Appendix C Massive Loop Integrals

The list of integrals given in this appendix is sufficient for the calculation of one-loop diagrams with vanishing external momenta. The latter are only kept in the calculation of self-energy diagrams on the external lines, like the ones in Figure 6.8. All formulas have been derived by the author and checked by Tobias Theil.

C.1 Integrals with Two Propagators

$$I_1(m,M) = \int \frac{d^Dk}{(2\pi)^D} \frac{m}{[(k+p)^2-m^2][k^2-M^2]} = m\frac{i}{16\pi^2}\left[\frac{2}{\bar{\varepsilon}} + \frac{3}{2} + F_1(x) + F_2(x)\right], \tag{C.1}$$

$$I_2(m,M) = \int \frac{d^Dk}{(2\pi)^D} \frac{(p+k)_\mu}{[(k+p)^2-m^2][k^2-M^2]} = p_\mu\frac{i}{16\pi^2}\left[\frac{1}{\bar{\varepsilon}} + \frac{3}{4} + F_2(x)\right], \tag{C.2}$$

where p_μ is an external momentum, and we set $p^2 = 0$. $F_{1,2}(x)$ are given by

$$F_1(x) = -\frac{1}{2(1-x)^2}\left[x^2\log x - 2x\log x - x(1-x)\right], \tag{C.3}$$

$$F_2(x) = -\frac{1}{2(1-x)^2}\left[x^2\log x + (1-x)\right], \tag{C.4}$$

and ($D = 4 - 2\varepsilon$)

$$x = \frac{m^2}{M^2}, \qquad \frac{1}{\bar{\varepsilon}} = \frac{1}{2\varepsilon} + \frac{1}{2}\left[\log 4\pi - \gamma_E + \log\left(\frac{\mu^2}{M^2}\right)\right]. \tag{C.5}$$

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C.2 Integrals with Three Propagators

$$I_3(m_1,m_2,M) = \int \frac{d^4k}{(2\pi)^4} \frac{1}{[k^2-m_1^2][k^2-m_2^2][k^2-M^2]} \tag{C.6}$$

$$= \frac{i}{16\pi^2}\frac{1}{M^2}\left[\frac{x_1\log x_1}{(1-x_1)(x_1-x_2)} + \frac{x_2\log x_2}{(1-x_2)(x_2-x_1)}\right], \tag{C.7}$$

where

$$x_i = \frac{m_i^2}{M^2}, \qquad x = \frac{m^2}{M^2}. \tag{C.8}$$

Special cases:

$$I_3(m,m,M) = \frac{i}{16\pi^2}\frac{1}{M^2}\left[\frac{\log x}{(1-x)^2} + \frac{1}{(1-x)}\right], \tag{C.9}$$

$$I_3(m,M,M) = -\frac{i}{16\pi^2}\frac{1}{M^2}\left[\frac{x\log x}{(1-x)^2} + \frac{1}{(1-x)}\right], \tag{C.10}$$

$$I_3(M,M,M) = -\frac{i}{32\pi^2}\frac{1}{M^2}. \tag{C.11}$$

$$I_4(m_1,m_2,M) = \int \frac{d^Dk}{(2\pi)^D} \frac{k_\mu k_\nu}{[k^2-m_1^2][k^2-m_2^2][k^2-M^2]} \tag{C.12}$$

$$= \frac{ig_{\mu\nu}}{32\pi^2}\left[\frac{1}{\bar{\varepsilon}} + \frac{3}{4} + \frac{x_1^2\log x_1}{2(1-x_1)(x_1-x_2)} + \frac{x_2^2\log x_2}{2(1-x_2)(x_2-x_1)}\right]. \tag{C.13}$$

Special cases:

$$I_4(m,m,M) = \frac{ig_{\mu\nu}}{32\pi^2}\left[\frac{1}{\bar{\varepsilon}} + \frac{3}{4} + F_1(x)\right], \tag{C.14}$$

$$I_4(m,M,M) = \frac{ig_{\mu\nu}}{32\pi^2}\left[\frac{1}{\bar{\varepsilon}} + \frac{3}{4} + F_2(x)\right], \tag{C.15}$$

$$I_4(M,M,M) = \frac{ig_{\mu\nu}}{32\pi^2}\left[\frac{1}{\bar{\varepsilon}}\right], \tag{C.16}$$

where $F_1(x)$ and $F_2(x)$ are given in (C.3) and (C.4), respectively.

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C.3 Integrals with Four Propagators

$$I_5(m_1,m_2,M_1,M_2) = \int \frac{d^4k}{(2\pi)^4}\frac{1}{[k^2-m_1^2][k^2-m_2^2][k^2-M_1^2][k^2-M_2^2]} \tag{C.17}$$

$$= -\frac{i}{16\pi^2}\frac{1}{M_1^4}\left[F(x_1;x_2,\eta)+F(x_2;x_1,\eta)+F(\eta;x_1,x_2)\right], \tag{C.18}$$

where

$$F(x;y,z) = \frac{x\log x}{(x-1)(x-y)(x-z)}, \qquad x_i = \frac{m_i^2}{M_1^2}, \qquad \eta = \frac{M_2^2}{M_1^2}. \tag{C.19}$$

Special cases:

$$I_5(m_1,m_2,M,M) = -\frac{i}{16\pi^2}\frac{1}{M^4}\left[H(x_1,x_2)+H(x_2,x_1)+\frac{1}{(1-x_1)(1-x_2)}\right], \tag{C.20}$$

$$I_5(m,m,M,M) = -\frac{i}{16\pi^2}\frac{1}{M^4}\left[\frac{(1+x)\log x}{(1-x)^3}+\frac{2}{(1-x)^2}\right], \tag{C.21}$$

where

$$H(x_1,x_2) = \frac{x_1\log x_1}{(1-x_1)^2(x_1-x_2)}. \tag{C.22}$$

$$I_6(m_1,m_2,M_1,M_2) = \int \frac{d^4k}{(2\pi)^4}\frac{k_\mu k_\nu}{[k^2-m_1^2][k^2-m_2^2][k^2-M_1^2][k^2-M_2^2]} \tag{C.23}$$

$$= -\frac{ig_{\mu\nu}}{64\pi^2}\frac{1}{M_1^2}\left[\tilde{F}(x_1;x_2,\eta)+\tilde{F}(x_2;x_1,\eta)+\tilde{F}(\eta;x_1,x_2)\right], \tag{C.24}$$

where

$$\tilde{F}(x;y,z) = \frac{x^2\log x}{(x-1)(x-y)(x-z)}, \qquad x_i = \frac{m_i^2}{M_1^2}, \qquad \eta = \frac{M_2^2}{M_1^2}. \tag{C.25}$$

Special cases:

$$I_6(m_1,m_2,M,M) = -\frac{ig_{\mu\nu}}{64\pi^2}\frac{1}{M^2}\left[\tilde{H}(x_1,x_2)+\tilde{H}(x_2,x_1)+\frac{1}{(1-x_1)(1-x_2)}\right], \tag{C.26}$$

$$I_6(m,m,M,M) = -\frac{ig_{\mu\nu}}{64\pi^2}\frac{1}{M^2}\left[\frac{2x\log x}{(1-x)^3}+\frac{1+x}{(1-x)^2}\right], \tag{C.27}$$

where

$$\tilde{H}(x_1,x_2) = \frac{x_1^2\log x_1}{(1-x_1)^2(x_1-x_2)}. \tag{C.28}$$

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C.4 More Complicated Integrals

C.4.1 Preliminaries

The integrals in previous sections have the general structure

$$I = \int \frac{d^D k}{(2\pi)^D} f(k^2)\,, \tag{C.29}$$

with the integrand depending only on k^2. This is the case of one-loop diagrams in which external momenta are set to zero. Such integrals are much easier to evaluate than those with external momenta like self-energy diagrams or vertex diagrams, which we calculated in the context of renormalization of QCD and QED in Chapter 4. In particular shifts in momenta and Feynman integrals can be avoided. Integrating over angular variables one finds first

$$I = \frac{i}{(4\pi)^{D/2}} \frac{1}{\Gamma(D/2)} \int_0^\infty ds s^{1-\varepsilon} f(k^2 = -s) \tag{C.30}$$

with $D = 4 - 2\varepsilon$. It is useful to perform subsequently a change of variables:

$$s = rM^2, \qquad ds s^{1-\varepsilon} = dr r^{1-\varepsilon} (M^2)^{2-\varepsilon} \tag{C.31}$$

so that the calculation is reduced to the evaluation of the following r-integrals

$$R = \int_0^\infty dr F(r, x_i), \qquad x_i = \frac{m_i^2}{M^2} \tag{C.32}$$

with m_i denoting the masses of particles different than the one having the mass M. These particles can be scalars, fermions, and gauge bosons. Depending on the diagram considered, one obtains a function

$$F(r, x_i) = \frac{r^{n-\varepsilon}}{[r+a][r+b]...[r+e]}\,, \tag{C.33}$$

where in most cases the parameters $(a, b, c, ..)$ are just equal to different x_i or 1. See examples at the end of this appendix. The number of factors in the denominator could be larger than four considered in Appendix C.3. This is the case if calculations are done not in the Feynman gauge but in R_ξ gauges or unitary gauge. For these cases the list of integrals in Appendix C is insufficient. Therefore, following we give a list of r-integrals that should allow the evaluation of any box and Z-penguin diagram in any gauge and in any theory at one-loop level. We will also give integrals relevant for self-energy diagrams.

The basic technique to evaluate all integrals given here is to do first the decomposition

$$\frac{r^n}{[r+a][r+b]\cdots[r+e]} = \frac{A}{[r+a]} + \frac{B}{[r+b]} + ...\frac{E}{[r+e]}, \tag{C.34}$$

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where n is smaller than the number of factors in the denominator and

$$A = \frac{(-a)^n}{(b-a)(c-a)(d-a)(e-a)}\,, \tag{C.35}$$

$$B = \frac{(-b)^n}{(a-b)(c-b)(d-b)(e-b)}\,, \tag{C.36}$$

$$E = \frac{(-e)^n}{(a-e)(b-e)(c-e)(d-e)} \tag{C.37}$$

with analogous expressions for other terms. They have, for instance, $(-c)^n$ and $(-d)^n$ in the numerator.

Inserting (C.34) into (C.32) one obtains a sum of terms each multiplied by $r^{-\varepsilon}$. They are evaluated by using

$$\int_0^\infty dr \frac{r^{-\varepsilon}}{[r+a]} = \frac{1}{\varepsilon} - \gamma_E - \ln a\,. \tag{C.38}$$

To give compact expressions for the integrals listed here, we introduce the functions:

$$G_1(x) = \frac{x \ln x}{(y_1 - x)}, \qquad G_2(x) = \frac{x \ln x}{(y_1 - x)(y_2 - x)}\,, \tag{C.39}$$

$$G_3(x) = \frac{x \ln x}{(y_1 - x)(y_2 - x)(y_3 - x)}\,, \qquad G_4(x) = \frac{x^2 \ln x}{(y_1 - x)(y_2 - x)(y_3 - x)(y_4 - x)}\,, \tag{C.40}$$

$$G_5(x) = \frac{x^4 \ln x}{(y_1 - x)(y_2 - x)(y_3 - x)(y_4 - x)(y_5 - x)}\,. \tag{C.41}$$

Here x stands for one particular variable in the set $\{a, b, c, d, ..\}$, and y_i denote the remaining variables in this set. As an example, $G_5(c)$ given here reads

$$G_5(c) = \frac{c^4 \ln c}{(a-c)(b-c)(d-c)(e-c)(f-c)}\,. \tag{C.42}$$

C.4.2 Integrals for Box Diagrams

These integrals are defined as follows:

$$\mathcal{I}_1^{(n)} \equiv \int_0^\infty dr \frac{r^{n-\varepsilon}}{[r+a][r+b][r+c][r+d]}, \qquad n = 1, 2\,, \tag{C.43}$$

$$\mathcal{I}_2^{(n)} \equiv \int_0^\infty dr \frac{r^{n-\varepsilon}}{[r+a][r+b][r+c][r+d][r+e]}, \qquad n = 2, 3\,, \tag{C.44}$$

$$\mathcal{I}_3^{(n)} \equiv \int_0^\infty dr \frac{r^{n-\varepsilon}}{[r+a][r+b][r+c][r+d][r+e][r+f]}, \qquad n = 4\,. \tag{C.45}$$

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We find

$$\mathcal{I}_1^{(1)} = G_3(a) + G_3(b) + G_3(c) + G_3(d)\,, \tag{C.46}$$

$$\mathcal{I}_1^{(2)} = -aG_3(a) - bG_3(b) - cG_3(c) - dG_3(d)\,, \tag{C.47}$$

$$\mathcal{I}_2^{(2)} = -G_4(a) - G_4(b) - G_4(c) - G_4(d) - G_4(e)\,, \tag{C.48}$$

$$\mathcal{I}_2^{(3)} = aG_4(a) + bG_4(b) + cG_4(c) + dG_4(d) + eG_4(e)\,, \tag{C.49}$$

$$\mathcal{I}_3^{(4)} = -G_5(a) - G_5(b) - G_5(c) - G_5(d) - G_5(e) - G_5(f)\,. \tag{C.50}$$

C.4.3 Integrals for Z-Penguins

These integrals are defined as follows:

$$\mathcal{J}_1^{(n)} \equiv \int_0^\infty dr \frac{r^{n-\varepsilon}}{[r+a][r+b][r+c]}, \qquad n = 1,2,3\,, \tag{C.51}$$

$$\mathcal{J}_2^{(n)} \equiv \int_0^\infty dr \frac{r^{n-\varepsilon}}{[r+a][r+b][r+c][r+d]}, \qquad n = 1,2,3,4\,, \tag{C.52}$$

$$\mathcal{J}_3^{(n)} \equiv \int_0^\infty dr \frac{r^{n-\varepsilon}}{[r+a][r+b][r+c][r+d][r+e]}, \qquad n = 4\,. \tag{C.53}$$

We find

$$\mathcal{J}_1^{(1)} = G_2(a) + G_2(b) + G_2(c)\,, \tag{C.54}$$

$$\mathcal{J}_1^{(2)} = -aG_2(a) - bG_2(b) - cG_2(c) + \frac{1}{\varepsilon}\,, \tag{C.55}$$

$$\mathcal{J}_1^{(3)} = a^2G_2(a) + b^2G_2(b) + c^2G_2(c) - \frac{(a+b+c)}{\varepsilon}\,, \tag{C.56}$$

$$\mathcal{J}_2^{(1)} = \mathcal{I}_1^{(1)}, \qquad \mathcal{J}_2^{(2)} = \mathcal{I}_1^{(2)}, \qquad \mathcal{J}_3^{(3)} = \mathcal{I}_2^{(3)}, \tag{C.57}$$

$$\mathcal{J}_2^{(3)} = a^2G_3(a) + b^2G_3(b) + c^2G_3(c) + d^2G_4(d) + \frac{1}{\varepsilon}\,, \tag{C.58}$$

$$\mathcal{J}_2^{(4)} = -a^3G_3(a) - b^3G_3(b) - c^3G_3(c) - d^3G_3(d) - \frac{(a+b+c+d)}{\varepsilon}\,, \tag{C.59}$$

$$\mathcal{J}_3^{(4)} = -a^2G_4(a) - b^2G_4(b) - c^2G_4(c) - d^2G_4(d) - e^2G_4(e) + \frac{1}{\varepsilon}\,. \tag{C.60}$$

C.4.4 Integrals for Self-Energies

These integrals are defined as follows:

$$\mathcal{K}_1^{(n)} \equiv \int_0^\infty dr \frac{r^{n-\varepsilon}}{[r+a][r+b]}, \qquad n = 1,2,3\,. \tag{C.61}$$

$$\mathcal{K}_2^{(n)} \equiv \int_0^\infty dr \frac{r^{n-\varepsilon}}{[r+a][r+b][r+c]}, \qquad n = 2\,. \tag{C.62}$$

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We find

$$\mathcal{K}_1^{(1)} = G_1(a) + G_1(b) + \frac{1}{\varepsilon}\,, \qquad \mathcal{K}_2^{(2)} = \mathcal{J}_1^{(2)}\,, \tag{C.63}$$

$$\mathcal{K}_1^{(2)} = -aG_1(a) - bG_1(b) - \frac{(a+b)}{\varepsilon}\,, \tag{C.64}$$

$$\mathcal{K}_1^{(3)} = a^2G_1(a) + b^2G_1(b) + \frac{(a^2+b^2+ab)}{\varepsilon}, \tag{C.65}$$

$$\mathcal{K}_2^{(2)} = \mathcal{J}_1^{(2)}. \tag{C.66}$$

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