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Reflection of an electromagnetic pulse incident on a magnetoactive nonlinear medium

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Abstract. The effect of an external magnetic field $B_{\rm ext}$ on the propagation of an electromagnetic wave pulse incident on a nonlinear medium is studied. It is shown that all the incident energy can be reflected in a large domain of $B_{\rm ext}$ (self-screening), if the incident wave is in pulsed form. Having a complicated behavior, the reflection coefficient shows total transmission or self-screening in the intermediate range of the external magnetic field.

1. Introduction

Much work has been carried out studying the interaction of high-energy laser pulses with nonlinear materials [1]. In these materials, nonlinear processes such as parametric instabilities, self-focusing, specular reflection, and self-screening appear owing to the propagation of powerful laser pulses through them. In an earlier work [2], we investigated the reflection and transmission of powerful laser pulses through a nonlinear medium in the absence of an external magnetic field, and we looked at self-screening effects, time-dependent reflection coefficients, and reversed polarization of the reflected wave. In the present paper, the effect of an external magnetic field on the transmission and reflection of a high-energy pulsed electromagnetic wave through a nonlinear material is studied. Since the dielectric response function shows the behavior of the nonlinear material due to the incident electromagnetic waves [3], we want to calculate the displacement vector for such materials. In this type of investigation, the displacement vector is usually decomposed into two or more terms, where the first term represents the linear part and the others represent different orders of the nonlinear terms. However, in this article, we consider the displacement vector $\mathbf{D}$ to be in the form $\phi[\mathscr{F},\mathscr{G}]$,where $\mathscr{F}$ and $\mathscr{G}$ are functions of E and B, and using this method we generalize the work of [2] to calculate the reflection coefficient R, and show that it is a function of time [4]. Other notable results of this work are as follows:

(a) The amplitude and polarization of the reflected wave vary with time.

(b) There is complete reflection in the case of a large magnetic field (self-screening).

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(c) There is complicated behavior in the intermediate domain of the external magnetic field.

(d) There is consistency with the result of [2] for very small values of B_{ext}.

(e) There is radiation from the nonlinear medium in the presence of an external magnetic field; this behavior also occurs in nonstationary plasmas [4–6].

2. Calculations

We start by considering a linearly polarized transverse electromagnetic wave propagating in the z direction into a lossless magnetoactive nonlinear dielectric. The direction of the external magnetic field is taken to be the y axis for simplicity. It is assumed that the displacement vector $\mathbf{D}$ is a function of the electric and magnetic fields, which are both functions of z and t. The Maxwell equations

$$\boldsymbol{\nabla} \times \mathbf{E}(z,t) + \frac{\partial \mathbf{B}_t(z,t)}{\partial t} = 0, \tag{1}$$

and

$$\boldsymbol{\nabla} \times \mathbf{B}_t(z,t) - \frac{1}{c^2}\frac{\partial \mathbf{D}(E,B_t)}{\partial t} = 0, \tag{2}$$

where $\mathbf{B}_t$ is the sum of the external and internal magnetic fields, with a hodograph transformation [5], are used to calculate the propagation of the electromagnetic wave through the nonlinear medium. In addition, using the results of [2] to introduce a function $\Psi = \Psi(E, B_t)$ through $t \equiv -\partial\Psi/\partial E$, and $z \equiv \partial\Psi/\partial B_t$, we obtain the following equation:

$$\frac{\partial^2\Psi}{\partial E^2} - \frac{\kappa^2}{c^2}\frac{\partial^2\Psi}{\partial B_t^2} = 0, \tag{3}$$

where

$$\kappa^2 = \frac{\partial D}{\partial E} - \frac{\dfrac{\partial^2\Psi}{\partial E \partial B_t}}{\dfrac{\partial^2\Psi}{\partial B_t^2}}\frac{\partial D}{\partial B_t}. \tag{4}$$

Changing the variables E and Ψ through $\tau \equiv (1/E_n)\int_0^E \varrho(E)\,dE$ and $\Psi \equiv S/\sqrt{\varrho}$, we obtain an equation that governs the propagation of the electromagnetic wave through the magnetoactive nonlinear medium:

$$\frac{\partial^2 S}{\partial \tau^2} - \frac{\kappa^2 E_n^2}{c^2\varrho^2}\frac{\partial^2 S}{\partial B_t^2} = \alpha^2 S, \tag{5}$$

where

$$\alpha^2 = \left(\frac{E_n}{2}\right)^2 \left[2\varrho^{-3}\frac{\partial^2\varrho}{\partial E^2} - 3\varrho^{-4}\left(\frac{\partial\varrho}{\partial E}\right)^2\right]. \tag{6}$$

To solve (5), we constrain our solutions to special functions $D(E, B_t)$ such that α^2 and κ^2/ϱ^2 are constant (we denote the latter by n_0^2). Then (4) and (6) yield

$$n_0^2\varrho^2 = \frac{\partial D}{\partial E} - \frac{\dfrac{\partial^2\Psi}{\partial E \partial B_t}}{\dfrac{\partial^2\Psi}{\partial B_t^2}}\frac{\partial D}{\partial B_t} \tag{7}$$

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and

$$\varrho(\varepsilon) = (1 + q_1\varepsilon + q_2\varepsilon^2)^{-1}, \tag{8}$$

where $\varepsilon \equiv E/E_n$ with $0 \leqslant \varepsilon < 1$, E_n is a normalization constant, and q_1 and q_2 are constants that describe the medium. By using proper assumptions about the normalization factors [2], one can obtain expressions for ϱ and τ:

$$\varrho = \frac{1}{1 - \varepsilon^2} \tag{9}$$

and

$$\tau = \tanh^{-1}\varepsilon. \tag{10}$$

Normalizing the magnetic field, by $b = cB_t/n_0E_n$, we can change (5) to a Klein–Gordon equation with the following solution:

$$S = A\cosh[M\tau - (M^2 - 1)^{1/2}b], \tag{11}$$

where A and M (> 1) are constants. Finally, the function Ψ can be written as

$$\Psi = A(1 - \varepsilon^2)^{1/2}\cosh\eta, \tag{12}$$

where $\eta \equiv M\tau - (M^2 - 1)^{1/2}b$.

By solving our partial differential equation (7) [7], we conclude that the displacement vector $D(E, B_t)$ must be a function of $\mathscr{F}$ and $\mathscr{G}$:

$$\phi[\mathscr{F}(\varepsilon, b), \mathscr{G}(\varepsilon, D)] = 0, \tag{13}$$

where $\mathscr{F}$ and $\mathscr{G}$ are defined by

$$\mathscr{F}(\varepsilon, b) = \frac{1}{2(M^2 - \varepsilon^2)}[(\varepsilon - M)\log(-1 - \tanh\eta) + (M + \varepsilon)\log(1 - \tanh\eta)$$
$$-2\varepsilon\log(M - \varepsilon\tanh\eta)] - \tanh^{-1}\eta = c_1 \tag{14}$$

and

$$\mathscr{G}(\varepsilon, D) = D + E_n n_0^2\left[\frac{\varepsilon}{2(1 - \varepsilon^2)} + \tfrac{1}{4}\log\left(\frac{\varepsilon + 1}{\varepsilon - 1}\right)\right] = c_2, \tag{15}$$

and c_1 and c_2 are constants. Let us consider an electromagnetic plane wave incident from vacuum ($z < 0$) on the nonlinear medium ($z \geqslant 0$). Using the relation $z = \partial\Psi/\partial B_t|_{z=0} = 0$, the boundary condition $M\tau_0 - (M^2 - 1)^{1/2}b_0 = 0$ is obtained at the vacuum–medium interface. We rewrite this boundary condition, by using (10), as

$$\varepsilon_0 = \tanh\left[\frac{cB_{0t}}{n_0E_n}(1 - M^{-2})^{1/2}\right], \tag{16}$$

where the time-dependent parameters $\varepsilon_0, B_{0t}, \tau_0$, and b_0 have all been computed on the vacuum–medium boundary ($z = 0$). Since the tangential component of the electric and total magnetic field are continuous across the boundary, we can write

$$\frac{1 + r}{1 - r} = \frac{E_0}{cB_{0w}}, \tag{17}$$

Based on (16) and (17) and the relation $B_{0t} = B_{0w} + B_{\text{ext}}$, we obtain the reflection

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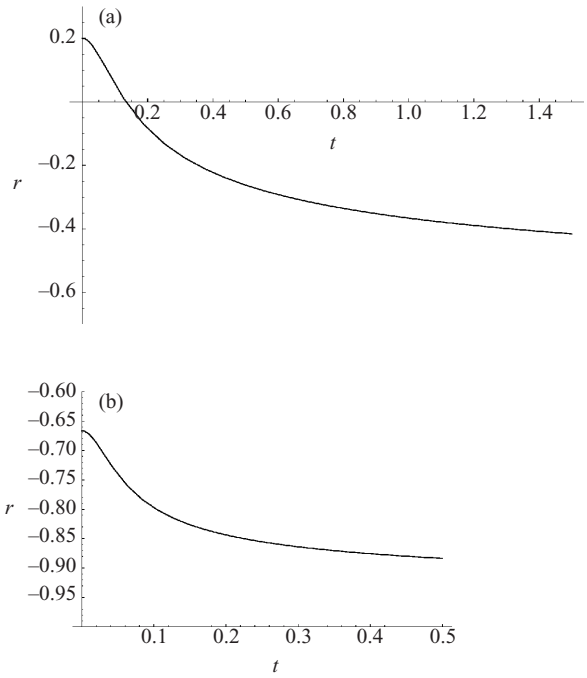


Figure 1. The reflection coefficient as a function of time when the external magnetic field is zero: (a) $v/c = 1.5$; (b) $v/c = 0.2$.

coefficient R ($\equiv |r^2|$) as

$$r = \frac{(v/c) - G(\varepsilon_0, B_{\text{ext}})}{(v/c) + G(\varepsilon_0, B_{\text{ext}})}, \tag{18}$$

where B_{0w} is the magnetic field of the incident wave at the boundary and $v \equiv (c/n_0)(1 - M^{-2})^{1/2}$ is the phase velocity of the wave. The function G is given by

$$G(\varepsilon_0, B_{\text{ext}}) = \frac{\tanh^{-1}\varepsilon_0}{\varepsilon_0} - \frac{vB_{\text{ext}}}{E_n\varepsilon_0}. \tag{19}$$

It can be checked that if $B_{\text{ext}} \to 0$, we will have the same results as obtained in [2], i.e.

$$G(\varepsilon_0) = \frac{\tanh^{-1}\varepsilon_0}{\varepsilon_0}. \tag{20}$$

3. Results

We have plotted the reflection coefficient versus time when $B_{\text{ext}} = 0$, as shown in Fig. 1, and we see that its time dependence is strongly affected by the values of v/c, whereas the results of [2] do not show this dependence and are valid only when $v/c > 1$.

We have plotted the reflection coefficient in the presence of B_{ext} in different ranges, as shown in Fig. 2. When $B_{\text{ext}} \gg 1$, Fig. 2(c) indicates that the magnetoactive nonlinear medium has strong self-screening and radiation, that is $r \leqslant -1$, which means that it can act as a power limiter, and, owing to our assumptions, our special medium is practically unstable. We also notice that in this case the amplitude as

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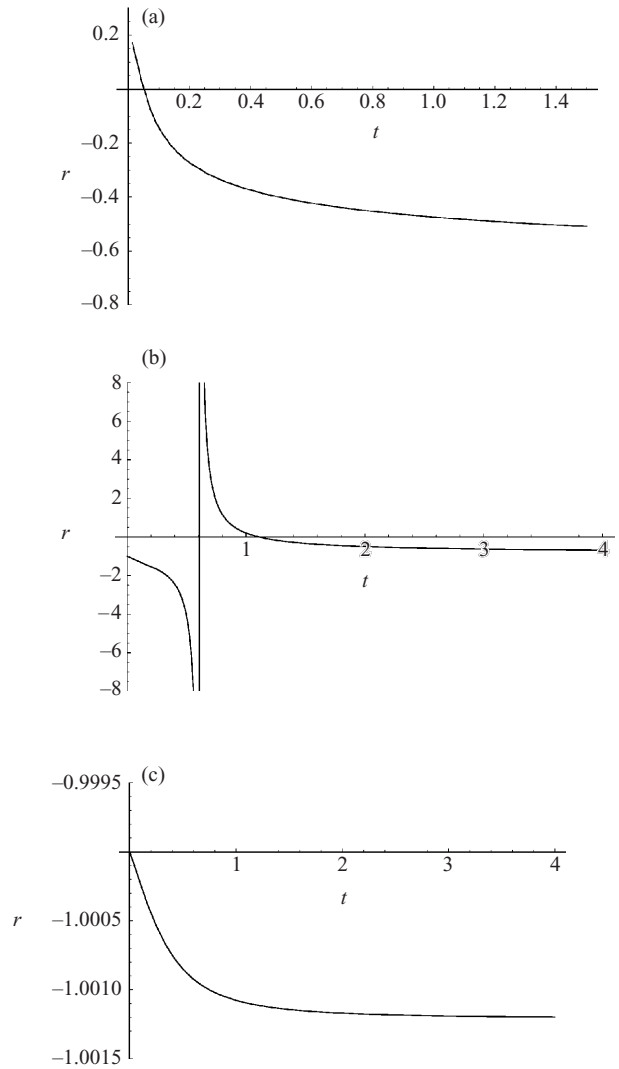


Figure 2. The reflection coefficient as a function of time: (a) $B_{\text{ext}} \ll 1$; (b) intermediate range of B_{ext}; (c) $B_{\text{ext}} \gg 1$.

well as the polarization of the reflected wave are changed. Figure 2(b) indicates a complicated behavior of the medium in the intermediate domain of the external magnetic field.

References

[1] Kruer, W. L., *Physics of Laser Plasma Interactions*. Redwood City, CA: Addison-Wesley, 1980.

[2] Shvartsburg, A. B., Stenflo, L. and Shukla, P. K., *Phys. Rev.* **E56**, 7315 (1997).

[3] Jackson, J. D., *Classical Electrodynamics*, 3rd edn. New York: Wiley, 1999.

[4] Alexandrov, A. F., Bogdankevich, L. S. and Rukhadze, A. A., *Principles of Plasma Electrodynamics*. Heidelberg: Springer-Verlag, 1984.

https://doi.org/10.1017/S0022377801001386 Published online by Cambridge University Press

[5] Shvartsburg, A. B., *Time-Domain Optics of Ultrashort Waveforms*. Oxford: Clarendon Press, 1996.

[6] Gradov, O. M., Larsson, J., Lindgren, T., Stenflo, L., Tegeback, R. and Uddholm, P., *Physica Scripta* **22**, 151 (1980).

[7] Moon, P. and Spencer, D. E., *Partial Differential Equations*. Lexington, MA: Raytheon Education Company, 1969.

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