J. Appl. Prob. **53,** 1271–1272 (2016)
*doi:*10.1017/jpr.2016.80
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CORRECTION

BIARD, R. AND SAUSSEREAU, B. (2014). Fractional Poisson process: long-range dependence and applications in ruin theory. *J. Appl. Prob.* **51,** 727–740.

There is a mistake in the proof of Theorem 1 of our paper 'Fractional Poisson process: long-range dependence and applications in ruin theory'. However, our result holds and we propose another proof here.

We follow the framework and notation of the above paper. In Section 1 we fix the gap in the proof of the main result of the above paper and, consequently, the fractional Poisson process has the long-range dependence property as stated in Theorem 1 of the above paper.

1. True proof of Theorem 1 of the above paper

The proof of Theorem 1 of the above paper is correct until the bottom of page 732 where we have used the following false inequality:

$$\int_{1-1/t}^{1} (1-u)^h u^{h-1}\,\mathrm{d}u \geq \mathcal{B}(1+h,h), \tag{1}$$

where $\mathcal{B}$ denotes the beta function.

In order to fix this gap, we have to replace (10) of page 733. The following proof indicates how we can easily correct our original error.

Proof. We start again our proof after (9) of page 732. By (7) and (8) from the above paper, we also may write

$$\begin{aligned}\operatorname{var}(N_h(tm)-N_h(tm-m)) &= 2h\left(\frac{\lambda}{\Gamma(1+h)}\right)^2 \int_{tm-m}^{tm} (tm-r)^h r^{h-1}\,\mathrm{d}r \\ &\quad + \frac{\lambda}{\Gamma(1+h)}((tm)^h-(tm-m)^h) \\ &\quad - \left\{\frac{\lambda}{\Gamma(1+h)}((tm)^h-(tm-m)^h)\right\}^2.\end{aligned}$$

Since

$$\int_{tm-m}^{tm} (tm-r)^h r^{h-1}\,\mathrm{d}r = (tm)^{2h}\int_{1-1/t}^{1} (1-u)^h u^{h-1}\,\mathrm{d}u,$$

we obtain

$$\begin{aligned}&\operatorname{var}(N_h(tm)-N_h(tm-m)) \\ &\quad = \left(\frac{\lambda}{\Gamma(1+h)}\right)^2\left[2h\int_{1-1/t}^{1}(1-u)^h u^{h-1}\,\mathrm{d}u - \left\{1-\left(1-\frac{1}{t}\right)^h\right\}^2\right](tm)^{2h} \\ &\qquad + \frac{\lambda}{\Gamma(1+h)}\left(1-\left(1-\frac{1}{t}\right)^h\right)(tm)^h.\end{aligned}$$

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This yields

$$\Delta_t^{(m)} \geq \left\{ t^h \frac{\lambda/\Gamma(1+h)}{2\lambda/\Gamma(1+h)+1} \left[2h \int_{1-1/t}^{1} (1-u)^h u^{h-1}\,\mathrm{d}u - \left\{ 1 - \left(1 - \frac{1}{t}\right)^h \right\}^2 \right] m^h + \frac{1-(1-1/t)^h}{2\lambda/\Gamma(1+h)+1} \right\} \frac{1}{1 - C_{t,\lambda,h} m^{h-1}}.$$

We denote f as the fuction defined from the interval $[0,1]$ to $\mathbb{R}$ by

$$f(x) = 2h \int_x^1 (1-u)^h u^{h-1}\,\mathrm{d}u - (1-x^h)^2.$$

We have $f(1) = 0$ and $f'(x) = -2hx^{h-1}\left((1-x)^h - 1 + x^h\right) < 0$ for $x \in (0,1)$, so f is positive on $[0, 1)$. Indeed, the function g defined from the interval $[0,1]$ to $\mathbb{R}$ by

$$g(x) = (1-x)^h - 1 + x^h$$

is such that $g(0) = g(1) = 0$, and for $x \in (0,1)$, $g'(x) = h[x^{h-1} - (1-x)^{h-1}]$, which is positive on $(0, \frac{1}{2})$ and negative on $(\frac{1}{2}, 1)$. So g is positive on $(0,1)$ and as a consequence, f' is negative on $(0,1)$.

Then

$$t \mapsto f\left(1 - \frac{1}{t}\right) = 2h \int_{1-1/t}^{1} (1-u)^h u^{h-1}\,\mathrm{d}u - \left\{ 1 - \left(1 - \frac{1}{t}\right)^h \right\}$$

is positive for $t \geq 1$. Thus, $\lim_{m\to\infty} \Delta_t^{(m)} = +\infty$ and, consequently, the long-range dependence property holds. □

Acknowledgements

The authors thank Professors Maheshwari and Vellaisamy for pointing out the wrong inequality (1) employed in the above paper.

https://doi.org/10.1017/jpr.2016.80 Published online by Cambridge University Press