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Rossby-wave driven zonal flows in the ionospheric E-layer

T. D. KALADZE[1,2], D. J. WU[1], O. A. POKHOTELOV[3,4], R. Z. SAGDEEV[5], L. STENFLO[6] and P. K. SHUKLA[7]

[1]Purple Mountain Observatory, Chinese Academy of Sciences, Nanjing, 210008, People's Republic of China

[2]I. Vekua Institute of Applied Mathematics, Tbilisi State University, 2 University Street, 0143 Tbilisi, Georgia

[3]Automatic Control and Systems Engineering, University of Sheffield, Sheffield, UK

[4]Institute of Physics of the Earth, 123995 Moscow, 10 B. Gruzinskaya Street, Russia

[5]Department of Physics, University of Maryland, College Park, MD 20740, USA

[6]Department of Physics, Umeå University SE-90 187 Umeå, Sweden

[7]Institut für Theoretische Physik IV, Ruhr–Universität Bochum, D-44780 Bochum, Germany

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Abstract. A novel mechanism for the generation of large-scale zonal flows by small-scale Rossby waves in the Earth's ionospheric E-layer is considered. The generation mechanism is based on the parametric excitation of convective cells by finite amplitude magnetized Rossby waves. To describe this process a generalized Charney equation containing both vector and scalar (Korteweg–de Vries type) nonlinearities is used. The magnetized Rossby waves are supposed to have arbitrary wavelengths (as compared with the Rossby radius). A set of coupled equations describing the nonlinear interaction of magnetized Rossby waves and zonal flows is obtained. The generation of zonal flows is due to the Reynolds stresses produced by finite amplitude magnetized Rossby waves. It is found that the wave vector of the fastest growing mode is perpendicular to that of the magnetized Rossby pump wave. Explicit expression for the maximum growth rate as well as for the optimal spatial dimensions of the zonal flows are obtained. A comparison with existing results is carried out. The present theory can be used for the interpretation of the observations of Rossby-type waves in the Earth's ionosphere.

1. Introduction

Zonal flows are a consequence of the zero-frequency modes propagating in the atmospheres of many planets. Both ground-based and satellite observations clearly show that, at different layers of the ionosphere, there are zonal flows with nonuniform velocities along the meridians (Gershman 1974; Gossard and Hooke 1975; Kamide 1988). It is known (e.g. Petviashvili and Pokhotelov 1992) that in the presence of velocity shear in the zonal flow, the nonlinear effects start to play a role in their dynamics. It is thus of interest to take into account the interaction of the planetary waves propagating in the ionosphere with the shear flows. In this way, the ionospheric medium builds up conditions that are favorable to the

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formation of nonlinear stationary solitary wave structures (Pokhotelov et al. 1996, 2001). In reality, several planetary ionospheres can support both propagating waves and zonal flows and they thus constitute dynamic systems that exhibit complex nonlinear interactions. It should be noted that zonal flows vary on time scales slower than those of the finite-frequency waves.

The generation of zonal flows is still not fully clarified. Recently, there has been renewed interest in examining the nonlinear coupling between coherent and incoherent drift waves and zonal flows (or convective cells) in non-uniform magnetoplasmas (e.g. Smolyakov et al. 2000; Manfredi et al. 2001; Shukla and Stenflo 2002). It has been found that pseudo-three-dimensional drift waves strongly couple with zonal flows that have their dynamics governed by the drift wave stresses driven Navier–Stokes equation. The latter is nonlinearly coupled with the Hasegawa–Mima equation in the drift wave-zonal flow theory. As there is a well-known analogy between drift waves and Rossby waves (e.g. Nezlin and Chernikov 1995) the idea of generation of zonal flows by Rossby waves was put forward by Shukla and Stenflo (2003). Their theory was further developed by Onishchenko et al. (2004). In these papers it was shown that zonal flows in a non-uniform rotating neutral atmosphere can be excited by finite amplitude Rossby waves. The driving mechanism of this instability is due to the Reynolds stresses that are inevitably inherent for finite-amplitude small-scale Rossby waves. Hence, these investigations provided an essential nonlinear mechanism for the transfer of spectral energy from small-scale Rossby waves to large-scale enhanced zonal flows in the Earth's neutral atmosphere. In addition, the zonal flow generation was considered within a simple model for Rossby-wave turbulence, using the classical nonlinear two-dimensional Charney equation to describe the dynamics of solitary vortex structures of the dipole type, i.e. a cyclone–anticyclone pair. This means that the wavelengths of the considered Rossby waves were small as compared with the Rossby radius r_{R} and the nonlinearity is therefore only due to the so-called vector, or Poisson bracket, nonlinearity. Hence, it corresponds to the quasi-geostrophic approximation in geophysical hydrodynamics for which structures are considered as purely two-dimensional, and the perturbations of the free surface of the liquid motion are considered as either absent or negligibly small.

In the present paper we will focus our attention at the Earth's ionosphere. A large amount of observational data has been stored up until now. These data verify the permanent existence of ultra-low frequency (ULF) planetary-scale perturbations in the E- and F-regions of the ionosphere (e.g. Lawrence and Jarvis 2003). Among them, special attention must be paid to large-scale Rossby-type perturbations propagating at a fixed latitude along the parallels around the Earth. Unlike the neutral atmosphere, the ionospheric E- and F-layers consist of neutral and charged particles which makes the ionosphere conductive. Therefore, the interaction of inductive currents with the inhomogeneous geomagnetic field (varying along the meridians) should be taken into account. Recently Kaladze and Tsamalashvili (1997), Kaladze (1998, 1999) and Kaladze et al. (2003) showed that the so-called magnetized Rossby waves can propagate in the E-layer of the ionosphere without perturbing the geomagnetic field. They have typically wavelengths larger than the Rossby radius r_{R}. For the ionosphere we have $r_{\mathrm{R}} \approx 1000$–$3000$ km. As shown by Kaladze et al. (2004), magnetized Rossby-wave turbulence should then be described by a more complex equation, namely the so-called generalized Charney equation that includes an additional scalar, Korteweg–de Vries (KDV) type, nonlinearity.

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This equation corresponds to the intermediate geostrophic approximation in geophysical hydrodynamics, for which the perturbation of the free surface of the atmosphere is taken into account.

The generation of zonal flows by magnetized Rossby waves in the ionospheric E-layer will be investigated below by taking into account all the above discussed basic typical features. The manuscript is organized in the following fashion: in Sec. 2 a brief description of magnetized Rossby waves is presented. A set of coupled equations describing the nonlinear interaction of magnetized Rossby waves and zonal flows is derived in Sec. 3. It is shown that this system admits the excitation of zonal flows. Our discussion and conclusions are presented in Sec. 4.

2. Linear magnetized Rossby waves in the ionospheric E-layer

Let us consider a weakly ionized E-layer that consists of electrons, ions and neutral particles. Due to the strong collisional coupling between the ionized particles and the neutrals the behavior of such a gas is mainly determined by its massive neutral component. The E-layer satisfies the condition $n/N \ll 1$, where n and N are the equilibrium number densities of the charged particles and the neutrals, respectively. The presence of charged particles makes the medium electrically conducting. For a typical ionization fraction in the E-layer, the Lorentz force is comparable to the Coriolis force. Hence, we must take into account the effects of the spatially inhomogeneous geomagnetic field $\mathbf{B}$ and the vertical component of the Earth's rotation $\mathbf{\Omega}$.

Magnetized Rossby waves (e.g. Kaladze and Tsamalashvili 1997; Kaladze 1998, 1999) represent the ionospheric generalization of tropospheric Rossby waves in the rotating atmosphere with a spatially inhomogeneous geomagnetic field. The theory of magnetized Rossby waves was developed by Kaladze et al. (2004). We introduce a local Cartesian coordinate system with the x-axis directed to the east, the y-axis to the north and the z-axis in the local vertical direction. Magnetized Rossby waves propagate in the middle-latitude E-layer of the ionosphere and their frequency is (Kaladze et al. 2004)

$$\omega_{\mathbf{k}} = -\frac{k_x(\alpha+\beta)r_{\mathrm{R}}^2}{1+k_\perp^2 r_{\mathrm{R}}^2} = \frac{k_x v_{\mathrm{R}}}{1+k_\perp^2 r_{\mathrm{R}}^2}. \tag{1}$$

Here $\omega_{\mathbf{k}}$ is the wave frequency, $\mathbf{k}$ the wave vector and $k_\perp = (k_x^2 + k_y^2)^{1/2}$ where k_x and k_y are the x and y components of the wave vector. The Rossby velocity v_{R} and the Rossby radius r_{R} are defined as

$$v_{\mathrm{R}} = -(\alpha+\beta)r_{\mathrm{R}}^2 = -r_{\mathrm{R}}^2 \frac{\partial}{\partial y}(f+\gamma), \tag{2}$$

and

$$r_{\mathrm{R}} = \frac{(gH_0)^{1/2}}{|f+\gamma|}, \tag{3}$$

where H_0 stands for the atmospheric reduced height, g is the gravitational acceleration and f is the Coriolis parameter that depends on latitude λ, i.e.

$$f = 2\Omega_{0z} = 2\Omega_0 \sin\lambda = f_0 + \beta y \tag{4}$$

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with

$$f_0 = 2\Omega_0 \sin\lambda_0 > 0 \quad \text{and} \quad \beta = \frac{\partial f}{\partial y} = \frac{1}{R}\frac{\partial f}{\partial \lambda} = \frac{2\Omega_0 \cos\lambda_0}{R} > 0. \tag{5}$$

Analogously, considering the inhomogeneous dipole geomagnetic field we write the geomagnetic field parameter as

$$\gamma = \frac{en}{\rho} B_{0z} = -\frac{2en}{\rho} B_{\text{eq}} \sin\lambda = \gamma_0 + \alpha y, \tag{6}$$

with

$$\gamma_0 = -\frac{2en}{\rho} B_{\text{eq}} \sin\lambda_0 < 0$$

and

$$\alpha = \frac{\partial \gamma}{\partial y} = \frac{en}{\rho}\frac{\partial B_{0z}}{\partial y} = -\frac{2enB_{\text{eq}}}{\rho R}\cos\lambda_0 < 0. \tag{7}$$

In (4)–(7) the quantities α, β, f_0 and γ_0 are related to the latitude λ_0, e is the magnitude of the electron charge, B_{eq} is the equatorial value of the geomagnetic field at a distance R from the Earth's center and $\rho = Nm$ is the neutral gas mass density. The factor $(\alpha+\beta)$ in (2) represents the generalized Rossby parameter, where α corresponds to the contribution from the Lorentz force. The parameters α and β are comparable in magnitude ($\beta \simeq -\alpha \simeq 10^{-11}$ m^{-1} s^{-1}) in the ionospheric E-layer and α depends on the ratio n/N. This ionization fraction is distinctly different at the nightside and dayside of the Earth. Thus, unlike the Rossby waves in a neutral atmosphere, the magnetized Rossby waves in the ionospheric E-layer can propagate both westwards and eastwards at a fixed latitude along the parallels.

The magnetized Rossby waves do not significantly perturb the geomagnetic field. They are induced by the latitudinal inhomogeneity of the Earth's angular velocity as well as of the geomagnetic field, determined by β and α, respectively. They are excited by the ionospheric dynamo electric field when the Hall effect due to the interaction with the ionized ionospheric component in the E-layer is included. The dynamics of propagation depends on the generalized Rossby parameter $(\alpha + \beta)$ and the modified Rossby radius. The Lorentz force counteracts the Coriolis force vorticity and partial or full compensation of the Coriolis deviation by that of the magnetic is therefore possible. Correspondingly, the phase velocities of the linear waves also decrease. The period of these waves is of the order of several days. A magnetized Rossby wave belongs to the ultra-low frequency range $(10^{-6}$–$10^{-5})$ s^{-1}, its wavelength is 1000 km or larger and its phase velocity is of the order of the velocity of the local winds, i.e. ∼1–100 m s^{-1}. Such waves correspond to longitudinal mode numbers less than 8–10. It has been established that the inhomogeneity (latitude variation) of the geomagnetic field and the Earth's rotation generates magnetized Rossby waves, which propagate along the parallels to the east as well as to the west. These large-scale waves are weakly damped. The features and the parameters of the theoretically investigated electromagnetic wave structures agree with those of the large-scale ULF mid-latitude long-period oscillations and the ionospheric wave perturbations observed in the ionosphere.

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3. Nonlinear interactions of magnetized Rossby waves and zonal flows in the E-layer

Considering large-scale structures with sizes $a \geqslant r_{\mathrm{R}}$, it was shown (e.g. Kaladze et al. 2004) that the magnetized Rossby-wave turbulence in the ionospheric E-layer could be described by the generalized Charney equation

$$\frac{\partial}{\partial t}\left(h - r_{\mathrm{R}}^2 \nabla_{\perp}^2 h\right) + v_{\mathrm{R}} \frac{\partial h}{\partial x} + v_{\mathrm{R}} h \frac{\partial h}{\partial x} - (f + \gamma) r_{\mathrm{R}}^4 J\left(h, \nabla_{\perp}^2 h\right) = 0. \tag{8}$$

Within this model, the ionosphere is treated as an incompressible shallow water fluid of depth $H = H_0(1 + h)$, where H_0 is the unperturbed constant depth and h stands for a dimensionless wave amplitude. The Poisson bracket operator $J(a, b) = \partial_x a \partial_y b - \partial_y a \partial_x b$ represents the vector nonlinearity. The new term in the generalized (8) is the scalar nonlinearity of the KDV type $\sim h \partial_x h$. We estimate from (8) that the vector nonlinearity can compensate the scalar nonlinearity only when the size a of the structure is less than r_{ig}, or

$$a < r_{\mathrm{ig}} = \left(r_{\mathrm{R}}^2 R\right)^{1/3}, \tag{9}$$

where we have introduced the so-called intermediate geostrophic radius r_{ig}. Here r_{R} is the Rossby radius defined by (3) and R is the Earth's radius, which is the scale of inhomogeneity of both the Earth's angular velocity and the geomagnetic field. In the problems considered by Shukla and Stenflo (2003) and Onishchenko et al. (2004) only the vector nonlinearity was kept. This means that only small size structures, satisfying the inequality (9), were considered in these papers. Below we will consider the generalized (8), i.e. we will study structures of arbitrary size.

The generalized Charney equation (8) for magnetized Rossby waves contains thus both scalar and vector nonlinearities and it can describe solitary vortex structures of arbitrary size. This equation corresponds to the so-called intermediate geostrophic approximation in geophysical hydrodynamics, where a perturbation of the free surface of the liquid plays a role (accordingly the two-dimensional divergence of the velocity differs from zero). The mechanism for self-organization of solitary structures is associated with the compensation of wave dispersion by both the scalar and vector nonlinearities. As a result, a solitary structure is in general intrinsically anisotropic and contains a circular vortex superimposed on a dipole perturbation. The degree of anisotropy increases sharply as the size of the vortex approaches the intermediate geostrophic size (9). The generalized Charney equation (8) for Rossby waves with $\alpha = \gamma = 0$ was first derived by Petviashvili (1980).

Owing to the presence of the scalar nonlinearity, (8) breaks the cyclone–anticyclone symmetry and predicts the existence of solitary waves (solitons) with monopole structures and defined signs; i.e. either cyclones or anticyclones. Such solitary structures were first found in laboratory modeling of solitary Rossby vortices by Antipov et al. (1982). A large-scale dipole vortex splits into two monopoles (a cyclone and an anticyclone), where a vortex of one polarity is long-lived whereas the vortex of the opposite polarity disperses. In case of magnetized Rossby waves, only those anticyclones survive that propagate faster then the maximum velocity of the corresponding linear waves. Thus, we emphasize that the presence of the scalar nonlinearity plays an additional role, similar to an instability, in forming new structures from former dipole structures.

Since the zonal flow varies on a much larger time scale than the comparatively small-scale magnetized Rossby waves, one can use a multi-scale expansion,

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assuming that there is a sufficient spectral gap separating the large- and small-scale motions. Following the standard procedure to describe the evolution of the coupled system (magnetized Rossby waves plus zonal flows), we decompose the perturbation of the dimensionless ionospheric depth h into its low- and high-frequency parts, that is $h = \hat{h} + \tilde{h}$, where $\hat{h}(y,t)$ refers to the large-scale zonal flow and $\tilde{h}(\mathbf{r},t)$ to the small-scale magnetized Rossby wave. Averaging (8) over the small spatial scales, we obtain the evolution equation for the mean flow

$$\frac{\partial}{\partial t}\big(\hat{h} - r_{\mathrm{R}}^2\nabla_{\perp}^2\hat{h}\big) = (f+\gamma)r_{\mathrm{R}}^4\langle J\big(\tilde{h},\nabla_{\perp}^2\tilde{h}\big)\rangle, \tag{10}$$

where the angular bracket denotes the averaging process. In (10) the term on the right-hand side describes the Reynolds stresses induced by the small-scale magnetized Rossby waves.

The nonlinear coupling of the magnetized Rossby waves with the zonal flow is governed by

$$\frac{\partial}{\partial t}\big(\tilde{h} - r_{\mathrm{R}}^2\nabla_{\perp}^2\tilde{h}\big) + v_{\mathrm{R}}\frac{\partial\tilde{h}}{\partial x} + v_{\mathrm{R}}\hat{h}\frac{\partial\tilde{h}}{\partial x} - (f+\gamma)r_{\mathrm{R}}^4\big[J\big(\hat{h},\nabla_{\perp}^2\tilde{h}\big) + J\big(\tilde{h},\nabla_{\perp}^2\hat{h}\big)\big] = 0. \tag{11}$$

The magnetized Rossby waves are considered as a superposition of a pump wave and two sidebands, that is $\tilde{h} = h_0 + \tilde{h}_+ + \tilde{h}_-$, where for the pump wave we have $h_0 = h_{\mathbf{k}}\exp(i(\mathbf{k}\cdot\mathbf{r} - \omega_{\mathbf{k}}t)) + h_{\mathbf{k}}^*\exp(-i(\mathbf{k}\cdot\mathbf{r} - \omega_{\mathbf{k}}t))$ with the frequency $\omega_{\mathbf{k}}$ given by (1).

The change in the zonal flow amplitude is given by $\hat{h} = h_{\mathbf{q}}\exp(i(\mathbf{q}\cdot\mathbf{r} - \Omega t)) + h_{\mathbf{q}}^*\exp(-i(\mathbf{q}\cdot\mathbf{r} - \Omega t))$, with $\mathbf{q} = q\hat{\mathbf{y}}$ where $\hat{\mathbf{y}}$ is the unit vector along the latitude. For the magnetized Rossby side bands we have $\tilde{h}_{\pm} = h_{\mathbf{k}_{\pm}}\exp(i(\mathbf{k}_{\pm}\cdot\mathbf{r} - \omega_{\mathbf{k}_{\pm}}t)) + h_{\mathbf{k}_{\pm}}^*\exp(-i(\mathbf{k}_{\pm}\cdot\mathbf{r} - \omega_{\mathbf{k}_{\pm}}t))$, where $\omega_{\mathbf{k}_{\pm}} = \omega_{\mathbf{k}} \pm \Omega$ and $\mathbf{k}_{\pm} = \mathbf{k} \pm \mathbf{q}$ are the frequencies and wave vectors of the magnetized Rossby sidebands.

Substituting h_0, $\hat{h}$ and $\tilde{h}_{\pm}$ into (10), we obtain

$$\Omega h_{\mathbf{q}} = -i\frac{(f+\gamma)r_{\mathrm{R}}^4}{1+q^2r_{\mathrm{R}}^2}k_x q\big[(k_+^2 - k^2)h_{\mathbf{k}_+}h_{\mathbf{k}}^* - (k_-^2 - k^2)h_{\mathbf{k}}h_{\mathbf{k}_-}^*\big], \tag{12}$$

where the expressions for the Fourier amplitudes $h_{\mathbf{k}_+}$ and $h_{\mathbf{k}_-}^*$, found from (11), are

$$h_{\mathbf{k}_+} = -\frac{i(f+\gamma)r_{\mathrm{R}}^4k_xq(k^2-q^2) - k_xv_{\mathrm{R}}}{(\Omega+\delta\omega_+)\big(1+k_+^2r_{\mathrm{R}}^2\big)}h_{\mathbf{q}}h_{\mathbf{k}}, \tag{13}$$

and

$$h_{\mathbf{k}_-}^* = \frac{i(f+\gamma)r_{\mathrm{R}}^4k_xq(k^2-q^2) - k_xv_{\mathrm{R}}}{(\Omega-\delta\omega_-)\big(1+k_-^2r_{\mathrm{R}}^2\big)}h_{\mathbf{q}}h_{\mathbf{k}}^*, \tag{14}$$

where

$$\delta\omega_{\pm} \equiv \omega_{\mathbf{k}} - \frac{k_xv_{\mathrm{R}}}{1+k_{\pm}^2r_{\mathrm{R}}^2}. \tag{15}$$

Substituting (13) and (14) into (12) we obtain

$$\Omega = i\frac{(f+\gamma)r_{\mathrm{R}}^4k_xq}{1+q^2r_{\mathrm{R}}^2}\big[i(f+\gamma)r_{\mathrm{R}}^4k_xq(k^2-q^2) - k_xv_{\mathrm{R}}\big]|h_0|^2$$
$$\times\left[\frac{k_+^2-k^2}{(\Omega+\delta\omega_+)\big(1+k_+^2r_{\mathrm{R}}^2\big)} + \frac{k_-^2-k^2}{(\Omega-\delta\omega_-)\big(1+k_-^2r_{\mathrm{R}}^2\big)}\right]. \tag{16}$$

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The dispersion relation (16) is in general too cumbersome for analysis, and it can thus only be solved numerically. In order to simplify it we consider the most interesting case, namely $q \ll k$, when the typical scales of the zonal flows are much larger then the scales of the magnetized Rossby waves. In this limiting case we can adopt the expansions

$$\delta\omega_{\pm} \simeq \mp q v_{\rm g} - \frac{q^2 v'_{\rm g}}{2}, \tag{17}$$

$$\left[\frac{k_+^2 - k^2}{(\Omega + \delta\omega_+)\left(1 + k_+^2 r_{\rm R}^2\right)} + \frac{k_-^2 - k^2}{(\Omega - \delta\omega_-)\left(1 + k_-^2 r_{\rm R}^2\right)}\right]$$

$$\simeq -\frac{q^2 \Omega v'_{\rm g}}{\omega_{\mathbf{k}} r_{\rm R}^2} \frac{1}{[(\Omega - q v_{\rm g})^2 - (q^2 v'_{\rm g}/2)^2]}, \tag{18}$$

where

$$v'_{\rm g} \equiv \frac{\partial v_{\rm g}}{\partial k_y} = \frac{\partial^2 \omega_{\mathbf{k}}}{\partial k_y^2} = -2\omega_{\mathbf{k}} r_{\rm R}^2 \frac{1 + k_x^2 r_{\rm R}^2 - 3k_y^2 r_{\rm R}^2}{\left(1 + k^2 r_{\rm R}^2\right)^2}. \tag{19}$$

Here

$$v_{\rm g} \equiv \frac{\partial \omega_{\mathbf{k}}}{\partial k_y} = -2k_y r_{\rm R}^2 \frac{\omega_{\mathbf{k}}}{1 + k^2 r_{\rm R}^2} \tag{20}$$

is the latitudinal (y-component) of the Rossby group velocity. We note that both $v_{\rm g}$ and $v'_{\rm g}$ can change sign when $k_x = \pm((3k_y^2 r_{\rm R}^2 - 1)/r_{\rm R}^2)^{1/2}$. This occurs on the Rossby wave caustics. Substituting (17) and (18) into (16), one finds

$$\Omega_{\pm} \simeq q v_{\rm g} \pm \left(-i \frac{(f+\gamma) r_{\rm R}^2 k_x^2 q^3}{1 + q^2 r_{\rm R}^2} \frac{v'_{\rm g}}{\omega_{\mathbf{k}}} |h_0|^2 [i(f+\gamma) r_{\rm R}^4 q k^2 - v_{\rm R}] + \left(\frac{q^2}{2} v'_{\rm g}\right)^2\right)^{1/2}. \tag{21}$$

Let us now investigate two special cases.

In the case of small-scale turbulence, when $a < r_{\rm ig}$, we obtain from (21)

$$\Omega_{\pm} \simeq q v_{\rm g} \pm \left(\frac{(f+\gamma)^2 r_{\rm R}^6 k_x^2 q^4 v'_{\rm g} k^2 |h_0|^2}{\left(1 + q^2 r_{\rm R}^2\right)\omega_{\mathbf{k}}} + \left(\frac{q^2}{2} v'_{\rm g}\right)^2\right)^{1/2}. \tag{22}$$

It is thus obvious that a necessary condition for instability is $v'_{\rm g}/\omega_{\mathbf{k}} < 0$. This condition is similar to the Lighthill criterion for modulation instability in nonlinear optics (Lighthill 1965). According to (19)

$$\frac{v'_{\rm g}}{\omega_{\mathbf{k}}} = -2r_{\rm R}^2 \frac{1 + k_x^2 r_{\rm R}^2 - 3k_y^2 r_{\rm R}^2}{\left(1 + k^2 r_{\rm R}^2\right)^2}, \tag{23}$$

which in the small wavelength limit case ($k r_{\rm R} \gg 1$) reduces to

$$\frac{v'_{\rm g}}{\omega_{\mathbf{k}}} = -2\frac{k_x^2 - 3k_y^2}{k^4}, \tag{24}$$

i.e.

$$\Omega_{\pm} \simeq q v_{\rm g} \pm \left(-\frac{2(k_x^2 - 3k_y^2)(f+\gamma)^2 r_{\rm R}^6 k_x^2 q^4 |h_0|^2}{k^2\left(1 + q^2 r_{\rm R}^2\right)} + \left(\frac{q^2}{2} v'_{\rm g}\right)^2\right)^{1/2}. \tag{25}$$

Instability occurs when $k_x^2 - 3k_y^2 > 0$, and the instability condition thus applies to magnetized Rossby pump waves with wave vectors in the cone

$$-\frac{k_x}{\sqrt{3}} < k_y < \frac{k_x}{\sqrt{3}}. \tag{26}$$

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The maximum growth is attained at the axis of the cone when $k_y = 0$. In this case the mode is purely growing with the growth rate

$$\gamma = -i\Omega_+ = \left(\frac{2(f+\gamma)^2 r_{\mathrm{R}}^6 k^2 q^4 |h_0|^2}{1+q^2 r_{\mathrm{R}}^2} - \frac{q^4 \omega_{\mathbf{k}}^2}{k^4} \right)^{1/2}. \tag{27}$$

Expression (27) describes the initial (linear) stage of zonal-flow growth due to the parametric instability of small-scale magnetized Rossby waves.

For the Rossby regime $f + \gamma \gg \omega_k$, the last term in the parenthesis of (27) is small. For $qr_{\mathrm{R}} \sim 1$ and $kr_{\mathrm{R}} \gg 1$ we can then estimate the growth rate to

$$\gamma \approx |f+\gamma|(kr_{\mathrm{R}})|h_0|. \tag{28}$$

This estimation shows that γ increases as k in the small wavelength limit ($kr_{\mathrm{R}} \gg 1$). Physically, this instability is the manifestation of an inverse cascade. It shows that the spectral energy of the small-scale magnetized Rossby wave turbulence is transferred into the large scales of the zonal flows, i.e. the magnetized Rossby-wave energy is converted into the energy of slow zonal motions. For the typical parameters of the Earth's atmosphere $(f+\gamma) \approx 10^{-4}$ s^{-1}, $kr_{\mathrm{R}} \simeq 10$ and $h_0 \simeq 10^{-2}$ we obtain $\gamma \simeq 10^{-5}$ s^{-1}. This estimate is consistent with existing observations. Thus, it is possible that the parametric instability of magnetized Rossby waves is responsible for the generation of mean flows in the ionosphere of our rotating Earth.

Onishchenko et al. (2004) investigated the special case $qr_{\mathrm{R}} \gg 1$ to obtain the expression for the maximum growth rate and to define the optimal parameters of the zonal flow. Here we will however consider the opposite case when $qr_{\mathrm{R}} \ll 1$. We then obtain

$$\gamma = \frac{q^2}{k^2} \sqrt{2(f+\gamma)^2 (kr_{\mathrm{R}})^6 |h_0|^2 - \omega_{\mathbf{k}}^2}. \tag{29}$$

In the Rossby regime $f + \gamma \gg \omega_{\mathbf{k}}$, and for the case $kr_{\mathrm{R}} \gg 1$, we thus have

$$\gamma \approx \frac{q^2}{k^2} |f+\gamma| (kr_{\mathrm{R}})^3 |h_0|. \tag{30}$$

In spite of the fact that the expression (30) contains a small factor q^2/k^2, owing to the high value of $(kr_{\mathrm{R}})^3 \gg 1$, the growth rate is significant. As we have mentioned above, the presence of the geomagnetic field causes reduction of the value of $(f+\gamma)$. Thus, the obtained growth rate (30) increases as $(f+\gamma)^{-2}$.

Let us now investigate the new term in (21) coming from the contribution of the scalar (KDV) nonlinearity. In this case of large-scale turbulence with $kr_{\mathrm{R}} \sim 1$, we estimate that when $q/k \sim 10^{-1}$

$$v_{\mathrm{R}} \sim (f+\gamma) r_{\mathrm{R}}^4 q k^2 \sim (f+\gamma) r_{\mathrm{R}} \frac{q}{k}. \tag{31}$$

Thus using the main term we obtain

$$\Omega_\pm \simeq q v_{\mathrm{g}} \pm q k_x r_{\mathrm{R}} |h_0| \sqrt{i(f+\gamma) q v_{\mathrm{R}} \frac{v_{\mathrm{g}}'}{\omega_{\mathbf{k}}}}. \tag{32}$$

This instability exists for any sign of $v_{\mathrm{g}}'/\omega_{\mathbf{k}}$ unlike the instability considered above. Substituting the expressions (2) and (23) into (32) one finds

$$\Omega_\pm \simeq q v_{\mathrm{g}} \pm \frac{q|h_0| k_x r_{\mathrm{R}}^3}{1+k^2 r_{\mathrm{R}}^2} \sqrt{2i(f+\gamma) q \big(1 + k_x^2 r_{\mathrm{R}}^2 - 3k_y^2 r_{\mathrm{R}}^2\big)(\alpha+\beta)}. \tag{33}$$

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Maximum growth rate is here attained again when $k_y = 0$, i.e.

$$\gamma = q|h_0|kr_{\mathrm{R}}^3\sqrt{(f+\gamma)q\frac{\alpha+\beta}{1+k^2r_{\mathrm{R}}^2}}. \tag{34}$$

An estimate of this growth rate when $kr_{\mathrm{R}} \sim 1$ gives

$$\gamma \sim \left(\frac{q}{k}\right)^{3/2} r_{\mathrm{R}}^{1/2}(f+\gamma)^{1/2}(\alpha+\beta)^{1/2}|h_0|. \tag{35}$$

For the typical parameters $(f+\gamma) \sim 10^{-4}$ s^{-1}, $\alpha+\beta \sim 10^{-11}$ m^{-1} s^{-1}, $r_{\mathrm{R}} \sim 10^6$ m, $|h_0| \sim 10^{-2}$ and $q/k \sim 10^{-1}$ of the Earth's ionosphere, we obtain $\gamma \sim 10^{-6}$ s^{-1}. Thus, this growth rate is ten times less than that obtained in the small-scale turbulence case (see (28)).

4. Discussion and conclusions

In the present study we have demonstrated how zonal flows in the shallow rotating ionospheric E-layer can be excited by finite-amplitude magnetized Rossby waves. The driving mechanism of this instability is due to the Reynolds stresses that are inevitably inherent for finite-amplitude small-scale magnetized Rossby waves. Hence, our investigation provides an essential nonlinear mechanism for the transfer of spectral energy from small-scale magnetized Rossby waves to large-scale enhanced zonal flows in the Earth's ionosphere. We have used the generalized Charney equation describing the turbulence of magnetized Rossby waves of arbitrary size. In the case of small-scale turbulence, when $kr_{\mathrm{R}} \gg 1$, only the vector nonlinearity is responsible for the parametric instability giving the most important growth rate. The peculiar feature of this instability is that it appears solely for magnetized Rossby waves that are localized in a cone bounded by the caustics for which $v_{\mathrm{g}}' = 0$. This can lead to the formation of a so-called caustic shadow in the spectrum of the magnetized Rossby waves. A typical value of the obtained growth rate is 10^{-5}–10^{-4} s^{-1}. However, the presence of the geomagnetic field (causing a contribution opposite to that of the Earth's angular velocity rotation) can increase the growth rate by one order.

In the case of large-scale turbulence, when $kr_{\mathrm{R}} \leqslant 1$, the KDV-type scalar nonlinearity gives the main contribution in forming the turbulence structures. The corresponding growth rate is positive for arbitrary sign of $v_{\mathrm{g}}'/\omega_{\mathbf{k}}$, but is one order less than in the case of the small-scale turbulence.

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