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Mechanical design for assembly of a 4-DOF robotic arm utilizing a top-down concept

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SUMMARY

This paper proposes an effective top-down design approach for the mechanical design for assembly of a four degree-of-freedom revolute jointed robotic arm. The design process begins by specifying top-level design criteria and passing down these criteria from the top level of the manipulator's structure to all of the subsequent components. With this proposed approach, the sequential design intents are captured, organized and implemented based on the entire system's objectives, as opposed to the conventional design process which aims at individual components' optimization. By considering the mechanical arm's performance objectives, the design starts with modeling the integration of all the individual links constituting the manipulator. During the design process, modifications are made based on the integrated information of kinematics, dynamics, and structural analyses of the desired robot configurations as a whole. An optimum assembly design is then achieved with workable subdesigns of the manipulator components. As a result, the proposed approach for manipulator design yields substantially less number of iterations, automatic propagation of design changes, and great saving of design efforts. Thus, it is suitable and can be used as a guideline for design automation purposes of complex systems such as robotic arms.

KEYWORDS: Top-down concept; Mechanical design; Robotic arm.

1. INTRODUCTION

Top-down design is a method of designing a product by specifying top-level design criteria and passing down these criteria from the top level of the product's structure to all the affected subsystems.[1–5] In addition, feature-based design for assembly (DFA) concepts has become an essential issue in recent research in the design community.[6-10]

In a typical mechanical design for assembly of engineering products, a component should be designed according to the overall system performance satisfying the desired functionalities of the engineered products.[2,3] That is, an optimum design of a component may not be a good design for the entire system. Component characteristics must be determined considering the system performance. Generally the mechanical design process is conducted in the bottom-up manner. Bottom-up design carries out the component design first and evaluates the system performance by substituting the component design results into the system analysis. The resulting design is analyzed to determine if it meets the specified design constraints. If it fails to do so, it is modified on the basis of further information obtained from the analysis. It is re-analyzed until the design meets the specifications, or until the designer realizes that the design is not feasible within the design constraints. Thus the bottom-up design approach has numerous iterative processes, because it uses the results of a design analysis as a basis for re-design decisions. It is a typical trial-and-error process which is, for most cases, a very tedious and timeconsuming process. On the other hand, top-down design uses the design analysis as a method to guide the design. In other words, the design is produced directly by analysis. This gives rise to a component that fits perfectly into the system and requires fewer modifications later on.

Libardi *et al.*[11] defined top-down assembly design as a recursive subdivision of an initial abstract concept into logical subassemblies until the level of components or parts is reached. Kikkawa *et al.*[12] have developed a product modeling system to support mechanism design that copes with two aspects of the design process, top-down design and preliminary design. They proposed a constraint based modeling system with default values, in order to support the top-down design of a mechanism from the beginning of the design process. According to their work, top-down design of a mechanism involves defining the mechanism configuration, which is abstracted in terms of kinematic pair relations between parts. The parts of the mechanism are then designed in detail, such as their shape, material and so on. Thus, the refinement of abstract model into a detailed one is, in their interpretation, a feature of top-down design. Mantripragada *et al.*[13] expressed the topdown design process as one that starts at the assembly or the final product level, defines datum schemes consistent with the important aspects of that assembly, and coordinates these datums down the assembly tree to the part level.

Recently, various researchers have proposed the development of software systems to implement the topdown assembly design methodology. Matripragada *et al.*[13] described a prototyped software system that implements the top-down approach to the concept stage of assembly design and analysis. They adopted a tool called Key Characteristics to capture a few important characteristics of the product, then differentiating them from the large number of unprioritized tolerances that appear on engineering drawings. The Key Characteristics are further used to identify

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important datum on parts and subassemblies, and to define relationships between them. They used a technique called Datum Flow Chain to assign hierarchy to datum and to define which parts assembled with other parts in the assembly.

Mantyla and Gui[14] present a general top-down assembly model, called Delta, that provides both a function-oriented and module-oriented view of the assembly structure. The concept of general assembly model has been based on the concept of product modeling technology, which aims at developing a neutral integrated product information model. The general assembly model is capable of supporting both conceptual design at high levels of abstraction, and feature modeling at low levels.

Mantlya[15] also presented a prototype modeling system called WAYT, to achieve the goals of top-down assembly design. The prototype consists of three major components, a Design Browser to represent hierarchical-part-of-graphs at several levels of detail, a Geometry Browser to associate geometric feature models with the design entities in a model created by the design browser and, a Constraint Browser to create and edit geometric constraints describing the mating conditions and other positional and dimensional restrictions among the parts of an assembly.

Among others, Koh and Park[2] presented the top-down design methodology for the design of an automobile exhaust system. The system is represented by lumped parameters for components. The parameters are optimized from the overall system design. Each component is designed according to the results of the system design. Design of bellows in an automobile exhaust system is presented to demonstrate the effectiveness of the system design.

The design of a robotic manipulator involves simultaneous participationa of different disciplines in the design process. To be able to conduct the design process with a simultaneous contribution of different disciplines, there should be an integrated design methodology. Such a methodology requires to provide common representation schemes for the design knowledge and intents, facilitates sharing of design knowledge and intents, as well as guides the design process based on cooperative strategies.[16]

In this paper, the top-down design methodology is employed for developing the design and assembly processes of a 4-degree-of-freedom robot manipulator. The design of the manipulator begins directly with the assembly model, and then based on the system requirements, the individual components are designed and modeled. The entire design, modeling and analysis of the 4-DOF manipulator have been carried out with Pro/Engineer® and MATLAB®.

2. TOP-DOWN DESIGN PROCESS OF THE 4-DOF MANIPULATOR

Top-down design process consists of some of the following steps,[1] namely,

(a) Defining design intents;
(b) Defining preliminary product configurations;
(c) Introducing skeleton models;
(d) Communicating design intents throughout the assembly structure;
(e) Continued population of the desired assembly.

The manipulator designed here is a 4 revolute-jointed serial arm robot. The main application of the robot is for drilling operation of aluminum parts. The manipulator design consists of the following major design and analysis steps, in line with the top-down design concept, listed below:

(i) Forward kinematic analysis;
(ii) Inverse kinematic analysis;
(iii) Motion kinematic analysis;
(iv) Force-torque analysis;
(v) Dynamic analysis; and
(vi) Structural analysis.

The design begins with a model of the integration of all the individual links of the manipulator. During the design process, modifications are made based on the results of the above analytical steps obtained from the Matlab routines. The top-down design steps are integrated with the design and analysis steps of the manipulator to obtain an optimum assembly design with much less number of iterations and design time reduction.

The step-by-step realizations of the 4-DOF manipulator top-down design process are described in the following section.

3. STEP-BY-STEP REALIZATION OF THE MANIPULATOR DESIGN

3.1. Defining the design intents

As mentioned previously, the main purpose of the manipulator is to drill holes in aluminum components. This capability can be realized by incorporating a pneumatic drill in link number 4, the end-effector of the manipulator. A mechanism for pushing the drill bit and retracting it from the component to be drilled needs to be incorporated in the end-effector itself.

To carry out the kinematics analysis, the underlying manipulator is modeled as a serial assemblage of links and joints. One end of the chain is connected to the ground, whereas the other end is attached with an end effector tool bit. The kinematic chain configuration is shown in Figure 1.

According to the design intents of this manipulator, it is required to perform a complete kinematics analysis to obtain pertinent configuration.

3.1.1. Kinematics analysis. To determine the forward kinematic equations of the manipulator, we emnloy the Denavit-Hartenberg (D-H) parameters and coordinate transformation conventions.[17]

D-H parameters completely specify the configuration of a frame i system relative to a frame $(i-1)$ system. The individual D-H coordinate transformation matrices, which relate the successive links in the manipulator, when combined, give the forward kinematics equations associated with the underlying manipulator. The D-H parameters for

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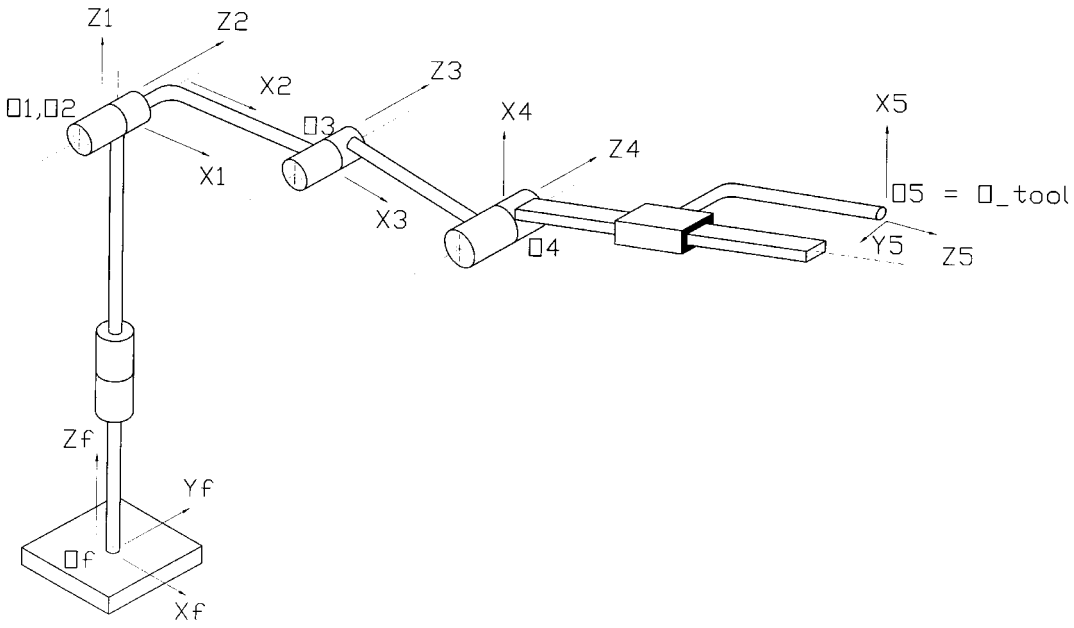


Fig. 1. 4-revolute-jointed manipulator modeled as an open kinematic chain.

the derived 4-R serial manipulator shown in Figure 1, are given in Table I.

where α_i = twist angle, b_i = displacement, θ_i = joint rotation angle, d_i = offset distance

It is noted here that the displacement (b_i) and offset distance (d_i) have been chosen arbitrarily at this stage of the design. These parameters may be modified as the design progresses. In addition, joint rotation angles (θ_i) specify one particular configuration of the manipulator.

The D-H configuration transformation matrix ($^{i-1}T_i$), relating the frame i coordinate system to frame $i-1$ coordinate system is given as follows,[12]

$$^{i-1}T_i = \begin{bmatrix} \cos(\theta_i) & -\sin(\theta_i) & 0 & b_i \\ \cos(\alpha_i)\cdot\sin(\theta_i) & \cos(\alpha_i)\cdot\cos(\theta_i) & -\sin(\alpha_i) & -d_i\cdot\sin(\alpha_i) \\ \sin(\alpha_i)\cdot\sin(\theta_i) & \sin(\alpha_i)\cdot\cos(\theta_i) & \cos(\alpha_i) & d_i\cdot\cos(\alpha_i) \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)$$

Based on the D-H parameters for the kinematic chain shown in Figure 1, the overall coordinate transformation matrix relating the base/fixed coordinate system (F), to the endeffector coordinate system (5) is given as follows,

Table I. D-H parameters for the 4-DOF manipulator.

Link No.	α_i	b_i*	θi*	d_i*
1	0°	0	90°	875
2	270°	0	−30°	0
3	0°	600	52°	0
4	0	800	248°	0
5	270	0	0	275

$$^{F}T_5 = (^{F}T_1)\cdot(^{1}T_2)\cdot(^{2}T_3)\cdot(^{3}T_4)\cdot(^{4}T_5)$$
$$= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1536.4 \\ 1 & 0 & 0 & 875 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} ^{F}R_5 & ^{F}P_5 \\ 0 & 1 \end{bmatrix} \quad (2)$$

where $^{F}R_5$ represents the orientation, and $^{F}P_5$ represents the position of the fifth coordinate system of the manipulator with respect to the base/fixed coordinate system.

3.2. Defining preliminary product configuration

The preliminary product structure is defined in Pro/Engineer®, which is used as a tool to carry out the top-down assembly design. It is noted that each of the links in itself is an assembly of a number of individual components. Thus, each link has been modeled separately as a subassembly. The product structure has been separately modeled for each of these subassemblies.

Figure 2 shows the product structure defined for link number 4 (end-effector). It lists the components of the subassembly and defines their hierarchy. Thus the end-effector assembly consists of the drill assembly, slider assembly, link structure assembly, hydraulic cylinder assembly, end cap assembly and actuator assembly. It is noteworthy that these components have been added to the assembly model without them having to exist, or being actually assembled, which demonstrates the design flexibility of the approach.

Having a product structure before actually modeling the components has several advantages.[1] It ensures that the components are placed consistently, with similar assembly features and constraints. For the end-effector subassembly, there are certain motion constraints between the individual components. These constraints are modeled based on certain standard features of each component model, such as datum planes and axes. Thus, it ensures that the constraints are maintained even if the design is modified later, since the

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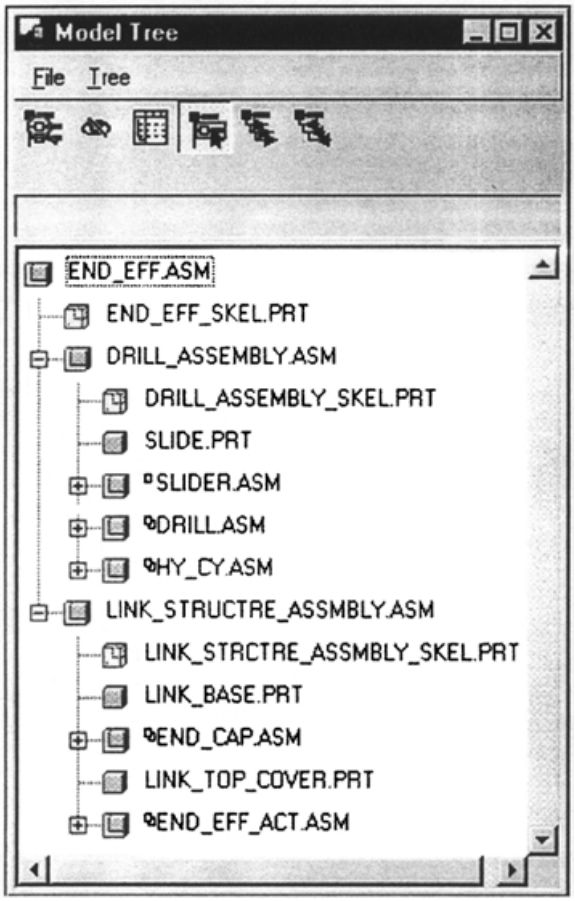


Fig. 2. Model tree showing the preliminary product structure of end-effector subassembly.

datum planes and axes remain the same. Another advantage of defining the preliminary product structure is the fact that much of the non-geometrical information that defines the entire design can be communicated in the product structure very early in the design process without actually creating any detailed geometry. This saves tremendous trial-and-error efforts required in conventional design processes.

3.3. Introducing skeleton models

In this step, the skeleton model of the end-effector is to be built. Figure 3 shows the skeleton model of the end-effector.

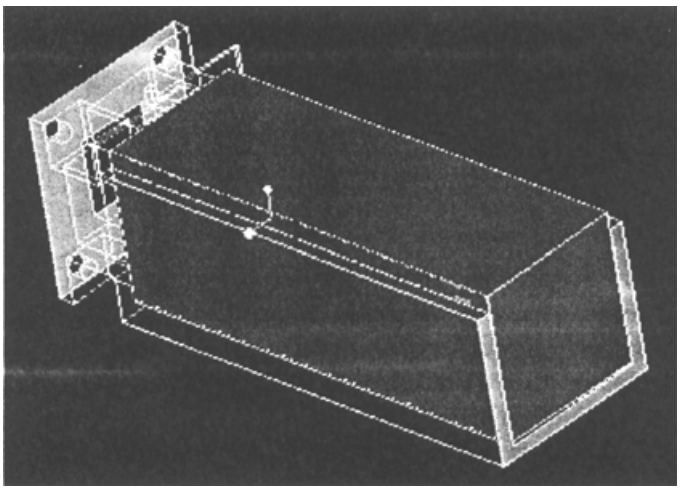


Fig. 3. Skeleton model of the end-effector.

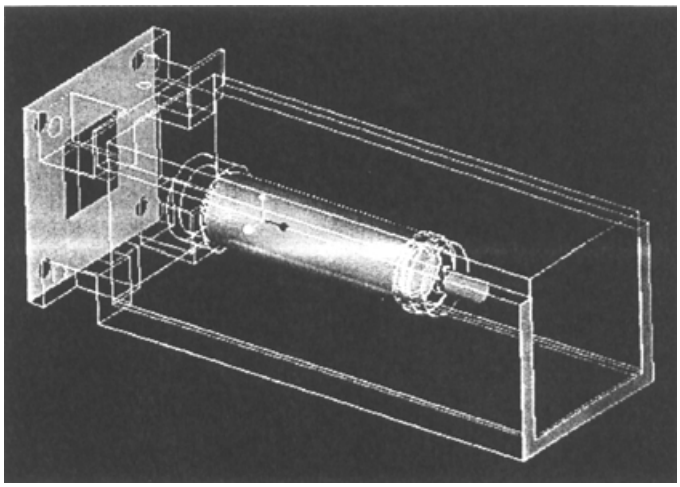


Fig. 4. Skeleton model of the pneumatic drill, with end-effector skeleton seen as wire-frame model.

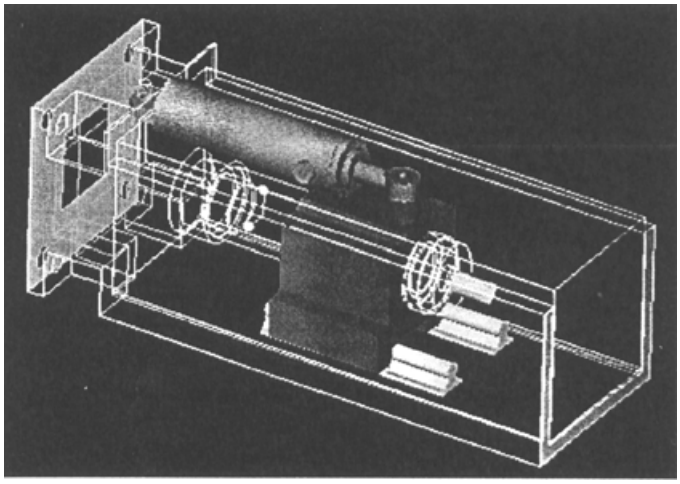


Fig. 5. Slider mechanism – the slider, slides and hydraulic cylinder.

The skeleton model helps to maintain the space and size constraints for the underlying link[18] The size of the link is based on the kinematic considerations for the manipulator.

While modeling other components of the end-effector subassembly, it is required that they fit inside the outer surfaces of the skeleton model. This allows the space and size constraints to be in effect. The dimensions of the link structure are referenced to the boundary surfaces of the skeleton model, thus if any changes are made to the design, only the dimensions of the end-effector skeleton need to be changed. However, all the related dimensions are modified automatically by the propagated feature of the top-down design.

Next, a slider mechanism is incorporated in the end-effector to push and retract the drill bit from the component to be drilled.[19] The slider dimensions are referenced to the pneumatic drill skeleton (Figure 4). Thus, any changes in the size of the drill will automatically modify the slider dimensions.

Figure 5 shows the slider mechanism of the end-effector, along with the slides and the hydraulic cylinder to push and retract the slider and the drill bit during a drilling operation.

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The skeleton of the pneumatic drill is shown in a wire-frame model.

The skeleton models here are also used to simulate the motion between individual components. During an actual operation, the hydraulic cylinder pushes the slider, which in turn pushes the pneumatic drill and the drill bit into the components to be drilled. Figure 6 in the next section shows the results of motion simulation and interference check of the endeffector link.

3.3.1 Differential kinematics analysis. The differential kinematics equations associated with the manipulator, express the velocity relations between Cartesian variables such as $x(t)$ and $y(t)$ whose desired behavior is known, and the corresponding link variables such as $\theta(t)$ which must be directly controlled in order to achieve the desired Cartesian motion. To derive the equations of motion kinematics we represent the complete end effector configuration (denoted by X) of the given manipulator as a function of (n) link values, denoted by θ. The vector X is a rank six column vector in a general three-dimensional case. θ is an n-vector, with n denoting the number of movable links. Each element X_i of X is a function of θ_j of θ. By differentiating X with respect to time, we obtain the Cartesian configuration velocity vector $\dot{X}$, and a matrix of partial derivatives, which relates $\dot{X}$ to θ, the vector of corresponding link velocities.[20] This yields

$$\dot{X} = \begin{bmatrix} \dot{p}_x \\ \dot{p}_y \\ \dot{p}_z \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = [J] \cdot \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \\ \dot{\theta}_4 \end{bmatrix} = [J] \cdot [\dot{\theta}] \tag{3}$$

where [J] = the Jacobian.

For the 4-DOF manipulator considered here, based on the D-H parameters summarized in Table I, the Jacobian matrix can be expressed as follows.

$$[J] = \begin{bmatrix} (d_5 \cdot s_1 \cdot s_{2+3+4} - s_1 \cdot L) & (-d_5 \cdot c_1 \cdot c_{2+3+4} - c_1 \cdot M) \\ (-d_5 \cdot c_1 \cdot s_{2+3+4} + c_1 \cdot L) & (-d_5 \cdot s_1 \cdot c_{2+3+4} - s_1 \cdot M) \\ 0 & (d_5 \cdot s_{2+3+4} - L) \\ 0 & -s_1 \\ 0 & c_1 \\ 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} (-d_5 \cdot c_1 \cdot c_{2+3+4} - c_1 \cdot b_4 \cdot s_{2+3}) & -d_5 \cdot c_1 \cdot c_{2+3+4} \\ (-d_5 \cdot s_1 \cdot c_{2+3+4} - s_1 \cdot b_4 \cdot s_{2+3}) & d_5 \cdot s_{2+3+4} \\ (d_5 \cdot s_{2+3+4} - b_4 \cdot c_{2+3}) & d_5 \cdot s_{2+3+4} \\ -s_1 & -s_1 \\ c_1 & c_1 \\ 0 & 0 \end{bmatrix} \tag{4}$$

where

$$L = (b_4 \cdot c_{2+3} + c_2 \cdot b_3) \quad \text{and} \quad c_1 = \cos(\theta_1)$$
$$M = (b_4 \cdot s_{2+3} + s_2 \cdot b_3) \quad \text{and} \quad c_{2+3} = \cos(\theta_2 + \theta_3)$$

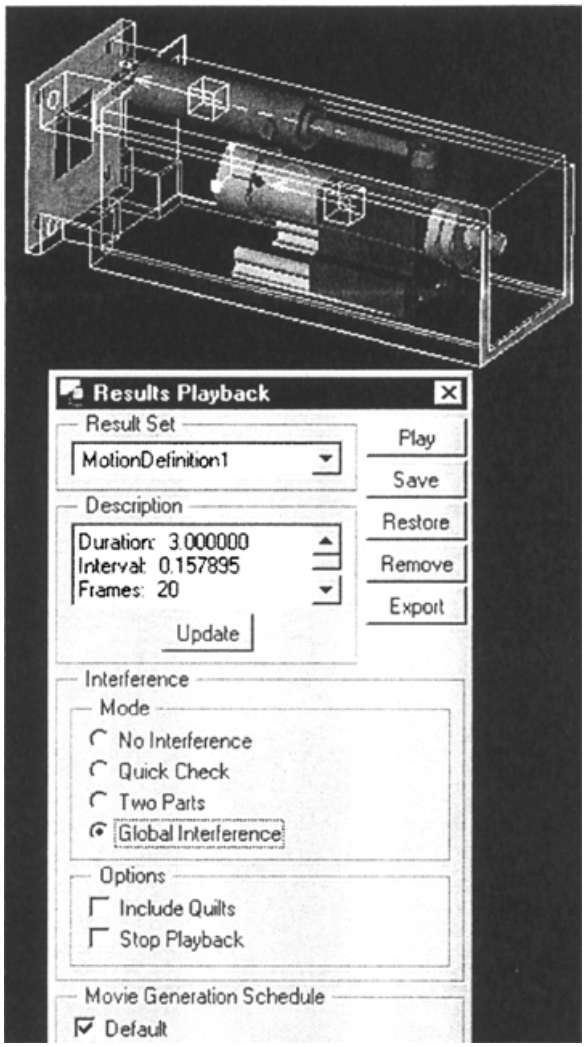


Fig. 6. Running motion simulation and interference check on the end-effector model.

3.3.2. Force-torque analysis. The contact between the manipulator and the environment results in interactive forces and moments at the manipulator/environment interface. The external force and moment thus produced, must be compensated for by internal joint torques and forces if the manipulator is to remain in its static equilibrium. To develop a relationship between the external forces and moments applied at the drill tip and the internal joint torques and forces they induce, we utilize the concept of Jacobian matrix.

For the 4-DOF manipulator considered, external force vector is given by,

$$F_{tool} = F_5 = \begin{bmatrix} F_t \\ M_t \end{bmatrix} = \begin{bmatrix} f_{tx} \\ f_{ty} \\ f_{tz} \\ m_{tx} \\ m_{ty} \\ m_{tz} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -f_{tz} \\ " \\ " \\ -m_{tz} \end{bmatrix} \tag{5}$$

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The generalized torque vector [T] can be calculated using the following equation:

$$[T] = [J]^T \cdot [F] \tag{6}$$

or

$$\begin{bmatrix} m_{1z} \\ m_{2z} \\ m_{3z} \\ m_{4z} \end{bmatrix}_{4\times 1} = [J]^T_{4\times 6} \cdot \begin{bmatrix} 0 \\ 0 \\ -f_z \\ \prime\prime \\ \prime\prime \\ -m_z \end{bmatrix}_{6\times 1} \tag{7}$$

where $[J]^T$ is the transpose of the Jacobian given by Equation (4).

3.4. Communicating design intents throughout the assembly structure

In this step, critical design intents of the manipulator is passed from one level of design to the next, to execute the top-down design concept. This critical information is defined in the high-level skeleton model, (the end-effector skeleton in this case) and passed down to other consequent skeleton models in the robot structure assembly, illustrated in the model tree in Figure 2.

The solid model of the link is referenced to the skeleton model, thus any design changes made to the skeleton model are reflected automatically in the subassembly model and its components. Since the components are initially modeled keeping in mind the motion and space constraints, it is required that they comply with these constraints even after design changes and modifications are made. This cuts down the design time significantly, by reducing the number of iterative steps mandated in the conventional bottom-up design approach.

3.5. Continued detailing the additional features of the assembly

This step involves adding detailed parts and subassemblies to the model. Components to be designed are created directly in the context of the assembly, whereas standard bought out components like gears, motors etc. are modeled separately and then added onto the main assembly model. Designed components are defined in terms of spatial relationships in the assembly, so that the design is based on clearances instead of independent dimensions. A library of standard parametric features of the entire assembly has been created, so that the desired shape and size of components can be obtained by selecting desired parameters. This saves tremendous time for the follow up designs and/or modifications.

Figure 7 shows the completed model of the end-effector consisting of all the components defined in the product structure shown in Figure 2.

Similar procedure is adopted for the design of the remaining three links of the 4-DOF manipulator. The completed model of the manipulator is shown in Figure 8 with a solid model view and Figure 9 with a wireframe model view.

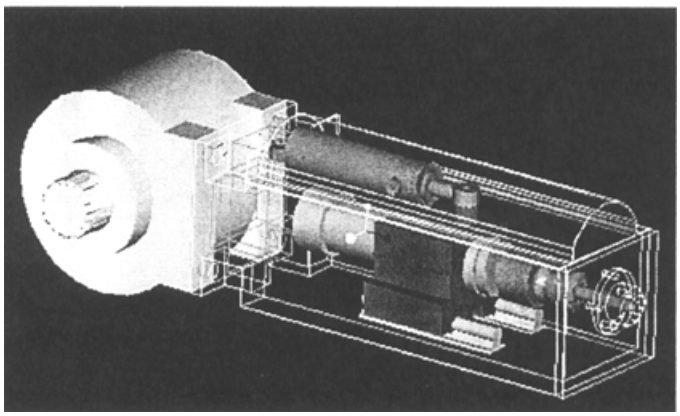


Fig. 7. Competed model of end-effector.

To complete the structural analysis, the Finite Element Analysis and evaluation with Pro/Mechanica® of Pro/Engineer® has been conducted for the robot manipulator. The results were not presented here due to the space limitation. The FEA results are used to further perform the design sensitivity analysis and design optimization investigation in order to minimize the mass of all of the manipulator links as well as the associated parts. This analysis is required in that any modifications made to

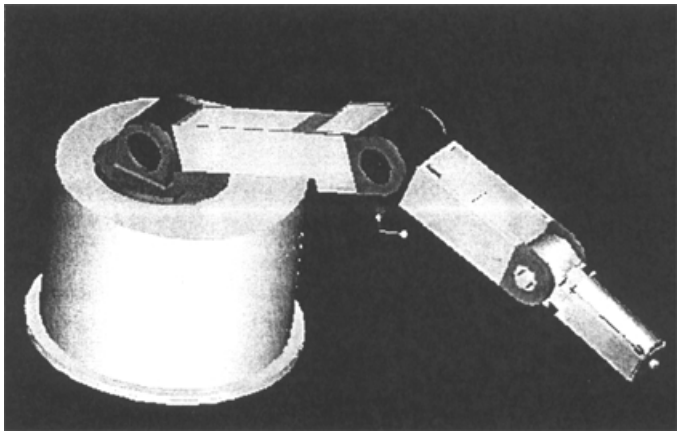


Fig. 8. Competed model of the manipulator.

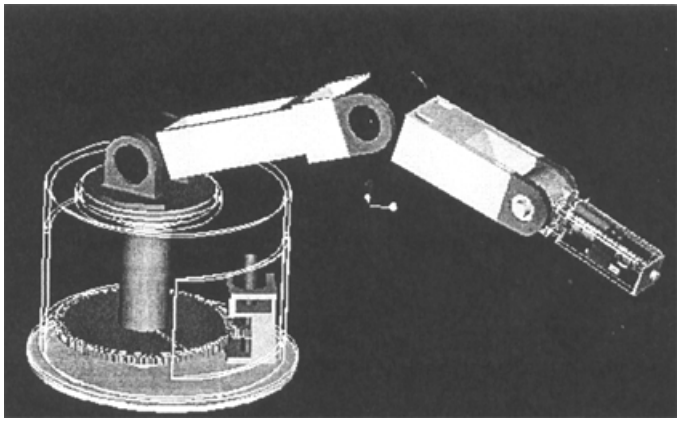


Fig. 9. Wire-frame view of the competed model.

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certain link structure are automatically reflected for associated adjustments for all relevant components in the manipulator assembly model. Apparently, it greatly cuts down the re-design/re-modeling time if required.

4. CONCLUSION

Top-down design methodology is a powerful design technique when designing large assemblies having a number of size, shape and motion constraints among the system's components. This paper proposed and implemented a top-down design methodology for developing a generic 4-d.o.f., revolutejointed robotic manipulator for its initial design configuration. The methodology was implemented by a parametric and feature-based solid modeling and analysis package through a step-by-step procedure. It was evidenced that the methodology is simple to employ and very effective. The design efforts and number of iterations in design and/or modeling by conventional bottom-up approaches would be obviously cut down considerably due to the logical flow of design intents. Therefore, the proposed top-down design procedure of the underlying manipulator, though specific, can provide a guideline for the effective and efficient design for assembly consideration of complex mechanical systems involving multiple components assemblages such as robotic arms.

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