8
Electroweak interactions

Although the main topic of this book is the study of QCD, it is in practice impossible to ignore the electroweak interactions. From an experimental point of view they are inextricably linked with QCD. The success of the present day e^+e^- colliders in the study of QCD is due to the large event rates which occur at the Z boson resonance. In addition, many of the event signatures at hadronic colliders involve either the production of vector bosons or leptons coming from the weak decays of heavy quarks. This chapter describes the Standard Electroweak Model [1], which, together with the discovery of QCD, was the triumph of the 1960s and 1970s. The model provides a unified description of the weak and electromagnetic interactions in terms of a renormalizable, gauge invariant field theory. We shall present the Lagrangian of the model and the Feynman rules which follow from it, together with some consequences of the standard model couplings. A more complete treatment of the electroweak sector of the Standard Model can be found, for example, in ref. [2].

8.1 Gauge boson interactions

In QED the Lagrangian density has the property of local gauge invariance. The analogous property for QCD was explained in Section 1.3. The Lagrangian for the photon field A^μ coupled to an electron field ψ is given by

$$\mathcal{L} = -\tfrac{1}{4}F^{\mu\nu}F_{\mu\nu} + \bar{\psi}(i\not{D} - m_e)\psi, \quad F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu \,. \tag{8.1}$$

D is the covariant derivative, which in QED takes the form

$$D_\alpha = \partial_\alpha + ieA^\alpha \,. \tag{8.2}$$

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The Lagrangian (8.1) is invariant under independent redefinitions of the phase of the field ψ at each space-time point,

$$\psi'(x) = \exp i\theta(x)\ \psi(x), \tag{8.3}$$

provided that the gauge field A^μ transforms as

$$A'_\mu(x) = A_\mu(x) - \frac{1}{e}\partial_\mu\theta(x)\ . \tag{8.4}$$

The Lagrangian thus possesses a local gauge invariance under transformations of the field belonging to the U(1) gauge group, Eq. (8.3). There is a single gauge coupling e corresponding to this gauge group.

The Standard Electroweak Model is based on the more complicated group SU(2)⊗U(1). Initially the Lagrangian of this model contains three massless bosons, $W^i (i = 1, 2, 3)$, associated with the gauge group SU(2) and one massless boson, B, associated with the U(1) gauge group. The Lagrangian of the gauge bosons is written as

$$\mathcal{L} = -\tfrac{1}{4}W^{i\,\mu\nu}W^i_{\mu\nu} - \tfrac{1}{4}B^{\mu\nu}B_{\mu\nu}\ , \tag{8.5}$$

where the field strength tensors of the U(1) gauge field B and the SU(2) gauge fields W^i are

$$W^i_{\mu\nu} = \partial_\mu W^i_\nu - \partial_\nu W^i_\mu - g_W\epsilon^{ijk}W^j_\mu W^k_\nu \tag{8.6}$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu\ , \tag{8.7}$$

g_W being the SU(2) gauge coupling. Note that the vector bosons W^i already have interactions because of the non-Abelian nature of their symmetry group, SU(2). This is entirely analogous to the fact that gluons carry colour charge in QCD.

The coupling of the gauge fields to fermionic matter fields is implemented using the covariant derivative, which is

$$D^\mu = \delta_{ij}\partial^\mu + ig_W(T\cdot W^\mu)_{ij} + iY\delta_{ij}g'_W B^\mu \tag{8.8}$$

where g'_W is the U(1) gauge coupling. The matrices T are a representation of the SU(2) *weak isospin* algebra and the U(1) charge Y is called the *weak hypercharge*. In order to specify the coupling to matter we therefore have to choose the SU(2) representation, T, and the U(1) gauge charge, Y, for the matter fields.

The tensor ϵ^{ijk} appears in Eq. (8.6) because its components are the structure constants of SU(2):

$$[T^i, T^j] = i\epsilon^{ijk}T^k,\ \epsilon^{123} = 1\ . \tag{8.9}$$

Defining $W^\pm_\mu = (W^1_\mu \mp iW^2_\mu)/\sqrt{2}$ and $T^\pm = T^1 \pm iT^2$ we have

$$W_\mu\cdot T = W^3_\mu T_3 + \frac{1}{\sqrt{2}}W^+_\mu T^+ + \frac{1}{\sqrt{2}}W^-_\mu T^- \tag{8.10}$$

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where the matrices $T^{\pm}$and T^3 satisfy the relations

$$\begin{aligned} [T^+, T^-] &= 2T^3 \\ [T^3, T^\pm] &= \pm T^\pm \,. \end{aligned} \tag{8.11}$$

T^+ and T^- are the weak isospin raising and lowering operators. For example, in the doublet representation of SU(2) we have

$$T^3 = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} \end{pmatrix}, \; T^+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \; T^- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} . \tag{8.12}$$

The Lagrangian of Eq. (8.5) evidently describes four massless vector bosons forming a singlet (B) and a triplet ($W^{\pm}, W^3$) under weak isospin. The model cannot yet describe the real world because there is only one massless vector boson observed in nature, the photon. The addition of explicit mass terms for three of the bosons would violate local gauge invariance and spoil the renormalizability of the theory. It is necessary to introduce a mechanism of symmetry breaking which gives mass to all but one of the vector bosons and retains only one conserved charge, the electric charge. This can be achieved by spontaneous symmetry breaking similar to that described in Chapter 1 for the case of chiral symmetry breaking in QCD. In that case, we saw that the breaking gave rise to a massless Goldstone boson (pion) for each broken symmetry generator. In the electroweak theory, the Goldstone degrees of freedom are not manifest as massless particles but instead combine with the corresponding gauge bosons to give them mass, as will be explained below.

8.2 The Higgs mechanism

In the Standard Electroweak Model, the fundamental SU(2)⊗U(1) electroweak symmetry is broken by the *Higgs mechanism* [3]. A single complex doublet of scalar Higgs fields is introduced

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} . \tag{8.13}$$

The upper and lower components will be identified *a posteriori* as having electric charge one and zero respectively, but for the moment the superscripts only serve to distinguish the two fields. These fields transform as a doublet under SU(2). By assigning them a weak hypercharge of one half we fix their transformation properties under U(1). A Lagrangian invariant under local SU(2)⊗U(1) transformations is obtained by coupling the field ϕ to the vector bosons using the covariant derivative defined in Eq. (8.8),

$$\mathcal{L} = (\partial^\mu \phi^\dagger + i g_W W^\mu \cdot T \phi^\dagger + \tfrac{1}{2} i g'_W B^\mu \phi^\dagger)$$

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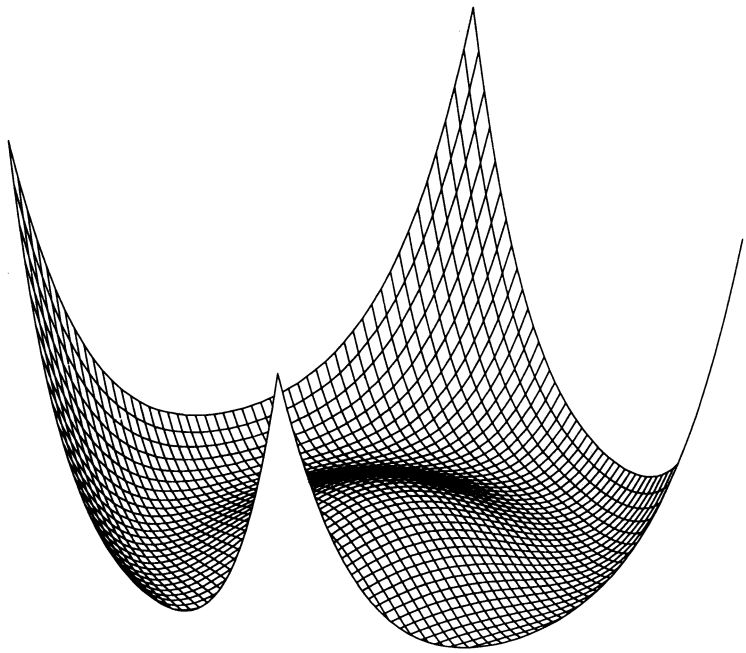


Fig. 8.1. The scalar potential which gives rise to spontaneous symmetry breaking in the Standard Model.

$$\times (\partial_\mu \phi - i g_W W_\mu \cdot T\phi - \tfrac{1}{2} i g'_W B_\mu \phi) - \mathcal{V}(\phi^\dagger \phi) \,. \qquad (8.14)$$

The Higgs potential $\mathcal{V}$ is chosen to be of the form

$$\mathcal{V}(\phi^\dagger \phi) = \lambda \, (\phi^\dagger \phi)^2 - \mu^2 \phi^\dagger \phi \,. \qquad (8.15)$$

Note that this potential has a different sign for the mass term from that usually chosen. As a result, with the parameters $\lambda, \mu^2 > 0$, this potential has a classical minimum which is not at $\phi = 0$.

To illustrate this phenomenon it is helpful to replace the complex fields in Eq. (8.13) by four real components

$$\begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \begin{pmatrix} \phi_1 + i\phi_3 \\ \phi_2 + i\phi_4 \end{pmatrix} . \qquad (8.16)$$

Since the potential $\mathcal{V}$ is a function of

$$\phi^\dagger \phi = \phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2 \,, \qquad (8.17)$$

it is clearly invariant under four-dimensional rotations. The situation is illustrated in Fig. 8.1 where, in order to represent a four-dimensional rotation in a two-dimensional plot, we arbitrarily set ϕ_3 and ϕ_4 equal to zero. Fig. 8.1 then shows the potential $\mathcal{V}(\phi^\dagger \phi)$ in the ϕ_1, ϕ_2 plane. There

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is evidently a circle of degenerate minima, corresponding to

$$|\phi| = \sqrt{\frac{\mu^2}{2\lambda}} \equiv \frac{v}{\sqrt{2}} \,. \tag{8.18}$$

Transitions between field configurations on this circle cost no energy and correspond to massless excitations.

We next choose a particular direction in the internal SU(2) space for the minimum of ϕ,

$$\langle\phi\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} . \tag{8.19}$$

The direction chosen is of no consequence; the important feature is the residual U(1) invariance. Transformations generated by $T^3 + Y$ leave the vacuum expectation value invariant, *i.e.*

$$(T^3 + Y)\,\langle\phi\rangle = 0 \,. \tag{8.20}$$

This combination is the single unbroken generator which is identified with the electric charge,

$$Q \equiv T^3 + Y = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} . \tag{8.21}$$

The choice of the vacuum expectation value in Eq. (8.19) breaks the SU(2)⊗U(1) symmetry, since it identifies a particular direction in the internal group space. We shall need to consider the fluctuations around this new minimum, so we introduce a reparametrization of the Higgs field,

$$\phi = U^{-1}(\xi) \begin{pmatrix} 0 \\ (H+v)/\sqrt{2} \end{pmatrix} \tag{8.22}$$

$$U(\xi) = \exp(-iT \cdot \xi/v) \,. \tag{8.23}$$

Note that we still have four real degrees of freedom, (three ξs and one H), equivalent to the two complex fields. We now make a gauge transformation of the form

$$\phi \;\rightarrow\; U(\xi)\phi \tag{8.24}$$

$$T \cdot W^\mu \;\rightarrow\; UT \cdot W^\mu U^{-1} + \frac{i}{g_W}(\partial^\mu U)U^{-1} \,. \tag{8.25}$$

The ξ degrees of freedom no longer appear in the Higgs Lagrangian, Eq. (8.14). As we shall see below, they will reappear as the longitudinal modes of the massive gauge bosons. This gauge, in which there are no unphysical degrees of freedom, is called the *unitary gauge*. The Higgs boson H is the only remaining dynamical field. The Higgs Lagrangian

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now becomes

$$\begin{aligned}\mathcal{L} &= \tfrac{1}{2}\partial_\mu H \partial^\mu H - \mathcal{V}\left(\frac{(v+H)^2}{2}\right) \\ &\quad + \frac{(v+H)^2}{8}\chi^\dagger(2g_W T\cdot W_\mu + g'_W B_\mu)(2g_W T\cdot W^\mu + g'_W B^\mu)\chi\,,\end{aligned} \tag{8.26}$$

where χ is a unit vector along the direction of the vacuum expectation value (8.19)

$$\chi = \begin{pmatrix} 0 \\ 1 \end{pmatrix}. \tag{8.27}$$

We therefore have three generators of the SU(2)⊗U(1) symmetry which are spontaneously broken. As described in Chapter 1, Goldstone's theorem would lead us to expect three massless bosons. However, this theorem does not apply if the theory contains long-range vector fields with the same quantum numbers as the would-be Goldstone bosons. When such fields are present, the massless Goldstone modes provide the extra longitudinal degrees of freedom necessary to change the quanta of the vector fields from massless to massive bosons.

To see how this works in practice, we examine the terms quadratic in the vector boson fields:

$$\mathcal{L}_{\rm M} = \frac{v^2}{8}\left[(g_W W^3_\mu - g'_W B_\mu)(g_W W^{3\,\mu} - g'_W B^\mu) + 2g_W^2 W^-_\mu W^{+\,\mu}\right]. \tag{8.28}$$

The propagator for the B and W^3 fields is not diagonal. We therefore redefine the electrically neutral fields by introducing new fields A_μ and Z_μ which propagate independently,

$$\begin{pmatrix} W^3_\mu \\ B_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix}\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix}, \tag{8.29}$$

where the angle θ_W (the Weinberg or electroweak mixing angle) is fixed by the relative strengths of the coupling constants:

$$\sin^2\theta_W = \frac{g'^2_W}{g_W^2 + g'^2_W} \simeq 0.23\,. \tag{8.30}$$

After this rotation the quadratic terms in the vector boson fields become

$$\mathcal{L}_{\rm M} = \frac{g_W^2 v^2}{4} W^+_\mu W^{-\,\mu} + \frac{(g_W^2 + g'^2_W)v^2}{8} Z_\mu Z^\mu\,. \tag{8.31}$$

We therefore find that the W and Z bosons have acquired masses, given by

$$M_W = \tfrac{1}{2} v g_W, \tag{8.32}$$

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$$M_Z = \tfrac{1}{2}v\sqrt{g_W^2 + g_W'^2} \equiv \frac{M_W}{\cos\theta_W} . \tag{8.33}$$

The photon remains massless because there are no terms quadratic in the field A in Eq. (8.31). As we shall see below, low-energy weak interactions allow us to make the identification

$$\frac{g_W^2}{8M_W^2} \equiv \frac{1}{2v^2} = \frac{G_F}{\sqrt{2}} \tag{8.34}$$

where G_F is the Fermi constant. The measured value, $G_F = 1.166 \times 10^{-5}$ GeV^{-2}, therefore implies that the vacuum expectation value of the Higgs field, v, is 246 GeV.

In summary, the Higgs field ϕ acquires a non-zero vacuum expectation value at a particular point on the circle of minima away from $\phi = 0$ and the symmetry is spontaneously broken. However, this does not give rise to Goldstone bosons: instead, three of the four scalar field components get 'eaten' by the gauge particles to form massive vector bosons ($W^{\pm}, Z^0$), and there remains a single physical neutral scalar particle — the Higgs boson (H).

The self-couplings of the Higgs boson are

$$\mathcal{L}_{\text{Higgs}} = \tfrac{1}{2}\partial_\mu H \partial^\mu H - \mu^2 H^2 - \lambda v H^3 - \tfrac{1}{4}\lambda H^4 , \tag{8.35}$$

and its mass M_H is therefore

$$M_H = \sqrt{2}\mu \equiv \sqrt{2\lambda}v . \tag{8.36}$$

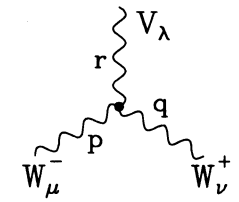
One combination of the parameters in the Higgs potential, Eq. (8.15), $\mu^2/\lambda = v^2$ is fixed by the measured parameters of the electroweak theory. The other parameter, which is related to the physical Higgs mass, is essentially arbitrary.

This is the 'minimal model'. Many extensions of the Standard Model (supersymmetry, grand unification, ...) retain the Higgs mechanism as the primary method for mass generation for gauge bosons, but with more complicated Higgs sectors and more Higgs bosons.

8.2.1 Boson couplings

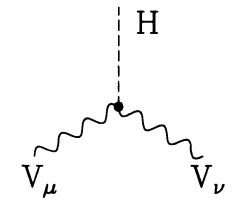
The fundamental couplings between the bosons are shown in Fig. 8.2. Note that the Higgs boson couples to weak gauge bosons with the normal weak interaction strength.

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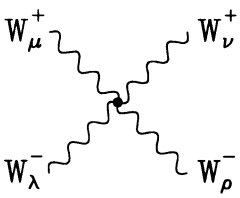
$$+ig_V\,[(p-q)_\lambda g_{\mu\nu}+(q-r)_\mu g_{\nu\lambda}+(r-p)_\nu g_{\lambda\mu}]$$

(all momenta incoming, $g_A=e$, $g_Z=g_W\cos\theta_W$)

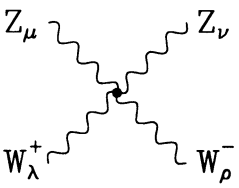


$$+ig_{VH}M_W g_{\mu\nu}$$

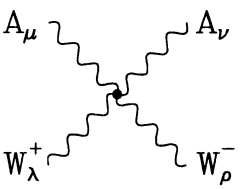
($g_{WH}=g_W$, $g_{ZH}=g_W/\cos^2\theta_W$)



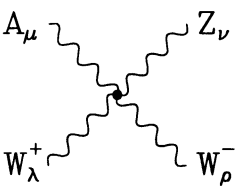
$$+ig_W^2[2g_{\mu\nu}g_{\lambda\rho}-g_{\mu\lambda}g_{\nu\rho}-g_{\mu\rho}g_{\nu\lambda}]$$



$$-ig_W^2\cos^2\theta_W[2g_{\mu\nu}g_{\lambda\rho}-g_{\mu\lambda}g_{\nu\rho}-g_{\mu\rho}g_{\nu\lambda}]$$



$$-ie^2[2g_{\mu\nu}g_{\lambda\rho}-g_{\mu\lambda}g_{\nu\rho}-g_{\mu\rho}g_{\nu\lambda}]$$



$$-ieg_W\cos\theta_W[2g_{\mu\nu}g_{\lambda\rho}-g_{\mu\lambda}g_{\nu\rho}-g_{\mu\rho}g_{\nu\lambda}]$$

Fig. 8.2. Feynman rules for boson interactions.

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As an example of the use of these Feynman rules, we calculate the partial width of the Standard Model Higgs boson into W pairs, which will be needed in Chapter 11. We find that the tree-level invariant matrix element squared for the process

$$H \to W^+W^- \,, \tag{8.37}$$

summed over final-state polarizations, is given by

$$\sum |\mathcal{M}|^2 = \frac{g_W^2 M_H^4}{4M_W^2}\left(1 - 4\frac{M_W^2}{M_H^2} + 12\frac{M_W^4}{M_H^4}\right) . \tag{8.38}$$

The Lorentz-invariant phase space for decay into two particles is

$$d\Phi_2(P, p_1, p_2) = \frac{1}{8\pi}\frac{2|p_1|}{M_P}\frac{d\Omega}{4\pi} \,, \tag{8.39}$$

where p_1 is the three-momentum of the decay products in the rest frame of the decaying particle, which has mass M_P, and $d\Omega$ is the element of solid angle. Including the initial-state normalization factor of $1/(2M_H)$, we obtain the final result for the Higgs partial width into W bosons:

$$\Gamma(H \to W^+W^-) = \frac{g_W^2 M_H^3}{64\pi M_W^2}\left(1-\frac{4M_W^2}{M_H^2}\right)^{\frac{1}{2}}\left(1-\frac{4M_W^2}{M_H^2}+\frac{12M_W^4}{M_H^4}\right). \tag{8.40}$$

Note that for $M_H \gg M_W$ the tree-level Higgs width grows and eventually becomes larger than its mass,

$$\Gamma(H \to W^+W^-) = \left(\frac{M_H^3}{M_U^2}\right), \quad M_U = 1.74 \text{ TeV} \,. \tag{8.41}$$

The scale at which $\Gamma_H > M_H$ is set by M_U. This growth arises from the decay into longitudinally polarized Ws, as will be discussed in more detail later.

8.2.2 The ρ parameter

From the masses of the vector bosons and the value of the weak mixing angle one can define the parameter ρ [4],

$$\rho = \frac{M_W^2}{M_Z^2 \cos^2\theta_W} \,. \tag{8.42}$$

This parameter is equal to unity in the Standard Electroweak Model at the tree-graph level, see Eq. (8.33). Using the measured values $M_W = 80.22 \pm 0.26$ GeV [5], $M_Z = 91.1884 \pm 0.0022$ GeV [6] and $\sin^2\theta_W = 0.2325 \pm 0.0013$ obtained from the measured couplings of the Z [6], we find that the measured value of ρ is indeed equal to unity to an accuracy of better than 1 percent ($\rho = 1.008 \pm 0.007$). This is an impressive verification of a Standard Model prediction. However this relationship is

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not a consequence of spontaneous symmetry breaking *per se* but rather of the particular form of the Higgs potential. For example, the ρ parameter would not be equal to unity in a model with an isospin triplet of Higgs fields [4].

In order to investigate this point further, it is useful to construct an alternative form for the Higgs Lagrangian in which its extra global symmetries are apparent [7]. As noted in Eq. (8.17), the potential is a function of

$$\phi^\dagger\phi = \phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2 \,, \tag{8.43}$$

and hence we have an SO(4) symmetry. However, it is more convenient to express this SO(4) symmetry as the product SU(2)⊗SU(2), the group SO(4) being isomorphic to SU(2)⊗SU(2).

Let us construct a 2×2 matrix out of the doublet Higgs field and its complex conjugate

$$\Phi = \begin{pmatrix} \phi^{0\,*} & \phi^+ \\ -\phi^{+\,*} & \phi^0 \end{pmatrix} . \tag{8.44}$$

After the removal of a normalization factor, the matrix Φ is a special unitary matrix. It is made up of the columns $i\tau_2\phi^*$ and ϕ, where τ_j is the j-th Pauli matrix. It is easy to show that if ϕ transforms as a doublet,

$$\phi \to \exp(i\theta\tau \cdot n)\phi = (\cos\theta + i\tau \cdot n \sin\theta)\phi \tag{8.45}$$

then so does $i\tau_2\phi^*$, using the identity $\tau_2\tau_j^*\tau_2 = \tau_j$.

The potential $\mathcal{V}$ is a function of $\mathrm{Tr}(\Phi^\dagger\Phi)$ and is therefore invariant under the transformation

$$\Phi \to U_L \Phi U_R^\dagger \tag{8.46}$$

where U_L and U_R are special unitary 2×2 matrices. The left one, U_L, is the familiar gauged SU(2) invariance. The Higgs Lagrangian becomes

$$\mathcal{L} = \tfrac{1}{2}\left\{\mathrm{Tr}\left[(D_\mu\Phi)^\dagger D^\mu\Phi\right] + \mu^2\mathrm{Tr}\left[\Phi^\dagger\Phi\right] - \lambda\mathrm{Tr}\left[\Phi^\dagger\Phi\Phi^\dagger\Phi\right]\right\} \,, \tag{8.47}$$

where the covariant derivative in the present notation is

$$D_\mu\Phi = \partial_\mu\Phi + \tfrac{1}{2}ig_W W_\mu \cdot \tau\Phi - \tfrac{1}{2}ig'_W B_\mu\Phi\tau_3 \,. \tag{8.48}$$

The last term breaks the $\mathrm{SU}(2)_R$ symmetry, so we consider first the limit $g'_W = 0$. Then the Lagrangian including the gauge fields has the full $\mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R$ symmetry if the W field transforms as a triplet under the gauged $\mathrm{SU}(2)_L$ and as a singlet under the global $\mathrm{SU}(2)_R$. The vacuum expectation value of the Higgs field is

$$\langle\Phi\rangle = \frac{v}{\sqrt{2}}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} . \tag{8.49}$$

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After symmetry breaking, the group is broken down to a single SU(2) (or equivalently O(3)):

$$\langle\Phi\rangle \to U\,\langle\Phi\rangle\,U^\dagger = \langle\Phi\rangle\,, \tag{8.50}$$

where the left and right transformations are now the same. The residual O(3) symmetry means that the quadratic terms in the W field in Eq. (8.31) must take the form

$$W^1_\mu W^{1\,\mu} + W^2_\mu W^{2\,\mu} + W^3_\mu W^{3\,\mu} = 2W^-_\mu W^{+\,\mu} + W^3_\mu W^{3\,\mu}\,, \tag{8.51}$$

from which it follows that $M_W = M_Z$, *i.e.* $\rho = 1$ in the limit we are considering, $g'_W = 0$. Now suppose that $g'_W \neq 0$. The SU(2) symmetry of the Higgs field ensures that the mass difference between the W and the Z arises only from the symmetry-breaking term in Eq. (8.48). The net effect is to replace the W^3 field in (8.51) by $W^3 - (g'_W/g_W)B$, and therefore the relation between the couplings and the boson masses in Eqs. (8.32,8.33), corresponding to $\rho = 1$, is preserved. As a consequence ρ is constrained to be unity before the inclusion of symmetry-breaking effects from other sectors of the Standard Model. For this reason the residual symmetry is referred to as 'custodial SU(2)' [7].

The fermion mass sector of the Standard Model, to be discussed in the next section, does not respect the custodial SU(2) symmetry. Higher-order radiative corrections involving fermions will thus cause calculable corrections to the value of the ρ parameter. In fact, the largest effect comes from the top quark mass [4]:

$$\rho - 1 \approx \frac{3G_F}{8\sqrt{2}\pi^2} m_t^2 = 0.01\,\left(\frac{m_t}{175\ \text{GeV}}\right)^2\,. \tag{8.52}$$

8.3 Fermion multiplets and interactions

The coupling of the gauge bosons to fermions which preserves gauge symmetry is specified by the covariant derivative, Eq. (8.8). Before symmetry breaking, the coupling of the fermions to the vector bosons is given by

$$\mathcal{L} = \bar\psi_R i(\partial\!\!\!/ + ig'_W Y_R B\!\!\!/)\psi_R + \bar\psi_L i(\partial\!\!\!/ + ig_W \cdot T W\!\!\!/ + ig'_W Y_L B\!\!\!/)\psi_L\,. \tag{8.53}$$

The U(1) charges of the left- and right-handed fermions, Y_L and Y_R, are chosen to satisfy the relation $Q = T^3 + Y$, so that after symmetry breaking we obtain the correct values of the electric charges, Q. For the case of leptons the left-handed fields are put into doublets

$$\psi_L = \gamma_L \begin{pmatrix} \nu_e \\ e^- \end{pmatrix},\ \gamma_L \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix},\ \gamma_L \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix} \tag{8.54}$$

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Fermion			T_L^3	Y_L	T_R^3	Y_R	Q_f
u	c	t	$+\frac{1}{2}$	$+\frac{1}{6}$	0	$+\frac{2}{3}$	$+\frac{2}{3}$
d	s	b	$-\frac{1}{2}$	$+\frac{1}{6}$	0	$-\frac{1}{3}$	$-\frac{1}{3}$
ν_e	ν_μ	ν_τ	$+\frac{1}{2}$	$-\frac{1}{2}$	-	-	0
e^-	μ^-	τ^-	$-\frac{1}{2}$	$-\frac{1}{2}$	0	-1	-1

Table 8.1. Weak isospin and hypercharge assignments.

where as before

$$\gamma_R = \tfrac{1}{2}(1+\gamma_5), \quad \gamma_L = \tfrac{1}{2}(1-\gamma_5). \tag{8.55}$$

The right-handed fields are all SU(2) singlets:

$$\psi_R = \gamma_R e^-, \ \gamma_R \mu^-, \ \gamma_R \tau^- \ . \tag{8.56}$$

Similarly the quarks form three left-handed doublets

$$\psi_L = \gamma_L \begin{pmatrix} u \\ d' \end{pmatrix}, \ \gamma_L \begin{pmatrix} c \\ s' \end{pmatrix}, \ \gamma_L \begin{pmatrix} t \\ b' \end{pmatrix}, \tag{8.57}$$

and the right-handed quarks are again singlets. The primes on the down-type quarks in Eq. (8.57) will be explained in Section 8.3.3. The hypercharges are assigned so that the quarks and leptons have the charges given in Table 8.1.

We see that the fermions fall naturally into three families (sometimes called *generations*). Each family is made up of a left-handed doublet of quark fields (in each of three colours), a left-handed doublet of leptons and all the corresponding right-handed fields.

8.3.1 Anomaly cancellation

The theoretical motivation for grouping the quarks and leptons as shown in Table 8.1 is that complete families are required for the cancellation of anomalies in the currents which couple to gauge fields. Hence, for example, the top quark must exist as the partner of the b, τ and ν_τ, completing the third family.

Anomalies occur in a theory if symmetries present at the classical level are destroyed by quantum effects [8]. They typically involve contributions to the divergence of a current which is conserved at the classical level. If the gauge currents are anomalous, the Ward identities, which are vital for the proof that the gauge theory is renormalizable, are invalidated.

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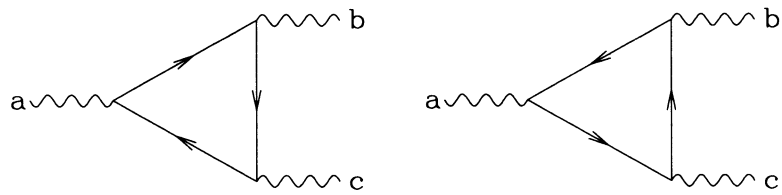


Fig. 8.3. Diagrams responsible for the simplest anomalies.

An important class of anomalies arises from the simple triangle diagram with two vector currents and one axial vector current, as shown in Fig. 8.3. Elimination of the anomalies for a particular current in the lowest-order triangle diagram is sufficient to ensure that the current remains anomaly free, even after the inclusion of more complicated diagrams [9].

If the gauge currents a, b and c which interact at the three corners of the triangle couple to the matrices L^a, L^b and L^c for the left-handed fields, and to the matrices R^a, R^b and R^c for the right-handed fields, the vector-vector-axial vector triangle anomaly is proportional to

$$A = \mathrm{Tr}\,(R^a\{R^b, R^c\}) - \mathrm{Tr}\,(L^a\{L^b, L^c\}) \tag{8.58}$$

where the trace means a sum over all possible fermion triangles. We are imagining here that all the families of left- and right-handed fermions are assembled in a single column vector, on which the matrices L^a and R^a act.

In order to show that the combination given in Eq. (8.58) vanishes in the Standard Model, we shall first demonstrate a pair of useful results:

$$i) \quad \mathrm{Tr}\,(T^a\{T^b, T^c\}) = 0\,, \tag{8.59}$$

$$ii) \quad \mathrm{Tr}\,(Y_L^3 - Y_R^3) = 3\mathrm{Tr}\,(Y_L[T^3]^2)\,. \tag{8.60}$$

In these equations T is a matrix which operates on all left-handed fields. Because all such fields are in doublets, the matrices T are block diagonal and each block is made up of a doublet representation of SU(2). The matrices T therefore obey the same SU(2) algebra as the individual blocks. Eq. (8.59) follows because there is no SU(2) tensor to which the trace could be proportional: the only available tensor is ϵ^{abc}, which is antisymmetric. The second result follows from the two facts that all left-handed fields are in doublets and that there are no massless charged fields, *i.e.* each left-handed charged field has a corresponding right-handed field, so that

$$\mathrm{Tr}\,(Q_L^3 - Q_R^3) \equiv \mathrm{Tr}\,([T^3 + Y_L]^3 - Y_R^3) = 0\,. \tag{8.61}$$

Expanding the second of these equations, we have $\mathrm{Tr}\,([T^3]^3) = 0$ by Eq. (8.59), and $\mathrm{Tr}\,(T^3 Y_L^2) = 0$ since the trace of each block of T^a with a common value of Y_L is zero. Hence we obtain Eq. (8.60).

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Having established these identities we now proceed to demonstrate the cancellation of anomalies in the Standard Electroweak Model. Substituting the Standard Model couplings into Eq. (8.58), with all combinations of the SU(2) matrices T^a or the U(1) matrices Y, we obtain the form of the anomaly for the various gauge currents of the theory. Two of the resulting traces of the couplings vanish for each fermion doublet (*i.e.* separately for leptons and quarks):

$$\text{Tr}\,(T^a\{T^b, T^c\}) = 0, \quad \text{Tr}\,(T^a Y_L^2) = 0\,. \tag{8.62}$$

The other two traces vanish only for a complete family [10]

$$\text{Tr}\,(Y_R^3 - Y_L^3) = 0, \quad \text{Tr}\,(Y_L\{T^a, T^b\}) = 0\,. \tag{8.63}$$

Note that because of Eq. (8.60) the two conditions in Eq. (8.63) are equivalent to a single constraint. To show the anomaly cancellation it is therefore sufficient to demonstrate that

$$\text{Tr}\,(Y_L\{T^a, T^b\}) = \text{Tr}\,(Q_L\{T^a, T^b\}) = 0\,. \tag{8.64}$$

The first form of this equation shows that a non-zero contribution can only occur for $a = b$, otherwise there are cancellations within a single doublet. The second form makes it clear that for $a = b$ the condition reduces to the single condition that the charge matrix for the left-handed fields must be traceless,

$$\text{Tr}\,Q_L = 0\,. \tag{8.65}$$

This condition is satisfied in the Standard Model for each complete family. This remarkable equation implies a relation between the fact that quarks occur in three colours and that they have fractional charges, that is,

$$3(Q_u + Q_d) + Q_e = 0\,. \tag{8.66}$$

The Standard Model by itself provides no explanation of this fact.

It should be noted that there are still anomalies in global (non-gauged) currents in the Standard Model. Such anomalies are harmless since they do not destroy renormalizability. For example, the strong isospin current corresponds to a global symmetry (in the approximation $m_u = m_d$) and is anomalous. It is this anomaly which is responsible for π^0 decay, as discussed in Chapter 1.

8.3.2 Fermion masses and couplings

The Higgs field ϕ gives masses to the fermions via Yukawa interactions with the fermion fields. After spontaneous symmetry breaking, a Yukawa interaction of the form $g_f \bar{\psi}_f \psi_f \phi$ leads to a fermion mass, $m_f = g_f v/\sqrt{2}$, and residual $\bar{\psi}_f \psi_f H$ interactions of strength $g_f = \sqrt{2} m_f / v$. The values

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g_u	2×10^{-5}	g_c	9×10^{-3}	g_t	1
g_d	4×10^{-5}	g_s	8×10^{-4}	g_b	3×10^{-2}
g_e	3×10^{-6}	g_μ	6×10^{-4}	g_τ	1×10^{-2}

Table 8.2. Yukawa couplings.

Fermions			Q_f	V_f	A_f
u	c	t	$+\frac{2}{3}$	$(+\frac{1}{2} - \frac{4}{3}\sin^2\theta_W) \sim +0.191$	$+\frac{1}{2}$
d	s	b	$-\frac{1}{3}$	$(-\frac{1}{2} + \frac{2}{3}\sin^2\theta_W) \sim -0.345$	$-\frac{1}{2}$
ν_e	ν_μ	ν_τ	0	$\frac{1}{2}$	$+\frac{1}{2}$
e	μ	τ	-1	$(-\frac{1}{2} + 2\sin^2\theta_W) \sim -0.036$	$-\frac{1}{2}$

Table 8.3. Couplings of fermions to the Z boson. The numerical values of V_f are for $\sin^2\theta_W = 0.232$.

of the Yukawa couplings required to reproduce the observed pattern of masses are shown in Table 8.2. The Standard Model offers no explanation for the wide range of Yukawa couplings required.

The interaction Lagrangian can be expressed in terms of physical fields by substituting for B and W^3 using Eq. (8.29):

$$\begin{aligned}\mathcal{L} &= \sum_f \bar{\psi}_f \left(i\partial\!\!\!/ - m_f - g_W \frac{m_f H}{2M_W} \right) \psi_f \\ &- \frac{g_W}{2\sqrt{2}} \sum_f \bar{\psi}_f (\gamma^\mu(1-\gamma_5)T^+ W_\mu^+ + \gamma^\mu(1-\gamma_5)T^- W_\mu^-)\psi_f \\ &- e\sum_f q_f \bar{\psi}_f A\!\!\!/ \psi_f - \frac{g_W}{2\cos\theta_W} \sum_f \bar{\psi}_f \gamma^\mu (V_f - A_f\gamma_5)\psi_f Z_\mu \ . \quad (8.67)\end{aligned}$$

The couplings of the fermions to the Z boson are

$$\begin{aligned} V_f &= T_f^3 - 2Q_f \sin^2\theta_W \ , \\ A_f &= T_f^3 \ , \end{aligned} \quad (8.68)$$

where Q_f is the charge of the fermion in units of the positron electric charge e. The values of e and the weak SU(2) charge g_W are related by

$$e = g_W \sin\theta_W \ . \quad (8.69)$$

These couplings are summarized in Table 8.3. They have been measured experimentally; Fig. 8.4 shows the constraints on the couplings from the

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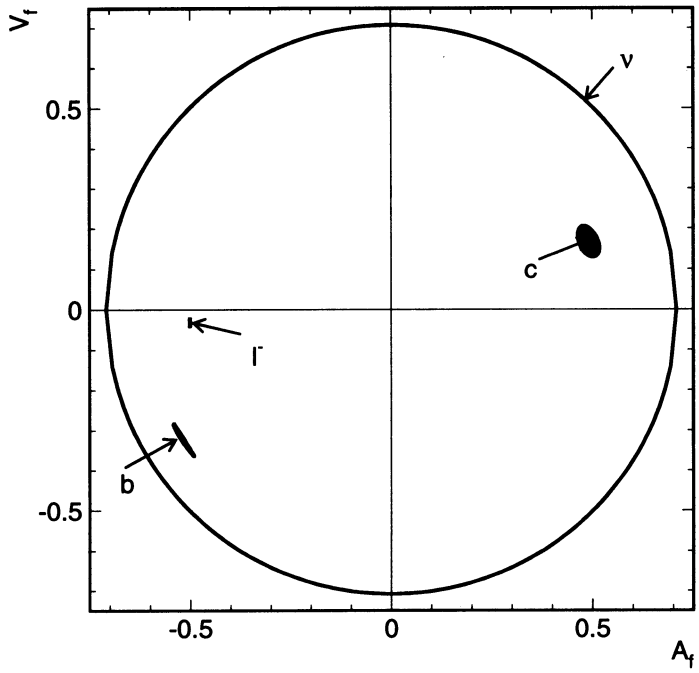


Fig. 8.4. Fermion couplings to the Z boson as measured at LEP. Figure from ref. [11].

LEP data. The charged lepton couplings all lie within the small rectangle indicated, and are consistent with lepton universality. The width of the Z boson into neutrinos, obtained by subtracting the observed channels from the total width, measures the quantity, $V_\nu^2 + A_\nu^2$. These measurements constrain it to lie on the circle in Fig. 8.4. Small deviations from the values given in Table 8.3 are due to electroweak radiative corrections.

Fig. 8.5 shows the Feynman rules derived from Eq. (8.67). Also shown are the propagators of the vector boson fields in the unitary gauge defined by Eq. (8.24). In this gauge only real physical particles propagate. To relate the electroweak parameters to those of low-energy weak interactions we consider the amplitude for the decay $\mu^- \to e^- \bar{\nu}_e \nu_\mu$, which according to the Feynman rules given in Fig. 8.5 is

$$\frac{-ig_W^2}{2} \bar{u}(\nu_\mu)\gamma^\alpha \gamma_L u(\mu)\ \bar{u}(e)\gamma^\beta \gamma_L v(\overline{\nu_e})\ \left(-g_{\alpha\beta} + \frac{k_\alpha k_\beta}{M_W^2}\right) \frac{1}{k^2 - M_W^2}\,. \quad (8.70)$$

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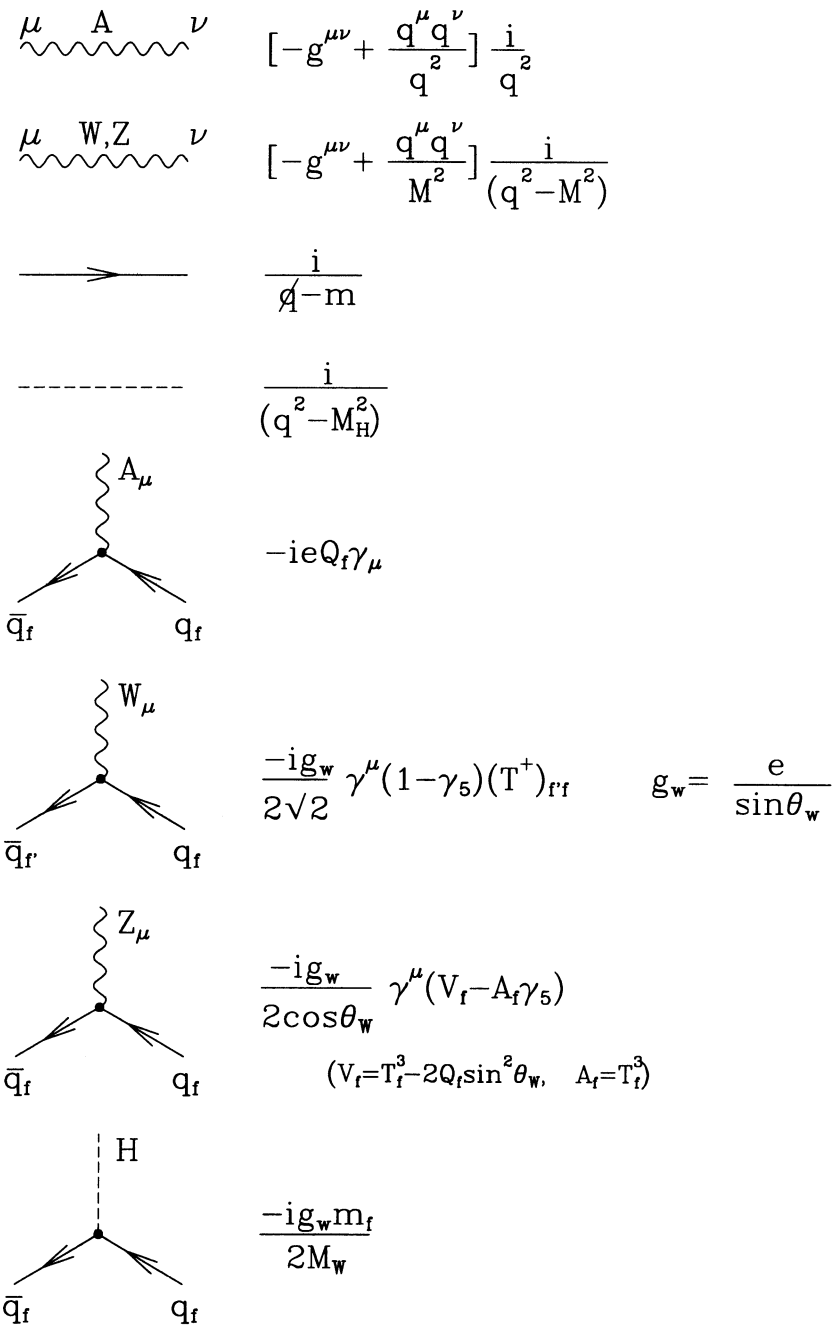


Fig. 8.5. Propagators and Feynman rules for fermion interactions.

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At low energies this reproduces the Fermi model,

$$\frac{iG_F}{\sqrt{2}}\bar{u}(\nu_\mu)\gamma^\alpha(1-\gamma_5)u(\mu)\;\bar{u}(e)\gamma_\alpha(1-\gamma_5)v(\overline{\nu_e})\;, \tag{8.71}$$

provided that the weak charge g_W is related to the Fermi constant G_F as follows:

$$\frac{G_F}{\sqrt{2}} = \frac{g_W^2}{8M_W^2}\;. \tag{8.72}$$

Note that whereas the Higgs boson couples to the W and Z with a standard weak coupling strength, the coupling to the fermions is proportional to the fermion mass. In other words, the Higgs boson prefers to couple to the heaviest available fermions. This is manifest in the partial decay widths (see Chapter 11), which are readily obtained from the diagrams of Figs. 8.2 and 8.5.

The Standard Model thus contains a large number of arbitrary Yukawa couplings to fix the quark and charged lepton masses (plus the quark mixing matrix, discussed below), together with four more basic parameters: the two couplings g_W, g'_W, the Higgs vacuum expectation value v and the Higgs mass M_H. The influence of the Higgs mass on low-energy observables is rather mild. Thus at low energy, neglecting fermion masses, we have to supply the values of three physical observables to make predictions with the Standard Model. These are normally taken to be the value of the low-energy fine structure constant, α, the value of the Fermi constant as measured in muon decay, and the mass of the Z boson, measured with great accuracy at LEP. Their values are [5,6]

$$\alpha^{-1} = 137.035\,989\,5(61) \tag{8.73}$$
$$G_F = 1.166\,39(2)\times 10^{-5}\ \mathrm{GeV}^{-2} \tag{8.74}$$
$$M_Z = 91.188\,5(22)\ \mathrm{GeV} \tag{8.75}$$

At tree level their relationships to the fundamental parameters of the Standard Electroweak Model are

$$\begin{aligned} \alpha &= \frac{g_W^2\sin^2\theta_W}{4\pi}\;, \\ G_F &= \frac{1}{\sqrt{2}v^2}\;, \\ M_Z &= \frac{\frac{1}{2}g_W v}{\cos\theta_W}\;. \end{aligned} \tag{8.76}$$

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8.3.3 The CKM matrix

The primed down-type quarks q' in Eq. (8.57) differ from the mass eigenstate combinations q by a unitary transformation, V:

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = V \begin{pmatrix} d \\ s \\ b \end{pmatrix} . \tag{8.77}$$

The matrix V is known as the Cabibbo-Kobayashi-Maskawa (CKM) matrix [12]

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} . \tag{8.78}$$

The constraint of unitarity is $V^\dagger V = 1$. Written out explicitly, this imposes nine conditions

$$\sum_{j=1}^{3} |V_{ij}|^2 = \sum_{i=1}^{3} |V_{ij}|^2 = 1 , \tag{8.79}$$

$$\sum_{k=1}^{3} V_{ki}^* V_{kj} = 0 \quad (i \neq j) . \tag{8.80}$$

It is important to establish how many independent parameters the CKM matrix contains. In general, an $n \times n$ unitary matrix depends on n^2 real parameters. Since a real unitary matrix is an orthogonal matrix, with $\frac{1}{2}n(n-1)$ independent elements, it follows that there are $\frac{1}{2}n(n-1)$ parameters that can be represented by real mixing angles. The remaining $\frac{1}{2}n(n+1)$ parameters are phases. However, $2n-1$ of these can be absorbed into arbitrary phase rotations of the quark fields q and q', leaving $\frac{1}{2}(n-1)(n-2)$ phases. This means that for $n = 3$ we have three independent mixing angles and one phase.

The standard parametrization of the CKM matrix is

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \tag{8.81}$$

with $s_{ij} = \sin\theta_{ij}$, $c_{ij} = \cos\theta_{ij}$. In the Standard Model the phase δ is responsible for CP violation. Note that if we had only two generations there would be one mixing angle (the Cabibbo angle) and no phases, and hence no possibility for CP violation arising from the mixing matrix.

The experimental limits on the magnitudes of individual matrix ele-

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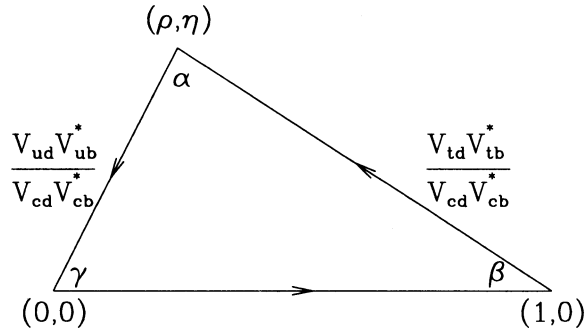


Fig. 8.6. Unitarity triangle.

ments, taking into account constraints derived from unitarity, are [5]

$$V = \begin{pmatrix} 0.9745 - 0.9757 & 0.219 - 0.224 & 0.002 - 0.005 \\ 0.218 - 0.224 & 0.9736 - 0.9750 & 0.036 - 0.046 \\ 0.004 - 0.014 & 0.034 - 0.046 & 0.9989 - 0.9993 \end{pmatrix} . \quad (8.82)$$

It can be seen that the mixing angles are small, and satisfy a hierarchy

$$1 \gg \theta_{12} \gg \theta_{23} \gg \theta_{13} . \quad (8.83)$$

Thus it makes sense to use a small-angle approximation to the general mixing matrix, Eq. (8.81). This leads to a representation due to Wolfenstein [13]. Identifying $s_{12} = \lambda$ we may write $V_{cb} \simeq s_{23} = A\lambda^2$ and $V_{ub} = s_{13}e^{-i\delta} = A\lambda^3(\rho - i\eta)$ with $A \simeq 1$ and $|\rho - i\eta| < 1$. Then, retaining terms up to $O(\lambda^3)$,

$$V \simeq \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} . \quad (8.84)$$

From Eq. (8.82) we see that the poorly determined elements are V_{td} and V_{ub}. Therefore the most interesting relationship that can be derived from unitarity is an equation relating these two elements. In fact choosing the values db for ij in Eq.(8.80) we obtain

$$V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0 . \quad (8.85)$$

This relation between complex numbers defines a triangle in the complex plane, known as a unitarity triangle, shown in Fig. 8.6. Since V_{ud} and V_{tb} are both close to one we may approximate Eq. (8.85) by

$$V_{ub}^* + V_{cb}^* V_{cd} + V_{td} = 0 . \quad (8.86)$$

The accurate determination of the angles and lengths of the sides of this unitarity triangle remains a challenge for experiment.

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8.4 W and Z decay properties

The Standard Model predictions for the partial decay widths of the W and Z bosons into the various fermion-antifermion final states can be tested with great precision at present-day high-energy e^+e^- and $p\bar{p}$ colliders. In this section we derive some simple results for the decay widths. At leading order in electroweak perturbation theory, the partial width for the decay of the Z boson into a fermion-antifermion pair,

$$Z(q) \to f(p)\bar{f}(p') \,, \tag{8.87}$$

is given (in the Standard Model) by

$$\frac{-ig_W}{2\cos\theta_W}\bar{u}(p)\gamma^\mu(V_f - A_f\gamma_5)v(p')\varepsilon_\mu(q). \tag{8.88}$$

The invariant matrix element squared for this process summed (averaged) over final (initial) polarizations is given by

$$\overline{\sum}|\mathcal{M}|^2 = \frac{8}{3}\frac{G_F M_Z^4}{\sqrt{2}}(|V_f|^2 + |A_f|^2) \,. \tag{8.89}$$

The sum over the three polarizations of the Z boson is performed using

$$\sum_i \varepsilon_\mu^{(i)}(q)\varepsilon_\nu^{(i)\,*}(q) = -g_{\mu\nu} + \frac{q_\mu q_\nu}{M_Z^2} \,. \tag{8.90}$$

The two-particle phase space is given by Eq. (8.39). The final result for the partial width is

$$\begin{aligned}\Gamma(Z^0 \to f\bar{f}) &= C\frac{G_F M_Z^3}{6\sqrt{2}\pi}(|V_f|^2 + |A_f|^2) \\ &= 332\; C(|V_f|^2 + |A_f|^2)\ \text{MeV} \,,\end{aligned} \tag{8.91}$$

where C is a colour normalization factor which is 1 for leptons and 3 for quarks. The corresponding result for the W boson is

$$\Gamma(W^+ \to f\bar{f}') = C\frac{G_F M_W^3}{6\sqrt{2}\pi} \,. \tag{8.92}$$

The W^+ decay rate refers to the sum of the decays to a given quark of charge $\frac{2}{3}$ and all antiquarks of charge $\frac{1}{3}$, *e.g.* $W^+ \to u\bar{d} + u\bar{s} + u\bar{b}$. For any individual mode $W \to f_i\bar{f}_j$ there is an additional factor $|V_{ij}|^2$ from the CKM mixing matrix.

Using these results for the partial widths of the W and the Z we can calculate the branching ratios for the various fermion-antifermion decay modes shown in Table 8.4. Note the sizeable branching fraction for decay into b quarks.

There is also a large branching fraction of the Z boson into neutrinos, which are not detectable. The measurement of the width for decay of

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Process	Relative coupling	Branching Ratio
$W^+ \to e^+\bar{\nu}, \mu^+\bar{\nu}, \tau^+\bar{\nu}$	1	$3 \times 11.1\%$
$W^+ \to u\bar{d} + u\bar{s} + u\bar{b}$	3	33.3%
$W^+ \to c\bar{d} + c\bar{s} + c\bar{b}$	3	33.3%
$Z \to \nu_e\bar{\nu}_e, \nu_\mu\bar{\nu}_\mu, \nu_e\bar{\nu}_\tau$	$\frac{1}{2}$	$3 \times 6.8\%$
$Z \to e^+e^-, \mu^+\mu^-, \tau^+\tau^-$	$\frac{1}{4} + (\frac{1}{2} - 2\sin^2\theta_W)^2$	$3 \times 3.4\%$
$Z \to u\bar{u}, c\bar{c}$	$3\,[\frac{1}{4} + (\frac{1}{2} - \frac{4}{3}\sin^2\theta_W)^2]$	$2 \times 11.8\%$
$Z \to d\bar{d}, s\bar{s}, b\bar{b}$	$3\,[\frac{1}{4} + (\frac{1}{2} - \frac{2}{3}\sin^2\theta_W)^2]$	$3 \times 15.2\%$

Table 8.4. Vector boson decay into fermion-antifermion.

the Z boson into invisible particles counts the number of light neutrino species. The theoretical result for the ratio of the width into one species of neutrino, divided by the width into one species of lepton is

$$\frac{\Gamma(Z \to \nu\bar{\nu})}{\Gamma(Z \to l^+l^-)} = \frac{2}{1 + (1 - 4\sin^2\theta_W)^2} \approx 1.99\,. \tag{8.93}$$

The experimental result [6] for the ratio of 'invisible' width to the width into one type of lepton pair is

$$\frac{\Gamma(Z \to \text{inv})}{\Gamma(Z \to l^+l^-)} = 5.956 \pm 0.031\,, \tag{8.94}$$

This measurement fixes the number of light neutrino species to be 2.991 ± 0.016 [6].

At hadron colliders one can normally only detect the decays of the vector bosons into charged leptons. Although the hadronic decay modes are enhanced relative to the leptonic modes, at a hadron collider there is a very serious background to the former from normal QCD two-jet production[1].

8.5 Vector boson scattering

The underlying gauge theory structure of the Standard Electroweak Model leads to strong cancellations in the high-energy behaviour of amplitudes for the scattering of gauge bosons [15]. In this section we illustrate these

[1] A statistically significant signal for the hadronic decay of the W and Z, at the expected level, has been found by the UA2 collaboration [14].

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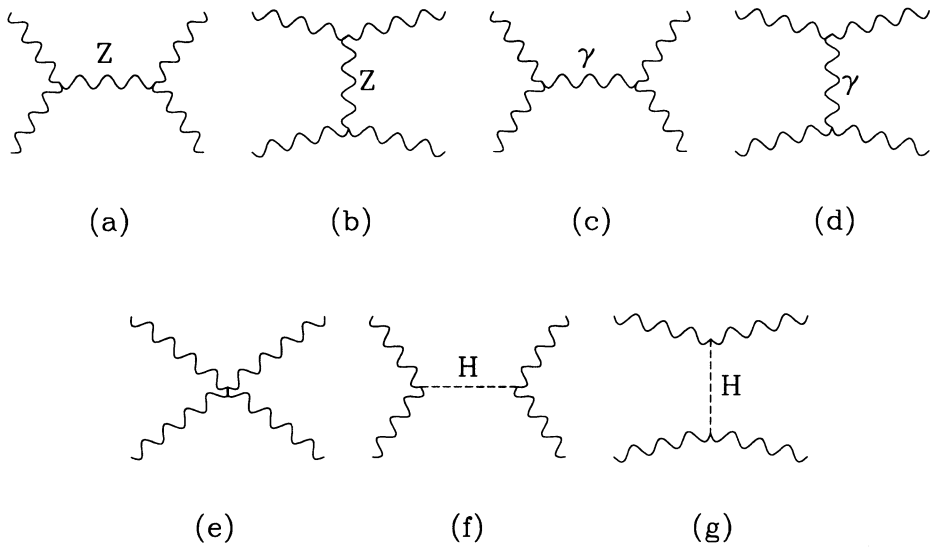


Fig. 8.7. Tree diagrams for $W^+ + W^- \to W^+ + W^-$ scattering.

cancellations by considering the scattering of W-bosons. The relevant diagrams, shown in Fig. 8.7, can be calculated using the Feynman rules of Fig. 8.2. The calculation will also allow us to display the role of the physical Higgs boson in WW scattering.

Consider the process

$$W^+(p_+) + W^-(p_-) \to W^+(q_+) + W^-(q_-) \tag{8.95}$$

in the centre-of-mass frame in which the incoming W bosons travel along the z axis. In this frame the kinematics of the reaction may be written as

$$\begin{aligned} p_\pm &= (E, 0, 0, \pm p)\,, \\ q_\pm &= (E, 0, \pm p\sin\theta, \pm p\cos\theta)\,, \end{aligned} \tag{8.96}$$

where $E^2 - p^2 = M_W^2$ and θ is the centre-of-mass scattering angle. The vector bosons can have longitudinal or transverse polarizations. Here we consider only the scattering of longitudinal bosons, which is responsible for the leading behaviour at high energy. The longitudinal polarization vectors for the W bosons are

$$\begin{aligned} \varepsilon_L(p_\pm) &= \Big(\frac{p}{M_W}, 0, 0, \pm\frac{E}{M_W}\Big)\,, \\ \varepsilon_L(q_\pm) &= \Big(\frac{p}{M_W}, 0, \pm\frac{E}{M_W}\sin\theta, \pm\frac{E}{M_W}\cos\theta\Big)\,. \end{aligned} \tag{8.97}$$

The polarizations satisfy the Lorentz condition $\varepsilon(k)\cdot k = 0$ and are normalized so that $\varepsilon^2 = -1$. In the high-energy limit we find that the result

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for the diagrams in Fig. 8.7 involving the three-boson vertex is

$$T^{a-d} = g_W^2 \left\{ \frac{p^4}{M_W^4}\left[3 - 6\cos\theta - \cos^2\theta\right] + \frac{p^2}{M_W^2}\left[\frac{9}{2} - \frac{11}{2}\cos\theta - 2\cos^2\theta\right]\right\}, \tag{8.98}$$

neglecting terms not enhanced by factors of p^2/M_W^2. The corresponding result for the diagram with the four-boson vertex is

$$T^e = g_W^2 \left\{ \frac{p^4}{M_W^4}\left[-3 + 6\cos\theta + \cos^2\theta\right] + \frac{p^2}{M_W^2}\left[-4 + 6\cos\theta + 2\cos^2\theta\right]\right\}. \tag{8.99}$$

Finally, the graphs involving Higgs exchange give

$$T^{f-g} = g_W^2 \left\{ \frac{p^2}{M_W^2}\left[-\frac{1}{2} - \frac{1}{2}\cos\theta\right] - \frac{M_H^2}{4M_W^2}\left[\frac{s}{s - M_H^2} + \frac{t}{t - M_H^2}\right]\right\}. \tag{8.100}$$

Because of the longitudinal polarizations, the high-energy behaviour of individual graphs in Fig. 8.7 grows with the centre-of-mass momentum like

$$T(s,t) = A\left(\frac{p}{M_W}\right)^4 + B\left(\frac{p}{M_W}\right)^2 + C\,. \tag{8.101}$$

On summing over graphs, the term A cancels without including the Higgs graphs, (f) and (g). The full cancellation of the B term involves the Higgs boson in an essential way. On summing all graphs the result is

$$T(s,t) = -g_W^2 \frac{M_H^2}{4M_W^2}\left[\frac{s}{s - M_H^2} + \frac{t}{t - M_H^2}\right] \tag{8.102}$$

$$\equiv -4\left[\lambda + \frac{(\lambda v)^2}{s - 2\lambda v^2} + \frac{(\lambda v)^2}{t - 2\lambda v^2}\right]. \tag{8.103}$$

Eq. (8.103) results from substituting for M_W and M_H using Eqs. (8.32) and (8.36). It is no accident that this expression looks like the amplitude for the scattering of charged scalar particles. The longitudinal components of the W are derived from the charged components of the Higgs doublet ϕ. The result of Eq. (8.103) can be simply calculated from the theory without gauge couplings in which there are three Goldstone bosons interacting with one massive scalar:

$$\mathcal{L}_{\text{Goldstone}} = \tfrac{1}{2}\partial_\mu H \partial^\mu H + \tfrac{1}{2}\partial_\mu z \partial^\mu z + \partial_\mu w^+ \partial^\mu w^- - \lambda v H^2$$
$$-\lambda v H(2w^+w^- + z^2 + H^2) - \tfrac{1}{4}\lambda(2w^+w^- + z^2 + H^2)^2\,. \tag{8.104}$$

The two charged massless Goldstone bosons are denoted by $w^\pm$ and the diagrams are shown in Fig. 8.8.

Although the leading-order WW scattering amplitude does not diverge at high energy, it might still be too large to be consistent with the fundamental physical limitation due to unitarity of the scattering matrix. To

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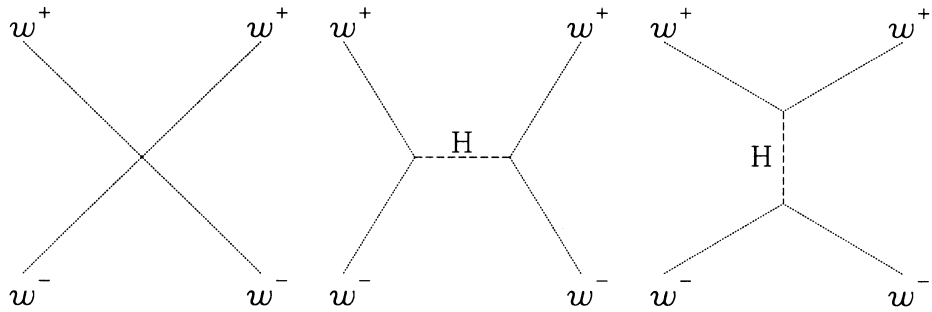


Fig. 8.8. Tree diagrams for $w^+ + w^- \to w^+ + w^-$ scattering.

derive the unitarity limit on the high-energy behaviour it is convenient to perform a partial wave expansion,

$$T(s,t) = 16\pi \sum_J (2J+1) a_J(s) P_J(\cos\theta) \,, \tag{8.105}$$

where P_J are the Legendre polynomials, $P_0(x) = 1$, $P_1(x) = x$ and $P_2(x) = \frac{1}{2}(3x^2 - 1)$ etc., and $\cos\theta = 1 + 2t/s$ at high energy. The partial wave expansion decomposes the amplitudes into the contributions from various total angular momenta J. The differential cross section, neglecting particles masses, is given by

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s} |T(s,t)|^2 \,. \tag{8.106}$$

Inserting Eq. (8.105) and using the orthogonality property of the Legendre polynomials,

$$\int_{-1}^{1} dx \; P_J(x) P_K(x) = \delta_{JK} \, \frac{2}{2J+1} \,, \tag{8.107}$$

we find that the total cross section is given by

$$\sigma = 16\pi \sum_J (2J+1) |a_J(s)|^2 \,. \tag{8.108}$$

Using the optical theorem

$$\sigma = \frac{1}{s} \mathrm{Im} \; T(s,0) \,, \tag{8.109}$$

we find that

$$|a_J|^2 = \mathrm{Im} \; a_J \tag{8.110}$$

and hence the modulus of a_J cannot exceed one. Since J is conserved, the modulus of each partial wave amplitude cannot exceed its initial value, which for a plane-wave incoming state is one.

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From Eq. (8.102), we find that the result for the $J = 0$ partial wave amplitude is

$$\begin{aligned} a_0(s) &= -\frac{G_F M_H^2}{8\pi\sqrt{2}}\left[2 + \frac{M_H^2}{s - M_H^2} - \frac{M_H^2}{s}\ln\left(1 + \frac{s}{M_H^2}\right)\right] \\ &\to -\frac{G_F M_H^2}{4\pi\sqrt{2}} \,. \end{aligned} \tag{8.111}$$

In the second line of this equation we have taken the limit $s \gg M_H^2$. Since unitarity will be violated if $|a_0| > 1$, this implies that $M_H^2 < 4\sqrt{2}\pi/G_F$. A slightly more sophisticated analysis [16], involving consideration of all elastic boson-boson scattering channels, gives the limit

$$M_H < \left(\frac{8\sqrt{2}\pi}{3G_F}\right)^{\frac{1}{2}} \approx 1 \text{ TeV} \,. \tag{8.112}$$

We therefore find that if the limit in Eq. (8.112) is exceeded then weak interactions will become strong, *i.e.* the high-energy scattering of weak bosons will not be adequately described by perturbation theory. It is important to stress the nature of the limit that we have derived. If Eq. (8.112) is violated the perturbative methods which we have used to derive the bound cannot be trusted. The Higgs mass can exceed the value given by the bound, but in that case its properties cannot be analysed using perturbation theory. Therefore if we can investigate the scattering of longitudinal vector bosons at a high-energy hadron collider, we will either discover the Higgs boson or see strong interactions of the electroweak bosons, or both. This provides a compelling motivation for the construction of such a machine.

8.6 Vector boson pair production

We have seen that the Standard Model makes specific predictions for the couplings of vector bosons, which follow from the non-Abelian symmetry and the local gauge invariance of the Lagrangian. It is important to test these predictions as precisely as possible. As described above, gauge cancellations amongst the couplings have an enormous effect on the high-energy behaviour of WW scattering. A process which will be subjected to experimental scrutiny in the more immediate future is the production of W pairs in e^+e^- annihilation.

We consider the process $e^-e^+ \to W^-W^+$ with Feynman diagrams shown in Fig. 8.9. For simplicity we set the electron mass to zero in which case Fig. 8.9(d) makes no contribution. The outgoing vector bosons can have longitudinal or transverse polarizations. Here we consider only

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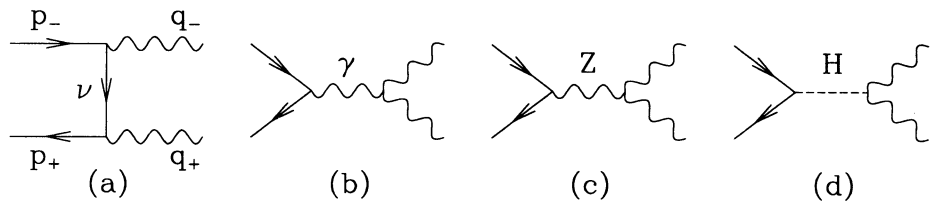


Fig. 8.9. Diagrams for $e^-e^+ \to W^-W^+$.

the longitudinal polarization, which is responsible diagram-by-diagram for the leading behaviour at high energy. We choose a frame in which the outgoing momenta are directed along the z axis. Then the polarization vectors are

$$\begin{aligned} \varepsilon_L(q_\pm) &= \left(\frac{q}{M_W}, 0, 0, \pm\frac{E}{M_W}\right) \\ &\sim \frac{1}{M_W}\left(q_\pm - \frac{M_W^2}{2E^2}q_\mp\right) . \end{aligned} \tag{8.113}$$

Because of these terms, the individual diagrams shown in Fig. 8.9 will lead to amplitudes which grow like E^2. If we are only interested in the leading terms we may contract each vector boson with the leading term in Eq. (8.113). The contribution of diagram (a) is given by

$$M = \frac{(-ig_W)^2}{8}\bar{v}(-p_+)\not{\varepsilon}(q_+)(1-\gamma_5)\frac{i}{\not{p}_- - \not{q}_-}\not{\varepsilon}(q_-)(1-\gamma_5)u(p_-) . \tag{8.114}$$

Inserting the leading term for the polarizations from Eq. (8.113) and dropping non-leading terms, we obtain

$$M = -i\frac{(-ig_W)^2}{8M_W^2}\bar{v}(-p_+)(\not{q}_+ - \not{q}_-)(1-\gamma_5)u(p_-) . \tag{8.115}$$

The contribution of the photon exchange diagram (b), using the Feynman rules of Figs. 8.2 and 8.5, is

$$\begin{aligned} M &= (-ig_W)(ig_W)Q_e \sin^2\theta_W \bar{v}(-p_+)\gamma^\rho u(p_-)\frac{-ig_{\rho\alpha}}{(q_+ + q_-)^2} \\ &\times V^{\alpha\beta\delta}(q_+ + q_-, -q_-, -q_+)\varepsilon_\beta(q_-)\varepsilon_\delta(q_+) . \end{aligned} \tag{8.116}$$

The contribution of the Z exchange diagram (c) is

$$\begin{aligned} M &= \frac{(-ig_W)(ig_W)}{2}\bar{v}(-p_+)\gamma^\rho(V_e - A_e\gamma_5)u(p_-)\frac{-ig_{\rho\alpha}}{(q_+ + q_-)^2 - M_Z^2} \\ &\times V^{\alpha\beta\delta}(q_+ + q_-, -q_-, -q_+)\varepsilon_\beta(q_-)\varepsilon_\delta(q_+) \end{aligned} \tag{8.117}$$

where

$$V^{\alpha\beta\delta}(p,q,r) = g^{\alpha\beta}(p^\delta - q^\delta) + g^{\beta\delta}(q^\alpha - r^\alpha) + g^{\delta\alpha}(r^\beta - p^\beta) . \tag{8.118}$$

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Hence we find, using the approximate longitudinal polarization vector of Eq. (8.113),

$$V^{\alpha\beta\delta}(q_+ + q_-, -q_-, -q_+)\varepsilon_\beta(q_-)\varepsilon_\delta(q_+)$$
$$= -\frac{(q_+ + q_-)^2}{2M_W^2}\left[q_+^\alpha - q_-^\alpha\right] + O(1)\,. \tag{8.119}$$

The result for the sum of the Z and γ exchange contributions, after inserting the values for V_e and A_e, is

$$M = i\frac{(-ig_W)^2}{8M_W^2}\bar{v}(-p_+)(\not{q}_+ - \not{q}_-)(1-\gamma_5)u(p_-)\,, \tag{8.120}$$

which cancels exactly with Eq. (8.115). So we see that the leading high-energy behaviour is cancelled, as a result of the relationship between the three-boson couplings and the couplings to fermions imposed by the gauge structure.

The total cross section for W^+W^- production by a massless fermion-antifermion pair is given by [17]

$$\sigma = \frac{2\pi\alpha^2}{s}\left\{\left[\left(Q_i + \frac{V_i}{2x_W}\frac{s}{s-M_Z^2}\right)^2 + \left(\frac{A_i}{2x_W}\frac{s}{s-M_Z^2}\right)^2\right]A(y_W,\beta_W)\right.$$
$$-\frac{\mathrm{sign}(Q_i)}{2x_W}\left(Q_i + \frac{V_i + A_i}{2x_W}\frac{s}{s-M_Z^2}\right)\sum_j |V_{ij}|^2\, I(y_W,\beta_W)$$
$$\left.+\frac{1}{8x_W^2}\left(\sum_j |V_{ij}|^2\right)^2 E(y_W,\beta_W)\right\}, \tag{8.121}$$

where $x_W = \sin^2\theta_W, y_W = M_W^2/s$ and $\beta_W = \sqrt{1-4y_W}$. The matrix V_{ij} here refers to the CKM matrix, necessary in the case of initial quarks. The three terms in Eq. (8.121) correspond to the square of the s-channel exchange diagram, the interference between s- and t-channel exchange diagrams and the square of the t-channel exchange diagram, respectively. The functions A, I and E are given by

$$A(y,\beta) = \frac{\beta^3}{y^2}\left[\frac{1}{24} + \frac{5}{6}y + \frac{1}{2}y^2\right]$$
$$I(y,\beta) = \frac{\beta}{y^2}\left[\frac{1}{24} + \frac{3}{4}y - \frac{7}{6}y^2 - y^3\right] - 4y(1+\frac{1}{2}y)\ln\left(\frac{1+\beta}{1-\beta}\right)$$
$$E(y,\beta) = \frac{\beta}{y^2}\left[\frac{1}{24} + \frac{5}{6}y - 2y^2\right] + 2(1-2y)\ln\left(\frac{1+\beta}{1-\beta}\right). \tag{8.122}$$

Because of the singular nature of these functions at small y, individual terms lead to contributions to the cross section which grow like s, due

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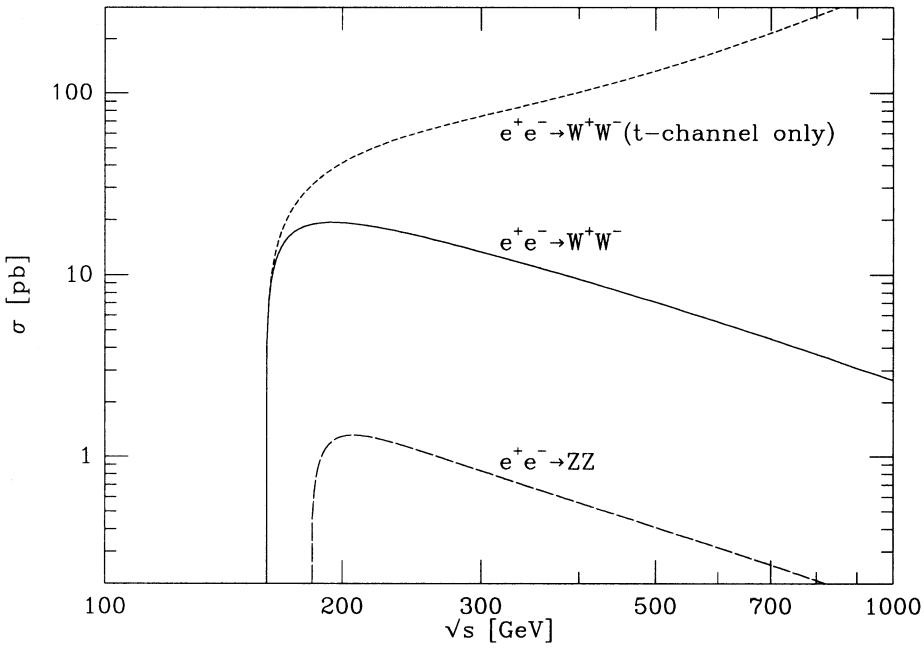


Fig. 8.10. W and Z pair cross sections in e^+e^- annihilation.

to the production of longitudinal gauge bosons. This behaviour is cancelled on summing over all diagrams because of the gauge structure of the underlying Lagrangian. Fig. 8.10 shows the W pair production cross section. To illustrate the strength of the cancellation we also show the contribution from the t-channel exchange diagram, *i.e.* the third line of Eq. (8.121) calculated using Fig. 8.9(a) alone.

The corresponding formula for Z pair production is

$$\sigma = \frac{\pi\alpha^2}{4sx_W^2(1-x_W)^2}(V_i^4 + A_i^4 + 6V_i^2A_i^2)\left[\frac{1+4y_Z^2}{1-2y_Z}\ln\left(\frac{1+\beta_Z}{1-\beta_Z}\right) - \beta_Z\right] \tag{8.123}$$

where $y_Z = M_Z^2/s$ and $\beta_Z = \sqrt{1-4y_Z}$. Fig. 8.10 also shows the Z pair production cross section as a function of energy.

Vector boson pair production can also be studied at hadronic colliders. The techniques for calculating single vector boson production in hadron-hadron reactions will described in the next chapter. Fig. 8.11 shows the cross sections for the production of pairs of vector bosons calculated at lowest order in perturbation theory [2]. Higher-order QCD corrections have

[2] In this figure we assume that the weak bosons have zero width. In practice the non-zero widths will allow the production of off-shell bosons and modify the threshold region.

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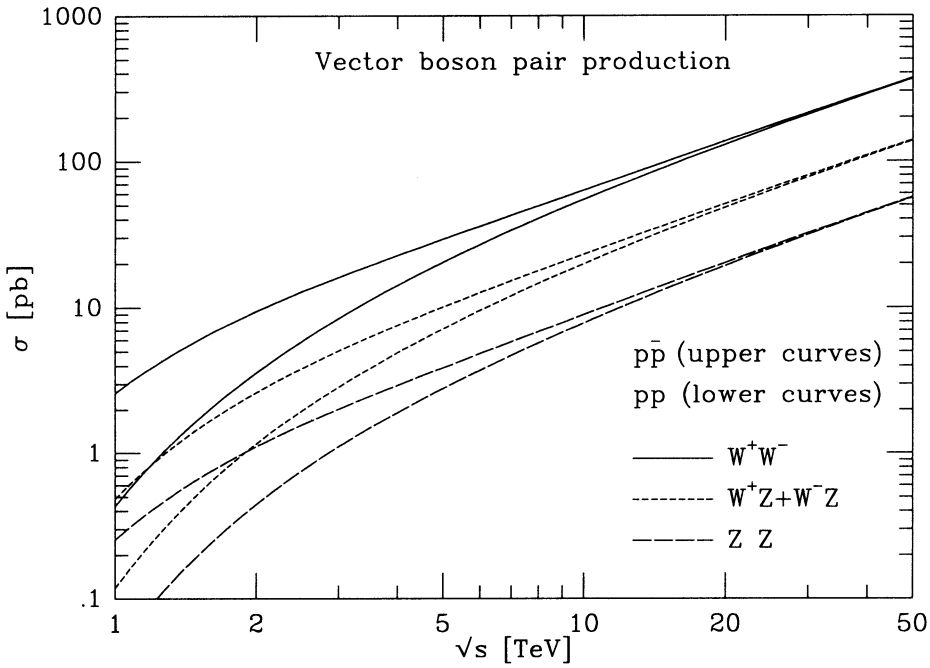


Fig. 8.11. Vector boson pair production cross sections in $p\bar{p}$ and pp collisions.

been calculated to all of these processes and tend to increase the cross sections. For details we refer the reader to refs. [18-20].

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