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ARTICLE

The Squared Mass-Ratios Puzzle for Comparativism

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Abstract

Comparativists (about mass) eliminate absolute masses from the fundamental ontological picture by virtue of a principle of economy, the "Comparative Razor," which requires that only mass-relations, which are invariant under (metrical) symmetries, be considered fundamental. I show how this weapon backfires. If mass-relations are endowed with a standard (multiplicative) concatenation structure, power-transformations become (metrical) symmetries, leaving comparativists prima facie unable to distinguish a possible world and its duplicate where mass-relations are uniformly squared. Then, I considered possible exit strategies, which unfortunately either rely on hidden absolutist assumptions, or leave comparativists and absolutists on equal footing.

1. Absolutism under comparativist fire

A physical quantity, such as mass,[1] is a determinable property that is associated with two different arrays of determinates: monadic, "absolute" masses ("being 3 kg in mass") and dyadic, "comparative" mass-relations ("being thrice as massive as") (Dasgupta 2013). Absolutism is the view that mass-relations are (metaphysically) dependent on the more fundamental distribution of absolute masses among objects. Comparativists deny this and take mass to be fundamentally relational: mass-relations are all that (fundamentally) exists, and absolute masses, if real, depend on them. Here, my aim is to defend absolutism by putting comparativism under the pressure of a new argument, "the squared mass-ratios puzzle" (section 2). I show, basically, how the main argument against absolutism, based on a principle of economy, backfires and gets comparativists into deep trouble. To set the scene for this argument, it will be useful, in section 1, to

[1] Throughout the article I use mass as a working example, but the argument could easily be extended to any quantity admitting a ratio scale.

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replay the comparativist attack on absolutism. Sections 3 and 4 will then be devoted to examining and rejecting possible exit strategies.

Let me first outline the position known as "absolutism (about mass)," before turning to the comparativist attack. In speaking of absolute masses, first, I posit a set M of monadic properties m_i with cardinality $\aleph_1$, that is as many absolute masses as positive values in our common mass scales. Aristotelian absolutists, like Armstrong (1988), refuse to admit *uninstantiated* absolute masses, while Martens (2021) and Platonist absolutists like Mundy (1987) readily do. But nothing in what follows crucially pivots on this in-house dispute.[2]

Second, I assume that M is endowed with the canonical structure of extensive measurement: a total weak order ($\gtrsim_A$) and a weakly associative, monotonous concatenation relation ($\circ_A$), interpreted as addition.[3] I won't ask whether these relations supervene on the intrinsic nature of the absolute masses (Armstrong 1988) or are genuine, second-order relations (Mundy 1987; Eddon 2013); nor whether absolute masses have a quiddistic identity (Martens 2021) or are only individuated through second-order relations (Wolff 2020; Jacobs 2023). But I will assume that absolute masses have transworld identities to help formulate the interworld Leibniz rescaling scenarios.

Absolute masses thus have a structure $\mathscr{A} = \langle \mathrm{M}, \gtrsim_A, \circ_A \rangle$ that allows M to be numerically represented by a function φ: $\mathrm{M} \to \mathbb{R}^+$, unique up to multiplication by a scalar, interpreted as unit change (Krantz et al. 1971, 74). Following Stevens's typology, the quantity M is then said to have a *ratio scale*, in which the ratios among mass-values are preserved through the admissible transformations $\varphi' = k\varphi$ ($k > 0$) (1946). In the following, I call "additive" quantities that have such ratio scales. Finally, under a fixed φ, mass-values ("3 kg") are taken to represent the corresponding absolute masses m_i.

I take absolutism to be the conjunction of the three following claims:

(1_A) Absolute masses M are real and form the structure $\mathscr{A}$.

(2_A) In a world W, an object x has its mass in virtue of the absolute mass m_i it instantiates. Mass-absolute facts of the form "$m_i x$" are the fundamental mass-facts.

This has a nice consequence regarding the distribution of mass-values. Call m a function that attributes to any x in W a mass-value in a unit. Once a scaling φ for the absolute masses is chosen (by a unit-definition),[4] then for any x in W and any m_i of M, if $m_i x$ then $m(x) = \varphi(m_i)$.

[2] Similarly, nothing here depends on how these properties are metaphysically construed, either as universals (Armstrong 1988; Mundy 1987) or as points in a "quality space" (Wolff 2020; Jacobs 2023).

[3] For a similar treatment, see Martens (2021, 2519; 2022, 3; 2024, 13). A complete axiomatization typically includes an Archimedean condition (Krantz et al. 1971, 73).

[4] This is how absolutists typically interpret unit definitions. The sentence "the unit of mass is the mass of this object" is understood as: "this object has the absolute mass m_i associated by φ with the number "1" (compare with footnote 13).

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(3_A) In a world W, mass-relational facts depend on mass-absolute facts. Noting "r_ixy" the fact that x is in the mass-relation r_i with y, r_ixy obtains in virtue of m_ix and m_jy.

This means that absolutists don't need *primitive* mass-relations: With a mosaic of mass-absolute facts and the structure $\mathscr{A}$, determinate mass-relations among objects come for free. Now, call m_R the function that attributes to any two x and y in W a real, positive value corresponding to just how much more massive x is than y. Then, for a fixed φ, if m_ix and m_jy then $m_R(x,y) = \varphi(m_i)/\varphi(m_j)$. The mass-ratio $m_R(x,y)$ can be directly expressed as the ratio of the values for their respective absolute masses.

Absolutism is under heavy comparativist attack. Dasgupta (2013) argues that because monadic mass-values ("3 kg") only express disguised mass-relations to a chosen standard, absolute masses are metrically undetectable (at least, prior to dynamic considerations)[5] and altogether dispensable. This translates into what Martens calls "Kinematic Comparativism": Because mass is a dimensionful quantity with unit-dependent values, the mass of an object "can only be reported or expressed, non-dynamically, in terms of how this magnitude relates to the magnitude of another particle having the same determinable property" (2021, 2520). From a representational viewpoint, it means that the mass-values $m(x)$ entirely depend on the choice of a φ and are *not invariant* under $\varphi' = k\varphi$ ($k > 0$) (*passive* rescalings, i.e., unit changes). Because only the mass-ratios $m_R(x,y)$ are preserved, only these ratios should be considered as representing something physical, that is the mass-relations. Then, by virtue of a principle of economy that Martens calls the "Occamist norm" (2021, 2522), absolute masses are considered redundant surplus and eliminated from the *fundamental* ontological furniture.

There is another way of framing the same argument, that will prove useful later on. Call an "initial world" W_0 the complete state of a world at a given time, conventionally[6] chosen as "t_0"; an "initial world" is typically what is described in the *initial conditions* of a problem of physics: a distribution of fundamental properties and relations on a given set of objects. Because they consider absolute masses as fundamental, absolutists hold that two initial worlds W_0 and W'_0 that differ only by a (uniform) doubling of absolute masses fundamentally differ. Comparativists deny this. To put this more precisely, define:

Active Leibniz (Mass) *k*-rescaling: for all x of W_0, with m_ix, give its counterpart x' in W'_0 the absolute mass m_j such that $\varphi(m_j) = k\varphi(m_i)$ ($k > 0$), to the effect that $m'(x') = km(x)$, *ceteris paribus*.[7]

Absolutists hold that active k-rescalings generate distinct initial worlds (Martens 2021). Starting from W_0, uniformly multiplying all mass-values *in the same unit* mirrors

[5] Dasgupta would certainly disagree with this clause. See his 2013 and 2020.

[6] Nothing in what follows depends on t_0 being the real beginning of W.

[7] The transformation is "active" because it moves the concrete objects within the space of absolute masses. It is not a mere "passive" change of unit. Unlike Martens (2021, 2523), I avoid speaking of a "multiplication of absolute masses": first, because it's still unclear what this could mean (Wolff 2020, 149), and second, to prevent Leibniz scalings from being confused with automorphisms on the structure of absolute masses.

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a redistribution of fundamental absolute masses and thus makes for a distinct initial world W'_0. Comparativists deny this: Uniform multiplications of mass-values only make for representational differences, that is, different numerical descriptions (*in different units*) of fundamentally the same initial world. This can be put in the form of a *reductio*: first, suppose an *active* k-rescaling, yielding two different distributions of mass-values $m(.)$ and $m'(.)$, and second, if it can be compensated by an appropriate change of units, so that any metric difference is canceled out and the mass-ratios preserved, then the rescaling doesn't generate a distinct initial world (or equivalently, it is not an *active* one). To put this more formally, define:

Passive Absorption: For any active k-rescaling ($k > 0$), there is a passive rescaling $\varphi \rightarrow \varphi' = \varphi/k$ such that $m'(x') = \varphi'(m_j) = \varphi(m_j)/k = k\varphi(m_i)/k = km(x)/k = m(x)$, to the effect that the change in mass-values is absorbed by a change of unit.

Because all active Leibniz k-rescalings correspond to metrical symmetries of mass, that is, a symmetries of its ratio scale, they can all be passively absorbed as mere unit changes. Then, define:

Comparativist Razor: Any active rescaling that can be passively absorbed shall not be taken as generating fundamentally distinct possible initial worlds.

The "Razor" is a close cousin to Martens's "Occamist norm." It relies on the very same principle that only that which is invariant (under admissible transformations) should be considered fundamental, this time applied to the world possibilities generated by Leibniz k-rescalings. It dictates to treat metrical symmetries as "ontic symmetries" (Baker 2020), that is, transformations that leave the fundamental structure of the world untouched.[8] By applying this Razor (or this chainsaw, one might say), comparativists significantly reduce the range of possible initial worlds by numerically identifying all those that differ only by Leibniz k-rescalings.

Absolutists have several responses at their disposal. Following Jacobs (2024), they can argue that absolute masses, although metrically idle, are relevant to (metaphysically) explain why mass-relations obey the "Ratio Multiplication Principle" (Roberts 2016): for any x, y, z, $m_R(x,y).m_R(y,z) = m_R(x,z)$. Mass-relations behave "as if" they were ratios of underlying absolute masses, which would be a conspiracy on a cosmic scale if mass-relations were all there (fundamentally) is. Absolutists can also argue that, *pace* Martens, dimensionfulness doesn't imply "Kinematical Comparativism" (Jacobs 2021; Tricard forthcoming). Or, following Martens, they can fall back on *dynamic* considerations.

Taking Newtonian Dynamics (ND) as a sample theory, it is argued that absolute masses are indispensable in determining the dynamics of a system (Baker 2020; Martens 2021). Because the equations of ND are traditionally formulated in terms of monadic mass $m(x)$, applying an active Leibniz k-rescaling on W_0 does generate an (initial) world W'_0 that evolves differently, in an empirically detectable way. For example, particles that escaped their gravitational pull in W_0 now collide in W'_0. But

[8] See also Jalloh (2024).

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initial worlds that evolve differently should be counted as fundamentally distinct,[9] which comparativists cannot do.

In response, comparativists may retort that relying on the text-book version of ND is question begging. If one first believes that mass is fundamentally relational, then one can argue that a "reduced" version of ND should first be formulated, one that quantifies over mass-relations as the fundamental ingredients (Dasgupta 2013; Roberts 2016; Jalloh 2024). If this can be done—and I will assume it can—then *k*-rescalings are restored as dynamic symmetries, thus taking the wind out of the absolutist argument.[10]

The dynamic turn in the debate will not play a major role in what follows. My focus is on undermining the initial victory of comparativism *at the kinematic level*, which largely rests on the Comparativist Razor. The puzzle I will now present shows how it can be used against comparativism.

2. The squared mass-ratios puzzle for comparativism

Let me first outline the basic tenets of comparativism. First, consider a set R of mass-relations r_i, also with cardinality $\aleph_1$,[11] endowed with a total weak order ($\succsim_R$) and a weakly associative, monotonous concatenation relation ($\circ_R$). This time, $\circ_R$ is interpreted as multiplication, which means, intuitively, that mass-relations behave like ratios.[12] Note also that $\circ_R$ is a closed operation on R, has an identity element (the mass-relation "being as massive as") and includes an inverse for each mass-relation. Thus, the structure $\mathscr{R} = \langle R, \succsim_R, \circ_R \rangle$ is a totally ordered group isomorphic to the multiplicative positive reals (Martens 2024). It is often assumed that mass-relations are uniquely represented: for any two representation functions ψ, ψ' from R to $\mathbb{R}^+$, $\psi = \psi'$. As will shortly become clear, this is the crux of the problem that I intend to pose to comparativism.

I take comparativism to be the conjunction of these three claims:

(1_C) Mass-relations R are real and form the structure $\mathscr{R}$.

(2_C) In a world W, the mass-relational facts r_ixy are the only fundamental mass facts.

This implies that, once a ψ is chosen on R, if r_ixy then $m_R(x,y) = \psi(r_i)$.

(3_C) In a world W, absolute masses (if real) depend for their instantiation on the underlying network of instantiated mass-relations.

[9] This is an instance of Earman's "Symmetry Principle 2" that every kinematic symmetry is a dynamic symmetry (1989, 46).

[10] Dasgupta (2020) offers a more refined response, where W_0 and W'_0 are treated as distinct possible worlds in a "looser sense" of "possibility." Baker (2013) and Jacobs (2023) defend alternative stories where W_0 and W'_0's distinct evolutions are explained by a coscaling of G.

[11] This, again, is accepted by Platonist comparativists such as Bigelow and Pargetter (1988) but rejected by nominalists like Field (1980).

[12] Unlike Baker and Martens, and for a reason that will soon become clear, I shall refrain from calling them "mass-ratios" and save the expression for the *numerical* ratios.

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This requires some clarification, for the status granted to absolute masses in comparativism is not easy to grasp. For sure, comparativism in its basic form is simply an "Anti-realism about absolute masses" (Martens 2024), and there is simply no need of a claim such as (3_C). But for others like Dasgupta, absolute masses are real but depend on mass-relations, which raises a problem of metaphysical underdetermination: many distributions of absolute masses are compatible with a given network of mass-relations (Martens 2016, 10). This is reflected in the fact that, for a fixed ψ, if $r_i xy$ then there is a function m such that $\psi(r_i) = m(x)/m(y)$ that is unique only up to multiplication by $k > 0$. The choice of m can be done by setting $m(S) = 1$ for some object S that is designated as the mass-standard.[13] So, if mass-values $m(x)$ represent absolute masses, then an object can possess many different absolute masses, one for each mass-relations that it bears with any possible standard. So, the network of mass-relations seems to metaphysically underdetermine the distribution of absolute masses.

I see three possible options. First, one may adopt a *very thin* view according to which a x has an absolute mass whenever it can be attributed a mass-value $m(x)$, relative to a standard S. Just as a same object has several profiles depending on the point of view taken on it, it has several "monadic mass-profiles" depending on the chosen comparison point. But comparativists may wish to recover a *thicker* concept of absolute mass, so that an object can only have one absolute mass (at the same time). A second option, then, is to adopt Dasgupta's specific brand of structuralism, where absolute mass-facts are *collectively* grounded on the whole relational network (Dasgupta 2014). A third option is the view that Martens coined "Regularity Comparativism," where facts about absolute masses supervene on the entire world mosaic and are selected together with the lawlike dynamic regularities (Martens 2017). Because nothing in my argument depends on this, I will take "absolute masses" to refer to anything that corresponds to monadic mass-values in a comparative framework, be it nothing more than mass-relations, thin monadic profiles or thicker absolute masses.

So, here is the puzzling case. Suppose a comparativist initial world W_0, that is a fundamental distribution of mass-relations r_i among objects and a ψ such that, for any objects x and y, if $r_i xy$ then $m_R(x,y) = \psi(r_i)$. Then, generate a candidate for a distinct possible world W_0* by applying:

> **Active Leibniz β-power-transformation:** For all x, y of W_0 with $r_i xy$, duplicate them such that their counterparts x* and y* in W_0* are in a mass-relation r_j with $\psi(r_j) = \psi(r_i)^\beta$ ($\beta > 0$), to the effect that m_R*$(x^*,y^*) = m_R(x,y)^\beta$, *ceteris paribus*.

With $\beta = 2$, this generates W_0* by redistributing mass-relations such that all the mass-ratios are uniformly squared. The question is: Are W_0 and W_0* to be counted as two fundamentally different initial worlds?

The immediate answer is that they obviously are because W_0 and W_0* differ in the distribution of *fundamental* mass-relations, in a way that is even metrically detectable: Two objects that previously were measured as being twice as massive as one another are now four times as massive, and the difference doesn't seem to depend on

[13] This is how comparativists typically view the unit choice—compare with note 4. See Dasgupta (2014; 2020) for an alternative approach to unit definitions.

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representational choices. Also, besides being apparently the case, comparativists are *required* to treat W_0 and W_0* as two distinct initial worlds for dynamic reasons. Suppose indeed that both worlds obey the same comparativist ND in which the laws govern mass-ratios. Then W_0 and W_0* have different dynamic evolutions. Just like scalar multiplications are not dynamic symmetries of an absolutist ND, power transformations are not symmetries of a comparativist one. So, if comparativism, with its fundamental ontological furniture, failed to distinguish between W_0 and W_0*, the same problem would arise once again, as in the classical doubling scenario of Baker (2020) and Martens (2021).

I claim that, despite appearances, comparativism precisely fails to do just that. The core issue is the assumption that mass-relations have a unique numerical representation, each expressed by a single ratio.[14] This is unproblematic in absolutism: *Fundamental* absolute masses, endowed with a primitive additive structure, admit a *ratio* scale with invariant mass-ratios. But in comparativism, mass-relations are fundamental. Hence, to determine how uniquely they are represented, and by which type of scale, one must first and foremost look at *their* primitive structure. So, let me start with that, unfold the problem, before seeing how comparatists might respond to it.

By hypothesis, mass-relations are primitively imposed with a total weak order $\gtrsim_R$ and a weakly associative, monotonous, multiplicative concatenation relation $\circ_R$, that on R is closed, has an identity element and includes an inverse. Given this structure $\mathscr{R}$, they admit *many* numerical representations, that are unique only up to power transformations $\psi^* = \psi^\beta$ ($\beta > 0$), which form their *metrical* symmetry group. Define a function $\psi^* = f(\psi)$ with $f(x) = x^{1/\beta}$ ($\beta > 0$). Then:

(i) $\psi(r_i) \geq \psi(r_j)$ iff $\psi^*(r_i) \geq \psi^*(r_j)$ because f is strictly increasing;
(ii) $\psi(r_i \circ_R r_j) = \psi(r_i)\psi(r_j)$ iff $\psi^*(r_i \circ_R r_j) = \psi(r_i \circ_R r_j)^{1/\beta} = \psi(r_i)^{1/\beta}\psi(r_j)^{1/\beta} = \psi^*(r_i)\psi^*(r_j)$;
(iii) ψ^* has the same identity element as ψ and includes an inverse for all relations of R.

So ψ^* is also an equally good representation of R. Contrary to what is usually assumed, mass-relations are uniquely represented *only up to power transformations*, by what may be called "log-ratio scales,"[15] in which only the ratios $\log\psi(r_i)/\log\psi(r_j)$ are meaningful.

Given this, one can see the "squared mass-ratios puzzle" unfold. Simply redescribe the world W_0* by using ψ^* instead ψ as the conventional function for the mass-relations. Then for all x^* and y^* that stand in the relation r_j:

$$m_R^*(x^*,y^*) = \psi^*(r_j) = \psi(r_j)^{1/\beta} = (\psi(r_i)^\beta)^{1/\beta} = \psi(r_i) = m_R(x,y)$$

So, by changing the representation from ψ to ψ^*, it is possible to describe W_0* and W_0 with the same numerical ratios but in different "scales": Objects in W_0* that were

[14] Or, that they are "kinematically absolute" in Martens's terms (2024), and admit of an "absolute scale" in the language of the Representational Theory of Measurement (Luce et al. 2007, 113).

[15] There is no canonical name for this type of scale; this one is adapted from Stevens's "log-interval scales" (1959)—see the following text.

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previously described as being four times as massive as one another are now twice as massive, just like in W_0. The difference between W_0* and W_0 can be "passively absorbed." Hence, by virtue of the Comparativist Razor, they don't fundamentally differ. With its own tools, comparativism is prima facie unable to distinguish between the two worlds W_0 and W_0*.

To borrow Wüthrich's (2009) witty term, this is *abysmally embarrassing* (for comparativists). First, I assumed that comparativists were able to formulate a version of ND with active Leibniz *k*-rescalings as dynamic symmetries. But obviously, power transformations *are not* symmetries even of a comparativist ND. Initial worlds W_0 and W_0* are predicted by the theory to have different dynamic evolutions. Now, comparativists find themselves *again* in the situation of being unable to distinguish two worlds, W_0 and W_0*, that *should* be counted as distinct for dynamical reasons.

Here is a second, most undesirable consequence. If all this were true, then (monadic) mass would simply not be the additive quantity that we (empirically) know. Remember the function m such that, if $r_i xy$ then $m_R(x,y) = \psi(r_i) = m(x)/m(y)$. For a fixed "scaling" ψ, m is unique up to multiplication by $k > 0$. But ψ admits power transformations of the form $\psi^* = \psi^\beta$ ($\beta > 0$). This implies that, m is unique only up to transformations of the form: $m^* = km^\beta$ (k, $\beta > 0$). If those transformations were admissible, then monadic mass wouldn't have a *ratio* scale anymore, but what Stevens (1959, 31) calls a "log-interval scale." In log-interval scales, only the ratios of log-intervals $(\log m(x) - \log m(y))/(\log m(w) - \log m(z))$, and *not* the ratios $m(x)/m(y)$, are meaningful. And to measure monadic mass, it would then require to fix not one, but two points: the unit (the kilogram) and a free exponent β; mass simply wouldn't be an additive quantity anymore. So, one dimension of the puzzle is: How come empirical, monadic masses admit of a ratio scale, if all there fundamentally is the structure $\mathscr{R}$ of mass-relations? This is a metaphysical problem: Fundamental mass-relations with their primitive structure don't provide a sufficient ground for monadic masses with a strong *ratio* scale.

Comparativists would never accept such damaging consequences. To solve the puzzle, they can tackle one of the two premises of the puzzle:

(a) Mass-relations, given their structure $\mathscr{R}$, admit of a "log-ratio" scale with power transformations as their metrical symmetry group.
(b) Applying the Comparativist Razor, no active transformation that corresponds to a metrical symmetry can generate a fundamentally distinct world.

So, comparativists can either attack premise (b) and deny that the Comparativist Razor applies, or attack premise (a) and insist that mass-relations do have a *unique* numerical representation. Let me consider these strategies in turn.

3. Is comparativism an undeclared absolutism (about mass-relations)?

One strategy is to accept (a) but deny that (b) the Comparativist Razor applies, insisting that an active β-transformation makes for a distinct world possibility. A redistribution of the mass-relations makes a fundamental difference *even if it preserves their structure*. In other terms, comparativists now become "*absolutists*

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about mass-relations."[16] This is Robert's (2016) expression, which can be confusing because "absolutism" is usually defined as the commitment to *monadic* mass properties.[17] But it is dialectically useful to keep the term in a *generalized* sense, as the claim that two worlds that differ only by a relevant Leibniz transformation (here, a power transformation) are numerically distinct, being different in their fundamental, substructural ingredients (here, "absolute" mass-relations). Although this sounds like a solution, it is a very weak position from a dialectical viewpoint. In fact, why should this absolutist move be considered legitimate now, when it comes to mass-relations, when it wasn't earlier with absolute masses?

First, the very same reason for using the Comparativist Razor applies here. Remember that comparativists argued that the metrical symmetries of the ratio scale make mass-values representationally redundant and, prior to dynamic considerations, absolute masses uneconomical surplus. Because only the mass-ratios were meaningful, only the mass-relations that they represent were to be counted as fundamental. The same applies here: The symmetries of the "log-ratio" scale make the *values of mass-ratios* representationally redundant and "absolute mass-relations" uneconomical surplus. Then, by virtue of the Razor, comparativists *should* be asked to get rid of them, and to admit *relations among mass-relations* as the only fundamental ingredients, masswise (again, before the dynamics dictates otherwise). If not, then it is up to them to explain why suspend *now* their own principle of economy.

As a matter of fact, comparativists do have a good reason not to apply the Razor. According to their own comparativist ND, mass-relations do play a determinative and empirically detectable role in the subsequent evolutions of W_0 and W_0*, so they are right to be absolutist about mass-relations. But then, they must justify *why* dynamic considerations now prevail, to make *their* case, when they did not deem it sufficient to defend the absolutists' absolute masses—or, equivalently, they must explain why they no longer find it necessary to seek a reformulation of ND, one that quantifies on relations among mass-relations only. Without answers to these questions, they are in a dialectically weak spot.

Comparativists may also argue that the Razor, although legitimate, cannot be applied indefinitely without entering into a vicious regress. After all, there is no principled reason to stop here: Once *relations among mass-relations* only are admitted at the fundamental level, one may look at their own structure, find out that they have nontrivial symmetries too, apply the Razor once again and climb the ladder of algebraic abstraction one rung higher, and so on *ad infinitum*. This obviously has to stop somewhere, as recognized by Roberts (2016, 5). But why stop *here*, and be absolutist about mass-relations, and not before as the absolutists claimed we should? Again, I am afraid that any good reason for the comparativists to stop here will only recycle an equally good reason for the absolutists to stop one step earlier: either by

[16] Martens, in a footnote, anticipates this move by saying that comparativists should endow their fundamental mass-relations with a quiddistic identity, to avoid precisely the problem that I call the "squared mass-ratios puzzle" (2022, 4, fn. 7). However, I don't think that quidditism is the only way here: The comparativists may very well adopt a "sophisticated" absolutist stance about mass-relations, only individuated by their second-order relations.

[17] I thank an anonymous reviewer for pointing this out.

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relying on dynamic relevance à la Martens, or explanatory relevance à la Jacobs, and so forth (section 1).

So, the comparativists cannot without some inconsistency (or bad faith) resist the application of *their own* principle of economy in the at-hand case. Perhaps their hope then lies in the other strategy, in attacking the puzzle at its root: The fact that (a) the structure $\mathscr{R}$ of mass-relations has a "log-ratio" scale with power transformations ($\psi^* = \psi^\beta$, $\beta > 0$) as its metrical symmetries.

4. Are mass-relations uniquely representable?

The rejection of (a) is motivated by the persistent idea that mass-relations should be "kinematically absolute" (Martens 2024), expressed by unique and constant numerical ratios, independent of any free parameter such as a unit choice (Roberts 2016, 6–7). In what follows, I consider three ways in which this can be more precisely articulated: by claiming the scale-independence of mass-relations (4.1), or their kinematic absoluteness (4.2), or by arguing that mass-relations have *more structure* than $\mathscr{R}$, so as to suitably restrict their symmetries (4.3).

But let me first make a quick answer, which will set the direction for subsequent, more elaborate responses. As it is widely admitted, calling mass-relations "ratios" may be misleading (Martens 2016; Jacobs 2024). As Roberts also admits, "the dyadic relational quantity we call 'mass-ratio' is . . . a 'ratio' by courtesy only. Really, it's just a fundamental 2-place relational quantity, . . . for there is nothing for them to be ratios of" (2016, 13). But if that is the case, I don't see where fundamental mass-relations get enough structure to be uniquely representable. Calling them "ratios" won't change the fact that their structure $\mathscr{R}$ *is not* uniquely representable. Surely, one can impose that any function m representing monadic masses be such that $m_R(x,y) = m(x)/m(y)$. But for m_R to be unique, one also has to assume that m is a ratio scale (and not a log-interval scale), unique up to scalar multiplications *only*. That is, the monadic masses must be endowed with an additive structure, that nevertheless they cannot inherit from more fundamental mass-relations. So, the comparativist must either admit that mass-relations are not uniquely representable, and crash into the puzzle, or assume that monadic masses are additively structured *independently* of mass-relations, which is a serious concession to absolutists. Let me now unfold this dialectic.

4.1. Scale-independent mass-relations?

First, one may argue with Baker (2020) that mass-relations are *scale-independent*, and if it's the case, that their numerical representation doesn't rely on any scaling convention and is simply unique. As Baker sees it, scale-independence is an ontic notion, closely linked to the existence of ontic symmetries. If absolute masses were to be *actively* and uniformly doubled, then the mass-relations would admittedly remain the same. They are independent of how mass is "scaled" in reality. Ontic scale-independence is mirrored, on the representational side, by a value invariance: "[A] comparative relation for a quantity like mass is scale-independent iff, when the quantity is represented numerically, multiplying its values by a constant cannot change whether the relation holds" (2020, 81). Mass-ratios are preserved by any k-rescaling of the mass-values, so they represent scale-invariant aspects of reality.

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Yet, Baker plainly relies on the (familiar but) crucial assumption that the *admissible* transformations of the mass-values are of the right, multiplicative sort, that preserves mass-ratios. If, however, power transformations were allowed, then mass-values could be transformed in many ways that *do not* preserve the ratios. So, what decides the scope of the *legitimate* transformations? The answer, unsurprisingly: The structure of what is to be transformed, and the usual reason to admit the multiplicative sort only is that monadic masses have an additive structure. Now, this is of course unproblematic for absolutists, who posit fundamental absolute masses with the suitable structure $\mathscr{A}$. But how could comparatists do the same?

Take basic comparativism first, that is antirealism about absolute masses. Fundamental mass-relations with the structure $\mathscr{R}$ are all there is. And $\mathscr{R}$ allows for a transformation group much broader than multiplications by a positive scalar (the similarity group). Because there are simply no absolute masses, there is nothing else to rely on to further reduce the group. Monadic mass-values can therefore be redistributed according to power transformations: $m' = km^{\beta}$ ($k, \beta > 0$), under which mass-ratios are scale-*variant*.

However, comparativists may admit absolute masses, as long as they depend on the underlying mass-relations. But to solve the present puzzle, the comparativist needs absolute masses be delivered *with their additive structure*. Because this additive structure puts additional constraints on the way absolute masses can be distributed over the network of mass-relations, the additive structure excludes certain arrangements that are nevertheless permitted by the underlying $\mathscr{R}$ structure. Therefore, the latter simply cannot determine the former.

Here is another, last attempt. The additivity of monadic masses has an empirical meaning. The concatenation relation $\circ_A$ can be operationally interpreted, for instance, as the placing of two massive objects on the same pan of a balance, to compare them with a third. More generally, it is an empirical law that monadic masses behave in a regular, additive way (Mundy 1987). So, perhaps the required restriction (to multiplicative transformations) can be obtained, if not by dependence on the structure of fundamental mass-relations, then by "piggy-backing" on the additive behavior of empirical masses. By analogy with Martens's (dynamical) "Regularity Comparativism" (2017), the strategy may be called "Kinematical Regularity Comparativism," where the required kinematic properties of monadic mass, such as its additivity, are provided by the regularities selected by the theory that "best systematizes" the global mosaic.

However, on top of the problems from which "Regularity Comparativism" generally suffers,[18] this specific brand of comparativism has one major explanatory flaw. The additive behavior of masses is treated as a brut fact, left unexplained. Of course, any theory is allowed some unexplained primitives. But here, the brut, unexplained fact involves entities (absolute masses) that are *not primitive*, but supposedly dependent on mass-relations. And as already explained, the structure of

[18] In a nutshell: First, comparativism is in conflict with the separability thesis of the Humean view of the world mosaic—see Martens (2017, 1232); second, "[the piggy-back approach] would make life too easy for people who want to get rid of objects and properties that they do not like for some whimsical reason" (Arntzenius 2012, 170).

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mass-relations is not rich enough to force masses to behave in an additive way.[19] So, although nothing fundamental forces them to do so, they behave as if constrained by a structure of absolute masses. Here, comparativism is plagued with another sort of "cosmic conspiracy" (Jacobs 2024). Or, as Martens notes, "there is an obvious cry for inference to the best (i.e. only!) explanation: the comparativist worlds are constrained as if there were fictitious absolute masses exactly because there are absolute masses!" (2016, 111).

Thus, I simply don't see how comparativists could endow monadic masses with the required additive structure, for mass-relations to be scale-independent in Baker's sense. This road to representational uniqueness for mass-relations is closed off.

4.2. "Kinematically absolute" mass-relations?

Comparativists may also argue that mass-relations are "kinematically absolute." Whereas absolute masses (if real) can only be numerically reported by *being compared* one to another (Martens 2021, 2520), mass-relations are measured noncomparatively. For instance, the relation "being 5 times as massive as" between x and y is detected by balancing, on a pan balance, 5 exactly similar copies of y with x, without any standard or comparison point. Therefore, just like in the number of fingers of a glove (Martens 2021, 2521), there is really something "5-ish" in "being 5 times as massive as," and this is why mass-relations have a unique numerical representation.

In response, I will argue that mass-relations are indeed kinematically absolute, but again, *only if* absolute masses are posited and endowed with an independent additive structure.

First, let me consider this commonly held notion that, in ordinary measurement systems, it is mass-relations that are measured (Roberts 2016, 4). Is it truly the case? To *directly* measure mass-relations would require devising an operation that associates a (positive) real with *any* pair of objects (x,y). This simply is *not* what is done with common mass-measurement systems: They associate a (positive) real with pairs of (x,S), with "x" the only variable and "S" a logical constant, holding for a chosen mass-standard that remains the same within a same scaling. In other words, what is operationalized here is *monadic* mass ("the mass of x") with its traditional ratio scale. To obtain a value for the mass-relation between *any* x and y, one has to make *two* distinct measurements $m(x,\mathrm{S})$ and $m(y,\mathrm{S})$ *within the same scaling* and then take the ratio $m(x,\mathrm{S})/m(y,\mathrm{S})$. This ratio is undeniably constant across all change in the standard S. Therefore, the kinematical absoluteness of mass-relations here can *only* be secured *if* monadic masses are first granted the suitable, additive structure. This brings the comparativist back to the previous impasse: It is not possible to grant this without positing independently structured absolute masses.

Comparativists might object that the measurements $m(x,\mathrm{S})$ still stand for mass-relations. True enough, they can be expressed by relational predicates: "being k times as massive as S" instead of traditional monadic values ("being k S in mass").[20] Strictly speaking though because "x" is the only variable in $m(x,\mathrm{S})$, "being k times as

[19] In the next section, I examine the idea, which comes up naturally here, of *enriching* the structure of fundamental mass-relations beyond $\mathscr{R}$, to ground this restriction.

[20] I thank an anonymous reviewer for pointing out this objection.

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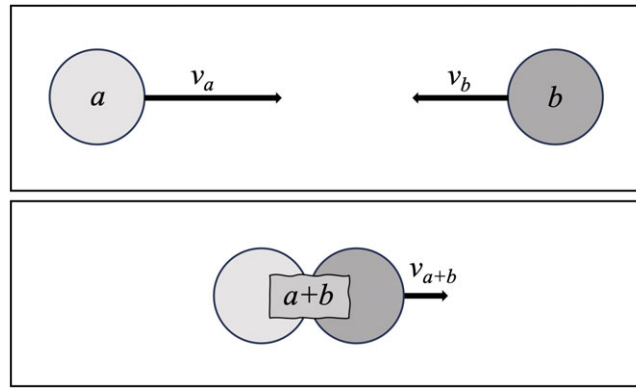


Figure 1. Weyl's operationalization of mass.

massive as S" expresses *relational properties* (of the *x*) rather than *relations*. So comparativists would then need to argue that these relational properties ultimately depend on the mass-relations (between the *x* and S), rather than on their absolute masses, so that very little is gained.

Yet, one may object that there are ways to *uniquely* measure mass-relations from (ratios of) quantities, such as velocity or acceleration, without assuming additive monadic mass. For instance, Martens declares that the structure $\mathscr{R}$ of mass-relations "corresponds nicely to Weyl's operational definition of (inertial) mass" (2021, 2519; 2024, 16). So, let's have a look at it, to see if they really provide an operationalization of mass-relations.[21]

Consider this experimental set-up: two bodies a and b (inertially) moving with inward velocities v_a and v_b (top box in Figure 1), before colliding inelastically and coalescing into the aggregate $a+b$, with velocity v_{a+b} (bottom box).

Weyl's operationalization of mass proceeds first by defining equality and order in mass (Weyl 1949, 139):

(1) a is as massive as b iff, with equal (opposite) velocities $v_a = -v_b$, $v_{a+b} = 0$
(2) a is more massive than b iff, with $v_a = -v_b$, $v_{a+b} \neq 0$ and $\text{sign}(v_{a+b}) = \text{sign}(v_a)$

Then Weyl defines a concatenation operation that "consists simply in joining the two bodies" (Weyl 1949, 139), which clearly shows that he is building nothing more than a traditional *additive* scale of monadic masses. As such, this in no way constitutes an operationalization of mass-relations. I suspect that Martens referred to Jammer's account of Weyl's ideas (2000, 10). Instead of a concatenation operation, Jammer defines the mass-ratios $m_R(a,b)$:

(3) If $v_{a+b} = 0$, $m_R(a,b) = -v_b/v_a$

[21] One could also think of Mach's famous operational definition of inertial mass, which relies on (observable) acceleration-ratios and Newton's third law—see Jammer (1961, 92). For present purpose, it is sufficiently similar to Weyl's for me to consider only one of them (Jammer 2000, 10).

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This relies on the principles of conservation of mass and momentum: $m_a v_a + m_b v_b = (m_a + m_b)v_{a+b}$, with m_a and m_b the monadic masses of a, b. By imposing $v_{a+b} = 0$, one easily obtains $m_a/m_b = -v_b/v_a$. Then, for all x and y, by independently measuring the velocities v_x and v_y, one is able to obtain a unique, positive real value for $m_R(x,y)$, thus providing the structure of mass-relations with a *unique* numerical representation. Values for monadic masses $m(x)$ can then be defined by choosing a as the mass standard ($m(a) = 1$), such that for all x, $m(x) = m_R(x,a)$. And $m(x)$ is unique only up to a multiplicative factor, typical of ratio scales.

So, runs the objection, the Weyl–Jammer definition provides a way to measure and *uniquely* represent mass-relations, *without* making any explicit assumption about the additive structure of monadic masses, but recovering it as a by-product. What more could one ask for?

First, remark that (*pace* Martens) the Weyl–Jammer definition is not, strictly speaking, an operationalization of the structure $\mathscr{R}$ because it does not provide ways to *compare* mass-relations. In fact, because (1) and (2) reduce to (3), it is nothing but a method to measure mass-relations derivatively from independent velocity measurements (with their own ratio scale). Still, the core of the objection remains: It provides a unique representation of mass-relations without explicitly assuming an additive structure for monadic mass.

But it does, implicitly. To see this, let me compare it with another way to measure mass-relations, which provides them with just the required "log-ratio scale" that fits their structure $\mathscr{R}$. What follows is adapted from what Luce and Tukey (1964) invented under the name "Simultaneous Conjoint Measurement."[22] The idea is to measure mass and velocity conjointly as components of a third quantity, momentum. This is done, basically, by building an ordering of momenta that allows to match mass-relations with velocity-relations. First, one must be able to order masses (with Weyl's (1) and (2)) and velocities:

(4) a goes as fast as b iff if they collide exactly in the middle of the segment defined by their initial positions
(5) a goes faster than b iff, if a is as massive as b, then $v_{a+b} \neq 0$ and $\text{sign}(v_{a+b}) = \text{sign}(v_a)$

Next, by varying the velocities and masses of bodies a and b, one produces an ordering of momentum $\succsim_p$:

(6) a has more momentum than b ($p_a \succsim_p p_b$) iff $v_{a+b} \neq 0$ and $\text{sign}(v_{a+b}) = \text{sign}(v_a)$

If these qualitative axioms are satisfied, then there are two positive-real-valued functions χ_m et χ_v such that:

[22] See Krantz et al. (1971, 267) and Aczél and Luce (2001, 5828) for an application to the present case and (Narens et Luce 1986) for treatment of similar cases. It requires some adaptation because Conjoint Measurement was not, originally, designed to operationalize relational quantities, but to measure quantities that lack a proper concatenation operation and are very common in psychology and in the social sciences (Michell 1990, 68).

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(7) $p_a \succsim_p p_b$ iff $\chi_m(a)\chi_v(a) \geq \chi_m(b)\chi_v(b)$

Such representation functions are unique up to power transformations: $\chi'_m = \alpha_1.\chi_m{}^{\beta}$ and $\chi'_v = \alpha_2.\chi_v{}^{\beta}$ ($\alpha_1, \alpha_2, \beta > 0$). Therefore, a conjoint measurement of mass and velocity yields *log-interval scales* for both quantities.[23] Now, instead of functions for monadic masses and velocities, one may want functions for mass- and velocity-*relations*. To that end, simply define $m_R(x,y) = \chi_m(x)/\chi_m(y)$ and $v_R(x,y) = \chi_v(x)/\chi_v(y)$, such that:

(8) $p_a \succsim_p p_b$ iff $m_R(a,b) \geq v_R(b,a)$

Such functions are unique up to power transformations of the form $m_R' = m_R{}^{\beta}$ and $v_R' = v_R{}^{\beta}$ ($\beta > 0$), which characterize them as (what I called) "log-ratio scales," which is exactly the type of scale that suits the structure $\mathscr{R}$. So, I take conjoint measurement of mass-relations (and velocity-relations) to provide just the type of operationalization required by their assumed structure.

Three remarks are in order. First, as Luce and Tukey insist, this method provides a *fundamental* measurement of the conjoint quantities (here, mass and velocity). This is the main difference with Weyl's method, where mass is measured *derivatively*, from velocity. Second, because mass (and velocity) can also be fundamentally measured with the traditional extensive methods, this means that a same quantity can be *fundamentally* measured in different ways.[24] So, the question naturally arises of how the conjoint measurements (of mass and velocity) χ_m and χ_v relate to their extensive measurements m and v. Krantz et al. (1971, 485) show that:[25]

(9) $\chi_m = \gamma_1.m^{\beta}$ and $\chi_v = \gamma_2.v^{\beta}$ ($\gamma_1,\gamma_2,\beta > 0$)

This means that the traditional ratio scale for monadic mass (velocity), obtained when extensively measured, is a special case (with $\beta = 1$) of the log-interval scale obtained when it is measured conjointly with velocity (mass) (Wolff 2020, 86). This also entails that:

(10) $m_R(x,y) = (m(x)/m(y))^{\beta}$ and $v_R(x,y) = (v(x)/v(y))^{\beta}$ ($\beta > 0$)

So, the apparently unique mass- and velocity-ratios obtained by taking the ratios of monadic masses and velocities is also a special case (with $\beta = 1$) of $m_R(x,y)$ and $v_R(x,y)$ when obtained by a conjoint measurement.

Third, one crucial condition (for the representation and uniqueness theorems) is of particular interest. Indeed, it has to be assumed that mass and velocity *independently*

[23] Proof in Krantz et al. (1971, 258). Additional conditions (independence, double cancellation, solvability, and an Archimedean condition) are also required—see also Michell (1990, 68 et sq.). I come back to the independence condition in the following text.

[24] For a useful classification of the different uses of "fundamental" and "derived" in the literature, see Krantz et al. (1971, 502).

[25] See also Narens and Luce (1986, 172) and Aczél and Luce (2001, 5828) for similar results.

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contribute to momentum, in the sense that a measure of momentum χ_p is a *noninteractive* function of χ_m and χ_v:[26]

(11) $\chi_p = \chi_m{}^{\beta}\chi_v{}^{\beta}$ $(\beta > 0)$

This differs from the way momentum is traditionally defined, as $p = mv$, where no free exponent β appears. But, as Krantz et al. explain, "[T]his difference is only apparent since we could rewrite all physics in terms of $p' = p^{\beta} = m^{\beta}v^{\beta}$, where $\beta > 0$. It is pure convention that we choose $\beta = 1$" (1971, 267).[27] What is not conventional, but *physically meaningful*, is that the exponent is the same for mass and velocity, that is that "the ratio of their exponents is 1 rather than some other number. This ratio establishes the trading relation . . . between m and v in their contributions to momentum" (ibid.). By conventionally setting $\beta = 1$, one ensures that the conjoint measurements of mass and velocities just match those obtained when both quantities are independently measured and represented by ratio scales—and therefore, that $m_R(x,y)$ just matches the unique ratios $m(x)/m(y)$ of additive mass. As Narens and Luce write:

> By selecting the exponent β to be 1 (or equivalently, by identifying $[\chi_v]$ with $[v]$), we have by fiat altered what is really a log-interval representation [of momentum] into one that appears to be a ratio scale. (This means that in order to force [momentum] actually to be a ratio scale, more physical structure than the ordering of the mass-[velocity] pairs is needed.) (1986, p. 172)[28]

This last point allows me to fully meet the challenge posed by the Weyl–Jammer operationalization of mass-relations. Remember that it is a method to derivatively measure mass-ratios from velocity-ratios. Apparently, it does recover unique mass-ratios (and a ratio scale for monadic masses) without making any assumption about the structure of monadic masses. But it makes an equivalent assumption! In Jammer's (3), momentum p is implicitly defined as mv, that is by setting β to 1 in (11). As Narens and Luce suggest, this is assuming that momentum has a ratio scale rather than a more general log-interval scale. Now, this can be justified in only two ways: either by the (independent) assumption that *both* velocity *and mass* have ratio scales or by assuming that momentum has an extensive structure, with a definable concatenation operation.

In the first case, assuming that *both* velocity *and mass* have ratio scales automatically sets β to 1. But clearly, this entails that, even though no *extensive* structure for masses has been explicitly posited in the Weyl–Jammer procedure, the ratio scale for masses is not a by-product of the measurement, but its precondition. This was presupposed as soon as momentum were defined as mv. In the second case, momentum is endowed with more structure than it has in conjoint measurement.

[26] f is noninteractive iff it is a monotonic transformation of an additive function—see Michell (1990, 77).

[27] See also Krantz et al. (1971, 487), Narens and Luce (1986, 172) and Ellis (1966, 118–126).

[28] In the quote, Luce and Narens do speak of the same exponent β as Krantz et al., i.e. the one in (9), (9), and (10). But they are treating a case of conjoint measurement of mass *and volume* through a *density* ordering. I adapt the quantity terms (and the symbolism) to suit the case at hand.

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Actually, the principle of momentum conservation that Jammer mobilizes can be interpreted as a concatenation operation: The momentum of the aggregate $a+b$ is the sum of a's and b's momenta. But this is empirically sound only if the masses are also conserved, that is the mass of a and b add up to the mass of the aggregate. So, this already presupposes that mass has an extensive structure and therefore, once again, a ratio scale.

I conclude that the Weyl–Jammer operationalization of mass-relations only yields unique numerical ratios because of a hidden assumption ($\beta = 1$) that, one way or another, imposes a ratio scale on monadic mass. As discussed in section 4.1, this assumption is not accounted for in a comparativist framework: From fundamental mass-relations exhibiting the structure $\mathscr{R}$, it simply doesn't follow that monadic masses have the structure required by ratio scales.

4.3. Enriching the structure of mass-relations?

There is one last road to explore: Why not enrich the structure $\mathscr{R}$ of mass-relations to restrict their metrical symmetries from the power group ($\psi^* = \psi^\beta$, $\beta > 0$) to the identity ($ß = 1$), so that each mass-relation corresponds to a unique real?

The most straightforward way is, I think, to rely on what is known as "Hölder's theorem"; but it is traditionally formulated in an absolutist framework, so let's see if it can be twisted for comparativist purposes. Basically, one assumes that the monadic masses M satisfy a total weak order ($\succsim_A$), a weakly associative, monotonous, additive concatenation relation ($\circ_A$), *plus* two qualitative conditions of order *density* and *unboundedness*, to reach "Dedekind-completeness." It is then possible to build infinite, additive *standard sequences* for all m_i of M: $S(m_i) = < m_i, 2m_i, 3m_i, \ldots >$, so that the ratio m_i/m_j is uniquely determined as a relation (the "Dedekind cut") between $S(m_i)$ and $S(m_j)$. Hölder's theorem states that the set of such ratios is isomorphic to the positive reals, so that each corresponds to a unique real number.[29]

To twist this adequately, comparativists need to identify the ratios m_i/m_j with the *fundamental* mass-relations. This implies reversing the order of dependency traditionally assumed, with the ratios determined by additive standard sequences of monadic masses. In the reversed view, mass-relations come first and primitively determine pairs of standard sequences, so that absolute masses be delivered *with their additive structure*.

Yet, on closer analysis, the problem encountered in section 4.1 "appears only to have been wished away."[30] The fact remains that the underlying $\mathscr{R}$ structure is *still* not rich enough to uniquely determine monadic masses with the required additive structure.[31] Simply saying that it does so "primitively" is not illuminating because it is not clear *by virtue of what* additional primitive structure.

Returning to the problem: To secure that mass-relations are *uniquely* represented by the positive reals, comparativists need to add *more primitive structure* to $\mathscr{R}$.

[29] Details can be found in Hölder (1901, 1996) and Michell (1994).

[30] I am paraphrasing van Fraassen here (1989, 107).

[31] A further reason for this can be found in Michell (1994, 396): Because no constraints are imposed on the type of additive operation that can be applied to monadic masses, it may be traditional numerical additivity (compatible with a ratio scale), but also many others obtained by exponentiation (*not compatible* with a ratio scale).

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Intuitively speaking, mass-relations need to be endowed with some sort of a "distance metric" that $\mathscr{R}$, as a multiplicative structure, lacks. This is certainly the most promising option for comparativists. Because it can be tried in many ways, I go with the simplest, hoping that my general answer will also cover the others.

The idea is to embrace Bigelow and Pargetter's "three levels theory" of quantities (1988), which posits (1) particular objects, standing in (2) whole arrays of *mass*-relations, themselves bearing (3) second-order *proportion relations*—a theory that (now) qualifies as a brand of comparativism.[32] That two mass-relations r_i and r_j stand in the n:m proportion means intuitively that r_i^m (r_i concatenated m times) amounts to r_j^n. Together these proportions determine a metric for mass-relations: Because "being as massive as," the identity element, is invariantly associated with the number 1, any relation r_i standing in the n:1 proportion with it is associated with the real n.[33]

Note, however, that these (second-order) proportions are external, superimposed on mass-relations.[34] But given their structure $\mathscr{R}$, the mass-relations can receive many different geometries. For instance, r_i and r_j may stand in the n/m but also in the $(n/m)^{ß}$ proportions ($ß > 0$). It is the same problem, over and over again. Why should one contingent mass-geometry, the one with $ß = 1$, be posited rather than another? One good reason is, again, to best explain the *empirical* additive behavior of monadic masses, and this time (compared to section 4.1), comparativists are equipped to do so.

I accept this as consistent way out of the puzzle, one which comparativists can take. But note that now, comparativists are far from their initial, economical stand of positing (as fundamental) *only* the mass-relations expressed by the invariant ratios of empirical masses. Now, faced with the fact that empirical masses exhibit constant numerical ratios, both absolutists and comparativists posit fundamental structures with unobservable ingredients: either absolute masses with their structure $\mathscr{A}$ (Mundy 1987), or mass-relations with their second-order geometry. It is then sufficient for my purpose to conclude that comparativists have lost all their initial advantage at the kinematic level.

5. Conclusion

That comparativists run into the problem I have presented here and are prima facie unable to distinguish between a world W and its duplicate W* where all mass-relations have been uniformly squared, is deeply puzzling. Until now, the picture seemed pretty clear. On the table were absolute masses, with their ratio scale, and mass-relations, represented by unique numerical ratios. Comparativists considered it more economical to posit only the latter, at the fundamental level, and to have them mirrored by the constant empirical ratios. What I showed, basically, is that

[32] Despite Bigelow's reservations (1988, 72).

[33] In Michell's terms, the (unique) "representation" is backed up by "instantiation." Contemporary comparatists may be reluctant to view real numbers as worldly, scale-independent proportion relations, instantiated by physical mass-relations. But if preferred, these proportions relations can be replaced by any suitable "qualitative" relations without changing the substance of my response in the following text.

[34] They are in Bigelow's account. But why couldn't they be *internal*, supervening on the nature of the mass-relations? Armstrong (1988) did try to make numbers *qua* relations supervene on magnitudes *qua* properties, with no success (Eddon 2007; Tricard 2022). I see no reason to expect more success with relations.

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comparativists had it too easy by assuming that these ratios were unique. As it happens, mass-relations with their structure $\mathscr{R}$ admit of many equivalent representations depending on the conventional choice of a free exponent.

To evade the puzzle, comparativists may become absolutist about mass-relations, but then have to explain why *they* have the right to do so. Or they may endow monadic masses with an additive structure, which they cannot inherit from the underlying mass-relations, and must thus possess primitively—which is precisely what comparativism initially rejected. Unless, of course, they supply their fundamental relations with *more* primitive, unobservable structure. But then, on what grounds do this, if not to account for the constant ratios of monadic masses, which absolutism does just as well?

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