Robotica (2001) volume 19, pp. 305–309. Printed in the United Kingdom © 2001 Cambridge University Press

A new parallel wrist using only revolute pairs: the 3-RUU wrist

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(Received in Final Form: November 10, 2000)

SUMMARY

Only one parallel wrist with three equal legs containing just revolute pairs has been already presented in the literature. This parallel wrist is overconstrained, i.e., it involves three degrees of freedom required to orientate the end effector by using repetitions of constraints. The overconstrained mechanisms have the drawback of jamming or undergoing high internal loads when geometric errors occur. This paper presents a new parallel wrist, named 3-RUU wrist. The 3-RUU wrist is not overconstrained. It has three equal legs just involving revolute pairs and actuators adjacent to the frame and uses an architecture (3-RUU) already employed to obtain manipulators that make the end effector translate. The 3-RUU wrist kinematic analysis is addressed. This analysis shows that the new parallel wrist can reach singular configurations (translation singularities) in which the spherical constraint between end effector and frame fails. The singularity condition that makes finding all the 3-RUU wrist singular configurations possible is written in explicit form and geometrically interpreted.

KEYWORDS: Parallel wrist; Revolute pairs; Three equal legs; Singularity condition.

1. INTRODUCTION

Many industrial applications need rigid body motions able to be performed by manipulators with three degrees of freedom (dof). For example, the single translation and the single orientation of workpieces are requested with a certain frequency. Furthermore, the motion requirements join themselves to precision and structural stiffness requirements well matched by parallel manipulators.

These needs pushed researchers to propose parallel manipulators either for purely translational motions[1–6] (translational manipulators) or for purely spherical motions[7–10] (parallel wrists) of the end effector.

Parallel manipulators consist of an end effector (platform) connected to the frame (base) by a number of kinematic chains (legs), usually, having only one actuated pair per leg. The platform's motion is controlled by acting on the legs and all the legs concur to carry the external loads applied to the platform.

Three dof parallel manipulators with three equal legs and only one actuated pair per leg simplify both the design procedures and the manufacturing process. Among these manipulators the ones having only revolute pairs are to be preferred because of the manufacturing and actuation simplicity these pairs exhibit.

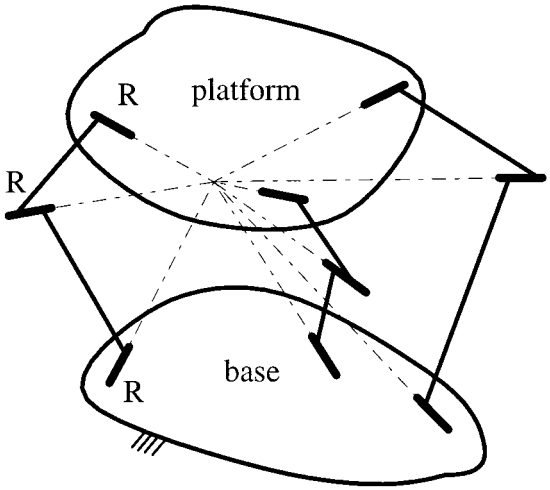


Fig. 1. Gosselin and Angeles' architecture.

In the literature, the parallel wrist of Gosselin and Angeles[7] (Figure 1) is the one with three equal legs containing only revolute pairs. The Gosselin and Angeles' wrist (Figure 1) has three leg of type RRR (R stands for revolute pair). This parallel wrist is overconstrained and it involves three dofs required to orientate the platform by using constraints that are repetitions of other constraints. Overconstrained mechanisms have the drawback of jamming or undergoing high internal loads when geometric errors occur.

Not overconstrained parallel mechanisms, having three equal legs and three dofs, must use leg architectures which leave five dofs (connectivity five) to the relative motion between platform and base.[10] The unique leg architecture with connectivity five involving just binary links and revolute pairs is the RRRRR architecture (Figure 2). The parallel mechanism with three legs of type RRRRR is the 3-RRRRR mechanism (Figure 3a).

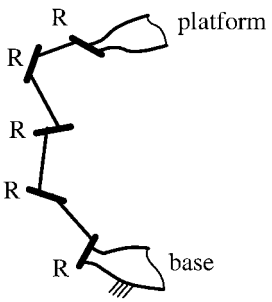


Fig. 2. Leg of type RRRRR.

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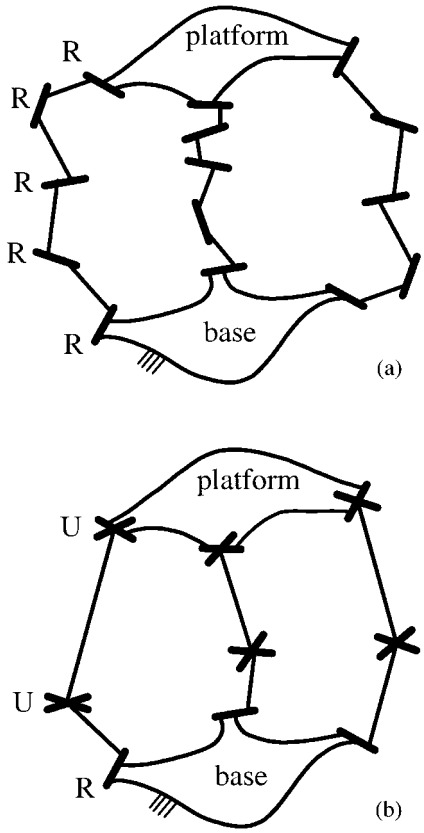


Fig. 3. (a) 3-RRRRR mechanism; (b) 3-RUU mechanism.

The 3-RRRRR mechanism makes platform's elementary spherical motion possible when at least three revolute pair axes per leg converge towards a unique point.[10] If this geometric condition (Figure 4) is kept during motion and the convergence point stands still in the base, the 3-RRRRR mechanism will make the platform perform finite spherical motions, i.e., it is a not overconstrained parallel wrist just having three equal legs and only revolute pairs.

A particular case of 3-RRRRR is the 3-RUU mechanism (U stands for universal joint) (Figure 3.b). It has three equal legs of type RUU. The 3-RUU architecture has been already proposed to obtain purely translational parallel mechanism.[6]

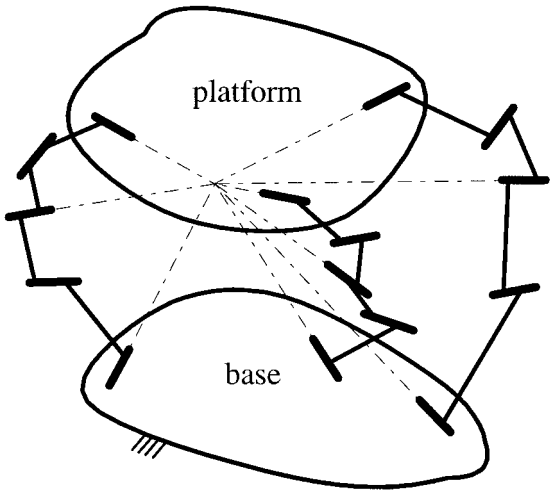


Fig. 4. 3-RRRRR configuration making elementary spherical motions possible.

This paper analyses the 3-RUU mechanism and demonstrates that the 3-RUU mechanism is a parallel wrist (3-RUU wrist) when some mounting and manufacturing conditions are matched. Moreover, it shows that the 3-RUU wrist can reach singular configurations (translation singularities) in which the spherical constraint between platform and base fails. Finally, the singularity condition that makes it possible to find all the 3-RUU wrist's singular configurations is provided in explicit form and geometrically interpreted.

2. KINEMATICS OF THE 3-RUU WRIST

Figure 5 shows a 3-RUU mechanism meeting the following mounting and manufacturing conditions:

(i) the three revolute pair axes fixed in the platform (base) must converge towards a point of the platform (base); (manufacturing condition)
(ii) the six revolute pair axes, embedded two by two in the three binary links adjacent to the base, must converge towards a base point; (manufacturing and mounting condition)
(iii) the platform point located in the intersection of the three revolute pair axes fixed in the platform (point P in Figure 5) must coincide with the base point located in the intersection of the three revolute pair axes fixed in the base; (mounting condition)
(iv) in each leg the two revolute pair axes which are not bound to converge towards a unique point must be parallel to each other and perpendicular to the straight line through the universal joints' centers (manufacturing condition).

In the following paragraphs, it will be shown that these mounting and manufacturing conditions are sufficient to make the platform perform finite spherical motions when the three revolute pairs adjacent to the base are actuated, provided singular configurations are not considered. Hereafter, the 3-RUU mechanisms matching these conditions will be called 3-RUU wrists.

With reference to Figure 5, the point P is the platform point located in the intersection of the three revolute pair

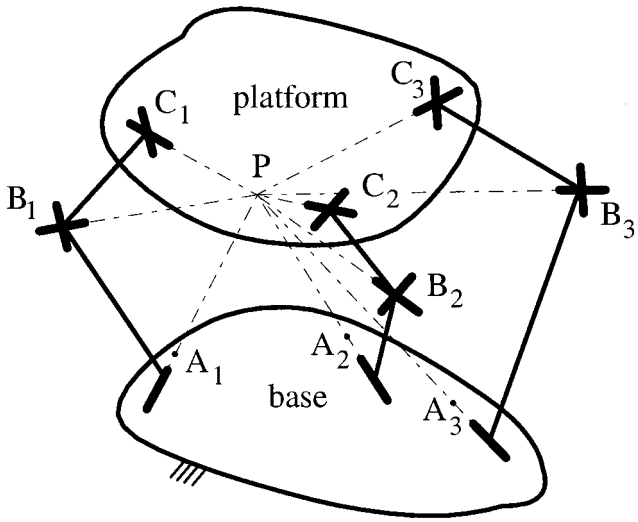


Fig. 5. 3-RUU wrist.

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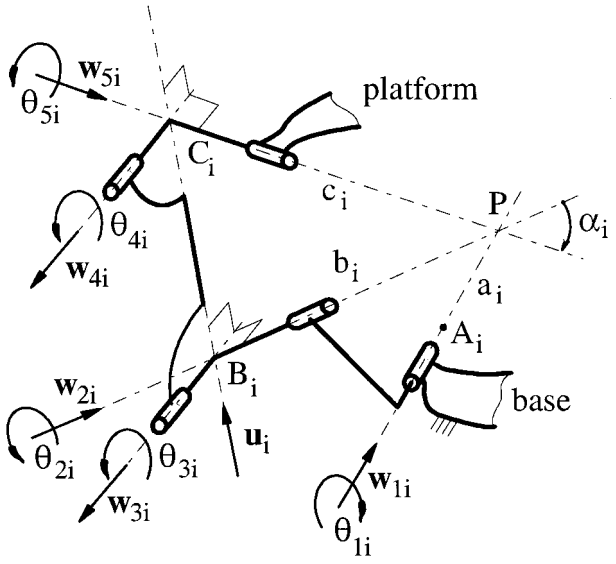


Fig. 6. i-th leg of the 3-RUU wrist.

axes fixed in the platform. The points B_i and C_i, i = 1,2,3, are the universal joints' centers and the point A_i, i = 1,2,3, are base points lying on the revolute pair axes fixed in the base.

Figure 6 shows the i-th leg, i = 1,2,3, of the 3-RUU wrist. With reference to Figure 6, by numbering the revolute pairs from the base to the platform, $\mathbf{w}_{ji}$, j = 1, . . . , 5, is the unit vector of the j-th revolute pair's axis; θ_{ji}, j = 1, . . . , 5, is the joint coordinate of the j-th revolute pair; $\mathbf{u}_i$ is the unit vector of the line through the universal joints; a_i, b_i and c_i are the constant lengths of the segments A_iP, B_iP and C_iP, respectively.

By considering separately the three legs and using the above defined notations, the platform angular velocity, ω, and the velocity $\dot{\mathbf{P}}$ of the platform's point P, measured in a reference system fixed in the base, can be written in the following three different ways (Figure 6)

$$\boldsymbol{\omega} = \dot{\theta}_{1i}\mathbf{w}_{1i} + \dot{\theta}_{2i}\mathbf{w}_{2i} + \dot{\theta}_{5i}\mathbf{w}_{5i} + (\dot{\theta}_{3i} + \dot{\theta}_{4i})\mathbf{w}_{3i} \qquad i = 1,2,3 \qquad (1.1)$$

$$\dot{\mathbf{P}} = \dot{\mathbf{C}}_i + \boldsymbol{\omega} \times (\mathbf{P} - \mathbf{C}_i) \qquad i = 1,2,3 \qquad (1.2)$$

where $\dot{\theta}_{ji}$, j = 1, . . . , 5, are time derivatives of the joint coordinates θ_{ji}, j = 1, . . . , 5, and $\dot{\mathbf{C}}_i$ is the velocity of the platform point C_i. The velocity $\dot{\mathbf{C}}_i$ can be expressed as follows (Figure 6)

$$\dot{\mathbf{C}}_i = \dot{\theta}_{1i}\mathbf{w}_{1i} \times (\mathbf{B}_i - \mathbf{A}_i) + (\dot{\theta}_{1i}\mathbf{w}_{1i} + \dot{\theta}_{2i}\mathbf{w}_{2i} + \dot{\theta}_{3i}\mathbf{w}_{3i}) \times (\mathbf{C}_i - \mathbf{B}_i) \qquad i = 1,2,3 \qquad (2)$$

Moreover, the following vector relationships hold (Figure 6)

$$(\mathbf{P} - \mathbf{C}_i) = c_i\,\mathbf{w}_{5i} \qquad i = 1,2,3 \qquad (3.1)$$

$$(\mathbf{B}_i - \mathbf{A}_i) = a_i\,\mathbf{w}_{1i} - b_i\,\mathbf{w}_{2i} \qquad i = 1,2,3 \qquad (3.2)$$

$$(\mathbf{C}_i - \mathbf{B}_i) = b_i\,\mathbf{w}_{2i} - c_i\,\mathbf{w}_{5i} \qquad i = 1,2,3 \qquad (3.3)$$

Taking into account the relationships (1.1), (3.1) and (2) in which the vectors $(\mathbf{B}_i - \mathbf{A}_i)$ and $(\mathbf{C}_i - \mathbf{B}_i)$ are substituted by the right sides of expressions (3.2) and (3.3) respectively, the relationships (1.2) become

$$\dot{\mathbf{P}} = b_i\dot{\theta}_{3i}\,\mathbf{w}_{3i} \times \mathbf{w}_{2i} + c_i\dot{\theta}_{4i}\,\mathbf{w}_{3i} \times \mathbf{w}_{5i} \qquad i = 1,2,3 \qquad (4)$$

Equations (4) constitute a linear homogeneous system of nine scalar equations in nine unknowns: $\dot{\theta}_{3i}$, $\dot{\theta}_{4i}$, i = 1,2,3, and the three components of $\dot{\mathbf{P}}$. If the coefficient matrix of the unknowns is not singular, the unique solution of system (4) will be

$$\dot{\mathbf{P}} = 0 \qquad (5.1)$$

$$\dot{\theta}_{3i} = 0 \qquad i = 1,2,3 \qquad (5.2)$$

$$\dot{\theta}_{4i} = 0 \qquad i = 1,2,3 \qquad (5.3)$$

Differentiating Eqs. (4) yields

$$\ddot{\mathbf{P}} = b_i\ddot{\theta}_{3i}\,\mathbf{w}_{3i} \times \mathbf{w}_{2i} + c_i\ddot{\theta}_{4i}\,\mathbf{w}_{3i} \times \mathbf{w}_{5i} + b_i\dot{\theta}_{3i}\,(\dot{\mathbf{w}}_{3i} \times \mathbf{w}_{2i} + \mathbf{w}_{3i} \times \dot{\mathbf{w}}_{2i}) + c_i\dot{\theta}_{4i}\,(\dot{\mathbf{w}}_{3i} \times \mathbf{w}_{5i} + \mathbf{w}_{3i} \times \dot{\mathbf{w}}_{5i}) \qquad i = 1,2,3 \qquad (6)$$

where $\ddot{\mathbf{P}}$ is the acceleration of P, whereas $\ddot{\theta}_{3i}$, $\ddot{\theta}_{4i}$, $\dot{\mathbf{w}}_{2i}$, $\dot{\mathbf{w}}_{3i}$ and $\dot{\mathbf{w}}_{5i}$ are time derivatives of $\dot{\theta}_{3i}$, $\dot{\theta}_{4i}$, $\mathbf{w}_{2i}$, $\mathbf{w}_{3i}$ and $\mathbf{w}_{5i}$ respectively.

By taking into account relationships (5.2) and (5.3), Eqs. (6) become

$$\ddot{\mathbf{P}} = b_i\ddot{\theta}_{3i}\,\mathbf{w}_{3i} \times \mathbf{w}_{2i} + c_i\ddot{\theta}_{4i}\,\mathbf{w}_{3i} \times \mathbf{w}_{5i} \qquad i = 1,2,3 \qquad (7)$$

Equations (7) constitute a linear homogeneous system of nine scalar equations in nine unknowns: $\ddot{\theta}_{3i}$, $\ddot{\theta}_{4i}$, i = 1,2,3, and the three components of $\ddot{\mathbf{P}}$. In system (7), the coefficient matrix of the unknowns is equal to the one of system (4). When this matrix is not singular, system (7) admits the following unique solution

$$\ddot{\mathbf{P}} = 0 \qquad (8.1)$$

$$\ddot{\theta}_{3i} = 0 \qquad i = 1,2,3 \qquad (8.2)$$

$$\ddot{\theta}_{4i} = 0 \qquad i = 1,2,3 \qquad (8.3)$$

Relationships (5.1) and (8.1) make the following conclusive statement possible:

> STATEMENT: If a 3-RUU mechanism performs an elementary motion starting from a not singular configuration matching the 3-RUU wrist's geometric condition, it will reach a new configuration in which the platform point P still is in the initial position (relationship (5.1)) and at rest (relationship (8.1)), i.e., in the 3-RUU wrist's geometric conditions are matched by the new configuration, too.

As a consequence, a 3-RUU wrist's platform has to perform a sequence of infinitesimal spherical motions with center P, i.e., it can only perform finite spherical motions with center P, until the 3-RUU wrist reaches a singular configuration. Since the 3-RUU wrist makes the platform perform finite spherical motions, it is a parallel wrist.

The 3-RUU wrist position analysis focused on the configurations keeping the platform point P at rest (Figure 5) is identical to the position analysis of the Gosselin and Angeles' wrist[7,11–13] (Figure 1) since relationships (5.2), (5.3), (8.2) and (8.3) state that the joint coordinates θ_{3i} and θ_{4i}, i = 1,2,3, do not change during motion. Therefore, with reference to the demonstrations reported in reference 12, on one side, the 3-RUU wrist admits at most two values of the actuated joint coordinates θ_{1i}, i = 1,2,3, compatible with a

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given platform orientation; on the other side, it admits at most eight platform orientations compatible with a given set of θ_{1i} values.

3. SINGULARITY CONDITIONS

The 3-RUU wrist singular configurations come out by analyzing the relationship between the platform velocities ω and $\dot{\mathbf{P}}$, on one side, and the actuated joint coordinates' time derivatives $\dot{\theta}_{1i}$, i = 1,2,3, on the other side. This relationship can be obtained from the linear system composed of the Eqs. (1.1) and (4) by linearly eliminating the unknowns $\dot{\theta}_{2i}$, $\dot{\theta}_{3i}$, $\dot{\theta}_{4i}$ and $\dot{\theta}_{5i}$, i = 1,2,3. In order to obtain this elimination the below reported procedure can be used.

The dot product of the i-th Eq. (4) by $\mathbf{w}_{5i}$ yields

$$\dot{\theta}_{3i} = \frac{\dot{\mathbf{P}} \cdot \mathbf{w}_{5i}}{b_i[\mathbf{w}_{3i} \cdot (\mathbf{w}_{2i} \times \mathbf{w}_{5i})]} \qquad i = 1,2,3 \tag{9}$$

The dot product of the i-th Eq. (4) by $\mathbf{w}_{2i}$ yields

$$\dot{\theta}_{4i} = -\frac{\dot{\mathbf{P}} \cdot \mathbf{w}_{2i}}{c_i[\mathbf{w}_{3i} \cdot (\mathbf{w}_{2i} \times \mathbf{w}_{5i})]} \qquad i = 1,2,3 \tag{10}$$

Taking into account expressions (9) and (10), Eqs. (1.1) become

$$\omega + \mathbf{w}_{3i} \left\{ \frac{(b_i \mathbf{w}_{2i} - c_i \mathbf{w}_{5i}) \cdot \dot{\mathbf{P}}}{b_i c_i [\mathbf{w}_{3i} \cdot (\mathbf{w}_{2i} \times \mathbf{w}_{5i})]} \right\} = \dot{\theta}_{1i}\, \mathbf{w}_{1i} + \dot{\theta}_{2i}\, \mathbf{w}_{2i} + \dot{\theta}_{5i}\, \mathbf{w}_{5i} \qquad i = 1,2,3 \tag{11}$$

Dot multiplying the i-th Eq. (11) by $\mathbf{w}_{2i} \times \mathbf{w}_{5i}$ and the i-th Eq. (4) by $\mathbf{w}_{3i}$ gives the following six equation system

$$\mathbf{n}_i \cdot \omega + \frac{(b_i \mathbf{w}_{2i} - c_i \mathbf{w}_{5i})}{b_i c_i} \cdot \dot{\mathbf{P}} = \dot{\theta}_{1i}\, \mathbf{w}_{1i} \cdot \mathbf{n}_i \qquad i = 1,2,3 \tag{12.1}$$

$$\mathbf{w}_{3i} \cdot \dot{\mathbf{P}} = 0 \qquad i = 1,2,3 \tag{12.2}$$

where

$$\mathbf{n}_i = \mathbf{w}_{2i} \times \mathbf{w}_{5i} \qquad i = 1,2,3 \tag{13}$$

System (12) is the sought-after relationship between the platform velocities ω and $\dot{\mathbf{P}}$, on one side, and the actuated joint coordinates' time derivatives $\dot{\theta}_{1i}$, i = 1,2,3, on the other side.

Since Eqs. (12.2) contain just $\dot{\mathbf{P}}$, they can be analyzed and solved separately.

When system (12.2) is singular the velocity $\dot{\mathbf{P}}$ is not determined and can be different from zero, i.e., the spherical constraint between platform and base fails. The 3-RUU wrist configurations making system (12.2) singular will be called translation singularities.

The i-th Eq. (12.2) expresses analytically the kinematic constraint imposed by the i-th leg of type RUU (Figure 6) on the platform motion. With reference to Figure 6, the i-th Eq. (12.2) states that one leg of type RUU, in which the unit vectors $\mathbf{w}_{1i}$, $\mathbf{w}_{2i}$ and $\mathbf{w}_{5i}$ converge towards a unique point and the unit vectors $\mathbf{w}_{3i}$ and $\mathbf{w}_{4i}$ are parallel to each other and perpendicular to the unit vector $\mathbf{u}_i$, forbids the displacement along the $\mathbf{w}_{3i}$ direction of the platform point P which instantaneously coincides with the intersection of the directions of the three unit vectors $\mathbf{w}_{1i}$, $\mathbf{w}_{2i}$ and $\mathbf{w}_{5i}$. When the point P goes to infinity[6], i.e. the unit vectors $\mathbf{w}_{1i}$, $\mathbf{w}_{2i}$ and $\mathbf{w}_{5i}$ are all parallel, the forbidden displacement of P along $\mathbf{w}_{3i}$ direction becomes a forbidden platform rotation around the direction of the free unit vector $\mathbf{w}_{2i} \times \mathbf{w}_{3i}$. In this case Eqs. (12) have to be replaced by other relationships[6] since they were obtained on condition that the point P exists.

The coefficient matrix of $\dot{\mathbf{P}}$ in system (12.2) has the following expression

$$\mathbf{M} = [\mathbf{w}_{31}, \mathbf{w}_{32}, \mathbf{w}_{33}]^T \tag{14}$$

where $(\cdot)^T$ denotes the transpose of $(\cdot)$. System (12.2) becomes singular when the determinant of $\mathbf{M}$, det($\mathbf{M}$), vanishes. By considering the definition (14) of $\mathbf{M}$, the following expression of det($\mathbf{M}$) arises:

$$\det(\mathbf{M}) = \mathbf{w}_{31} \cdot (\mathbf{w}_{32} \times \mathbf{w}_{33}) \tag{15}$$

Hence the analytic condition that identifies the translation singularities is

$$\mathbf{w}_{31} \cdot (\mathbf{w}_{32} \times \mathbf{w}_{33}) = 0 \tag{16}$$

When condition (16) is not matched, system (12.2) admits the unique solution $\dot{\mathbf{P}} = 0$ and Eqs. (12.1) become

$$\mathbf{n}_i \cdot \omega = \dot{\theta}_{1i}\, \mathbf{w}_{1i} \cdot \mathbf{n}_i \qquad i = 1,2,3 \tag{17}$$

If Eqs. (17) are linearly dependent, the platform angular velocity, ω, will not be determined. The 3-RUU wrist configurations making system (17) singular will be called rotation singularities. The coefficient matrix of ω in system (17) is

$$\mathbf{N} = [\mathbf{n}_1, \mathbf{n}_2, \mathbf{n}_3]^T \tag{18}$$

System (17) is singular when the determinant of $\mathbf{N}$, det($\mathbf{N}$), vanishes. Taking into account definition (18) yields

$$\det(\mathbf{N}) = \mathbf{n}_1 \cdot (\mathbf{n}_2 \times \mathbf{n}_3) \tag{19}$$

On the other hand, definitions (13) of the vectors $\mathbf{n}_i$, i = 1,2,3, and the geometric relationships among $\mathbf{w}_{3i}$, $\mathbf{w}_{2i}$ and $\mathbf{w}_{5i}$ (see Figure 6) yield

$$\mathbf{n}_i = -\mathbf{w}_{3i} \sin(\alpha_i) \tag{20}$$

where α_i, i = 1,2,3, is the constant angle formed by the directions of $\mathbf{w}_{2i}$ and $\mathbf{w}_{5i}$. Since the angles α_i, i = 1,2,3, are constant, relationships (19) and (20) make it possible to conclude that the analytic condition making det($\mathbf{N}$) vanish is condition (16) yet. As a consequence, if system (17) is singular, system (12.1) will be singular, too. Hence, the 3-RUU wrist configurations matching condition (16) are both translation singularities ($\dot{\mathbf{P}}$ undetermined) and rotation singularities (ω undetermined).

Condition (16) is satisfied when the unit vectors $\mathbf{w}_{3i}$, i = 1,2,3, are all parallel to a single plane.

From an analytic point of view, condition (16) is an equation that contains the three orientation parameters of the platform and the links' geometric constants. If the link geometry is given, condition (16) will become an equation in three unknowns: the three orientation parameters of the platform. Hence, the values of the unknowns, matching Eq.

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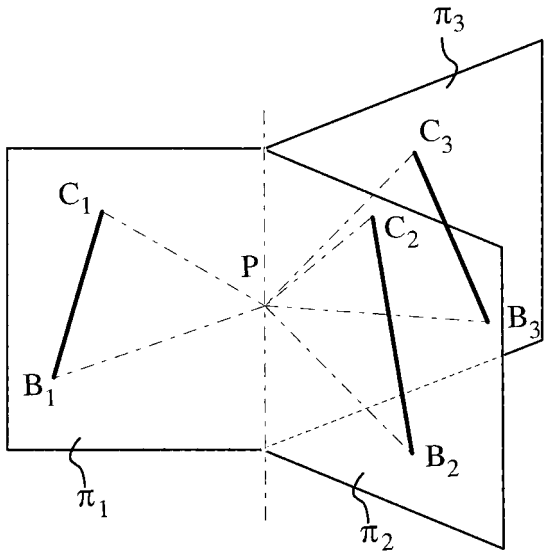


Fig. 7. Singularity condition.

(16), can be represented in a three-dimensional Cartesian diagram, having the unknown variables on the axes, with points lying on a fixed surface (singularity surface).

From a geometric point of view, condition (16) is satisfied when the three planes the triangles B_iPC_i, $i = 1,2,3$, lie on (see Figs. 5 and 6) have a straight line through P as common intersection (Figure 7). In fact, in this case, the unit vectors $\mathbf{w}_{3i}$, $i = 1,2,3$, are all perpendicular to the above mentioned line and are all parallel to every single plane perpendicular to the same line.

4. CONCLUSIONS

A new parallel wrist called 3-RUU wrist has been presented. The 3-RUU wrist is not overconstrained; it has three equal legs just involving revolute pairs and actuators adjacent to the frame. In addition, the 3-RUU wrist uses an architecture (3-RUU) already employed to obtain manipulators that make the end effector translate.

Moreover, the 3-RUU wrist kinematic analysis has been presented. This analysis shows that the new parallel wrist can reach singular configurations (translation singularities) in which the spherical constraint between end effector and frame fails.

Finally, the singularity condition that makes it possible to find all the 3-RUU wrist singular configurations has been obtained in explicit form and it has been geometrically interpreted.

Acknowledgments

The financial support of the Italian MURST is gratefully acknowledged.

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