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109.27 A weighted Hölder's inequality

Hölder's inequality [1, p. 19] is

$$\sum_{k=1}^{n} a_k b_k \leqslant \left(\sum_{k=1}^{n} a_k^p\right)^{1/p} \left(\sum_{k=1}^{n} b_k^q\right)^{1/q} \tag{1}$$

where $\frac{1}{p} + \frac{1}{q} = 1$ for positive p, q, a_k and b_k. Equality holds if, and only if,

$$\frac{a_i^p}{b_i^q} = \frac{a_j^p}{b_j^q}$$

for all i and j [2]. This short Note provides a weighted version of Hölder's inequality and gives a neat proof of that.

Theorem (*Weighted Hölder's inequality*)

If $\dfrac{1}{p} + \dfrac{1}{q} = 1$ and $\displaystyle\sum_{k=1}^{n} w_k = 1$ where $0 \leqslant w_k \leqslant 1$, then for all positive p, q, a_k and b_k

$$\sum_{k=1}^{n} w_k a_k b_k \leqslant \left(\sum_{k=1}^{n} w_k a_k^p\right)^{1/p} \left(\sum_{k=1}^{n} w_k b_k^q\right)^{1/q}. \tag{2}$$

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Equality holds if, and only if, $\frac{a_i^p}{b_i^q} = \frac{a_j^p}{b_j^q}$ for all i and j.

Our proof is similar to the traditional proof for Hölder's inequality based on Young's inequality [1, p. 15] or weighted AM-GM inequality [1, p. 19]. Recall that Young's inequality states:

$$ab \leqslant \frac{a^p}{p} + \frac{b^q}{q}$$

for all positive a and b.

Proof: Using Young's inequality,

$$\frac{w_k^{1/p} a_k}{\left(\sum_{k=1}^n w_k a_k^p\right)^{1/p}} \frac{w_k^{1/q} b_k}{\left(\sum_{k=1}^n w_k b_k^q\right)^{1/q}} \leqslant \frac{1}{p} \frac{w_k a_k^p}{\sum_{k=1}^n w_k a_k^p} + \frac{1}{q} \frac{w_k b_k^q}{\sum_{k=1}^n w_k b_k^q}. \tag{3}$$

By summing over k from 1 to n on both sides of (3), we obtain

$$\frac{\sum_{k=1}^n w_k a_k b_k}{\left(\sum_{k=1}^n w_k a_k^p\right)^{1/p} \left(\sum_{k=1}^n w_k b_k^q\right)^{1/q}} \leqslant \frac{1}{p} + \frac{1}{q} = 1, \tag{4}$$

which gives us (2).

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109.28 A trick with the gamma function

The gamma function is probably the most common of the special functions and has many applications in physics [1]. For the definition and basic properties of this function, readers may consult [2]. In some cases, it can also be used to rewrite singular integrals in terms of a series expansion which is more tractable for obtaining an asymptotic estimate of the singular behaviour by elementary means. Consider a function $f(x)$ defined by an integral as follows:

$$f(x) = \int_0^1 dt \frac{\sqrt{t}}{(1 - xt^2)^\beta},$$

which depends on real $x < 1$ and real $\beta > 1$. Note that $f(0) = \frac{2}{3}$ for all β. Some readers may recognise this as a type of hypergeometric function.

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