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Novel 6-DOF parallel manipulator with large workspace

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SUMMARY

The workspace of a parallel manipulator is usually smaller than the size of the robot itself. It is important to derive new structures that enjoy the advantages of parallel manipulators and also have a large workspace. In this paper we present two configurations of similar structures RRRS and RRSR with rotating links. The RRRS structure has a relatively large workspace—larger than the size of the robot itself—which is not common in parallel robots. The inverse and forward kinematics of the robots are presented. The workspaces of the robots are compared to similar and well-known structures, such as Eclipse, Alizade, Delta, and Hexa robots.

KEYWORDS: Parallel manipulator; Workspace; RRRS; RRSR.

Introduction

Parallel manipulators have many advantages over serial manipulators in terms of high pay-load/weight ratio, velocity, stiffness, accuracy, and low inertia. Their major drawback is their limited range of motion—workspace. In addition, numerous kinematic chains result in complex kinematic singularities inside the workspace where the manipulator becomes uncontrollable.

Numerous parallel manipulator structures have been proposed. Some of them are designed for a specific task,[8] others for a specific workspace,[7] and still others are just smart mechanic architectures.[5,6] A catalogue of a large variety of parallel configurations can be found in the book by Merlet[5] and on his Internet web site.[6]

When observing the different designs, one can see that each structure has its own advantages and disadvantages; most of them, however, share one shortcoming—the workspace of the manipulators is relatively small, usually much smaller than the size of the manipulator itself. For each structure it is possible to maximize the manipulator workspace by changing its link dimensions and parameters,[4,10] but the workspace is still limited by mechanic constraints. Mobile legs also enlarge the workspace of the robot,[9] but the base plate grows as well.

Determination of the workspace of parallel architectures is an important issue, which has been addressed by several research studies. Generally, the solutions of parallel robot kinematics lead to complex equations, so determining their workspaces is a challenging issue. Procedures for workspace evaluation of parallel manipulators have been formulated by determining extreme paths;[1] computing conditions occurring at the workspace boundaries,[3,11] or numerical computation.[2,3,12]

The current investigation presents two configurations of parallel manipulators, where three links are mounted on sliders, moving independently on a circular guide. The idea of this structure stems from circular sliders robots, such as Eclipse[2] and Alizade,[13] where the prismatic links of the Alizade structure are changed by two interconnected joints, as seen in Fig. 1.

Unlike the Delta[14] or Hexa[15] robots, we chose the inside joints configuration which reduces singularity conditions when the upper link is aligned with the moving plate or both links are aligned. Choosing the inside joint configuration reduces the angle between the link and the plate, while in outside configuration the angle is closer to 180° which leads to loss of degree of freedom. As will be shown later, the presented RRRS manipulator structure has significantly large workspace, larger than the robot itself.

Another RRRS manipulator has already been presented[16] where the axes are oriented differently than that in the proposed structure. The legs are also fixed to the basis, therefore resulting in a limited workspace.

The two suggested structures are presented next. Their forward and inverse kinematics solutions are presented in Section 'Kinematics of the manipulators.' In Section 'Workspace analysis and comparison,' the manipulators workspaces are evaluated and compared to similar and well-known structures.

The Manipulators Structure

The structures of the manipulators consist of three identical kinematic chains connecting the base and the moving platform. The kinematic chains are described by abbreviation of the joint types, starting from the base platform and ending at the moving platform. Each chain contains a lower link mounted on a circular slider and moving independently on a circular guide around the base center at radius R_b. The slider is connected to the moving platform by two linked joints.

In the RRRS robot the links are connected by a revolute joint and the upper link is connected to the moving platform by a spherical joint (see Fig. 2). Both revolute joints at the slider and the interconnected links are parallel and tangent to the circle; therefore, the links are always directed towards the center of the circle eliminating links collision. If the lengths of the links are equal, the robot can be folded to a plane.

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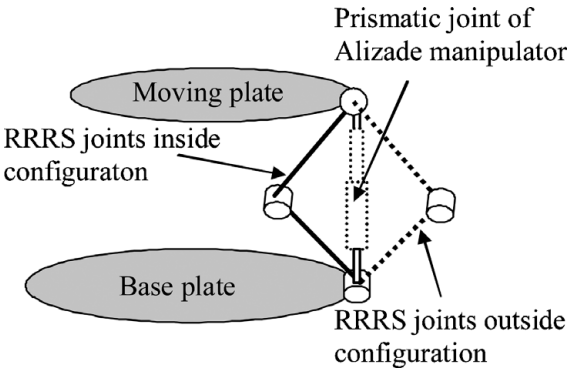


Fig. 1. Modification of Alizade manipulator.

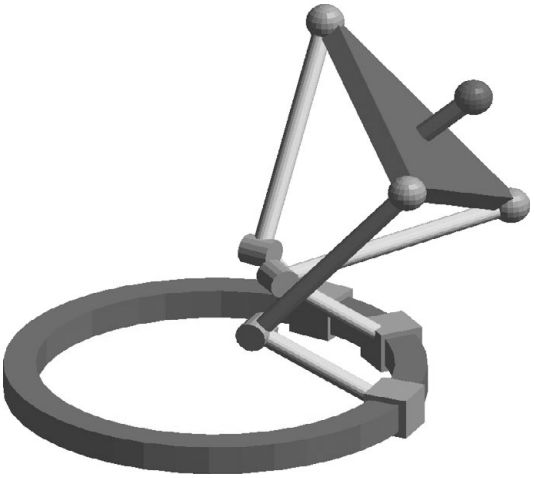


Fig. 2. RRRS manipulator.

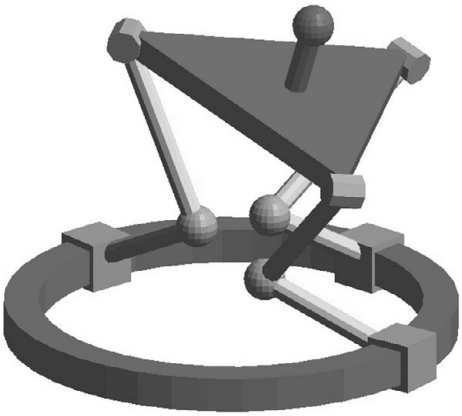


Fig. 3. RRSR manipulator.

In the RRSR robot, the links are connected by spherical joints and the upper link is connected to the moving plate by revolute joint (see Fig. 3).

Both actuators on each chain are located at the base of the robot which allows lightweight moving plate. One actuator is prismatic actuator sliding along the circular guide and the second controls the angle between first link and base plate (see Figs. 4 and 5). For simplicity we refer to the first prismatic actuator as revolute joint since its position is defined by an angle about the base center.

To prevent joints collision at the circle center, the lengths of the lower links are limited by the radius of the base circle.

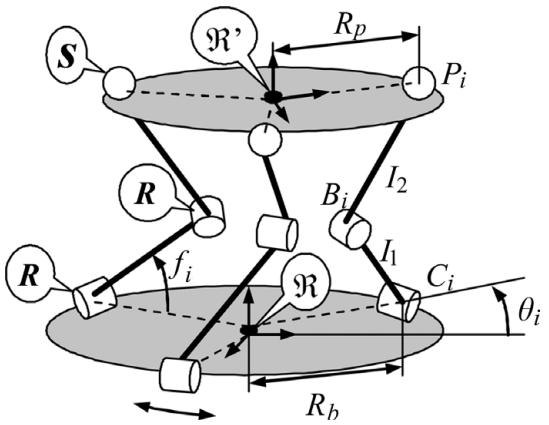


Fig. 4. RRRS manipulator structure and parameters.

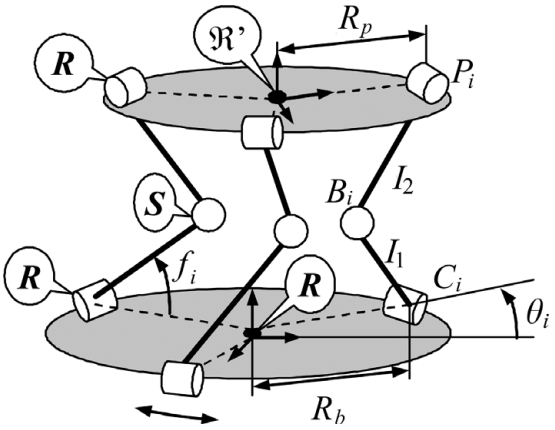


Fig. 5. RRSR manipulator structure and parameters.

Since the lower link always directed to the center of the circle, the links do not collide during movements.

Kinematics of the Manipulators

Mechanism kinematics deals with the study of the mechanism's motion as constrained by the geometry of the links. The study of mechanism kinematics is divided into two parts, inverse kinematics and forward kinematics. The inverse kinematics problem is to map a known position and orientation of a moving platform to a set of input joint variables that will achieve that position. The forward kinematics problem involves mapping from a known set of input variables to a set of positions and orientations of the moving platform. As the number of closed kinematics loops in the parallel mechanism increases, the difficulty of solving the forward kinematics relationships increases. The inverse and forward kinematics solutions of the two suggested parallel manipulators are described next.

The kinematic models of the manipulators are depicted in Figs. 4 and 5. A fixed reference system $\Re$: $O-xyz$ is attached to the base platform, with O defined as a circular guide center. Another reference frame, called the top frame $\Re' : O'-x'y'z'$ is located at the center of the moving platform. The vertices of the moving platform are denoted as platform joints P_i, the slider positions are denoted as C_i, and the two links are connected at points B_i $(i=1, 2, 3)$.

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The three kinematic chains of the links connecting the slider and the platform joint P_i are identical. The RRRS kinematic chain has a revolute joint B_i and spherical joint P_i, while the RRSR kinematic chain has a spherical joint at B_i and revolute joint at P_i; hence the solution is different.

Inverse kinematics

In the inverse kinematics problem, the position and orientation of the moving platform is known; therefore the coordinates of the equilateral triangle of the moving plate, which are the position of the plate joints, are known—P_i.

RRRS manipulator. Since the links of the RRRS manipulator are always aligned with the radial direction of the circular guide, the solution of θ_i is straightforward:

$$\theta_i = a\tan 2(P_{ix}, P_{iy}). \tag{1}$$

The second angle φ_i is found from the $P_iC_iB_i$ triangle plane. The point $B_i = C_i + [l_1\cos\varphi_i, l_1\sin\varphi_i]$, C_i is now known. And from the length of $\|P_i - B_i\| = l_2$, we get the angle φ_i. Two solutions are possible–legs inside and legs outside. We choose the legs inside solution.

RRSR manipulator. The solution of this manipulator is not as straightforward as the RRRS solution, because it has a spherical joint at the connection between the two links and because the coordinates of plate vertices P_i do not give the direct solution for θ_i. The actuator angles θ_i and φ_i can be found from the following two constraining equations:

$$\|B_i - P_i\| = l_2 \tag{2}$$

$$(B_i - P_i) \bullet \hat{S}_i = 0, \tag{3}$$

where $\hat{S}_i$ is a unit vector along the rotary joint at P_i. P_i and $\hat{S}_i$ are defined by the geometry of the moving plate and their coordinates in the basis coordinate system are obtained by multiplication by moving plate transformation. The second equation forces the upper link to be perpendicular to the axis of rotating joint at P_i. B_i is given by

$$B_i = [(R_b + l_1\cos\varphi_i)\cos\theta_i; (R_b + l_1\cos\varphi_i)\sin\theta_i; l_1\sin\varphi_i]. \tag{4}$$

We end up with two trigonometric equations:

$$((R_b + l_1\cos\varphi_i)\cos\theta_i - P_{xi})S_{xi} + ((R_b + l_1\cos\varphi_i)\sin\theta_i - P_{yi})S_{yi} + (l_1\sin\varphi_i - P_{zi})S_{zi} = 0, \tag{5}$$

$$((R_b + l_1\cos\varphi_i)\cos\theta_i - P_{xi})^2 + ((R_b + l_1\cos\varphi_i)\sin\theta_i - P_{yi})^2 + (l_1\sin\varphi_i - P_{zi})^2 - l_2^2 = 0. \tag{6}$$

Substituting $t_1 = tan(\theta_i/2)$, $t_2 = tan(\varphi_i/2)$, and using the elimination method,[18] we end up with an eight-degree polynomial in t_2; therefore, in the most global case, eight solutions are possible for t_2. For each t_2, there are two solutions for t_1.

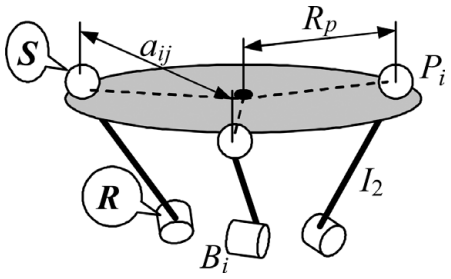


Fig. 6. Structure of a robot after reducing known geometry.

Direct kinematics

In direct kinematics, the actuators angles θ_i and φ_i are known, so the points B_i are also known. Then the problem is to find the orientation of the moving platform with the constraints of revolute and spherical joint location.

RRRS manipulator. The first two active joints are known from which the joints B_i can be calculated, so we remain with only the last RS joints location unknown. This structure is equivalent to the planarly actuated robot (see Fig. 6) described and solved by Ben-Horin *et al.*[17]

Assuming a rigid movable platform, the distances between two spherical joints i and j are constants a_{ij}

$$|P_j - P_i| = a_{ij}, i, j = 1, 2, 3, i \neq j. \tag{7}$$

For each link l_2 we define the projection angles α_i on the base plate and β_i on z-axis, then the equation is rewritten by substituting the known coordinates of the joints R_i.

$$(P_{j,x} - B_{i,x} + l_2\cos\beta_j\cos\alpha_j - l_i\cos\beta_i\cos\alpha_i)^2 + (P_{j,y} - B_{i,y} + l_2\cos\beta_j\sin\alpha_j - l_i\cos\beta_i\sin\alpha_i)^2 + (P_{j,z} - B_{i,z} + l_2\sin\beta_j - l_i\sin\beta_i)^2 - a_{ij} = 0. \tag{8}$$

These equations are then solved by substituting tangent half angle $t_i = \tan(\beta_i/2)$ which further results in three uncoupled quadratic equations. The solution of this nonlinear system of equations is obtained by ref. [18] which results in 16-order polynomial, which results in eight general solutions, four of which are reflections of the other. A more detailed solution of the above equations can be found in ref. [17].

RRSR manipulator. Constraining the first two joints leads to the SR structure, which is the inversion of RS. The solution is similar to the solution of the RRRS structure.

Workspace Analysis and Comparison

In this section, the procedure of computing the robots workspace is presented for the two new structures and for additional four similar and well-known robots: Eclipse (Fig. 7), Alizade (Fig. 8), Delta, and Hexa robots (Fig. 9); Eclipse and Alizade also have their first joint sliding on a circular guide. Delta and Hexa, being the most known structures, also have two joined links connecting base and moving platform.

In order to obtain a fair comparison, we chose similar manipulator dimensions under reasonable constraints. The basis for comparison is the diameter of the circular slider, which is common for all structures. We chose the following

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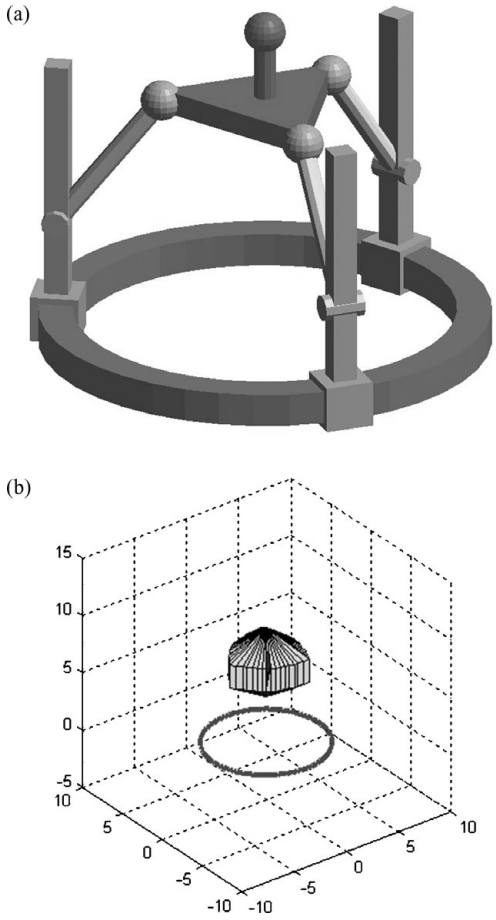


Fig. 7. Eclipse manipulator RPRS: (a) the structure; (b) workspace shape.

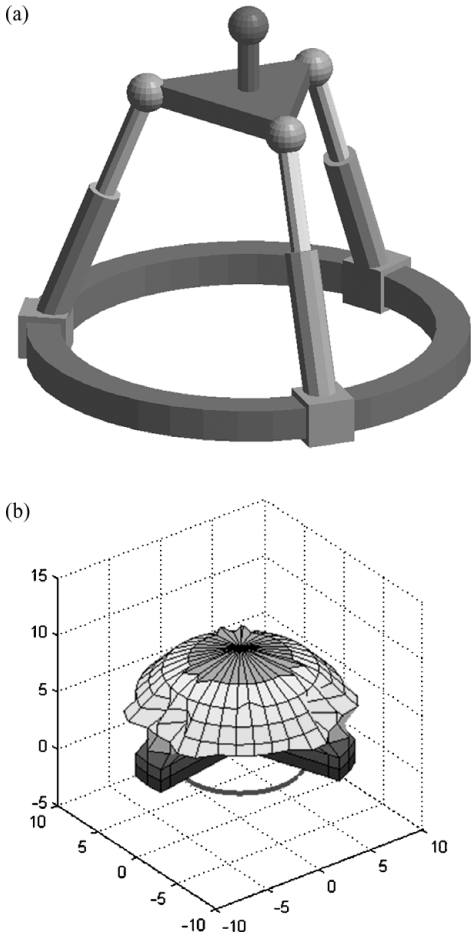


Fig. 8. Alizade manipulator RRPS: (a) the structure; (b) workspace shape.

dimensions for the manipulator components: $R_b = R_p = 5$ for all links lengths $l_1 = l_2 = 5$. For Eclipse, the constant length link $l_2 = 5$ and $R_p = 3$; the traveling length of the prismatic joint is 0–5, therefore the extended length of both joints is $5 + 5$, equal to the RRRS extended leg of 10. For Alizade, the prismatic joint varies between 5 and 10; again the extended length is equal to the RRRS extended leg of 10. Prismatic joint is less efficient for larger workspace since the joint can extend to its maximum length. The two links of the HEXA and Delta robots are also $5 + 5$.

We find the reachable points numerically by computing the inverse kinematics of the robots. We define a cylindrical grid surrounding the robot structure with radius changing from zero to three times the radius of the base circle and height from zero to the extended length of the two links. The radius changes from 0 to 15 with 0.5 increments, the angle from 0° to 360° with 5° increments and height from 0 to 10 with 0.5 increments. The inverse kinematics solution is then computed for each grid point. If the inverse kinematics solution exists, the point belongs to the workspace of the robot. The envelope of the workspace is then extracted from the collection of reachable points.

Fair comparison of different robots workspaces is generally a complex task. In general case a whole reachable workspace, those points that the moving platform can achieve in at least one orientation, is a better comparison index than just the workspace associated to one constant orientation. But since we choose to compare robots that exhibit structure and size similarity, for simplicity, all the workspace graphs correspond to the constant orientation of a moving platform with no rotations, i.e. the moving plate coordinate system is parallel to base circle coordinate system.

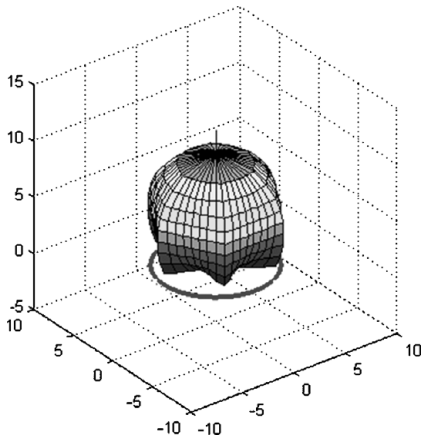


Fig. 9. Delta and Hexa manipulators, no rotation; workspace shape.

The blue thick circle below the workspace corresponds to the circular basis of the manipulator.

The Delta and Hexa manipulators have the same workspaces if the moving platform is not rotated. Their constant orientation workspace is given in Fig. 9.

Figures 10 and 11 represent the workspaces of the proposed RRSR and RRRS manipulators:

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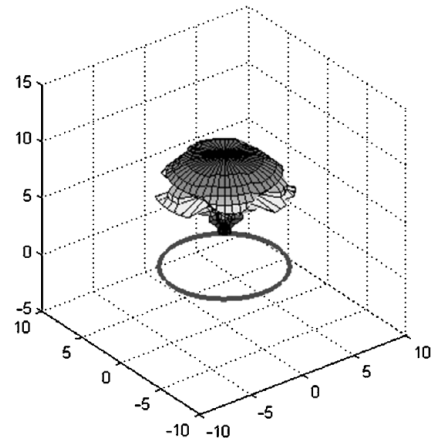


Fig. 10. RRSR manipulator, no rotation; workspace shape.

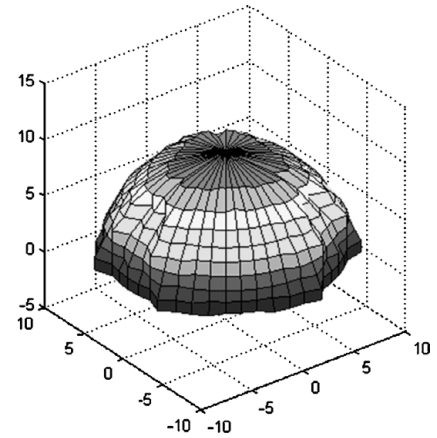


Fig. 11. RRRS manipulator, no rotation; workspace shape.

Table I. Workspace volumes comparison.

Eclipse	Alizade	Delta & Hexa	RRSR	RRRS
92.6	612.1	584.8	176.4	1980

It can be seen from Fig. 10 that the workspace of the RRSR manipulator is significantly smaller than that of the RRRS manipulator.

The volume of each workspace is summarized in Table I.

It can be seen that the RRRS structure has the considerably largest work volume—three times the volume of Alizade, Delta, and Hexa robots. On the contrary the RRSR robot has significantly smaller workspace even compared to Alizade or Hexa robots. This can be explained by the fact that the upper links connected to the moving plate by revolute joint are constrained to be pointed to the center of the plate. This limits their reach as opposed to RRRS configuration where spherical joints allow much wider range of motion.

Conclusion

In this paper, a new structure of 6-DOF parallel manipulator RRRS with large workspace is presented together with very similar RRSR structure. The inverse and forward kinematics are developed. The structure of the proposed manipulators results in relatively simple inverse and closed direct kinematics solutions. The robots are actuated by six revolute joints all of which are located at the basis allowing lightweight moving plate.

It was shown that the workspace of the proposed RRRS manipulator is significantly larger—by as much as three times—than workspaces of similar and well-known structures such as Eclipse, Alizade, Delta, and Hexa robots. Important to note that although the structure of the RRSR robot is very close to the RRRS, its workspace is much smaller.

Large workspace for parallel robot has to come on account of something else. Intuitively such structure has lower stiffness than other parallel robots compared in this paper having prismatic joints. Future work will include stiffness and singularity analysis of the RRRS manipulator.

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