" as true in this reading *only* "if the only figures existent externally were triangles": in this particular situation of the external world, the proposition would be true and would have an import. As to the proposition "every septagon is a figure", it is false in the externalist reading, if there are no septagons in the external world. Given that it is false in that external reading because the subject is non-existent, it has an import, by our definition above, for this means that the proposition, in that reading, requires the existence of the subject to be true. The same proposition is considered by Rāzī as false in the essentialist reading, which probably means that he takes the septagon to be a non-existent object. If so, the proposition has an import too in the essentialist reading, since it requires the existence of the subject to be true.

The essentialist reading seems to apply mostly to the mathematical entities according to Ṭūsī too, who takes Avicenna's own example and says:

> What we mean by the existence of the subject of an affirmative proposition is not only its existence in external reality, because in the sciences we make affirmative judgments of intelligible subjects even though we don't know whether or not they exist in external reality (just as we say, "the icosahedron is such and such"). Nor is its existence only in the intellect, because we also make judgments of external existents, about both the perpetual of existence and the non-perpetual of existence. What we mean by the existence of the subject, rather, is an existence more general than the external and the mental."[69]

The icosahedron is thus taken to exist as an "intelligible reality",[70] which gives such propositions an existential import when interpreted in the essentialist reading. But, as we saw above with the example of the griffin, the imaginary beings could not exist in Avicenna's theory and the same may be said about the impossible entities. So all affirmative propositions whose subjects are imaginary or impossible do not have an import, whether in the externalist or in the essentialist readings. This opinion seems to be widely shared by the post-Avicennan logicians, *except* Khunājī (d. 1251) and Abharī (d. 1265) in some texts.[71] The latter two logicians admit the existence of impossible entities, which leads to some strange and undesirable

[69] Ṭūsī, *Asās al-iqtibās fī al-manṭiq*, translation by Mullā Khusraw, edited by Ḥasan Shāfi'ī and Sa'īd Jamāl al-Dīn (Cairo, 2004), p. 110, 6–13; cited and translated by Tony Street, "Kātibī, Taḥtānī and the *Shamsiyya*", p. 16. In that same Street's article, we can see that Kātibī also defends the same position, for according to him, the proposition "every square is a figure" is true in the essentialist reading, but not in the externalist reading "if there are no squares in external existence", p. 15.

[70] Street, "Kātibī, Taḥtānī and the *Shamsiyya*", p. 16.

[71] See, for instance, Afḍal al-Dīn al-Khunājī, *Kashf al-Asrār 'an Ghawāmiḍ al-Afkār*, edited by Khaled El-Rouayheb (Tehran, 2010); cited in Street, "Kātibī, Taḥtānī and the *Shamsiyya*", p. 18, and Thom, "Abharī on the logic of conjunctive terms", p. 108.

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results, such as the falsity of all **E**-propositions and the truth of all **I**-propositions.[72] Since Avicenna does not admit the existence of the impossible and imaginary entities, he avoids these undesirable results. His theory gives an import to the affirmative propositions, whether they are interpreted in the externalist or the essentialist reading, because both readings give the same results with regard to the import, provided that the subject-term is a possible entity.[73]

As to the proposition "Possibly every animal is a man" (section 4 above), which is comparable to Rāzī's proposition "every figure is a triangle", Avicenna seems to interpret it in an *externalist* way, because he holds it true if the situation of the world is such that every single animal is a man. In that interpretation, it would have an import because the only reason why it is true is the fact that the only existent animals are men in that (future) situation. In that reading, the proposition has an import, because the subject is posited as existent in that particular future situation.

When interpreting the possible proposition "Every person is possibly a writer", Avicenna says explicitly that the subject "person" should refer to all existent persons, whether in the past, the present or the future.[74] So his reading of the subject-term seems to be externalist too, if we consider for instance, Abharī's definition of the externalist reading, which is the following:

> On the first usage, if it is said *every J is B*, what is meant is that every *J* in extramental existence is a *B* in extramental existence, whether the *J* is in the *present* or *before* it or *after*[75] [my emphasis].

In this case, it has an import if the future persons are said to be so-and-so (or possibly so-and-so, in Avicenna's example) *in so far as they are existent*, but it would not have an import, in Rāzī's account of the externalist reading above, if the future persons are not assumed to be existent (in that future situation). According to Paul Thom, this reading, as it is rendered by Abharī, extends "the *ut nunc* reading" of the "medieval Latin logicians", because it talks about "all actual *Js*, past, present and future".[76] If so, there is no incoherence in Avicenna's theory, for this possible proposition has an import in that particular reading, despite the fact that it talks about *future* persons too, *provided that* these persons are *posited as existent* in that future situation.

[72] Thom, "Abharī on the logic of conjunctive terms", p.109.
[73] Thom, "Abharī on the logic of conjunctive terms", p. 115.
[74] Avicenna, *al-Qiyās*, p. 172, 7, 15–p. 173, 2 (see the quotations in section 4 above).
[75] Cited and translated by Street, "Appendix: Readings of the subject-term", p. 119.
[76] Thom, "Abharī on the logic of conjunctive terms", p. 107.

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We may also interpret Avicenna's example in the essentialist reading, if we consider the following part of the quotation given above (section 4): "'every person' means everyone who is a person".[77] In that case, the subject-term may be seen as the "essence" of the person, for the essentialist reading is expressed as follows by Taḥtānī (d. 1365), for instance: "'Every J is B' is considered at times according to the essence (whereupon it's called 'essentialist', as though [the subject] is an essence in a proposition used in the sciences)...".[78] In this reading, the proposition has an import too, because its truth does not depend on the external existence of the persons, but on the existence of the essence.

So Avicenna seems to admit both the externalist and the essentialist readings. His opinion on the essentialist reading does not extend to the impossible objects, however, unlike Khunājī's opinion. So the propositions with an impossible object are false when they are affirmative and true when they are negative. In both the externalist and the essentialist readings, the affirmative propositions have an import, while the negative ones do not have an import.

But we find some confusions in his texts especially in his analysis of some kinds of negative propositions. For he gives an import to both the propositions containing 'sometimes not' and those containing 'always', But these propositions could not have the same import, because they contradict each other. Since "sometimes not" equals "not always", the propositions containing them, *e.g.* "People do not always laugh" could be analyzed as follows: "It is not the case that people always laugh". This makes them true even if no person exists. Avicenna's analysis of this kind of negative sentences is not therefore very convincing, nor logically correct.

Another confusion is related to the sentences containing 'as long as it is P', which are always true when they are affirmative. Because they are always true, they should not have an import. But Avicenna seems to give them an import because of their affirmative character. This does not match with the logical structure of these propositions which are tautological, that is, true in all cases, therefore even when the subject does not exist.

The analysis of the special negative assertorics where the temporal conditions are internal is also confused, for Avicenna does not give their right structures, as we saw in section 2. Consequently, their import is not determined clearly in his theory. But when one analyzes them in the right logical way, one finds that the special negative

[77] Avicenna, *al-Qiyās*, p. 172, 15–17.
[78] Taḥtānī, *Taḥrīr*, p. 94, 6–4; cited by Street, "Afḍāl al-Dīn al-Khunājī on the conversion of modal propositions", p. 462.

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assertorics do not have an import, while the affirmatives have an import.

As to the descriptional sentences, if we consider them for themselves, they do not require existential import, even when they are necessary, for "Every griffin is necessarily winged, as long as it is a griffin" may be hold true, even if griffins do not exist, given that the sentence says what follows: 'Every S is □P if it is S'. This is formalized as: '(x) [Sx ⊃ (Sx ⊃ □Px)]' which is equivalent to '(x)(Sx ⊃ □Px)' and does not seem to require the existence of griffins to be true, because one could hold it true only on the basis of the definition of the griffin, even if that object is non-existent. We may even give descriptional sentences that are true independently of the context, for instance, the following: "Every griffin is necessarily imaginary as long as it is a griffin". If so, the descriptional (universal) propositions do not behave as the substantive ones and do not require the existence of their subjects to be true.

CONCLUSION

Avicenna's theory is very rich, since he considers many kinds of assertoric propositions, which may be modalized. His general opinion is that the affirmatives have an import, while the negatives do not. This opinion holds for both the essentialist and the externalist readings, given that the impossible objects are not considered as existent in Avicenna's theory. But the general assertorics are said to have an import, whether they are affirmative or negative. Their contradictories, *i.e.* the perpetuals do not have import when they are negative, while they have an import when they are affirmative. The problem with this opinion is that the perpetual affirmatives are the contradictories of the general negative assertorics, which are said to have an import. This is untenable because the contradictories should not have the same import. In addition, the propositions containing the condition "as long as it is P" are said to have an import, whether negative or affirmative. This does not match with the logical structure of these affirmatives, and is not in accordance with Avicenna's general opinion about the negatives.

As to the modal propositions, it seems that the necessary affirmative propositions have an import in almost all cases, while the negative universal ones should not have an import and the possible affirmative propositions seem to have an import. However, we find some confusions in Avicenna's theory for, unlike what he says, the descriptional affirmative propositions (necessary or possible) should not have an import when they contain 'as long as it is P', because they are always true. The same are considered false when they are

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negative, thus Avicenna gives them an import, but the fact is that they should be without import, for they could be true if the Ss do not exist.

Avicenna gives the narrowest-possible an import in some particular conditions, *i.e.* when the reading of the subject-term is external and when this subject exists externally in the future situation considered. However, his analysis of the negative special assertorics containing internal temporal conditions as well as **I** special assertoric's negation is not logically correct. When they are analyzed in a right way, these propositions have an import when they are affirmative and do not have an import when they are negative.

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