Mathematical Structures in Computer Science (2025), **35**, e23, pp. 1–12
doi:10.1017/S0960129525100200

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PAPER

Errata to "Isomorphism theorems between models of mixed choice," fixes and consequences

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(Received 12 November 2024; revised 30 June 2025; accepted 1 August 2025)

Abstract

The results of Section 3.1 of the 2017 paper "Isomorphism Theorems between Models of Mixed Choice" need an additional assumption when $\bullet$ is "1." If $\bullet$ is nothing or "≤ 1," no change is needed. Also, the mistake only applies to the angelic cases, namely to the maps r_{AP} and $s^{\bullet}_{\mathrm{AP}}$; the demonic cases r_{DP} and $s^{\bullet}_{\mathrm{DP}}$ are unaffected. If $\bullet$ is "1," and in the angelic cases, instead of just assuming that $\mathcal{L}X$ is locally convex, we need to additionally assume that X is compact, or that $\mathcal{L}X$ is locally convex-compact, sober, and topological – for example, if X is core-compact – or that X is LCS-complete, namely, a homeomorph of a G_δ subspace of a locally compact sober space.

Keywords: Errata; previsions.

1. Introduction

The purpose of Goubault-Larrecq (2017) is to show that there are isomorphisms between certain spaces of previsions and certain powercones. I will not define the terms or motivate the constructions, which can be found in Goubault-Larrecq (2017); this note is meant to pinpoint mistakes and to fix them.[1]

The proof of Lemma 3.4 of Goubault-Larrecq (2017) is nonsense. In the cases where $\bullet$ is nothing or "≤ 1," the result still holds, and we will give a repaired proof; the assumption that X should be such that $[X \to \overline{\mathbb{R}}_{+\sigma}]$ is locally convex is unchanged. When $\bullet$ is "1," however, we need to impose additional conditions, and we call $\mathtt{AP}_1$-friendly any space satisfying these conditions, see Section 2 below. I do not know whether this requirement is necessary – I don't have a counterexample – but the repaired proof I will offer requires it. The fix is non-trivial, see Section 3.

The error propagates to all subsequent results of Section 3.1 of Goubault-Larrecq (2017). We will not restate them – "if $[X \to \overline{\mathbb{R}}_{+\sigma}]$ is locally convex" has to be replaced by "if X is $\mathtt{AP}_\bullet$-friendly" everywhere. As an exception to this rule, we will restate Proposition 3.11 and Corollary 3.12 of Goubault-Larrecq (2017) explicitly, since they summarize the other results of the section. We also state the repaired versions of Theorem 4.11 and of Propositions 4.19 and 4.21 of Goubault-Larrecq (2017), which are also affected by the error.

We will examine the collection of papers citing Goubault-Larrecq (2017) in Section 4 in order to detect whether the mistake has affected any of them. In short, the answer is: none is affected.

We will conclude with a section collecting a few typos in Section 5.

I am not aware of any other mistake. Notably, all the results about the demonic cases and the maps r_{DP} and $s^{\bullet}_{\mathrm{DP}}$ are correct as published, and do not need any form of amendment. This is

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important, for example, considering that the recent paper (Goubault-Larrecq 2025) depends on the latter, and not on the flawed results reported on (and fixed) here.

2. The New Assumption

I have said that we would not reintroduce the notions already introduced in Goubault-Larrecq (2017). However, some new ones need to be introduced, and we need to fix some notation, too.

Let $\mathbb{R}_+$ be the set of non-negative real numbers, $\overline{\mathbb{R}}_+$ be $\mathbb{R} \cup \{\infty\}$, ∞ being larger than all real numbers. $\overline{\mathbb{R}}_{+\sigma}$ is $\overline{\mathbb{R}}_+$ with the Scott topology of the usual ordering.

For every space X, the space $[X \to \overline{\mathbb{R}}_{+\sigma}]$ of lower semicontinuous functions from X to $\overline{\mathbb{R}}_+$ (namely, of continuous functions from X to $\overline{\mathbb{R}}_{+\sigma}$) is a cone under pointwise addition and pointwise scalar multiplication by non-negative real numbers. We will abbreviate it as $\mathcal{L}X$. We give it the Scott topology of its pointwise ordering, and this makes it a semitopological cone: while scalar multiplication is jointly continuous, addition is separately continuous. A *topological cone* is one in which addition is jointly continuous. It is sober if and only if it is sober as a topological space. We rely on Keimel (2008) for the theory of semitopological and topological cones.

A locally convex cone is a semitopological cone in which every element has a base of convex open neighborhoods. A *locally convex-compact* cone is a semitopological cone in which every element has a base of convex compact neighborhoods (Keimel 2008, Definition 4.9).

The assumption we need replaces "$\mathcal{L}X$ is locally convex" by the following notion. Please mind that the results of Goubault-Larrecq (2017, Section 3) might be true as stated: we do not have any counterexample or result showing that the extra condition of $\mathtt{AP}_1$-friendliness is needed in case $\bullet$ is "1." But the proof of Lemma 3.4 of Goubault-Larrecq (2017) is wrong, and the proofs below need $\mathtt{AP}_\bullet$-friendliness.

Definition 1 ($\mathtt{AP}_\bullet$-friendly). *An* $\mathtt{AP}$*-friendly space is a topological space X such that $\mathcal{L}X$ is a locally convex semitopological cone when given its Scott topology. A space is* $\mathtt{AP}_{\leq 1}$*-friendly if and only if it is* $\mathtt{AP}$*-friendly.*

An $\mathtt{AP}_1$*-friendly space is either:*

1. *a compact* $\mathtt{AP}$*-friendly space X,*
2. *or a space X such that $\mathcal{L}X$ is a locally convex-compact, locally convex sober topological cone,*
3. *or an LCS-complete space, namely, a space that is homeomorphic to a G_δ subspace of a locally compact sober space.*

We will use the adjective $\mathtt{AP}_\bullet$-friendly to denote any of the notions, where $\bullet$ is nothing, "≤ 1," or "1."

We discuss standard classes of $\mathtt{AP}_\bullet$-friendly spaces.

Given a space X, and a compact subset Q of X, the set $\blacksquare Q$ of open neighborhoods of Q is Scott-open in $\mathcal{O}X$, the lattice of open subsets of X. A space X is *consonant* if and only those sets $\blacksquare Q$ generate the Scott topology on $\mathcal{O}X$. Every locally compact space is consonant (see Exercise 5.4.12 in Goubault-Larrecq (2013)), every regular Čech-complete space is consonant (Dolecki et al., 1995, Theorem 4.1 and footnote 8), and every LCS-complete space is consonant (de Brecht et al. 2019, Proposition 12.1). The class of LCS-complete spaces contains M. de Brecht's quasi-Polish spaces (de Brecht 2013), which themselves contain all Polish spaces and all ω-continuous dcpos in their Scott topology; the LCS-complete spaces also contain all the continuous dcpos. See de Brecht et al. (2019) for the theory of LCS-complete spaces.

A space X is $\odot$-*consonant* if and only if every finite copower of X is consonant (de Brecht et al. 2019, Definition 13.1). Since locally compact spaces and LCS-complete spaces are closed under

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finite coproducts (in the category of topological spaces), those spaces are $\odot$-consonant; see de Brecht et al. (2019, Lemma 13.2) for the case of LCS-complete spaces.

By de Brecht et al. (2019, Lemma 13.6), for every $\odot$-consonant space X, $\mathcal{L}X$ is locally convex (in its Scott topology). We have just illustrated how large the class of $\odot$-consonant spaces is, and they are all AP-friendly.

We turn to $\mathtt{AP}_1$-friendliness, and specifically to conditions 2 and 3.

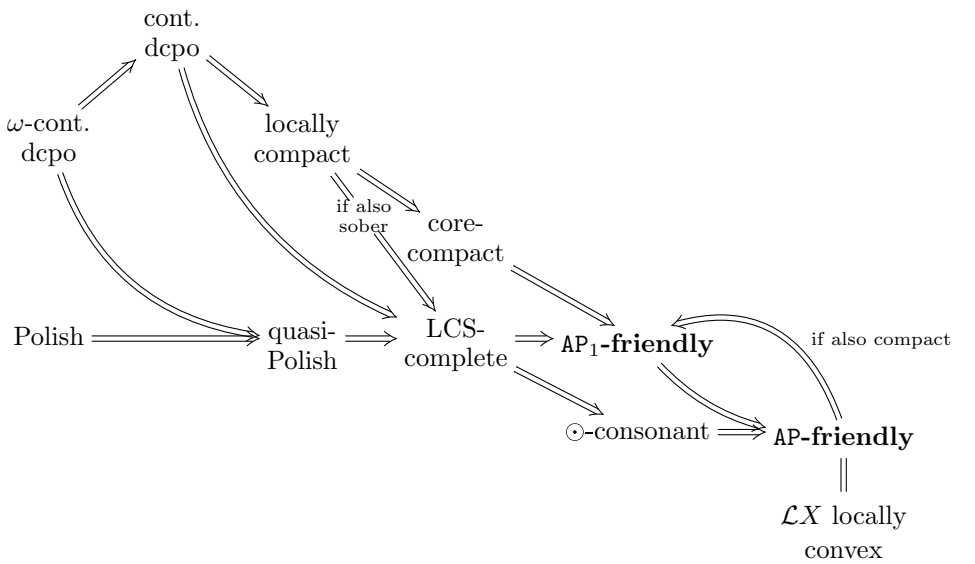
A space X is core-compact if and only if $\mathcal{O}X$ is a continuous dcpo. Every locally compact space is core-compact (Goubault-Larrecq 2013, Theorem 5.2.9), and every core-compact sober space is locally compact (Goubault-Larrecq 2013, Theorem 8.3.10).

Remark 2. For every core-compact space X, $\mathcal{L}X$ is a locally convex, locally convex-compact, and sober topological cone. In order to see this, we observe that $\mathcal{L}X$ is a continuous complete lattice. This was mentioned at the end of Section 16 of de Brecht et al. (2019), and can be deduced from Gierz et al. (2003, Proposition II-4.6), which says that if X is core-compact and L (here, $\overline{\mathbb{R}}_+$) is an injective T_0 space (i.e., a continuous complete lattice by Gierz et al. (2003, Theorem II-3.8)), then the poset $[X \to L]$ of continuous functions from X to L is a continuous complete lattice.

A continuous dcpo, in particular a continuous complete lattice, is always sober in its Scott topology (Goubault-Larrecq 2013, Proposition 8.2.12). It is also a c-space in its Scott topology (Goubault-Larrecq 2013, Proposition 5.1.37). Together with its cone operations, $\mathcal{L}X$ is then a c-cone in the sense of Keimel, which is then locally convex, locally convex-compact, and topological (Keimel 2008, Lemma 6.12). Hence every core-compact space X is $\mathtt{AP}_1$-friendly, in that it satisfies condition 2 of Definition 1. This applies in particular to all locally compact spaces X, and therefore to all continuous dcpos in their Scott topology.

Remark 3. As for condition 3, for every LCS-complete space X, we recall that X is $\odot$-consonant, hence that $\mathcal{L}X$ is locally convex in its Scott topology (de Brecht et al. 2019, Lemma 13.6). Therefore, every LCS-complete space X is AP-friendly, and clearly $\mathtt{AP}_1$-friendly, too. As another consequence, every $\mathtt{AP}_1$-friendly space is AP-friendly (and $\mathtt{AP}_{\leq 1}$-friendly).

We sum up this discussion (and perhaps a bit more) by the following diagram of implications.



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3. Fixing Lemma 3.4

A function from a space Z to $\overline{\mathbb{R}}_+$ is lower semicontinuous if and only if it is continuous from Z to $\overline{\mathbb{R}}_+$ with its Scott topology. $\mathbb{P}_{\mathtt{AP}}X$ is the collection of sublinear and lower semicontinuous maps from $\mathcal{L}X$ to $\overline{\mathbb{R}}_+$. $\mathbb{P}^{\leq 1}_{\mathtt{AP}}X$ (resp., $\mathbb{P}^{1}_{\mathtt{AP}}X$) is its subspace of subnormalized (resp., normalized) elements F, meaning that $F(a \cdot \mathbf{1} + h) \leq a + F(h)$ (resp., $=$) for every $h \in \mathcal{L}X$ and for every $a \in \mathbb{R}_+$; we write $\mathbf{1}$ for the constant function equal to 1. We write $\mathbb{P}^{\bullet}_{\mathtt{AP}}X$ for any one of those. Similarly, $\mathbb{P}^{\bullet}_{\mathtt{P}}X$ is obtained by taking only the (subnormalized, normalized) linear lower semicontinuous maps. Each of these spaces is given the weak topology, generated by subbasic open sets $[h > r]$ where $h \in \mathcal{L}X$ and $r \in \mathbb{R}_+$; $[h > r]$ denotes the collection of maps F in the relevant spaces such that $F(h) > r$.

For every $G \in \mathbb{P}_{\mathtt{P}}X$, G is subnormalized (resp., normalized) if and only if $G(\mathbf{1}) \leq 1$ (resp., $=$). We need to keep the more complicated definition for sublinear functions.

The following is Lemma 3.4 of Goubault-Larrecq (2017) in the case that $\bullet$ is nothing or "≤ 1." Its statement is unchanged.

Lemma 4. *Let $\bullet$ be nothing or "≤ 1," and X be an* AP*-friendly space. For every $F \in \mathbb{P}^{\bullet}_{\mathtt{AP}}X$, for every $h_0 \in \mathcal{L}X$, and every real number $r > 0$ such that $F(h_0) > r$, there is a $G \in \mathbb{P}^{\bullet}_{\mathtt{P}}X$ with $G \leq F$ and $G(h_0) > r$.*

Proof. By Corollary 9.5 of Keimel (2008), every lower semi-continuous sublinear map from a locally convex semitopological cone to $\overline{\mathbb{R}}_+$ is pointwise the supremum of lower semi-continuous linear maps. □

The situation is more complex when $\bullet$ is "1." We need a few technical lemmas. The first one deals with condition 1 of the definition of $\mathtt{AP}_1$-friendliness, namely when X is compact.

Lemma 5. *For every compact topological space X, for every $a \in \mathbb{R}_+$, the set $\{h \in \mathcal{L}X \mid \exists b > a, h \geq b \cdot \mathbf{1}\}$ is Scott-open in $\mathcal{L}X$. For every Scott-open subset U of $\mathcal{L}X$, the set $\{h \in \mathcal{L}X \mid \exists b > a, k \in U, h \geq b \cdot \mathbf{1} + k\}$ is open in $\mathcal{L}X$.*

Proof. We prove the second claim. The first claim will follow by taking $U \stackrel{\text{def}}{=} \mathcal{L}X$. Let $V \stackrel{\text{def}}{=} \{h \in \mathcal{L}X \mid \exists b > a, k \in U, h \geq b \cdot \mathbf{1} + k\}$. This is upwards-closed. Let $(h_i)_{i \in I}$ be a directed family with supremum $h \in V$. Then $h \geq b \cdot \mathbf{1} + k$ for some $b > a$ and some $k \in U$. In particular, $h \geq b \cdot \mathbf{1}$. Let us pick a $c \in]a, b[$.

We claim that there is an $i \in I$ such that $h_i \geq c \cdot \mathbf{1}$. For every $x \in X$, $h(x) \geq b > c$, so $h_i(x) > c$ for some $i \in X$. It follows that the sets $(h_i^{-1}(]c, \infty])_{i \in I}$ form an open cover of X. Also, if $h_i \leq h_j$, then $h_i^{-1}(]c, \infty]) \subseteq h_j^{-1}(]c, \infty])$, so the family $(h_i^{-1}(]c, \infty]))_{i \in I}$ is directed. Since X is compact, it is therefore included in $h_i^{-1}(]c, \infty])$ for some $i \in I$. Hence, for that choice of i, $h_i \geq c \cdot \mathbf{1}$.

We have $h \geq c \cdot \mathbf{1} + k$. Let h' map every $x \in X$ to $\max(h(x) - c, 0)$, and for every $j \in I$, let h'_j map every $x \in X$ to $\max(h_j(x) - c, 0)$. Then $h' = \sup_{j \in I} h'_j$, and $h' \geq k \in U$. Since U is Scott-open, some h'_j is in U. Let $\ell \in I$ be such that $h_\ell \geq h_i, h_j$. Then $h'_\ell \geq h'_j$, so h'_ℓ is in U. Also, $h_\ell \geq h_i \geq c \cdot \mathbf{1}$, so for every $x \in X$, $h'_\ell(x) = \max(h_\ell(x) - c, 0) = h_\ell(x) - c$. It follows that $h_\ell = c \cdot \mathbf{1} + h'_\ell$, so $h_\ell \in V$. □

Our second technical lemma will be used when conditions 2 or 3 of the definition of $\mathtt{AP}_1$-friendliness are met. Given two elements k and h of $\mathcal{L}X$, we write $[k, h]$ for the convex hull of $\{k, h\}$, namely the collection $\{a \cdot k + (1 - a) \cdot h \mid a \in [0, 1]\}$.

Lemma 6. *Let X be a topological space such that $\mathcal{L}X$ is a locally convex, locally convex-compact, and sober topological cone. Let $g_0 \geq g_1 \geq \cdots \geq g_n \geq \cdots$ be a descending sequence of elements of*

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$\mathcal{L}X$, and $h_0 \in \mathcal{L}X$. Let also W be a Scott-open neighborhood of h_0 in $\mathcal{L}X$ such that $[b \cdot g_n, h]$ is included in W for every $n \in \mathbb{N}$, every $b > 1$ and every $h \in W$. There is a convex Scott-open neighborhood V of h_0 in $\mathcal{L}X$ that contains every function $b \cdot g_n$ with $n \in \mathbb{N}$ and $b > 1$ and that is included in W.

Proof. Lemma 4.10 (c) of Keimel (2008) states that in a topological cone, the convex hull of a finite union of convex compact sets is compact; and therefore the upward closure of such a convex hull is compact saturated, and still convex. Since $\mathcal{L}X$ is topological, every set $\uparrow[h, k]$ is therefore convex and compact saturated.

We build convex compact saturated sets Q_n containing $\uparrow[(1 + 1/2^n) \cdot g_n, h_0]$ and included in W, so that Q_n is included in the interior $\mathrm{int}(Q_{n+1})$ of Q_{n+1} for every $n \in \mathbb{N}$, by induction on n, as follows.

We let $Q_0 \stackrel{\text{def}}{=} \uparrow[2 \cdot g_0, h_0]$. Since $h_0 \in W$, by assumption on W, $[2 \cdot g_0, h_0]$ is included in W. Since W is upwards-closed, $Q_0 \subseteq W$.

Having built $Q_n \subseteq W$, we let Q'_n be the upward closure of the convex hull of the union of $\{(1 + 1/2^{n+1}) \cdot g_{n+1}\}$ and Q_n. By Lemma 4.10 (c) of Keimel (2008), which we have already used above, Q'_n is convex and compact saturated. Additionally, every element h of Q'_n is above some element in $[(1 + 1/2^{n+1}) \cdot g_{n+1}, k]$ for some $k \in Q_n$ (hence $k \in W$). Indeed, every element of the convex hull of the union of two convex sets A and B can be written as $\alpha \cdot a + (1 - \alpha) \cdot b$ for some $\alpha \in [0, 1]$, $a \in A$, and $b \in B$.

By assumption on W, $[(1 + 1/2^{n+1}) \cdot g_{n+1}, k]$ is included in W, and since W is upwards closed, it follows that every element $h \in Q'_n$ is in W. Therefore, $Q'_n \subseteq W$. The first half of Proposition 10.6 of Keimel (2008) states that in a locally convex, locally convex-compact, and sober topological cone, every non-empty convex compact saturated set has a base of convex compact saturated neighborhoods. Hence, there is a convex compact saturated set Q_{n+1} included in W whose interior contains Q_n. The induction is complete.

Let $V \stackrel{\text{def}}{=} \bigcup_{n \in \mathbb{N}} Q_n$. V contains h_0, since $h_0 \in Q_0$. V is convex since it is a directed union of convex sets. Then, $V \subseteq \bigcup_{n \in \mathbb{N}} \mathrm{int}(Q_{n+1}) \subseteq \bigcup_{n \in \mathbb{N}} Q_{n+1} \subseteq V$, so $V = \bigcup_{n \in \mathbb{N}} \mathrm{int}(Q_{n+1})$, and therefore V is open. Since $Q_n \subseteq W$ for every $n \in \mathbb{N}$, V is also included in W. Finally, $(1 + 1/2^n) \cdot g_n$ is in Q_n for every $n \in \mathbb{N}$, hence in V. For $n \in \mathbb{N}$, $b > 1$, let us pick $m \in \mathbb{N}$ such that $m \geq n$ and $1 + 1/2^m \leq b$. Then $(1 + 1/2^m) \cdot g_m$ is in V, and $(1 + 1/2^m) \cdot g_m \leq b \cdot g_m \leq b \cdot g_n$, and since V is upwards-closed, $b \cdot g_n \in V$. □

Lemma 7. *Let X be a space satisfying condition 1 or 2 of the definition of $\mathtt{AP}_1$-friendliness. Let $h_0 \in \mathcal{L}X$ and W be a Scott-open neighborhood of h_0 in $\mathcal{L}X$ such that $[b \cdot \mathbf{1}, h]$ is included in W for every $b > 1$ and every $h \in W$. There is a convex Scott-open neighborhood V of h_0 in $\mathcal{L}X$ that contains every function $b \cdot \mathbf{1}$ with $b > 1$ and that is included in W.*

Proof. When X satisfies condition 2, namely when $\mathcal{L}X$ is a locally convex-compact, locally convex sober topological cone, the result is simply Lemma 7 with $g_n \stackrel{\text{def}}{=} \mathbf{1}$ for every $n \in \mathbb{N}$.

Hence we concentrate on the case where X satisfies condition 1, namely when $\mathcal{L}X$ is locally convex and X is compact. Since $\mathcal{L}X$ is locally convex and $h_0 \in W$, there is a convex Scott-open neighborhood U of h_0 included in W.

For every $a \in]0, 1[$, the set $(1 - a) \cdot U \stackrel{\text{def}}{=} \{(1 - a) \cdot h \mid h \in U\}$ is Scott-open, because it is equal to the inverse image of U by the map $h \mapsto 1/(1 - a) \cdot h$. Hence, the set $V_a \stackrel{\text{def}}{=} \{h \in \mathcal{L}X \mid \exists b > a, k \in (1 - a) \cdot U, h \geq b \cdot \mathbf{1} + k\}$ is Scott-open by Lemma 5, which applies since X is compact. When $a = 1$, we define V_1 as $\{h \in \mathcal{L}X \mid \exists b > 1, h \geq b \cdot \mathbf{1}\}$, and that is also Scott-open by the same lemma. When $a = 0$, we define V_0 as U. For uniformity of treatment, we note that V_a is equal to $\{h \in \mathcal{L}X \mid \exists b > 1, k \in U, h \geq ba \cdot \mathbf{1} + (1 - a) \cdot k\}$ for every $a \in [0, 1]$, including when $a = 0$ or $a = 1$. Let

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$V \stackrel{\text{def}}{=} \bigcup_{a\in[0,1]} V_a$. This is Scott-open, and we claim that V is convex. Let h and h' be two elements of V, say $h \geq ba \cdot \mathbf{1} + (1-a)\cdot k$ and $h' \geq b'a' \cdot \mathbf{1} + (1-a')\cdot k'$, with $b, b' > 1$, $k, k' \in U$, and $a, a' \in [0,1]$. Let $\alpha \in]0,1[$. Then $\alpha \cdot h + (1-\alpha)\cdot h' \geq (\alpha ba + (1-\alpha)b'a') \cdot \mathbf{1} + (\alpha(1-a)\cdot k + (1-\alpha)(1-a')\cdot k')$. Let $a'' \stackrel{\text{def}}{=} \alpha a + (1-\alpha)a'$. Then $a'' \in [0,1]$, and $\alpha ba + (1-\alpha)b'a' = b''a''$ for some $b'' > 1$. Indeed, $\alpha ba + (1-\alpha)b'a' \geq \alpha a + (1-\alpha)a' = a''$. If $a'' \neq 0$, then $a \neq 0$ or $a' \neq 0$, and since $\alpha \neq 0, 1$, the inequality is strict, so we can take $b'' \stackrel{\text{def}}{=} (\alpha ba + (1-\alpha)b'a')/a''$; if $a'' = 0$, then $a = a' = 0$, and any $b'' > 1$ fits. Also, $\alpha(1-a)\cdot k + (1-\alpha)(1-a')\cdot k' = (1-a'')\cdot k''$, where $k'' \stackrel{\text{def}}{=} \frac{\alpha(1-a)}{1-a''}k + \frac{(1-\alpha)(1-a')}{1-a''}k'$ if $a'' \neq 1$ (otherwise, we take for k'' an arbitrary element of U, e.g., h_0). Noting that $\alpha(1-a) + (1-\alpha)(1-a') = 1-a''$, k'' is a convex combination of k and k' (if $a'' \neq 1$), hence is in the convex set U. To sum up, $\alpha \cdot h + (1-\alpha)\cdot h' \geq b''a'' \cdot \mathbf{1} + (1-a'')\cdot k''$, where $b'' > 1$ and $k'' \in U$ (even if $a'' = 1$). It follows that $\alpha \cdot h + (1-\alpha)\cdot h'$ is in $V_{a''}$, hence in V. Therefore, V is convex.

Every function $b \cdot \mathbf{1}$ with $b > 1$ is in V_1, hence in V, and h_0 is in $V_0 = U$ hence in V. We claim that $V \subseteq W$. For every $h \in V$, there is an $a \in [0,1]$, a number $b > 1$, and a $k \in U$ such that $h \geq ba \cdot \mathbf{1} + (1-a)\cdot k$. The term $ba \cdot \mathbf{1} + (1-a)\cdot k$ is in $[b \cdot \mathbf{1}, k]$, which is included in W since $k \in U \subseteq W$, and by our assumption on W. Since W is open, hence upwards-closed, $h \in W$. □

We now deal with the case of LCS-complete spaces.

We will need the following bit of the theory of continuous valuations. A *continuous valuation* ν on a space Y is a Scott-continuous map $\nu : \mathcal{O}X \to \overline{\mathbb{R}}_+$ such that $\nu(\emptyset) = 0$ and $\nu(U) + \nu(U') = \nu(U \cup U') + \nu(U \cap U')$ for all open subsets U and U' of Y. There is a homeomorphism between $\mathbb{P}^\bullet_{\mathrm{P}} Y$ and the space $\mathbf{V}_\bullet Y$ of continuous valuations on Y (with the unique appropriate topology), for every space Y: in one direction, $\nu \in \mathbf{V}_\bullet Y$ is mapped to its *integration functional* ($h \in \mathcal{L}Y \mapsto \int_{y\in Y} h(y)\, d\nu$), and in the other direction any $G \in \mathbb{P}^\bullet_{\mathrm{P}} Y$ is mapped to ($U \in \mathcal{O}Y \mapsto G(\chi_U)$), where χ_U is the characteristic map of U. We note in particular that $\int_{y\in Y} \chi_U(y)\, d\nu = \nu(U)$. This was first proved by Kirch (Kirch 1993, Satz 8.1), under the additional assumption that Y is core-compact; Tix later observed that this assumption was unnecessary (Tix 1995, Satz 4.16).

For every continuous map $f : X \to Y$, for every continuous valuation ν on X, there is an *image* continuous valuation $f[\nu]$, which maps every open subset V of Y to $\nu(f^{-1}(V))$. There is a *change-of-variable formula*: for every $h \in \mathcal{L}Y$, $\int_{y\in Y} h(y)\, df[\nu] = \int_{x\in X} h(f(x))\, d\nu$ (Kirch 1993, Satz 4.4).

A continuous valuation ν on a space Y is *supported* on a subset X if and only if for all open subsets U and U' of Y such that $U \cap X = U' \cap X$, $\nu(U) = \nu(U')$ (Goubault-Larrecq 2021, Definition 4.5). Giving X the subspace topology, and writing i for the inclusion map $X \to Y$, Lemma 4.6 of Goubault-Larrecq (2021) states that a continuous valuation ν on Y is supported on X if and only if there is a (unique) continuous valuation μ on the subspace X such that $\nu = i[\mu]$.

The following lemma was already implicit in the proof of de Brecht et al. (2019, Theorem 1.1). A continuous valuation ν on Y is *bounded* if and only if $\nu(Y) < \infty$.

Lemma 8. *Let Y be an LCS-complete space, V_n ($n \in \mathbb{N}$) be countably many open subsets of Y, and ν be a bounded continuous valuation on Y. If ν is supported on every V_n, then ν is supported on $\bigcap_{n\in\mathbb{N}} V_n$.*

Proof. We check that ν is supported on the intersection $V \cap V'$ of any two open sets V and V' on which it is supported. Since ν is supported on V and on V', and since both are open, $\nu(V) = \nu(Y)$ and $\nu(V') = \nu(Y)$. By monotonicity, $\nu(V \cup V') = \nu(Y)$. Hence, $\nu(V \cap V') = \nu(V) + \nu(V') - \nu(V \cup V')$ (the minus sign is legitimate since ν is bounded) $= \nu(Y)$. For every open subset U of Y, $\nu(U \cup (V \cap V')) = \nu(Y)$ by monotonicity of ν, so $\nu(U) = \nu(U \cap (V \cap V')) + \nu(U \cup (V \cap V')) - \nu(V \cap V') = \nu(U \cap (V \cap V'))$; hence, for all open subsets U and U' of Y such that $U \cap$

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$(V \cap V') = U' \cap (V \cap V')$, $\nu(U) = \nu(U \cap (V \cap V')) = \nu(U' \cap (V \cap V')) = \nu(U')$, showing that ν is supported on $V \cap V'$.

Hence, ν is supported on $\bigcap_{i=0}^{n} V_i$ for every $n \in \mathbb{N}$. Without loss of generality, and replacing V_n $\bigcap_{i=0}^{n} V_i$ if necessary, we may therefore assume that $V_0 \supseteq V_1 \supseteq \cdots \supseteq V_n \cdots$.

We will use a precursor to de Brecht et al. (2019, Theorem 1.1): a theorem due to Mauricio Alvarez-Manilla (Alvarez-Manilla 2000, Theorem 3.27) states that on a locally compact sober space, every locally finite continuous valuation extends to a (unique τ-smooth) measure on the Borel σ-algebra. A continuous valuation is *locally finite* if and only if every point has an open neighborhood mapped to some finite number by the valuation. This applies to ν, which is locally finite, since bounded. We write the unique extension of ν we have just obtained as μ.

In Y, X is a countable intersection of open sets, hence lies in the Borel σ-algebra, so $\mu(U \cap X)$ makes sense for every open subset U of Y. We then have $\mu(U \cap X) = \mu(U \cap \bigcap_{n \in \mathbb{N}} V_n) = \mu(\bigcap_{n \in \mathbb{N}} (U \cap V_n)) = \inf_{n \in \mathbb{N}} \mu(U \cap V_n) = \inf_{n \in \mathbb{N}} \nu(U \cap V_n)$, since bounded measures map intersections of descending sequences of measurable sets to the infimum of their measures. Since ν is supported on V_n, and $(U \cap V_n) \cap V_n = U \cap V_n$, we have $\nu(U \cap V_n) = \nu(U)$ for every $n \in \mathbb{N}$. Hence, $\mu(U \cap X) = \nu(U)$. This entails that for all open subsets U and U' of Y such that $U \cap X = U' \cap X$, $\nu(U)$ and $\nu(U')$ are both equal to $\mu(U \cap X) = \mu(U' \cap X)$, so ν is supported on X. □

Finally, here is Lemma 3.4 of Goubault-Larrecq (2017) in repaired form. The conclusion is the same as originally published. The assumption that X is locally convex is replaced by $\mathtt{AP}_\bullet$-friendliness (Definition 1).

Lemma 3.4 of Goubault-Larrecq (2017) (repaired). *Let $\bullet$ be nothing, "≤ 1" or "1," and let X be an $\mathrm{AP}_\bullet$-friendly space. Let F be an element of $\mathbb{P}^\bullet_{\mathrm{AP}} X$. For every $h_0 \in \mathcal{L}X$, and every real number $r > 0$ such that $F(h_0) > r$, there is a $G \in \mathbb{P}^\bullet_{\mathrm{P}} X$ with $G \leq F$ and $G(h_0) > r$.*

Proof. The cases where $\bullet$ is nothing or "≤ 1" were dealt with in Lemma 4.

We now deal with the case where $\bullet$ is "1." This decomposes into subcases, according to whether X satisfies condition 1, 2 or 3 of Definition 1.

If condition 1 or 2 is satisfied. We let $W \stackrel{\text{def}}{=} F^{-1}(]1, \infty])$. We verify the assumptions of Lemma 7. For every $b > 1$ and every $h \in W$, we consider an arbitrary element $ba \cdot \mathbf{1} + (1-a)h$ of $[b \cdot \mathbf{1}, h]$, with $a \in [0, 1]$. Since F is normalized and positively homogeneous, $F(ba \cdot \mathbf{1} + (1-a) \cdot h) = ba + (1-a)F(h)$. If $a \neq 1$, using the fact that $h \in W = F^{-1}(]1, \infty])$, this is strictly larger than $ba + (1-a) \geq a + (1-a) = 1$; otherwise, this is equal to b, which is also strictly larger than 1. In any case, it follows that $ba \cdot \mathbf{1} + (1-a)h$ is in W. Hence, we may apply Lemma 7 (with $(1/r) \cdot h_0$ taking the place of h_0, so that $(1/r) \cdot h_0 \in W = F^{-1}(]1, \infty])$), and we obtain that there is a convex Scott-open neighborhood V of $(1/r) \cdot h_0$ in $\mathcal{L}X$ that contains every function $b \cdot \mathbf{1}$ with $b > 1$ and that is included in W.

We note that V is proper (namely, not the whole of $\mathcal{L}X$), since W is itself proper. Indeed, the constant 0 map $0 \cdot \mathbf{1}$ is not in V, because $F(0 \cdot \mathbf{1}) = 0$. We form the upper Minkowski functional q of V: by Keimel (2008, Proposition 7.6 (b)), q is a superlinear lower semicontinuous map from $\mathcal{L}X$ to $\overline{\mathbb{R}}_+$, such that $q^{-1}(]1, \infty]) = V$. In fact, taking upper Minkowski functionals defines an order-isomorphism between the collection of proper open convex sets and superlinear lower semicontinuous maps. Since $V \subseteq W = F^{-1}(]1, \infty])$, $q \leq F$.

Keimel's sandwich theorem (Keimel 2008, Sandwich Theorem 8.2) states that given any superlinear lower semicontinuous map $q \colon C \to \overline{\mathbb{R}}_+$ on a semitopological cone C, given any sublinear map (not necessarily lower semicontinuous) $p \colon C \to \mathbb{R}$ pointwise above q, there is a linear lower semicontinuous map $G \colon C \to \overline{\mathbb{R}}_+$ such that $q \leq G \leq p$. We apply this to q defined above and to $p \stackrel{\text{def}}{=} F$, and we obtained the corresponding G.

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The map $(1/r) \cdot h_0$ is in V, so $q((1/r) \cdot h_0) > 1$; then $q(h_0) > r$, and hence $G(h_0) > r$. We have $G(\mathbf{1}) \leq F(\mathbf{1}) = 1$ because F is normalized, so G is subnormalized. Finally, for every $b > 1$, $b \cdot \mathbf{1}$ is in V, so $q(b \cdot \mathbf{1}) \geq 1$, namely $q(\mathbf{1}) \geq 1/b$ for every $b > 1$. It follows that $q(\mathbf{1}) \geq 1$, so $G(\mathbf{1}) \geq 1$, hence G is normalized. Therefore, G is in $\mathbb{P}^1_{\mathrm{P}}X$.

If condition 3 is satisfied. Then X embeds as a G_δ subspace of a locally compact sober space Y. In particular, Y is core-compact, so $\mathcal{L}Y$ is a locally convex, locally convex-compact, and sober topological cone, by Remark 2. We equate X with the intersection $\bigcap_{n\in\mathbb{N}} V_n$ of a descending sequence of open subsets V_n of Y, $n \in \mathbb{N}$, and we write $i\colon X \to Y$ for the inclusion map.

Let $F'\colon \mathcal{L}Y \to \overline{\mathbb{R}}_+$ map every $h' \in \mathcal{L}Y$ to $F(h' \circ i)$. It is easy to see that F' is a normalized sublinear prevision on Y.

Every $h \in \mathcal{L}X$ extends to some $h^* \in \mathcal{L}Y$, in the sense that $h^* \circ i = h$, namely that $h^*(x) = h(x)$ for every $x \in X$. This is due to the fact that $\overline{\mathbb{R}}_+$ is a continuous lattice, and to Scott's theorem that the continuous lattices are exactly the injective spaces (Gierz et al. 2003, Theorem II-3.8); namely, the spaces R such that every continuous map from a subspace (here, X) of a space (here Y) to R has a continuous extension to the whole of Y.

We have $F'(h_0^*) = F(h_0^* \circ i) = F(h_0) > r$. Let also $W \stackrel{\text{def}}{=} F'^{-1}(]1, \infty])$. We will apply Lemma 6, this time with $g_n \stackrel{\text{def}}{=} \chi_{V_n}$ for every $n \in \mathbb{N}$. We note that $g_n \circ i = \mathbf{1}$ for every $n \in \mathbb{N}$, since $X \subseteq V_n$. We check that every function $ba \cdot g_n + (1-a) \cdot h$, where $n \in \mathbb{N}$, $b > 1$, $a \in [0, 1]$, and $h \in W$, is in W: $F'(ba \cdot g_n + (1-a) \cdot h) = F(ba \cdot (g_n \circ i) + (1-a) \cdot (h \cdot i)) = F(ba \cdot \mathbf{1} + (1-a) \cdot (h \cdot i)) = ba + (1-a)F'(h) > 1$, since $b > 1$ and $F'(h) > 1$. By Lemma 6, therefore, there is a convex Scott-open neighborhood V of $(1/r) \cdot h_0^*$ in $\mathcal{L}Y$ that contains every function $b \cdot \chi_{V_n}$ for every $n \in \mathbb{N}$ and $b > 1$, and such that $V \subseteq W$.

V is proper, since W is proper, because the constant 0 map is not in W. As before, there is a unique superlinear lower semicontinuous map $q\colon \mathcal{L}Y \to \overline{\mathbb{R}}_+$ such that $q^{-1}(]1, \infty]) = V$. By Keimel's sandwich theorem on $\mathcal{L}Y$, there is a lower semicontinuous linear map $G'\colon \mathcal{L}Y \to \overline{\mathbb{R}}_+$ such that $q \leq G' \leq F'$.

Hence, G' is the integration functional of a unique continuous valuation ν' on Y. For every $n \in \mathbb{N}$ and every $b > 1$, we have $G'(b \cdot \chi_{V_n}) \geq q(b \cdot \chi_{V_n}) > 1$, since $G' \geq q$ and $b \cdot \chi_{V_n} \in V = q^{-1}(]1, \infty])$. Hence $\nu'(V_n) = G'(\chi_{V_n}) > 1/b$ for every $b > 1$. Since $G' \leq F'$, $G'(\chi_{V_n}) \leq F'(\chi_{V_n}) = F(\chi_{V_n} \circ i) = F(\mathbf{1}) = 1$, so $\nu'(V_n) \leq 1$. It follows that $\nu'(V_n) = 1$, for every $n \in \mathbb{N}$. We also note that $\nu'(Y) = 1$: by a similar argument involving $G' \leq F'$, $\nu'(Y) \leq 1$, and $\nu'(Y) \geq \nu'(V_0) = 1$.

For every $n \in \mathbb{N}$, since $\nu'(V_n) = 1$, ν is supported on V_n. We argue as follows. We start with $1 = \nu'(V_n) \leq \nu'(U \cup V_n) \leq \nu'(Y) = 1$, so all terms in the sequence of inequalities are equal. In particular, $\nu'(U \cup V_n) = \nu'(V_n) = 1$. We use this with $\nu'(U \cup V_n) + \nu'(U \cap V_n) = \nu'(U) + \nu'(V_n)$ and we obtain that $\nu'(U \cap V_n) = \nu'(U)$, for every open subset U of Y. In particular, for all open subsets U and U' of Y such that $U \cap V_n = U' \cap V_n$, $\nu'(U) = \nu'(U')$. Using Lemma 8, then, ν' is supported on X, and is therefore equal to the image continuous valuation $i[\nu]$ of a unique continuous valuation ν on X.

Additionally, $\nu(X) = i[\nu](Y) = \nu'(Y) = 1$. Let G be the integration functional of ν on X, namely $G(h) \stackrel{\text{def}}{=} \int_{x\in X} h(x)\, d\nu$ for every $h \in \mathcal{L}X$. For all $a \in \mathbb{R}_+$ and $h \in \mathcal{L}X$, $G(a \cdot \mathbf{1} + h) = a\,\nu(X) + G(h) = a + G(h)$ for every $h \in \mathcal{L}X$, so G is normalized.

Next, $G'(h) = G(h \circ i)$ for every $h \in \mathcal{L}Y$. Indeed, $G'(h) = \int_{y\in Y} h(y)\, d\nu' = \int_{y\in Y} h(y)\, di[\nu] = \int_{x\in X} h(i(x))\, d\nu$ (by the change-of-variable formula) $= G(h \circ i)$. In particular, $G'(h_0^*) = G(h_0^* \circ i) = G(h_0)$. But $G' \geq q$, and since $(1/r) \cdot h_0^*$ is in $V = q^{-1}(]1, \infty])$, $q((1/r) \cdot h_0^*) > 1$, so $G'(h_0^*) \geq q(h_0^*) > r$. It follows that $G(h_0) > r$. Finally, $G' \leq F'$, so for every $h \in \mathcal{L}X$, $G'(h^* \circ i) \leq F'(h^* \circ i)$, meaning that $G(h) \leq F(h)$; whence $G \leq F$. □

All other results of Section 3.1 of Goubault-Larrecq (2017), except Lemma 3.10, need to be changed accordingly, replacing "if $[X \to \overline{\mathbb{R}}_{+\sigma}]$ is locally convex" by "if X is $\mathtt{AP}_\bullet$-friendly." Since

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they sum up all the results of that section, we state Proposition 3.11 and Corollary 3.12 of Goubault-Larrecq (2017) in their repaired form below.

Proposition 3.11 of Goubault-Larrecq (2017) (repaired). *Let $\bullet$ be the empty superscript, "≤ 1," or "1."*

Let X be a topological space. Then $r_{\mathtt{AP}}$ is a continuous map from $\mathcal{H}_{\mathtt{V}}(\mathbb{P}^{\bullet}_{\mathtt{P}\,wk}(X))$ to $\mathbb{P}^{\bullet}_{\mathtt{AP}\,wk}(X)$, and $s^{\bullet}_{\mathtt{AP}} \circ r_{\mathtt{AP}}$ is above the identity.

If X is $\mathrm{AP}_{\bullet}$-friendly, then $s^{\bullet}_{\mathtt{AP}}$ is a continuous map from $\mathbb{P}^{\bullet}_{\mathtt{AP}\,wk}(X)$ to $\mathcal{H}_{\mathtt{V}}(\mathbb{P}^{\bullet}_{\mathtt{P}\,wk}(X))$ such that $r_{\mathtt{AP}} \circ s^{\bullet}_{\mathtt{AP}}$ equals the identity.

Corollary 3.12 of Goubault-Larrecq (2017) (repaired). *Let $\bullet$ be nothing, "≤ 1" or "1." For every $\mathrm{AP}_{\bullet}$-friendly space X, $s^{\bullet}_{\mathtt{AP}}$ and $r_{\mathtt{AP}}$ together define an coembedding-coprojection pair of $\mathcal{H}_{\mathtt{V}}(\mathbb{P}^{\bullet}_{\mathtt{P}\,wk}(X))$ onto $\mathbb{P}^{\bullet}_{\mathtt{AP}\,wk}(X)$.*

In principle, the error would propagate to Proposition 3.33 (in Section 3.4, "Domain-Theoretic Consequences"), which is deduced from Proposition 3.11. Proposition 3.33 states that for a core-compact space X, $r_{\mathtt{AP}}$ defines a Scott retraction of $\mathcal{H}(\mathbb{P}^{\bullet}_{\mathtt{P}\,wk}(X))$ onto $\mathbb{P}^{\bullet}_{\mathtt{AP}}(X)$, with associated section $s^{\bullet}_{\mathtt{AP}}$, and that $r_{\mathtt{AP}} \circ s^{\bullet}_{\mathtt{AP}} \geq \mathrm{id}$. However, we have noticed that every core-compact space is $\mathtt{AP}_{\bullet}$-friendly in Remark 2.

Still, the error does propagate to Section 4. In its first paragraph, we find a sentence to the effect that "When $[X \to \overline{\mathbb{R}}_{+\sigma}]_{\sigma}$ is locally convex, we know that $r_{\mathtt{AP}} \circ s^{\bullet}_{\mathtt{AP}}$ is the identity map." This requires $\mathtt{AP}_{\bullet}$-friendliness instead. Then Theorem 4.11 should be the following.

Theorem 4.11 of Goubault-Larrecq (2017) (repaired). *Let $\bullet$ be nothing, "≤ 1," or "1." For every $\mathrm{AP}_{\bullet}$-friendly space X, for example, if X is core-compact or LCS-complete, $r_{\mathtt{AP}}$ defines a homeomorphism with inverse $s^{\bullet}_{\mathtt{AP}}$ between $\mathcal{H}^{cvx}_{\mathtt{V}}(\mathbb{P}^{\bullet}_{\mathtt{P}\,wk}(X))$ and $\mathbb{P}^{\bullet}_{\mathtt{AP}\,wk}(X)$ and an order-isomorphism between $\mathcal{H}^{cvx}(\mathbb{P}^{\bullet}_{\mathtt{P}\,wk}(X))$ and $\mathbb{P}^{\bullet}_{\mathtt{AP}}(X)$.*

Corollary 4.12 is unaffected, since a continuous dcpo is core-compact.

Similarly, Proposition 4.19 must be changed as follows.

Proposition 4.19 of Goubault-Larrecq (2017) (repaired). *Let $\bullet$ be nothing, "≤ 1," or "1." For every $\mathrm{AP}_{\bullet}$-friendly space X, for every non-empty closed subset A of $\mathbb{P}^{\bullet}_{\mathtt{P}\,wk}(X)$, $s^{\bullet}_{\mathtt{AP}}(r_{\mathtt{AP}}(A))$ is the closed convex hull $cl(conv(A))$ of A in $\mathbb{P}^{\bullet}_{\mathtt{P}\,wk}(X)$.*

No result on compact saturated sets and superlinear previsions is affected. No result on lenses and forks is affected by the mistake either, except for the final Proposition 4.21, which must be changed as follows, as it relies on Proposition 4.19.

Proposition 4.21 of Goubault-Larrecq (2017) (repaired). *Let $\bullet$ be nothing, "≤ 1," or "1." For every $\mathrm{AP}_{\bullet}$-friendly space X, for every lens L of $\mathbb{P}^{\bullet}_{\mathtt{P}\,wk}(X)$, $s^{\bullet}_{\mathtt{ADP}}(r_{\mathtt{ADP}}(L))$ is a lens, and is the smallest convex lens that contains L.*

4. Review of Papers Citing Goubault-Larrecq (2017)

Since the present paper fixes a mistake in Goubault-Larrecq (2017), we review whether and how this mistake propagates in the literature. To jump to the conclusion, let me say right away that no published paper is affected by the mistake as of June 30th, 2025.

As of June 30th, 2025, Google Scholar reports on 19 papers citing Goubault-Larrecq (2017). Most of them only cite Goubault-Larrecq (2017) without using its results, except for the following.

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- de Brecht et al. (2019) uses Theorem 4.11 of Goubault-Larrecq (2017) in its Corollary 13.7, which states that $r_{\mathtt{AP}}$ defines a homeomorphism with inverse $s^{\bullet}_{\mathtt{AP}}$ between $\mathcal{H}^{cvx}_{\mathtt{V}}(\mathbb{P}^{\bullet}_{\mathtt{P}\,wk}(X))$ and $\mathbb{P}^{\bullet}_{\mathtt{AP}\,wk}(X)$ when X is LCS-complete. Every LCS-complete space is $\mathtt{AP}_{\bullet}$-friendly, whatever $\bullet$ is, so Corollary 13.7 of de Brecht et al. (2019) is indeed true.
  We have used a few results from de Brecht et al. (2019) in the present revised version. The danger is of course that of circular reasoning. Those results are Proposition 12.1 of de Brecht et al. (2019), which states that every LCS-complete space is consonant, and Lemma 13.2 of de Brecht et al. (2019), which states that every LCS-complete space is $\odot$-consonant. Those are proofs that are entirely independent of any result of Goubault-Larrecq (2017).
- Goubault-Larrecq (2015) uses Propositions 3.41, 3.42, and 3.44 of Goubault-Larrecq (2017), which are unaffected.
- Goubault-Larrecq (2022b) collects several properties about $r_{\mathtt{AP}}$ and $s^{\bullet}_{\mathtt{AP}}$ at the beginning of its Section 3, in order to use them without mention later on. This includes the faulty Corollary 3.12 of Goubault-Larrecq (2017). But no result of Goubault-Larrecq (2022b) is affected, since $r_{\mathtt{AP}}$ and $s^{\bullet}_{\mathtt{AP}}$ are only used on continuous complete quasi-metric spaces in their d-Scott topology, and all such spaces are LCS-complete (de Brecht et al. 2019, Theorem 4.1).
- Goubault-Larrecq (2022a), which is a superset of Goubault-Larrecq (2022b), is similarly unaffected.
- Goubault-Larrecq (2024b) (versions 1 and 2) relies on the faulty results of Goubault-Larrecq (2017) in its Section 13. One has to replace the categories **K** by categories of $\mathtt{AP}_{\bullet}$-friendly spaces. This is corrected in version 3 of Goubault-Larrecq (2024b). Goubault-Larrecq (2024b) does not appear to be cited by any paper yet, according to Google Scholar.
- Lopez and Simpson (2018) use Goubault-Larrecq (2017) in its Lemma 22. This is unaffected by the mistake, since it only uses results on the demonic cases, namely, on $r_{\mathtt{DP}}$ and $s^{\bullet}_{\mathtt{DP}}$. Alternatively, the primary justification used in Lemma 22 is really Keimel and Plotkin (2017), which is unaffected by the mistake, for obvious reasons.
- Goubault-Larrecq (2025) is unaffected since it only uses results on the demonic cases.
- Goubault-Larrecq and Jia (2019) uses only one result from Goubault-Larrecq (2017), namely that $\mathcal{L}X = \mathcal{L}(X^s)$, where X^s is the sobrification of X, in its Example 3.14; this is unaffected.
- Goubault-Larrecq (2023) cites Goubault-Larrecq (2017) at several places. Lemma 3.24 of Goubault-Larrecq (2017) is used before Proposition 4.1; this result is unaffected by the mistake. The proof of Proposition 4.1 uses a variety of results from Goubault-Larrecq (2017), namely Lemma 3.28, Lemma 3.29, Proposition 4.5, and Proposition 4.3, none of which is affected by the mistake; it also references the faulty Corollary 3.12 of Goubault-Larrecq (2017), but only in order to inform the reader of the roots of the results of Goubault-Larrecq (2022b) used in the proof and, as we have seen, none of the results of Goubault-Larrecq (2022b) are affected. At any rate, Proposition 4.1 of Goubault-Larrecq (2023) requires not only $\mathcal{L}X$ to be locally convex (and to have an almost addition map) but also X to be compact, and that is condition 1 of the definition of $\mathtt{AP}_1$-friendliness, under which all results of Goubault-Larrecq (2017) hold. Remark 4.2 of Goubault-Larrecq (2023) makes use of Proposition 4.8 of Goubault-Larrecq (2017), which is not affected. We have used one result from Goubault-Larrecq (2021) in the present revised version. This is the fact that a continuous valuation ν on a space Y is supported on a subset X if and only if $\nu = i[\mu]$ for some (unique) continuous valuation μ on the subspace Y, where i is the inclusion map (Goubault-Larrecq 2021, Lemma 4.6). This does not rely on any result of Goubault-Larrecq (2017), avoiding the pitfall of circular reasoning, just like in the case of de Brecht et al. (2019).

Finally, I know about the following paper citing Goubault-Larrecq (2017) but not reported by Google Scholar:

https://doi.org/10.1017/S0960129525100200 Published online by Cambridge University Press

– Goubault-Larrecq (2024c) versions 1 through 3 are affected by the mistake, through the definition of the category $\mathbf{Top}^\flat$ (Definition 5.6; Definition 5.7 in version 4), which must consist of $\mathtt{AP}_\bullet$-friendly spaces, instead of spaces X such that $\mathcal{L}X$ is locally convex. This is corrected in version 4. Goubault-Larrecq (2024c) does not appear to be cited by any paper yet, according to Google Scholar.

5. Other Minor Corrections

– At various places throughout the paper (Goubault-Larrecq 2017), $s^\bullet_{\mathtt{AP}}$, $s^\bullet_{\mathtt{DP}}$, and $s^\bullet_{\mathtt{ADP}}$ appear without the $\bullet$ superscript.

– At the beginning of Section 3.1, in the statements of Lemma 3.10 and of Proposition 3.11, $r_{\mathtt{AP}}$ appears with a $\bullet$, but should not.

– In the statement of Lemma 3.21 and in the paragraph before Proposition 3.22, $r_{\mathtt{DP}}$ appears with a $\bullet$, but should not.

– In Definition 3.23, "$r_{\mathtt{ADP}}(E)$: " should be "$r_{\mathtt{ADP}}(E) \in$."

– In the proof of Lemma 4.4, line 4: C is undefined, and should be $\mathcal{L}X$; line 6: "$\Lambda = \Phi(h)$" should read "$\Lambda = G(h)$."

Acknowledgments. Thanks to the reviewer for having carefully checked this purely technical paper.

Competing interests. The author declares none.

Note

1 For a self-contained revision of the original paper, incorporating the present fixes, see Goubault-Larrecq (2024a).

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Cite this article: Goubault-Larrecq J (2025). Errata to "Isomorphism theorems between models of mixed choice," fixes and consequences. *Mathematical Structures in Computer Science* **35**(e23), 1–12. https://doi.org/10.1017/S0960129525100200

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