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Phase shift of long-wavelength mode due to drift-wave turbulence

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Abstract. The phase shift of long-wavelength modes due to drift-wave turbulence is studied in the Hasegawa–Mima model. Although the growth rate has been the main focus, it is equally crucial to understand the frequency shift from the linear characteristics. The test-field model is applied in order to properly treat the sweeping effect of the large scales on a small scale. The frequency shift is found to be small compared with the growth rate and the non-local triad mode-interaction time turns out to be determined by the energy of the long-wavelength modes.

1. Introduction

Flows of a large scale – for instance, zonal flows – play an important role in the steady state of turbulence. As for drift waves, many works exist that have analyzed the characteristics of short-scale turbulence (see, for example, Krommes and Kim 2000; Horton 1999). Although zonal flows on scales comparable to drift-wave scales have been observed in the computer simulations of Lin et al. (1998) and Hallatschek (2000), the assumption of separate scales is useful in making analytical progress. The eddy-damped quasi-normal Markovian (EDQNM) approximation has proven useful when dealing with such interactions on disparate scales. The effects of nonlinear interactions are both coherent and incoherent: they distort the response by changing the growth rate and frequency and impose a noise. While the growth rate is an important quantity for the level of fluctuations, especially the energy and the enstrophy, the frequency is another part of long-wavelength characteristics. Correlation functions of physical quantities at different times certainly depend on the frequencies.

The purpose of this paper is to derive the frequency shift of long-wavelength fluctuations due to couplings between short-wavelength fluctuations. Since, in the EDQNM closure, the integral information of the mode–mode interactions is couched in the triad mode-interaction time θ, it plays a critical role in determining the response function and the evolution of the energy. However, the EDQNM approximation has difficulty in predicting θ for triad interactions between disparate scales such as two small scales and one large scale ($\ell_q \gg \ell_k \sim \ell_p$). It tends to overestime the damping by non-local interactions and leads to a smaller value of θ. The deficiency is not related to the presence of waves and was originally pointed out by Kraichnan (1964) in the framework of the direct-interaction approximation (DIA). Triad mode-interactions between disparate scales are unrealistically dominated by advections by large scales approaching a uniform translation. In reality, large-scale motion

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advects triad mode-interactions nearly coherently and, thus, their effect should be miniscule.

The test-field model (TFM) fixes the above problem in a heuristic way (Kraichnan 1971). In the TFM, it is argued that θ represents the deformation of fluid elements, which is caused by the pressure gradient in the Navier–Stokes equation. Since the pressure gradient accounts for the incompressibility of the flow, we can imagine evaluating its effect by measuring the rate of growth of the solenoidal part of the flow when the assumption of incompressibility is relaxed. In the TFM, we proceed with the introduction of a test flow that is simply advected by the physical flow. The TFM was applied to two-dimensional flows with Rossby waves by Holloway and Hendershott (1977). In this paper, we follow the standard procedure and obtain the expressions for the triad mode-interaction time and frequency shift.

2. Frequency shift

Hasegawa and Mima (1978) (HM) developed an equation that has been used as a model for the nonlinear evolution of drift waves in the slab geometry. It can be derived as a simple limit of the nonlinear gyrokinetic (GK) equation as shown by Dubin et al. (1983) in which the ion temperature and the parallel motion are taken to vanish; upon taking the GK continuity equation for the ion gyrocenter density fluctuation and the GK Poisson equation, along with the assumption of adiabatic electrons for $k_\parallel \neq 0$, a generalization of the HM equation can be written as, in the Fourier space,

$$\partial_t \varphi_{\mathbf{k}} + i\Omega^{\mathrm{lin}}_{\mathbf{k}} \varphi_{\mathbf{k}} = \frac{1}{2} \sum_{\Delta} M_{\mathbf{k},\mathbf{p},\mathbf{q}} \varphi^*_{\mathbf{p}} \varphi^*_{\mathbf{q}}, \tag{2.1}$$

where

$$\Omega^{\mathrm{lin}}_{\mathbf{k}} \doteq \frac{\omega_*(\mathbf{k})}{\alpha_{\mathbf{k}} + k^2}, \tag{2.2}$$

$$M_{\mathbf{k},\mathbf{p},\mathbf{q}} \doteq \frac{(\hat{z} \cdot \mathbf{p} \times \mathbf{q})(\bar{q}^2 - \bar{p}^2)}{\bar{k}^2}, \tag{2.3}$$

$$\alpha_{\mathbf{k}} \doteq \begin{cases} 1 & (k_\parallel \neq 0), \\ 0 & (k_\parallel = 0), \end{cases} \tag{2.4}$$

$\omega_*(\mathbf{k}) \doteq k_y V_*$, $k \equiv |\mathbf{k}_\perp|$ (the shorthand notation $\bar{k}^2 \doteq \alpha_{\mathbf{k}} + k^2$ has been used) and $\sum_\Delta \equiv \sum_{\Delta(\mathbf{k};\mathbf{p},\mathbf{q})}$ denotes the sum over all $\mathbf{p}$ and $\mathbf{q}$ such that $\mathbf{k} + \mathbf{p} + \mathbf{q} = 0$. In (2.4), we adopt the approximation that, for $k_\parallel = 0$ fluctuations, the 1 is dropped because an electron parallel response is inhibited. The conservation properties, the general background physics and the extension of the HM equation are well documented by Krommes and Kim (2000), Horton (1999) and the references therein.

With the assumption of periodic boundary conditions and spatial homogeneity, the EDQNM closure yields the evolution equation for the generalized enstrophy $Z_{\mathbf{k}}$ of the potential φ_k:

$$\frac{\partial Z_{\mathbf{k}}}{\partial t} = 2 \sum_{\Delta} \theta^{\mathrm{r}}_{\mathbf{k},\mathbf{p},\mathbf{q}} (a_{\mathbf{kpq}} Z_{\mathbf{p}} Z_{\mathbf{q}} - 2 b_{\mathbf{kpq}} Z_{\mathbf{p}} Z_{\mathbf{k}}), \tag{2.5}$$

where $Z_{\mathbf{k}} \doteq \frac{1}{2} \bar{k}^4 \langle |\delta\varphi_{\mathbf{k}}|^2 \rangle$, $\theta^{\mathrm{r}}_{\mathbf{k},\mathbf{p},\mathbf{q}}$ is the real part ofthe complex triad mode-interaction

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time $\theta_{\mathbf{k},\mathbf{p},\mathbf{q}}$, and $Z_{\mathbf{k}}$ is the generalized enstrophy. The coefficients are

$$a_{\mathbf{kpq}} \doteq \frac{(\hat{z}\cdot\mathbf{p}\times\mathbf{q})^2(\bar{q}^2-\bar{p}^2)^2}{\bar{p}^4\bar{q}^4}, \tag{2.6}$$

$$b_{\mathbf{kpq}} \doteq \frac{(\hat{z}\cdot\mathbf{p}\times\mathbf{q})^2(\bar{q}^2-\bar{p}^2)(\bar{k}^2-\bar{p}^2)}{\bar{k}^2\bar{p}^4\bar{q}^2}. \tag{2.7}$$

It has been argued that, in dealing with the statistical closure, it is better to work with $Z_{\mathbf{k}}$ instead of other conservative quantities, such as the energy $E_{\mathbf{k}} \doteq \frac{1}{2}\bar{k}^2\langle|\delta\varphi_{\mathbf{k}}|^2\rangle$ and the potential enstrophy $W_{\mathbf{k}} \doteq \frac{1}{2}k^2\bar{k}^2\langle|\delta\varphi_{\mathbf{k}}|^2\rangle$ (see, for example, Krommes and Kim 2000; Horton 1999). We notice that (2.5) is an exact equation for the quadratic moment of a random field ζ_k driven by a Gaussian random force $\tilde{f}_{\mathbf{k}}$,

$$\frac{\partial\zeta_{\mathbf{k}}}{\partial t} + \left(i\Omega_{\mathbf{k}}^{\mathrm{lin}} + \eta_{\mathbf{k}}\right)\zeta_{\mathbf{k}} = \tilde{f}_{\mathbf{k}}, \tag{2.8}$$

where

$$\eta_{\mathbf{k}} = 2\sum_{\Delta}\theta_{\mathbf{k},\mathbf{p},\mathbf{q}}b_{\mathbf{kpq}}Z_{\mathbf{p}}, \tag{2.9}$$

$$\left\langle\tilde{f}_{\mathbf{k}}(t)\tilde{f}_{\mathbf{k}}^{*}(t')\right\rangle = \sum_{\Delta}\theta^{\mathrm{r}}_{\mathbf{k},\mathbf{p},\mathbf{q}}a_{\mathbf{kpq}}Z_{\mathbf{p}}Z_{\mathbf{q}}\delta(t-t'). \tag{2.10}$$

In the Langevin equation (2.8), $\eta_{\mathbf{k}}$ expresses the renormalized complex frequency, whose real and imaginary parts are the negative of the growth rate $\gamma_{\mathbf{k}}$ and the frequency shift $\Delta\omega_{\mathbf{k}}$, respectively. In the case of disparate interactions, Krommes and Kim (2000) have shown that a rigorous expansion in terms of the scale ratio $q/k \ll 1$ produces the growth rates for the long-wavelength modes:

$$\gamma_{\mathbf{q}} = -2q^4\sum_{\mathbf{k}\gg\mathbf{q}}\frac{k_y^2k_x\theta^{\mathrm{r}}_{\mathbf{q},\mathbf{k},-\mathbf{k}}}{(\alpha_k+k^2)(\alpha_{-k}+k^2)(\alpha_q+q^2)}\hat{q}\cdot\frac{\partial Z_{\mathbf{k}}}{\partial\mathbf{k}}. \tag{2.11}$$

Upon adopting the same procedure as used in Bowman and Krommes (1997) we obtain $\Delta\Omega_{\mathbf{q}}$ by substituting $\theta^{\mathrm{r}}_{\mathbf{q},\mathbf{k},-\mathbf{k}}$ for $\theta^{\mathrm{i}}_{\mathbf{q},\mathbf{k},-\mathbf{k}}$, the imaginary part of $\theta_{\mathbf{q},\mathbf{k},-\mathbf{k}}$,

$$\Delta\Omega_{\mathbf{q}} = 2q^4\sum_{\mathbf{k}\gg\mathbf{q}}\frac{k_y^2k_x\theta^{\mathrm{i}}_{\mathbf{q},\mathbf{k},-\mathbf{k}}}{(\alpha_k+k^2)(\alpha_{-k}+k^2)(\alpha_q+q^2)}\hat{q}\cdot\frac{\partial Z_{\mathbf{k}}}{\partial\mathbf{k}}. \tag{2.12}$$

Since (2.11) and (2.12) directly depend on the mode-interaction time, the linear characteristics of the drift waves do not have a direct impact on the phase shift or the growth of long-wavelength fluctuations, unlike in Diamond et al. (1998). Incidentally, exactly the same expression as (2.12) can be derived by using the heuristic algorithms developed in Krommes and Kim (2000), instead of rigorous expansions. Those algorithms are built around the second-order variations of the short-wavelength energy and the key quantity is the first-order wavenumber distension rate due to the nonlinear advection frequency that describes the evolution of the characteristic wavenumber of short-scale wave packets under a long-wavelength random modulation.

In order to close (2.5) or (2.8), we need to find the expression of $\theta_{\mathbf{k},\mathbf{p},\mathbf{q}}$. Since $\theta_{\mathbf{k},\mathbf{p},\mathbf{q}}$ is the integral time scale that plays the role of a relaxation time for phase correlations among the three modes, it can be constructed in terms of the (mean) response functions $G(t,t')$ of the random fields ζ, which are the inverses of the operators on

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the left-hand side of (2.8). However, the procedure has a problem with the random Galilean approximation as proposed by Kraichnan (1964), which becomes an issue when interactions between disparate scales are involved. Since $\theta_{\mathbf{k},\mathbf{p},\mathbf{q}}$ is related to the deformation of the fluid eddy of size k^{-1} due to other eddies of scale q^{-1} and p^{-1}, the sheer advection of small scale eddies by the large scale should not be counted. It is apparent in (2.8) and (2.9) that $\eta_{\mathbf{k}}$ only depends on the variance $Z_{\mathbf{k}}$, irrespective of the phase correlation. Large scales, thus, contribute positively to $\eta_{\mathbf{k}}$ leading to an unrealistically rapid decrease of $G_{\mathbf{k}}$ in time, which ends in a value smaller than the actual value for $\theta_{\mathbf{k},\mathbf{p},\mathbf{q}}$. One way to modify the mode coupling at small wavenumbers is by heuristically tying the decorrelation effects to the behavior of the solenoidal part of a test field advected by the turbulence, which we do in the following section.

3. Triad mode-interaction time

In this section, we use the test-field model to compute the triad relaxation time and frequency shift. Temporarily, we drop the drift-wave frequency, $\Omega_{\mathbf{k}}^{\mathrm{lin}} \to 0$. It is not crucial to have drift waves in the basic formulation and they will be restored in the final stage. It is useful to note that the generalized HM equation (2.1) in real space can be derived by taking the z component of the curl (or the vorticity) of the following Navier–Stokes-type equation:

$$\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla\right)(\mathbf{v}-\mathbf{v}') = -\nabla p + \nu\nabla^2(\mathbf{v}-\mathbf{v}'), \tag{3.1}$$

where $\mathbf{v} = \hat{z}\times\nabla\varphi$ and $\mathbf{v}'$ satisfies $\nabla\cdot\mathbf{v}' = 0$ and $\hat{z}\cdot\nabla\times\mathbf{v}' = \varphi$. The 'pressure' p is to be determined by the incompressible nature of the 'velocities' $\mathbf{v}$ and $\mathbf{v}'$, and the 'viscosity' ν is set to vanish. Note that, for pure HM dynamics, the advection term is $(\mathbf{v}\cdot\nabla)\mathbf{v}$. The importance of the additional advective nonlinearity in the generalized HM model (2.1) and in (3.1) has been stressed in Smolyakov and Diamond (1999) as it destroys the conservation of the potential enstrophy and instead favors the conservation of Z.

In computing the response function, we want to capture the deformation of the fluid eddies while excluding the simple advection due to a large-scale flow. As the distortion of the eddies is produced by the pressure gradient and the pressure gradient strictly enforces the incompressibility of the flow, Kraichnan (1971) postulated that the pressure effect can be estimated from the growth rate of the compressive part of the flow relaxing the constraint of incompressibility. This is accomplished by considering a test flow simply advected by a solenoidal physical flow. Let us represent the incompressible physical flow by the stream function (or the electrostatic potential) φ, as before, and the test flow by $\mathbf{u}_{\mathrm{t}} = \hat{z}\times\nabla\phi_{\mathrm{t}} + \nabla\psi_{\mathrm{t}}$. The advection equation for the test field is

$$\frac{\partial\mathbf{u}_{\mathrm{t}}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{u}_{\mathrm{t}} = 0, \tag{3.2}$$

and if the z component of the curl and the divergence of (3.2), respectively, are taken, the stream function ϕ_{t} and the velocity potential ψ_{t} evolve as

$$\frac{\partial}{\partial t}\nabla^2\phi_{\mathrm{t}} + \mathscr{K}(\varphi,\psi_{\mathrm{t}}) = 0, \tag{3.3}$$

$$\frac{\partial}{\partial t}\nabla^2\psi_{\mathrm{t}} - \mathscr{K}(\varphi,\phi_{\mathrm{t}}) = 0, \tag{3.4}$$

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where

$$\mathcal{K}(\varphi,\phi_{\rm t})=\frac{\partial^2\varphi}{\partial x^2}\frac{\partial^2\phi_{\rm t}}{\partial y^2}+\frac{\partial^2\varphi}{\partial y^2}\frac{\partial^2\phi_{\rm t}}{\partial x^2}-2\frac{\partial^2\varphi}{\partial x\partial y}\frac{\partial^2\phi_{\rm t}}{\partial x\partial y}. \tag{3.5}$$

Since we are focusing on the rate of mixing due to advection of $\phi_{\rm t}$ and $\psi_{\rm t}$ caused by φ, in (3.3) and (3.4), we neglect the cross terms between $\phi_{\rm t}$ and $\psi_{\rm t}$.

Because of the presence of the auxiliary 'velocity' $\mathbf{v}'$ in addition to $\mathbf{v}$ in (3.1), it is appropriate to separate $\phi_{\rm t}$ and $\psi_{\rm t}$ into two pieces: $\nabla^2\phi_{\rm t}=(\nabla^2-\breve{\alpha})\phi$ and $\nabla^2\psi_{\rm t}=(\nabla^2-\breve{\alpha})\psi$, where the Fourier transformation of $\breve{\alpha}\phi$ is $\alpha_{\mathbf{k}}\phi_{\mathbf{k}}$. In Fourier space, (3.3) and (3.4) become

$$\frac{\partial}{\partial t}\phi_k=\sum_{\Delta}M^G_{\mathbf{k},\mathbf{p},\mathbf{q}}\varphi^*_{\mathbf{p}}\psi^*_{\mathbf{q}}, \tag{3.6}$$

$$\frac{\partial}{\partial t}\psi_k=-\sum_{\Delta}M^G_{\mathbf{k},\mathbf{p},\mathbf{q}}\varphi^*_{\mathbf{p}}\phi^*_{\mathbf{q}}, \tag{3.7}$$

where

$$M^G_{\mathbf{k},\mathbf{p},\mathbf{q}}\doteq\frac{(\hat{z}\cdot\mathbf{p}\times\mathbf{q})^2\bar{q}^2}{\bar{k}^2q^2}. \tag{3.8}$$

Applying the EDQNM closure to (3.6) and (3.7) yields the spectral equation

$$\frac{\partial C^\phi_{\mathbf{k}}}{\partial t}=4\sum_{\Delta}\theta^{\rm r}_{\mathbf{k},\mathbf{p},\mathbf{q}}M^G_{\mathbf{k},\mathbf{p},\mathbf{q}}\left(M^G_{\mathbf{k},\mathbf{p},\mathbf{q}}C^\varphi_{\mathbf{p}}C^\psi_{\mathbf{q}}-M^G_{\mathbf{q},-\mathbf{p},\mathbf{k}}C^\varphi_{\mathbf{p}}C^\phi_{\mathbf{k}}\right) \tag{3.9}$$

and a similar equation for $C^\psi_{\mathbf{k}}$, where $C_{\mathbf{k}}$ are defined in such a way that $C^\phi_{\mathbf{k}}\doteq\frac{1}{2}\langle|\phi_k|^2\rangle$. It is convenient to write the spectral equations in terms of the 'enstrophies' $Z_{\mathbf{k}}\doteq\bar{k}^4C_{\mathbf{k}}$:

$$\frac{\partial Z^\phi_{\mathbf{k}}}{\partial t}=4\sum_{\Delta}\theta^{\rm r}_{\mathbf{k},\mathbf{p},\mathbf{q}}\frac{(\hat{z}\cdot\mathbf{p}\times\mathbf{q})^4}{\bar{p}^2q^2}\frac{Z^\varphi_{\mathbf{p}}}{\bar{p}^2}\left(\frac{Z^\psi_{\mathbf{q}}}{q^2}-\frac{Z^\phi_{\mathbf{k}}}{k^2}\right), \tag{3.10}$$

$$\frac{\partial Z^\psi_{\mathbf{k}}}{\partial t}=4\sum_{\Delta}\theta^{\rm r}_{\mathbf{k},\mathbf{p},\mathbf{q}}\frac{(\hat{z}\cdot\mathbf{p}\times\mathbf{q})^4}{\bar{p}^2q^2}\frac{Z^\varphi_{\mathbf{p}}}{\bar{p}^2}\left(\frac{Z^\phi_{\mathbf{q}}}{q^2}-\frac{Z^\psi_{\mathbf{k}}}{k^2}\right). \tag{3.11}$$

Note that (3.10) and (3.11) can be rigorously constructed from a Langevin model

$$\left(\frac{\partial}{\partial t}+\hat{\mu}_{\mathbf{k}}\right)\zeta_{\mathbf{k}}=\tilde{f}_{\mathbf{k}}, \tag{3.12}$$

where the damping rate $\hat{\mu}_{\mathbf{k}}$ is

$$\hat{\mu}_{\mathbf{k}}=2\sum_{\Delta}\theta^{\rm r}_{\mathbf{k},\mathbf{p},\mathbf{q}}\frac{(\hat{z}\cdot\mathbf{p}\times\mathbf{q})^4}{k^2\bar{p}^4q^2}Z^\varphi_{\mathbf{p}}, \tag{3.13}$$

which should be compared with the raw expression in (2.9). Now, we can generalize the above procedure to the drift-wave problem. Upon taking the drift wave into the formulation, the complex Green's function of the Langevin equation becomes

$$\left(\frac{\partial}{\partial t}+i\Omega^{\rm lin}_{\mathbf{k}}+\mu_{\mathbf{k}}\right)G_{\mathbf{k}}(t,t')=\delta(t-t'), \tag{3.14}$$

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where the complex decay rate is, instead of $\hat{\mu}_{\mathbf{k}}$,

$$\mu_{\mathbf{k}} = g \sum_{\Delta} 2\theta_{\mathbf{k},\mathbf{p},\mathbf{q}} \frac{(\hat{\mathbf{z}} \cdot \mathbf{p} \times \mathbf{q})^4}{k^2 \bar{p}^4 q^2} Z_{\mathbf{p}}^{\varphi}. \tag{3.15}$$

In (3.15), g is a phenomenological parameter of order 1 as shown by Kraichnan (1971). For later purposes, we simplify (3.15) for two interesting extreme cases upon utilizing the smallness of the scale ratio. For large k modes,

$$\mu_{\mathbf{k}} \approx g \sum_{p \ll k} \theta_{\mathbf{k},\mathbf{p},-\mathbf{k}} \sin^4 \beta p^2 (p^2 C_{\mathbf{p}} + k^2 C_{\mathbf{k}}) \tag{3.16}$$

$$\approx g \sum_{p \ll k} \theta_{\mathbf{k},\mathbf{p},-\mathbf{k}} \sin^4 \beta p^4 C_{\mathbf{p}}, \tag{3.17}$$

where β is the angle between $\mathbf{k}$ and $\mathbf{p}$ and $C_{\mathbf{k}} = \langle |\delta\varphi_{\mathbf{k}}|^2 \rangle$. In obtaining (3.17) we assume that $C_{\mathbf{k}}$ decays faster than k^{-2}, as is the case in the enstrophy cascade region. For small q modes,

$$\mu_{\mathbf{q}} \approx g q^2 \sum_{p \gg q} \theta_{\mathbf{p},\mathbf{q},-\mathbf{p}} \sin^4 \beta p^2 C_{\mathbf{p}}. \tag{3.18}$$

The triad mode-interaction time is computed using the Green's function $G_{\mathbf{k}}$ and is given by

$$\theta_{\mathbf{k},\mathbf{p},\mathbf{q}}(t) = \int_{-\infty}^{t} dt' \, G_{\mathbf{k}}(t,t') G_{\mathbf{p}}(t,t') G_{\mathbf{q}}(t,t'). \tag{3.19}$$

In the long-time limit, $\theta_{\mathbf{q},\mathbf{k},-\mathbf{k}}$, as it appears in (2.11) and (2.12), reduces to

$$\theta_{\mathbf{q},\mathbf{k},-\mathbf{k}} = \frac{1}{\left(2\mu_{\mathbf{k}}^{\mathrm{r}} + \mu_{\mathbf{q}}^{\mathrm{r}}\right) + i\left(\Omega_{\mathbf{q}}^{\mathrm{lin}} + \mu_{\mathbf{q}}^{\mathrm{i}}\right)} \tag{3.20}$$

$$\approx \frac{1}{2\mu_{\mathbf{k}}^{\mathrm{r}} + i\Omega_{\mathbf{q}}^{\mathrm{lin}}} \tag{3.21}$$

in *a priori* anticipation that the damping becomes stronger as the fluctuation scale gets shorter, $\mu_{\mathbf{k}}^{\mathrm{r}} \gg \mu_{\mathbf{q}}^{\mathrm{r}}$, and that the resonance broadening is smaller than the linear frequency itself, $\mu_{\mathbf{q}}^{\mathrm{i}} \ll \Omega_{\mathbf{q}}^{\mathrm{lin}}$ as $q \to 0$. If we further assume that $\Omega_{\mathbf{q}}^{\mathrm{lin}} \ll \mu_{\mathbf{k}}^{\mathrm{r}}$, then

$$\theta_{\mathbf{q},\mathbf{k},-\mathbf{k}} \approx \frac{1}{2\mu_{\mathbf{k}}^{\mathrm{r}}} \left(1 - i\frac{\Omega_{\mathbf{q}}^{\mathrm{lin}}}{2\mu_{\mathbf{k}}^{\mathrm{r}}}\right). \tag{3.22}$$

We realize that, in non-local triad interactions, the phase correlation time is inversely proportional to the decay rate $\mu_{\mathbf{k}}^{\mathrm{r}}$ of the short-scale fluctuation and that $\theta_{\mathbf{q},\mathbf{k},-\mathbf{k}}^{\mathrm{i}} \ll \theta_{\mathbf{q},\mathbf{k},-\mathbf{k}}^{\mathrm{r}}$ is proportional to $\Omega_{\mathbf{q}}^{\mathrm{lin}}$. Thus, we have checked that the *a priori* assumptions we used in reaching (3.21) are satisfied. Upon substituting (3.22) into (3.17), we find that $\mu_{\mathbf{k}}^{\mathrm{r}}$ is determined by the large-scale modes,

$$\mu_{\mathbf{k}}^{\mathrm{r}} \approx \sqrt{\frac{g}{2} \sum_{p \ll k} \sin^4 \beta p^4 C_{\mathbf{p}}}, \tag{3.23}$$

and, finally, we rewrite (2.11) and (2.12) for the growth rate $\gamma_{\mathbf{q}}$ and the frequency

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shift $\Delta\Omega_{\mathbf{q}} \ll \gamma_{\mathbf{q}}$ as

$$\gamma_{\mathbf{q}} = -q^4 \sum_{\mathbf{k} \gg \mathbf{q}} \frac{k_y^2 k_x}{\mu_{\mathbf{k}}^{\mathrm{r}} \bar{k}^4 \bar{q}^2} \hat{q} \cdot \frac{\partial Z_{\mathbf{k}}}{\partial \mathbf{k}}, \tag{3.24}$$

$$\Delta\Omega_{\mathbf{q}} = -\Omega_{\mathbf{q}}^{\mathrm{lin}} q^4 \sum_{\mathbf{k} \gg \mathbf{q}} \frac{k_y^2 k_x}{2\mu_{\mathbf{k}}^{\mathrm{r}\,2} \bar{k}^4 \bar{q}^2} \hat{q} \cdot \frac{\partial Z_{\mathbf{k}}}{\partial \mathbf{k}}. \tag{3.25}$$

4. Discussion

The principal goal of this paper was to give a complete description of short-scale contributions to the characteristics of long-wavelength modes. We have obtained the frequency shift and the triad mode-interaction time to augment the existing knowledge of the growth rate that was found in the literature. We found that the frequency shift was smaller than the growth rate and that the triad mode-interaction time was determined by the energy and/or the enstrophy of the long-wavelength modes. In order to achieve our goal we applied the test-field model to the generalized Hasegawa–Mima dynamics, which was shown to be derivable from a Navier–Stokes-type equation under the hypothesis that the sweeping effect of very large scales on the advection of small-scale structures would be eliminated by computing the effect of the 'pressure' gradient via the evolution of the compressive component of the 'velocity'.

In addition to the presence of an adjustable free parameter, the TFM has another difficulty: it does not guarantee the realizability in the pedagogical situation of a three-mode problem. Bowman and Krommes (1997) discussed the realizable TFM based on a fluctuation–dissipation ansatz. Since it was not proven that the original TFM formulation violated the realizability condition in drift-wave turbulence and since the realizable version of the TFM did not render the analytical approach doable, we did not pursue such remedies in the present work.

Recently, a work on the modulational instability of drift waves by Hallatschek and Diamond (2003) has come to our attention. Its analysis is based on the three-mode couplings: one drift-wave as a pump, another drift wave as a secondary and either zonal flow or a streamer. It explains the modulational instability in phase with the drift-wave packet as well as a new instability corresponding to the refraction of drift waves by the streamer. It, however, may have a different scope than the present work in that it has a monochromatic pump wave rather than a spectrum of waves, while the present work assumes the fully developed and well-separated drift-wave turbulence in the short scale.

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