Feedback

On 'Deleting Digits', Ioulia N. Baoulina, Martin Kreh, Jörn Steuding write:

The authors of [1] regret that there was an error in the statement of Theorem 5 and some inaccuracies in the proof. The revised statement and proof are given below. A version of our paper with corrected proof is also available in the arXiv (see http://arxiv.org/abs/1607.01548).

Theorem 5: Let ψ be the Dedekind ψ-function and

$$\psi(\mathbb{N}) = \{n \in \mathbb{N} \mid \exists x \in \mathbb{N} : \psi(x) = n\}.$$

Then

$$M(\psi(\mathbb{N})) = \{1,3,4,6,8,20,72,90,222,252,500,522,552,570,\\ 592,750,770,992,7000, \underbrace{55\ldots50}_{69}\}.$$

Proof:

First note that the elements given above are pairwise incomparable and we have

$$\begin{aligned}
\psi(1) &= 1, & \psi(2) &= 3, & \psi(3) &= 4, & \psi(5) &= 6,\\
\psi(7) &= 8, & \psi(19) &= 20, & \psi(71) &= 72, & \psi(89) &= 90,\\
\psi(146) &= 222, & \psi(251) &= 252, & \psi(499) &= 500, & \psi(521) &= 522,\\
\psi(411) &= 552, & \psi(569) &= 570, & \psi(511) &= 592, & \psi(625) &= 750,\\
\psi(769) &= 770, & \psi(991) &= 992, & \psi(6631) &= 7000
\end{aligned}$$

and

$$\psi\left(\underbrace{55\ldots549}_{68}\right) = \underbrace{55\ldots50}_{69},$$

since $\underbrace{55\ldots549}_{68}$ is a prime. All these numbers belong to $\psi(\mathbb{N})$.

Now let $n \in \psi(\mathbb{N})$ be arbitrary with at least two digits. If $d \lhd n$ with $d \in \{1,3,4,6,8\}$, then $n \notin M(\psi(\mathbb{N}))$. So n only contains the digits 2,5,7,9,0. It follows from the product formula $\psi(m) = \prod_{p \mid m}(p+1)p^{k_p-1}$ defining ψ that $\psi(m)$ is even for $m > 2$. First we note that 22, 50, 52, 70, 92, 292, 502, 550, 700, 922, 952 $\notin \psi(\mathbb{N})$. Now suppose n has at least four digits. Then one of the following cases holds:

- n ends with a 2 and contains a 7. Then $72 \lhd n$.
- n ends with a 2 and contains a zero and does not contain a 7. Then either $20 \lhd n$ or $90 \lhd n$ or $502 \lhd n$. In the latter case n contains one of the following strings:

  5002, 2502, 5202, 5022, 5502, 5052, 9502, 5902, 5092.

  Then $20 \lhd n$ or $90 \lhd n$ or $500 \lhd n$ or $522 \lhd n$ or $552 \lhd n$ or $592 \lhd n$.
- n ends with a 2 and contains only the digits 2, 5 and 9. Then $222 \lhd n$ or $252 \lhd n$ or $522 \lhd n$ or $552 \lhd n$ or $992 \lhd n$.
- n ends with a zero and contains a 2 or a 9. Then $20 \lhd n$ or $90 \lhd n$.

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- n ends with a zero and contains a 7 and a 5. Then $570 \lhd n$ or $750 \lhd n$.
- n ends with a zero and contains only the digits 7 and 0. Then $770 \lhd n$ or $7000 \lhd n$.
- n ends with a zero and contains only the digits 5 and 0. Then $500 \lhd n$ or $550 \lhd n$.

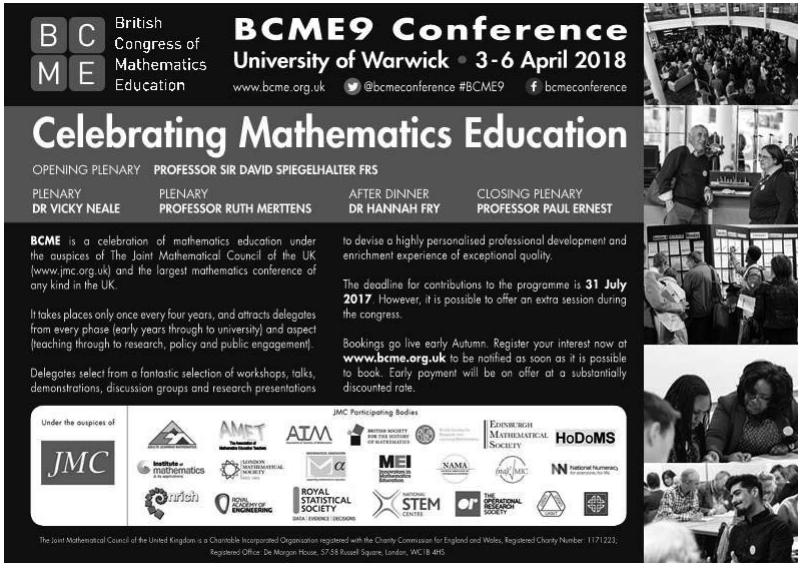
Thus, the only numbers n that need to be examined are of the form $5\ldots50$. It suffices to show that if $n = \underbrace{55\ldots5}_{\ell}0$ with $3 \leqslant \ell \leqslant 68$, then $n \notin \psi(\mathbb{N})$.

We note that, if m has k distinct odd prime factors, then $2^k \mid \psi(m)$. In our case, $n \equiv 2 \bmod 4$, so if there is $m \in \mathbb{N}$ with $\psi(m) = n$, then $m = p^k$ or $m = 2p^k$ with $p \in \mathbb{P}$ odd. Hence either $p^{k-1}(p+1) = n$ or $3p^{k-1}(p+1) = n$ with $k \in \mathbb{N}$ and odd $p \in \mathbb{P}$. Note that $4 \nmid (p+1)$. A computer search shows that if $p^{k-1} \mid n$ with $k \geqslant 3$ and $4 \nmid (p+1)$, then $k = 3$ and $p = 5$. This implies that either $n = 150$ or $n = 450$, which is a contradiction. Next, one can easily verify that there is no prime p with $n = p(p+1)$ or $3p(p+1)$, and so $k \neq 2$. Finally, note that neither $n - 1$ nor $\frac{1}{3}n - 1$ is a prime, which yields $k \neq 1$. This concludes the proof.

Reference

1. Ioulia N. Baoulina, Martin Kreh, Jörn Steuding, Deleting digits, *Math. Gaz.* **101** (March 2017) pp. 60-68.

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