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Bell Polynomials to the Kadomtsev-Petviashivili Equation with Self-Consistent Sources

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Abstract. Bell Polynomials play an important role in the characterization of bilinear equation. Bell Polynomials are extended to construct the bilinear form, bilinear Bäcklund transformation and Lax pairs for the Kadomtsev-Petviashvili equation with self-consistent sources.

AMS subject classifications: 02.30.Ik, 05.45.Yv

Key words: Bell polynomials, bilinear form, bilinear Bäcklund transformation, Lax pair.

1 Introduction

There are some techniques can be used to solve the soliton equation with self-consistent sources [1–5]. Among this methods, the bilinear method and bilinear Bäcklund transformation have proved particularly powerful. Through the dependent variable transformations, some soliton equation with self-consistent sources can be transformed into bilinear forms. Applying the bilinear method developed by Hirota [6], we can obtain the soliton solutions. The construction of the bilinear Bäcklund transformation [7] by using Hirota method relies on a particular skill in using appropriate exchange formulas which are connected with the linear presentation of the system. Yet, the construct of bilinear Bäcklund transformation is complicated. Recently, Lambert, Gilson et al. [8–10] proposed an alternative procedure based on the use of the Bell polynomials which enable one to obtain parameter families of bilinear Bäcklund transformation and Lax pairs for the soliton equations in a lucid and systematic way. Fan et al. [11–14] have applied this method for some soliton equations. But there are not any research in the soliton equation with self-consistent sources.

In this paper, we will extend the Binary Bell polynomials to deal the Kadomtsev-Petviashvili equation with self-consistent sources (KPESCS). First, we derive the bilinear

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form for the KPESCS by the binary Bell polynomials. Second, the bilinear Bäcklund transformation and Lax pairs are obtained in a quick and natural manner.

2 The bilinear form for the mKP equation

The mail tool used here is a class of generalized multi-dimensional Bell polynomials. First, we give some notations on the Bell polynomials to easily understand our presentation.

Lambert et al. proposed a generalization of the Bell polynomial [8–10]. Let $n_k \geq 0$, $k=1,\cdots,l$, denote arbitrary integers, $f=f(x_1,\cdots,x_l)$ be a C^∞ multi-variable function, the following polynomials

$$\mathcal{Y}_{n_1x_1\cdots n_lx_l}(f)=\exp(-f)\partial_{x_1}^{n_1}\cdots\partial_{x_l}^{n_l}\exp(f) \tag{2.1}$$

is called multi-dimensional Bell polynomial (generalized Bell polynomial or $\mathcal{Y}$-polynomials). If all partial derivatives $f_{r_1x_1\cdots r_lx_l}=\partial_{x_1}^{r_1}\cdots\partial_{x_l}^{n_l}$ $(r_k=0,\cdots,n_k,\ k=1,\cdots l)$ are taken as different variable elements, then the generalized Bell polynomial $\mathcal{Y}_{n_1x_1,\cdots,n_lx_l}(f)$ is the multivariable polynomial with respect to these variable elements $f_{r_1x_1,\cdots,r_lx_l}$. The subscripts in the notation $\mathcal{Y}_{n_1x_1,\cdots,n_lx_l}(f)$ denote the highest order derivatives of f with respect to the variable x_k, $k=1,\cdots,l$ respectively.

For the special case $f=f(x,t)$, the associated two-dimensional Bell polynomials defined by (2.1) read

$$\mathcal{Y}_x(f)=f_x,\mathcal{Y}_{2x}(f)=f_{2x}+f_x^2,\qquad \mathcal{Y}_{3x}(f)=f_{3x}+3f_xf_{2x}+f_x^3, \tag{2.2a}$$

$$\mathcal{Y}_{x,t}(f)=f_{x,t}+f_xf_t,\qquad \mathcal{Y}_{2x,t}(f)=f_{2x,t}+f_{2x}f_t+2f_{x,t}f_x+f_x^2f_t. \tag{2.2b}$$

Base on the use of above Bell polynomials (2.1), the multidimensional binary Bell polynomials ($\mathcal{Y}$-polynomials) can be defined as follows

$$\begin{aligned}\mathcal{Y}_{n_1x_1,\cdots,n_lx_l}(v,w)&=\mathcal{Y}_{n_1x_1,\cdots,n_lx_l}(f)|_{f_{r_1x_1,\cdots,r_lx_l}}\\&=\begin{cases}v_{r_1x_1,\cdots,r_lx_l}, & r_1+\cdots+r_l \text{ is odd},\\ w_{r_1x_1,\cdots,r_lx_l}, & r_1+\cdots+r_l \text{ is even},\end{cases}\end{aligned} \tag{2.3}$$

which is a multivariable polynomials with respect to all partial derivatives $v_{r_1x_1,\cdots,r_lx_l}(r_1+\cdots+r_l$ odd) and $w_{r_1x_1,\cdots,r_lx_l}(r_1+\cdots+r_l$ even), $r_k=0,\cdots,n_k$, $k=0,\cdots,l$.

The binary Bell polynomials also inherit the easily recognizable partial structure of the Bell polynomials. The lowest order binary Bell polynomials are

$$\mathcal{Y}_x(v)=v_x,\qquad \mathcal{Y}_{2x}(v,w)=w_{2x}+v_x^2, \tag{2.4a}$$

$$\mathcal{Y}_{x,t}(v,w)=w_{xt}+v_xv_t,\qquad \mathcal{Y}_{3x}(v,w)=v_{3x}+3v_xw_{2x}+v_x^3. \tag{2.4b}$$

The link between binary Bell polynomials $\mathcal{Y}_{n_1x_1,\cdots,n_lx_l}(v,w)$ and the standard Hirota bilinear equation $D_{x_1}^{n_1}\cdots D_{x_l}^{n_l}F\cdot G$ can be given by an identity

$$\mathcal{Y}_{n_1x_1,\cdots,n_lx_l}(v=\ln F/G,w=\ln FG)=(FG)^{-1}D_{x_1}^{n_1}\cdots D_{x_l}^{n_l}F\cdot G, \tag{2.5}$$

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in which $n_1+n_2+\cdots+n_l\geq 1$, and operators $D_{x_1},\cdots,D_{x_l}$ are classical Hirota bilinear operators defined by

$$D_{x_1}^{n_1}\cdots D_{x_l}^{n_l}F\cdot G=(\partial_{x_1}-\partial_{x_1'})^{n_1}\cdots(\partial_{x_l}-\partial_{x_l'})^{n_l}F(x_1,\cdots,x_l)\times G(x_1',\cdots,x_l')|_{x_x'=x_1,\cdots,x_l'=x_l}. \quad (2.6)$$

In the particular case when $F=G$, the formula (2.5) becomes

$$\begin{aligned} G^{-2}D_{x_1}^{n_1}\cdots D_{x_l}^{n_l}G\cdot G&=\mathcal{Y}_{n_1x_1\cdots,n_lx_l}(0,q=2\ln G)\\ &=\begin{cases} 0, & n_1+\cdots+n_l \text{ is odd},\\ P_{n_1x_1,\cdots,n_lx_l}(q), & n_1+\cdots+n_l \text{ is even}, \end{cases} \end{aligned} \quad (2.7)$$

in which the P-polynomials can be characterized by an equally recognizable even part partitional structure

$$P_{2x}(q)=q_{2x},\quad P_{x,t}(q)=q_{xt},\quad P_{4x}=q_{4x}+3q_{2x}^2,\quad P_{6x}(q)=q_{6x}+15q_{2x}q_{4x}+15q_{2x}^2. \quad (2.8)$$

The formulaes (2.5), (2.7) and (2.8) will prove particular useful in connecting nonlinear equations with their corresponding bilinear equations. This means that once a nonlinear equation is expressible as a linear combination of the P-polynomials, then it can be transformed into a linear equation.

The binary Bell polynomials $\mathcal{Y}_{n_1x_1,\cdots,n_lx_l}(v,w)$ can be separated into P-polynomials and $\mathcal{Y}$-polynomials

$$\begin{aligned} &(FG)^{-1}D_{x_1}^{n_1}\cdots D_{x_l}^{n_l}F\cdot G\\ =&\mathcal{Y}_{n_1x_1,\cdots,n_lx_l}(v,w)|_{v=\ln F/G,w=\ln FG}\\ =&\mathcal{Y}_{n_1x_1,\cdots,n_lx_l}(v,v+q)|_{v=\ln F/G,q=2\ln G}\\ =&\sum_{n_1+\cdots+n_l=even}\sum_{r_1=0}^{n_1}\cdots\sum_{r_l=0}^{n_l}\prod_{i=1}^{l}\binom{n_i}{r_i}P_{r_1x_1,\cdots,r_lx_l}(q)\mathcal{Y}_{(n_1-r_1)x_1\cdots(n_l-r_l)x_l}(v). \end{aligned} \quad (2.9)$$

The key property of the multi-dimensional Bell polynomials

$$\mathcal{Y}_{n_1x_1,\cdots,n_lx_l}(v)|_{v=\ln\psi}=\psi_{n_1x_1,\cdots,n_lx_l}/\psi, \quad (2.10)$$

implies that the binary Bell polynomials $\mathcal{Y}_{n_1x_1,\cdots,n_lx_l}(v,w)$ can still be linearized by means of the Hopf-Cole transformation $v=\ln\psi$, that is, $\psi=F/G$

$$\begin{aligned} &(FG)^{-1}D_{x_1}^{n_1}\cdots D_{x_l}^{n_l}F\cdot G|_{G=exp(q/2),F/G=\psi}\\ =&\psi^{-1}\sum_{r_1=0}^{n_1}\cdots\sum_{r_l=0}^{n_l}\prod_{i=1}^{l}\binom{n_i}{r_i}P_{r_1x_1,\cdots,r_lx_l}(q)\psi_{(n_1-r_1)x_1\cdots(n_l-r_l)x_l}(v). \end{aligned} \quad (2.11)$$

The formulaes (2.9) and (2.11) will then provide the shortest way to the associated Lax system of nonlinear equations.

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In this paper, we consider the KP equation with self-consistent sources

$$u_t+u_{xxx}+6uu_x+3\partial^{-1}u_{yy}+\frac{1}{2}\sum_{j=1}^{N}(\phi_j\psi_j)_x=0, \tag{2.12a}$$

$$\phi_{j,y}=\phi_{j,xx}+u\phi_j, \tag{2.12b}$$

$$-\psi_{j,y}=\psi_{j,xx}+u\psi_j. \tag{2.12c}$$

In order to detect the bilinear form, we introduce

$$u=q_{2x},\quad \phi_j=\frac{h_j}{f},\quad \psi_j=\frac{g_j}{f}. \tag{2.13}$$

Substituting (2.13) into (2.12a) and integrating with respect to x yields

$$E(q)=q_{xt}+q_{4x}+3q_{2x}^2+3q_{2y}+\frac{1}{2}\sum_{j=1}^{N}\frac{g_jh_j}{f^2}=0, \tag{2.14}$$

the equation is cast into a combination form of P-polynomials

$$E(q)=P_{xt}(q)+P_{4x}(q)+3P_{2y}(q)+\frac{1}{2}\sum_{j=1}^{N}\frac{g_jh_j}{f^2}=0. \tag{2.15}$$

Making a change of dependent variable

$$q=2\ln f,\quad u=2(\ln f)_{2x}, \tag{2.16}$$

and noting the property (2.7) and (2.8), Eq. (2.15) gives the bilinear representation as follows

$$D_xD_tf\cdot f+D_x^4f\cdot f+3D_y^2f\cdot f+\sum_{j=1}^{N}g_jh_j=0. \tag{2.17}$$

In the following, we consider the bilinear form for Eqs. (2.12b) and (2.12c). They can be transformed into

$$(\ln\phi_j)_y=(\ln\phi_j)_{xx}+(\ln\phi_j)_x^2+u, \tag{2.18a}$$

$$-(\ln\psi_j)_y=(\ln\psi_j)_{xx}+(\ln\psi_j)_x^2+u, \tag{2.18b}$$

by the transformations (2.13) and (2.16), we have

$$\left(\ln\frac{h_j}{f}\right)_y=\left(\ln\frac{h_j}{f}\right)_{xx}+\left(\ln\frac{h_j}{f}\right)_x^2+2(\ln f)_{xx}=(\ln h_jf)_{xx}+\left(\ln\frac{h_j}{f}\right)_x^2, \tag{2.19a}$$

$$-\left(\ln\frac{g_j}{f}\right)_y=\left(\ln\frac{g_j}{f}\right)_{xx}+\left(\ln\frac{g_j}{f}\right)_x^2+2(\ln f)_{xx}=(\ln g_jf)_{xx}+\left(\ln\frac{g_j}{f}\right)_x^2. \tag{2.19b}$$

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Let

$$v_j=\ln\frac{h_j}{f},\quad \omega_j=\ln h_jf,\quad \alpha_j=\ln\frac{g_j}{f},\quad \beta_j=\ln g_jf. \tag{2.20}$$

Eqs. (2.19a) and (2.19b) can be expressed as

$$v_{j,y}=\omega_{j,xx}+v_{j,x}^2, \tag{2.21a}$$

$$-\alpha_{j,y}=\beta_{j,xx}+\alpha_{j,x}^2. \tag{2.21b}$$

By the binary Bell polynomials (2.3)-(2.4b), Eqs. (2.21a)and (2.21b) can be written as the combination of $\mathcal{Y}$-polynomial

$$\mathcal{Y}_y(v_j,\omega_j)=\mathcal{Y}_{2x}(v_j,\omega_j), \tag{2.22a}$$

$$-\mathcal{Y}_y(\alpha_j,\beta_j)=\mathcal{Y}_{2x}(\alpha_j,\beta_j). \tag{2.22b}$$

By application of the identity (2.5), Eqs. (2.22a) and (2.22b) lead to the bilinear form

$$D_yh_j\cdot f=D_x^2h_j\cdot f, \tag{2.23a}$$

$$-D_yg_j\cdot f=D_x^2g_j\cdot f. \tag{2.23b}$$

Then Eqs. (2.17), (2.23a) and (2.23b) are the bilinear presentation for the KPESCS.

3 The bilinear Bäcklund transformation and Lax pairs for the KPESCS

In the section, we search for the bilinear Bäcklund transformation for the KPESCS.

Let

$$q=2\ln f,\quad \tilde{q}=2\ln\tilde{f},\quad \phi_j=\frac{h_j}{f},\quad \psi_j=\frac{g_j}{f},\quad \tilde{\phi}_j=\frac{\tilde{h}_j}{\tilde{f}},\quad \tilde{\psi}_j=\frac{\tilde{g}_j}{\tilde{f}},$$

be two different solutions for the Eqs. (2.14), (2.18a) and (2.18b). Then

$$v_j=\ln\frac{h_j}{f},\quad \omega_j=\ln h_jf,\quad \tilde{v}_j=\ln\frac{\tilde{h}_j}{\tilde{f}},\quad \tilde{\omega}_j=\ln\tilde{h}_j\tilde{f},$$

and

$$\alpha_j=\ln\frac{g_j}{f},\quad \beta_j=\ln h_jf,\quad \tilde{\alpha}_j=\ln\frac{\tilde{g}_j}{\tilde{f}},\quad \tilde{\beta}_j=\ln\tilde{g}_j\tilde{f},$$

are two different solutions for (2.21a) and (2.21b) respectively. We associate two-field condition

$$\begin{aligned}E(q)-E(\tilde{q})=&(q-\tilde{q})_{xt}+(q-\tilde{q})_{4x}+3(q-\tilde{q})_{2y}+3(q-\tilde{q})_{2x}(q+\tilde{q})_{2x}\\&+\frac{1}{2}\sum_{j=1}^{N}\left(\frac{g_jh_j}{f^2}-\frac{\tilde{g}_j\tilde{h}_j}{\tilde{f}^2}\right)=0,\end{aligned} \tag{3.1a}$$

$$(v_j-\tilde{v}_j)_y=(\omega_j-\tilde{\omega}_j)_{2x}+(v_j-\tilde{v}_j)_x(v_j+\tilde{v}_j)_x, \tag{3.1b}$$

$$-(\alpha_j-\tilde{\alpha}_j)_y=(\beta_j-\tilde{\beta}_j)_{2x}+(\alpha_j-\tilde{\alpha}_j)_x(\alpha_j+\tilde{\alpha}_j)_x. \tag{3.1c}$$

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Introduce the new variables

$$v=\frac{q-\tilde{q}}{2}=\ln\frac{f}{\tilde{f}},\quad \omega=\frac{q+\tilde{q}}{2}=\ln f\tilde{f}, \tag{3.2a}$$

$$v_j-\tilde{v}_j=\ln\frac{h_j}{f}-\ln\frac{\tilde{h}_j}{\tilde{f}}=\ln\frac{h_j}{\tilde{h}_j}-\ln\frac{f}{\tilde{f}}=\mu_j-l, \tag{3.2b}$$

$$\omega_j-\tilde{\omega}_j=\ln h_jf-\ln\tilde{h}_j\tilde{f}=\ln\frac{h_j}{\tilde{h}_j}+\ln\frac{f}{\tilde{f}}=\mu_j+l, \tag{3.2c}$$

$$v_j+\tilde{v}_j=\ln\frac{h_j}{f}+\ln\frac{\tilde{h}_j}{\tilde{f}}=\ln h_j\tilde{h}_j-\ln f\tilde{f}=\rho_j-\tau, \tag{3.2d}$$

$$\alpha_j-\tilde{\alpha}_j=\ln\frac{g_j}{f}-\ln\frac{\tilde{g}_j}{\tilde{f}}=\ln\frac{g_j}{\tilde{g}_j}-\ln\frac{f}{\tilde{f}}=\gamma_j-l, \tag{3.2e}$$

$$\beta_j-\tilde{\beta}_j=\ln g_jf-\ln\tilde{g}_j\tilde{f}=\ln\frac{g_j}{\tilde{g}_j}+\ln\frac{f}{\tilde{f}}=\gamma_j+l, \tag{3.2f}$$

$$\alpha_j+\tilde{\alpha}_j=\ln\frac{g_j}{f}+\ln\frac{\tilde{g}_j}{\tilde{f}}=\ln g_j\tilde{g}_j-\ln f\tilde{f}=\sigma_j-\tau, \tag{3.2g}$$

and rewrite the conditions (3.1a)-(3.1c) into the following

$$\frac{E(q)-E(\tilde{q})}{2}=v_{xt}+v_{4x}+3v_{2y}+6v_{2x}\omega_{2x}+\frac{1}{4}\sum_{j=1}^{N}\left(\frac{g_jh_j}{f^2}-\frac{\tilde{g}_j\tilde{h}_j}{\tilde{f}^2}\right)=0, \tag{3.3a}$$

$$\mu_{j,y}-\rho_{j,2x}-\mu_{j,x}^2-l_y+\tau_{2x}+l_x^2=(\mu_j-\rho_j)_{2x}+(l+\tau)_{2x}+(l-\mu_j)_x(l+\mu_j-\rho_j+\tau)_x, \tag{3.3b}$$

$$-\gamma_{j,y}+\sigma_{j,2x}+\gamma_{j,x}^2+l_y-\tau_{2x}-l_x^2=(\gamma_j+\sigma_j)_{2x}+(l-\tau)_{2x}+(\gamma_j-l)_x(\sigma_j-\tau+l+\gamma_j)_x. \tag{3.3c}$$

Possible choice of constraints maybe

$$\mathcal{Y}_y(l,\tau)=\mathcal{Y}_{2x}(l,\tau), \tag{3.4a}$$

$$\left(\ln\frac{g_j}{\tilde{f}}\right)_x=-\frac{\tilde{g}f}{g_j\tilde{f}}, \tag{3.4b}$$

$$\left(\ln\frac{\tilde{h}_j}{f}\right)_x=\frac{h_j\tilde{f}}{\tilde{h}_jf}. \tag{3.4c}$$

Then (3.3a)-(3.3c) can be written in the form

$$\partial_x\left[\mathcal{Y}_t(l,\tau)+\mathcal{Y}_{3x}(l,\tau)+3\mathcal{Y}_{xy}(l,\tau)+\frac{1}{4}\sum_{j=1}^{N}\frac{g_j\tilde{h}_j}{f\tilde{f}}\right]=0, \tag{3.5a}$$

$$\mathcal{Y}_y(\mu_j,\rho_j)=\mathcal{Y}_{2x}(\mu_j,\rho_j), \tag{3.5b}$$

$$-\mathcal{Y}_y(\gamma_j,\sigma_j)=\mathcal{Y}_{2x}(\gamma_j,\sigma_j). \tag{3.5c}$$

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By application of the identity (2.5), the system (3.5a)-(3.5c) lead to the bilinear Bäcklund transformation

$$D_t f\cdot\tilde{f}+D_x^3 f\cdot\tilde{f}+3D_xD_y f\cdot\tilde{f}+\frac{1}{4}\sum_{j=1}^{N}g_j\tilde{h}_j=0, \tag{3.6a}$$

$$D_y f\cdot\tilde{f}=D_x^2 f\cdot\tilde{f}, \tag{3.6b}$$

$$D_y h_j\cdot\tilde{h}_j=D_x^2 h_j\cdot\tilde{h}_j, \tag{3.6c}$$

$$D_y g_j\cdot\tilde{g}_j=D_x^2 g_j\cdot\tilde{g}_j, \tag{3.6d}$$

$$D_x\tilde{h}_j\cdot f=h_j\tilde{f}, \tag{3.6e}$$

$$D_x g_j\cdot\tilde{f}=-\tilde{g}_j f. \tag{3.6f}$$

Taking $f=1$, $g=1$, $h=1$, we have the one soliton solution for the KPESCS

$$\tilde{f}=e^{\xi_1}+e^{\eta_1}, \qquad \tilde{h}_1=\frac{1}{k_2}e^{\xi_1}-\frac{1}{l_1}e^{\eta_1}, \qquad \tilde{g}=k_1e^{\xi_1}-l_1e^{\eta_1}, \tag{3.7a}$$

$$\xi_1=k_1x-k_1^2y-4k_1^3t+\frac{1}{4k_1}t, \qquad \eta_1=-l_1x-l_1^2y+4l_1^3t-\frac{1}{4l_1}t. \tag{3.7b}$$

By the transformation $\phi=f/\tilde{f}$, $u=2(\ln\tilde{f})_{xx}$, it follows from the formula (2.10) that, Eqs. (3.6a) and (3.6b) become

$$\phi_t+\phi_{3x}+3p_{2x}(q)\phi_x+3\phi_{xy}+3p_{xy}(q)\phi+\frac{1}{4}\sum_{j=1}^{N}\frac{g_j\tilde{h}_j}{f\tilde{f}}\phi=0, \tag{3.8a}$$

$$\phi_y=\phi_{xx}+2(\ln\tilde{f})_{xx}\phi=\phi_{xx}+u\phi. \tag{3.8b}$$

By the relation (3.8b), Eq. (3.8a) can be written into

$$\phi_t+4\phi_{3x}+6u\phi_x+3u_x\phi+3\partial^{-1}u_y\phi_y+\frac{1}{4}\sum_{j=1}^{N}\frac{g_j\tilde{h}_j}{f\tilde{f}}\phi=0. \tag{3.9}$$

Let $\phi_j=\tilde{h}_j/\tilde{f}$, $\psi_j=\tilde{g}_j/\tilde{f}$, then use the relation (3.6f)

$$(\frac{g_j}{f}\phi)_x=\left(\frac{g_j}{f}\frac{f}{\tilde{f}}\right)_x=\left(\frac{g_j}{\tilde{f}}\right)_x=\frac{D_xg_j\cdot\tilde{f}}{\tilde{f}^2}=-\frac{\tilde{g}_jf}{\tilde{f}^2}=-\psi_j\phi. \tag{3.10}$$

So (3.9) becomes

$$\phi_t+4\phi_{3x}+6u\phi_x+3u_x\phi+3\partial^{-1}u_y\phi-\frac{1}{4}\sum_{j=1}^{N}\phi_j\partial^{-1}\psi_j\phi=0, \tag{3.11}$$

where ϕ_j, ψ_j satisfy (2.12b) and (2.12c). Thus, we get the Lax pairs (3.8b) and (3.11) for the KPESCS.

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