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OPTIMAL TARIFFS IN A TWO-COUNTRY R&D-BASED GROWTH MODEL

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This paper examines the effect of a tariff on long-run growth and welfare in a two-country innovation-led growth model. We show that although raising the home country's tariff reduces the growth and GDP of the foreign country, it will backfire by depressing R&D and growth of the home country. The Nash equilibrium tariffs can be positive, and they are larger when the government expenditure is more beneficial to private production and/or when the productivity of innovation is higher. The presence of positive Nash equilibrium tariffs provides a theoretical explanation for why countries have incentives to implement a tariff policy regardless of its negative effect on growth. Finally, the Nash equilibrium tariffs are higher than the globally optimal tariffs, that is, the levels that maximize the joint welfare of both countries.

Keywords: Optimal tariffs, Economic growth, R&D, Trade in intermediate goods

1. INTRODUCTION

In this study, we examine the effect of a tariff on long-run growth and welfare in an innovation-led endogenous growth model. The literature on trade and growth generally suggests that openness to trade has a positive effect on growth.[1] Despite

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this, tariffs are still commonly implemented in countries over the world. This paper aims to address the following question: are those tariff policies optimal, both from an individual country's perspective and from a global perspective? The main contribution of this study to the literature is that we provide the normative analysis of optimal tariffs, given that the existing open-economy endogenous growth models primarily focus on the growth effect of the tariff policy.

We build up a two-country version of the seminal R&D-based growth model developed by Romer (1990), in which R&D expands the varieties of intermediate goods.[2] In our open economy, intermediate goods can be traded between countries, and each country imposes a tariff on imports of intermediate goods.[3] The productive government expenditure is financed by the tariff.[4] Within this framework, we examine the effect of a unilateral increase in the tariff of the home country on both countries' innovation and growth. To provide welfare implications, we examine whether each country has an incentive to implement a positive tariff under the Nash equilibrium setting in which each country only cares about its own residents' welfare. Moreover, we derive a globally optimal level of tariffs, that is, one that maximizes the joint welfare of both countries, and compare it with the level of the Nash equilibrium tariffs.

The main findings and contributions of this paper are as follows. First, raising the tariff hinders the growth rate and GDP of the other country, and it will backfire by inducing a lower growth rate in the home country. The intuition is as follows. A higher tariff in the home country decreases the demand for imported foreign intermediate goods. Accordingly, the GDP level of the foreign country falls in response. To eliminate the trade deficit, the foreign country also decreases its import of intermediate goods from the home country, which depresses the profit of the home country's intermediate-good firms. Given that the incentive for R&D comes from intermediate-good firms' profits, this consequently reduces R&D (and hence the growth rate) in the home country. As is clear from the preceding intuitive illustration, an important feature of our model is that, through trade in intermediate goods, a country's growth rate positively depends on R&D in both countries. Therefore, a tariff that hinders R&D can be harmful to economic growth in both countries.

Second, our result is capable of capturing the "international technology spillovers" effect. As remarked by Keller (2004), international technology spillovers occur when the R&D of one firm is positively correlated with the total factor productivity (TFP) of another firm. In our model, when country A increases its R&D, it will expand the varieties and export more intermediate goods to country B. Consequently, the demand for intermediate goods of country B also increases because final goods production uses both domestic and foreign intermediate goods. This gives more incentives for country B to conduct R&D. Moreover, the increase of country A's R&D will, through exporting more intermediate goods, enhance the TFP of final-goods firms of country B. Our analysis thus complements the literature in providing a way to elaborate on how intermediate goods trade serves as a channel of international technology spillovers.

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Third, despite the negative growth effect, an individual country that acts non-cooperatively to maximize its residents' welfare may still have an incentive to impose a positive tariff. This is because the country needs to finance its government expenditure. As a result, the more productive the public infrastructure is, the more likely the Nash equilibrium tariffs are positive. This result can provide a theoretical explanation for why tariffs have commonly adopted the world over even though the prevailing view is that imposing tariffs is harmful to economic growth.

Finally, we derive the globally optimal level of tariffs, and find that it is lower than the Nash equilibrium level. Intuitively, raising the tariff has a negative effect on the growth rate of the foreign country, which will not be taken into account by the home country under the Nash equilibrium setting. However, in view of the global optimum, this external cost should be endogenized. Therefore, the globally optimal level of tariffs should be lower than the Nash equilibrium tariffs. By calibrating the model to the USA data, our numerical analysis shows that, given reasonable parameter values, the globally optimal level of tariffs can be greater than zero. The result thus indicates that (totally) free trade may not be favorable in the context of global welfare.

The remainder of this paper is organized as follows. Section 2 provides a review of the related literature. Section 3 establishes the two-country version of the Romer (1990)-type R&D-based growth model. Section 4 analytically examines the growth effect of an increase in the home country's tariffs on both countries' R&D. Section 5 deals with the welfare analysis. We compare the levels of the Nash equilibrium tariffs and the globally optimal tariffs. Section 6 provides a numerical analysis to quantify our results. The final section concludes this paper.

2. LITERATURE REVIEW

There is a substantial literature dealing with the relation between trade and growth in an endogenous growth model framework, which can be traced back to the seminal work of Grossman and Helpman (1990). Grossman and Helpman develop a two-country R&D-based growth model to examine how external trading environment and trade policies affect long-run growth. Our model structure and objective are close to theirs; hence, it is worthwhile to discuss the main differences between these two models. A key difference is that their model assumes perfect capital mobility so that one country can have sustainable trade surplus/deficit at a steady state. Nonetheless, by virtue of capital mobility, in their model, both countries must share a common long-run growth rate. Instead, we assume that trade is balanced between two countries. This salient feature allows for different growth rates in two countries at the steady state. More importantly, with this feature, a unilateral change in tariff or an increase in R&D investment can have *different* long-term impacts on the two countries' growth rates. Second, while their study focuses mainly on the positive analysis shedding light on how trade policies affect long-run growth, our paper attempts to explore the normative analysis of optimal

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tariffs. Finally, their study emphasizes the role of comparative advantage and it is the main factor driving the impact of trade policy on growth. Our paper, by contrast, highlights the channel of trade in intermediate goods through which R&D in one country can exhibit a beneficial effect on the other country's R&D and production.

In addition to Grossman and Helpman (1990), there are a number of important contributions in this literature. Using the Romer (1990)'s variety-expanding model, Devereux and Lapham (1994) show that freer trade may impede innovations in poorer countries.[5] With firm-level heterogeneity, Baldwin and Forslid (2000) show that trade liberalization can stimulate growth via a pro-competitive effect in the R&D sector. Baldwin and Robert-Nicoud (2008) find that the effect of free trade on growth and welfare is uncertain, depending on the model specifications regarding the innovation process. Under the setting of a semi-endogenous growth model, Gustafsson and Segerstrom (2010) show that, in the case where the size of intertemporal knowledge spillovers in R&D is large, trade liberalization reduces growth in the short run and worsens country welfare in the long run. Ourens (2016) revisits and modifies the results in Baldwin and Robert-Nicoud (2008), therefore providing more comprehensive conditions under which free trade can improve growth and welfare. Another branch of literature focuses on the effect of tariffs on growth, and the results are inconclusive. Some papers find a negative relation between tariffs and growth [e.g. Osang and Pereira (1996) and Naito (2006*a*)], while others suggest an ambiguous (possibly positive) relation between them [e.g. Naito (2006*b*) and Lee (2009)].[6]

Our paper is also related to earlier studies that examine the trade–growth nexus in AK-type endogenous growth models. Young (1991) and Grossman and Helpman (1995) examine this issue in endogenous growth models that feature learning-by-doing externalities. Young (1991) finds that trade may lead to lower (higher) GDP growth in the less (more) developed country even though both countries may gain higher welfare from trade. Grossman and Helpman (1995) find that whether trade openness will stimulate or stifle growth depends on the country's specialization and the scope of the spillover effects.

To sum up, our paper departs from previous studies in that we focus on the normative analysis of optimal tariffs.[7] In particular, we compare, in an open economy of endogenous growth, the Nash equilibrium tariffs chosen by noncooperative countries with the tariffs that maximize global welfare. Our results may contribute to the literature by providing an explanation for why countries have incentives to impose a positive tariff.

Finally, the model structure and analytical approach adopted in this paper are also related to a group of papers that compare noncooperative and cooperative policies in a two-country R&D-based growth model. Chu and Peng (2011) examine the effects of strengthening patent protection on growth and income inequality, and they also compare the patent protection policies under noncooperative and cooperative settings. Chu *et al.* (2015) focus on the monetary policies and find that the Nash equilibrium inflation rates are in general higher than the optimal inflation rates chosen by cooperative governments. These studies, however, do

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not deal with the issue of the tariff policy. Our paper contributes to this strand of the literature by examining the tariff policy.

3. MODEL

In this section, we describe our theoretical model. Throughout the paper, we will use the superscript h to denote variables for home country and f for foreign country. To simplify expressions, we will only present equations for the home country and omit those for the foreign country unless necessary. It is useful to keep in mind that, for each equation we present, there is an analogous equation for the foreign country.

3.1. The households

In each country, there is a representative household who is immobile across countries. The lifetime utility function of the representative household in country h is given by

$$U^h = \int_0^\infty (\ln C_t^h) e^{-\rho t} dt, \tag{1}$$

where C_t^h denotes consumption of final goods. The parameter $\rho > 0$ is the subjective time preference, which is identical for both countries.

The household maximizes (1) subject to the following budget constraint:

$$\dot{a}_t^h = r_t^h a_t^h + w_t^h L_t^h - P_t^h C_t^h, \tag{2}$$

where $a_t^h \equiv A_t^h V_t^h$ is the value of equity shares of monopolistic firms owned by the household, A_t^h is the number of equity shares (i.e. the number of varieties of intermediate goods), V_t^h is the value of an invented variety, r_t^h is the interest rate, w_t^h is the wage rate, L_t^h is the exogenous labor supply which we assume to be unity (i.e. $L_t^h = 1$), and P_t^h is the price of final goods. The final goods of the home country serve as the *numéraire* so that $P_t^h = 1$. Labor is homogeneous and perfectly mobile among sectors within a country so that a uniform wage rate holds.

The household's optimization problem gives the optimality condition of consumption:

$$\frac{1}{C_t^h} = \eta_t^h, \tag{3}$$

where η_t^h is the co-state variable of the current value Hamiltonian associated with (2). The familiar intertemporal optimality condition is

$$\frac{\dot{C}_t^h}{C_t^h} = r_t^h - \rho. \tag{4}$$

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3.2. Final goods

In country h, there is a unique final good, Y_t^h, produced by competitive firms using a Cobb–Douglas aggregator given by

$$Y_t^h = \frac{(y_t^{h,h})^{1-\alpha}(y_t^{h,f})^{\alpha}}{(1-\alpha)\alpha}, \tag{5}$$

where $y_t^{h,h}$ is the final input produced with a continuum of domestic intermediate goods and $y_t^{h,f}$ is the final input produced with a continuum of intermediate goods imported from country f. The parameter $\alpha \in [0, 1]$ determines the importance of imported goods in domestic final-good production.

From profit maximization, the conditional demand functions for the two final inputs are

$$y_t^{h,h} = (1-\alpha)Y_t^h / p_{y,t}^{h,h}, \tag{6}$$

$$y_t^{h,f} = \alpha Y_t^h / p_{y,t}^{h,f}, \tag{7}$$

where $p_{y,t}^{h,h}$ and $p_{y,t}^{h,f}$ are the prices of $y_t^{h,h}$ and $y_t^{h,f}$, respectively.

Final inputs are produced in a competitive market according to the following production functions:

$$y_t^{h,h} = \left(G_t^h\right)^{\psi} \left(l_{y,t}^{h,h}\right)^{1-\beta} \int_0^{A_t^h} \left(x_t^{h,h}(i)\right)^{\beta} di, \tag{8}$$

$$y_t^{h,f} = \left(G_t^h\right)^{\psi} \left(l_{y,t}^{h,f}\right)^{1-\beta} \int_0^{A_t^f} \left(x_t^{h,f}(j)\right)^{\beta} dj, \tag{9}$$

where G_t^h is the productive government expenditure and $\psi > 0$ reflects the extent of its productivity.[8] $l_{y,t}^{h,h}$ and $l_{y,t}^{h,f}$ are labor used for the production of each final input. $x_t^{h,h}(i)$ $(i \in [0, A_t^h])$ denotes a continuum of differentiated intermediate goods produced domestically, and A_t^h is the number of domestic intermediate-good firms (varieties). Similarly, $x_t^{h,f}(j)$ $(j \in [0, A_t^f])$ denotes a continuum of differentiated intermediate goods that are imported from the foreign country and A_t^f is the number of foreign intermediate-good firms.

Two points regarding the specification in equations (8) and (9) should be mentioned here. First, in line with the viewpoint proposed by Aschauer (1988, 1989), Barro (1990), and Turnovsky and Fisher (1995), government spending on infrastructure has a positive external effect on private production. Second, we follow the R&D-based models proposed by Peretto (2007) and Iwaisako (2013) in assuming that the productive government spending benefits the production of final inputs.[9] It is generally recognized that public infrastructure, such as the development and construction of industrial parks, logistics parks, railways, ports, and water supply as well as the provision of medical and educational services, is effective in improving the productivity of all intermediate inputs. Thus, the specification in

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equations (8) and (9) implies that the extent to which public infrastructure investment G_t^h can raise the productivity level is the same for all $x_t^{h,h}(i)$ and $x_t^{h,f}(j)$. We will examine the case where the government spending has different effects on domestic and imported intermediates sectors in Section 7.2.

The firm's profit functions that produce $y_t^{h,h}$ and $y_t^{h,f}$ can then be, respectively, written as

$$\pi_{y,t}^{h,h} = p_{y,t}^{h,h} y_t^{h,h} - \int_0^{A_t^h} p_{x,t}^{h,h}(i) x_t^{h,h}(i)\, di - w_t^h l_{y,t}^{h,h},$$

$$\pi_{y,t}^{h,f} = p_{y,t}^{h,f} y_t^{h,f} - \int_0^{A_t^f} (1+\tau^h) p_{x,t}^{h,f}(j) x_t^{h,f}(j)\, dj - w_t^h l_{y,t}^{h,f},$$

where τ^h denotes the tariff imposed by the government of country h on the purchase of imported intermediate goods $x_t^{h,f}(j)$. $p_{x,t}^{h,h}(i)$ and $p_{x,t}^{h,f}(j)$ are the prices of $x_t^{h,h}(i)$ and $x_t^{h,f}(j)$, respectively.

Competitive firms that produce $y_t^{h,h}$ maximize profits $\pi_{y,t}^{h,h}$ subject to (8) and that produce $y_t^{h,f}$ maximize profits $\pi_{y,t}^{h,f}$ subject to (9), leading to the conditional demand functions for intermediate goods and the inverse demand functions for labor:

$$x_t^{h,h}(i) = \left(\beta \frac{p_{y,t}^{h,h}}{p_{x,t}^{h,h}(i)}\right)^{\frac{1}{1-\beta}} \left(G_t^h\right)^{\psi/(1-\beta)} l_{y,t}^{h,h}, \tag{10}$$

$$x_t^{h,f}(j) = \left(\beta \frac{p_{y,t}^{h,f}}{(1+\tau^h) p_{x,t}^{h,f}(j)}\right)^{\frac{1}{1-\beta}} \left(G_t^h\right)^{\psi/(1-\beta)} l_{y,t}^{h,f}, \tag{11}$$

$$w_t^h = (1-\beta) p_{y,t}^{h,h} \left(G_t^h\right)^{\psi} \left(l_{y,t}^{h,h}\right)^{-\beta} \int_0^{A_t^h} \left(x_t^{h,h}(i)\right)^{\beta} di, \tag{12}$$

$$w_t^h = (1-\beta) p_{y,t}^{h,f} \left(G_t^h\right)^{\psi} \left(l_{y,t}^{h,f}\right)^{-\beta} \int_0^{A_t^f} \left(x_t^{h,f}(j)\right)^{\beta} dj. \tag{13}$$

Let $L_{y,t}^h \equiv l_{y,t}^{h,h} + l_{y,t}^{h,f}$ denote total labor for the production of final inputs. Based on equations (6)–(9), (12), and (13), we can derive $l_t^{h,h}/l_t^{h,f} = (1-\alpha)/\alpha$. Then, by putting this expression and the definition $L_{y,t}^h \equiv l_{y,t}^{h,h} + l_{y,t}^{h,f}$ together, we can obtain the shares of labor used in final inputs production: $l_{y,t}^{h,h} = (1-\alpha) L_{y,t}^h$ and $l_{y,t}^{h,f} = \alpha L_{y,t}^h$.

3.3. Intermediate goods

In country h, there is a continuum of intermediate-good firms indexed $i \in [0, A_t^h]$, each of which owns a perpetually protected patent for a specific intermediate good. Intermediate-good firm i hires labor to produce intermediate goods and sell the products to domestic downstream firms, denoted by $x_t^{h,h}(i)$, as well as

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exporting to country f, denoted by $x_t^{f,h}(i)$. Producing one unit of intermediate good requires one unit of labor, that is, $l_{x,t}^h(i) = x_t^{h,h}(i) + x_t^{f,h}(i)$, where $l_{x,t}^h(i)$ is labor employed by intermediate-good firm i. The profit function of intermediate-good firm i in country h is

$$\pi_{x,t}^h(i) = p_{x,t}^{h,h}(i)x_t^{h,h}(i) + p_{x,t}^{f,h}(i)x_t^{f,h}(i) - w_t^h l_{x,t}^h(i) - bp_{x,t}^{f,h}(i)x_t^{f,h}(i), \tag{14}$$

where $p_{x,t}^{f,h}(i)$ is the price of $x_t^{f,h}(i)$. When selling goods to the other country, the intermediate-good firms need to bear an international transportation cost $bp_{x,t}^{f,h}(i)x_t^{f,h}(i)$, where the parameter $b \in (0, 1)$ represents the magnitude of this cost.

The firm maximizes (14) subject to $l_{x,t}^h(i) = x_t^{h,h}(i) + x_t^{f,h}(i)$, (10), and the analogous equation of (11), that is:

$$x_t^{f,h}(i) = \left(\beta \frac{p_{y,t}^{f,h}}{(1+\tau^f)p_{x,t}^{f,h}(i)}\right)^{\frac{1}{1-\beta}} \left(G_t^f\right)^{\psi/(1-\beta)} l_{y,t}^{f,h}. \tag{15}$$

The optimal pricing rules are

$$p_{x,t}^{h,h}(i) = \frac{1}{\beta} w_t^h, p_{x,t}^{f,h}(i) = \frac{1}{\beta(1-b)} w_t^h. \tag{16}$$

Denoting $mc_t^{h,h}(i) = w_t^h$ and $mc_t^{f,h}(i) = \frac{1}{1-b} w_t^h$ as the marginal costs of producing one unit of $x_t^{h,h}(i)$ and $x_t^{f,h}(i)$, we can rewrite condition (16) as $p_{x,t}^{h,h}(i) = \frac{1}{\beta} mc_t^{h,h}$ and $p_{x,t}^{f,h}(i) = \frac{1}{\beta} mc_t^{f,h}$; namely, the "unconstrained" markup of the monopolistic intermediate-good firms is $1/\beta$. Given that the capital share β is about one-third, this suggests a monopolistic markup equal to 3. However, the empirical value of the monopolistic markup lies within the range of 1.05–1.4, far below $1/\beta \approx 3$ [see Jones and Williams (2000)]. An explanation for this gap is that the monopolistic firms may face the threat of imitation that reduces their markup. To characterize this fact, here, we follow a commonly adopted approach in the literature and introduce a parameter capturing patent breadth, $\mu^h > 1$. This parameter can be considered as a policy instrument determined by the patent authority [e.g. Li (2001), Goh and Olivier (2002), Chu and Cozzi (2014), and Yang (2021)]. Specifically, stronger patent protection increasing the patent breadth (i.e. a larger μ^h) raises the cost of imitators and allows monopolistic firms to charge a higher markup without losing market share to potential imitators. That is, the parameter μ^h serves as an upper bound on the markup of the monopolistic firms, that is, $p_{x,t}^{h,h}(i) = \mu^h \times mc_t^{h,h}$ and $p_{x,t}^{f,h}(i) = \mu^h \times mc_t^{f,h}$, where $\mu^h \in [1, 1/\beta]$. Throughout the analysis, we will specify μ^h as an exogenous variable by assuming that the patent breadth is treated as a policy parameter.[10] Accordingly, we have

$$p_{x,t}^{h,h}(i) = \mu^h w_t^h, p_{x,t}^{f,h}(i) = \mu^h \frac{w_t^h}{(1-b)}. \tag{17}$$

The above conditions indicate that intermediate-good firms are symmetric and that they set the same price. Thus, we have $p_{x,t}^{h,h}(i) = p_{x,t}^{h,h}$, $p_{x,t}^{f,h}(i) = p_{x,t}^{f,h}$,

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$x_t^{h,h}(i) = x_t^{h,h}$, $x_t^{f,h}(i) = x_t^{f,h}$, $l_{x,t}^h(i) = l_{x,t}^h$, and $\pi_{x,t}^h(i) = \pi_{x,t}^h$. By inserting (17) into (14) and after some calculations, we can obtain the total amount of monopolistic profits in country h as

$$\int_0^{A_t^h} \pi_{x,t}^h(i)di = A_t^h \pi_{x,t}^h = (\mu^h - 1) w_t^h L_{x,t}^h, \tag{18}$$

where $L_{x,t}^h \equiv \int_0^{A_t^h} l_{x,t}^h(i)di = A_t^h l_{x,t}^h$ is total labor employed in the intermediate goods sector.

3.4. R&D

New blueprints are developed by competitive R&D firms by using labor input and the existing stock of blueprints. Each blueprint creates a new type of intermediate good (or, equivalently, a new intermediate-good firm). In line with Romer (1990), the production function of new varieties is given by

$$\dot{A}_t^h = \varphi^h A_t^h L_{A,t}^h, \tag{19}$$

where $\varphi^h > 0$ is an R&D productivity parameter and $L_{A,t}^h$ denotes R&D labor in country h. In addition, the value of a blueprint V_t^h is determined by the standard no-arbitrage condition:

$$r_t^h V_t^h = \pi_{x,t}^h + \dot{V}_t^h. \tag{20}$$

This condition states that for each variety, the rate of return on an invention equals the sum of the monopolistic profit and capital gain/loss. The profit function of R&D firms can be written as $\pi_{A,t}^h = V_t^h \dot{A}_t^h - w_t^h L_{A,t}^h$. Given free entry into the R&D sector, the zero-profit condition of R&D implies:

$$\varphi^h A_t^h V_t^h = w_t^h. \tag{21}$$

3.5. Government

Since the government spending is financed by the tariff revenues, the balanced budget constraint is given by[11]

$$\int_0^{A_t^f} \tau^h p_{x,t}^{h,f}(j) x_t^{h,f}(j)dj = G_t^h. \tag{22}$$

3.6. Decentralized equilibrium

The decentralized equilibrium is a time path of allocations $\{C_t^h, C_t^f, Y_t^h, Y_t^f y_t^{h,h}, y_t^{h,f}, y_t^{f,h}, y_t^{f,f}, x_t^{h,h}(i), x_t^{f,h}(i), x_t^{h,f}(j), x_t^{f,f}(j), L_{x,t}^h, L_{x,t}^f, l_{y,t}^{h,h}, l_{y,t}^{f,h}, l_{y,t}^{h,f}, l_{y,t}^{f,f}, L_{A,t}^h, L_{A,t}^f, G_t^h, G_t^f\}_{t=0}^{\infty}$, prices $\{P_t^f, p_{y,t}^{h,h}, p_{y,t}^{f,h}, p_{y,t}^{h,f}, p_{y,t}^{f,f}, p_{x,t}^{h,h}(i), p_{x,t}^{f,h}(i), p_{x,t}^{h,f}(j), p_{x,t}^{f,f}(j), w_t^h, w_t^f, V_t^h, V_t^f\}_{t=0}^{\infty}$, policies $\{\tau^h, \tau^f\}$, such that at any instant of time:

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- the representative household in countries $k = h, f$ maximizes lifetime utility taking prices as given;
- competitive final-good firms in country h choose $\left\{y_t^{h,h}, y_t^{h,f}\right\}$ to maximize profit-taking prices as given;
- competitive final-good firms in country f choose $\left\{y_t^{f,h}, y_t^{f,f}\right\}$ to maximize profit-taking prices as given;
- competitive firms that produce final inputs in country h choose $\left\{l_{y,t}^{h,h}, x_{y,t}^{h,h}(i)\right\}$ and $\left\{l_{y,t}^{h,f}, x_{y,t}^{h,f}(j)\right\}$ to maximize profit-taking prices and policies as given;
- competitive firms that produce final inputs in country f choose $\left\{l_{y,t}^{f,f}, x_{y,t}^{f,f}(j)\right\}$ and $\left\{l_{y,t}^{f,h}, x_{y,t}^{f,h}(i)\right\}$ to maximize profit-taking prices and policies as given;
- monopolistic intermediate-good firms $i \in [0, A_t^h]$ in country h choose $\left\{x_t^{h,h}(i), x_t^{f,h}(i)\right\}$ to maximize profit taking w_t^h and patent breadth μ^h as given;
- monopolistic intermediate-good firms $j \in [0, A_t^f]$ in country f choose $\left\{x_t^{f,f}(j), x_t^{h,f}(j)\right\}$ to maximize profit taking w_t^f and patent breadth μ^f as given;
- R&D firms in countries $k = h, f$ choose $L_{A,t}^k$ to maximize profit taking $\left\{w_t^k, V_t^k\right\}$ as given;
- the labor market clears in both countries $k = h, f$, that is, $L_{A,t}^k + L_{x,t}^k + L_{y,t}^k = 1$, where $L_{y,t}^h \equiv l_{y,t}^{h,h} + l_{y,t}^{h,f}$ and $L_{y,t}^f \equiv l_{y,t}^{f,f} + l_{y,t}^{f,h}$.
- the total value of trade in intermediate goods is balanced such that $\int_0^{A_t^f} p_{x,t}^{h,f}(j) x_t^{h,f}(j) dj = \int_0^{A_t^h} p_{x,t}^{f,h}(i) x_t^{f,h}(i) di$.

In addition, as the total value of trade in intermediate goods is balanced, we can derive the resource constraint in this economy $Y_t^h = C_t^h + G_t^h + T_t^h$ where $T_t^h \equiv b \int_0^{A_t^h} p_{x,t}^{f,h}(i) x_t^{f,h}(i) di$ is the total transportation cost in terms of final goods. To keep the exposition simple, from the resource constraint, we can derive the relationship $C_t^h = \Lambda^h Y_t^h$ where $\Lambda^h = 1 - \alpha\beta + \frac{\alpha\beta(1-b)}{1+\tau^h}$ is a positive composite parameter.[12] Based on the relationship $C_t^h = \Lambda^h Y_t^h$, the term Λ^h can thus be interpreted as the proportion of final-good consumption to GDP, which is decreasing in the tariff.[13]

We are now ready to analyze the dynamics of the economy. In Appendix B, we show that since the dynamic system has one positive characteristic root coupled with one jump variable, the economy thus will jump immediately to a unique and stable balanced-growth path.[14] This result can be described by the following lemma:

LEMMA 1. *The aggregate economy always jumps immediately to a unique and stable balanced-growth path.*

Proof. See Appendix B. ■

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4. GROWTH EFFECT

In this section, we examine the effect of a tariff on long-run growth. At the steady state, the allocations of labor are stationary, which is described by the following lemma:

LEMMA 2. *The equilibrium allocations of labor in country h are given by*

$$L_x^h = \frac{1+\frac{\rho}{\varphi^h}}{\mu^h}\left(\frac{\Lambda^h-(1-\beta)}{\Lambda^h}\right), \tag{23}$$

$$L_A^h = 1-\left(1+\frac{(\mu^h-1)(1-\beta)}{\Lambda^h}\right)\frac{1+\frac{\rho}{\varphi^h}}{\mu^h}, \tag{24}$$

$$L_y^h = \frac{(1+\frac{\rho}{\varphi^h})(1-\beta)}{\Lambda^h}, \tag{25}$$

where $\Lambda^h = 1-\alpha\beta+\frac{\alpha\beta(1-b)}{1+\tau^h}$.

Proof. See Appendix C. ■

From Lemma 2, we can easily see that a rise in the tariff decreases labor in both the intermediate-goods and R&D sectors, that is, $\frac{\partial L_x^h}{\partial \tau^h}<0$, and $\frac{\partial L_A^h}{\partial \tau^h}<0$, while it increases final-good labor, that is, $\frac{\partial L_y^h}{\partial \tau^h}>0$.

The underlying economic intuition can be explained as follows. When the domestic tariff is raised, the demand for the imported intermediate goods decreases. Imposing the higher tariff reduces the foreign intermediate-good firms' revenues, which in turn causes a trade surplus for the home country. Given the assumption of trade balance, this trade surplus will be immediately recovered by a decrease in the home country's exports of intermediate goods. The reason why the home country exports less is that the foreign country's GDP falls because of the tariff, and thus it imports fewer intermediate goods from the home country. As a consequence, the domestic intermediate-good firms also sell less so that they hire less labor ($\frac{\partial L_x^h}{\partial \tau^h}<0$). As for the R&D sector, the incentive for R&D originates from the forward-looking behavior of the domestic intermediate-good firms that maximizes the expected profit. As the higher tariff negatively impacts the production of intermediate-good firms, it also depresses R&D, causing the R&D sector to employ less labor ($\frac{\partial L_A^h}{\partial \tau^h}<0$). Finally, when the tariff is raised, the foreign intermediate goods become more costly. This gives the home country's final input firms that import foreign intermediate goods an incentive to substitute labor for intermediate goods. Thus, we have $\frac{\partial L_y^h}{\partial \tau^h}>0$.

Substituting equations (8) and (9) into (5) and applying the symmetry condition in the intermediate-good sector, the final output can then be expressed as

$$Y_t^h = \frac{\left[\left(l_{y,t}^{h,h}\right)^{1-\beta}\left(x_t^{h,h}\right)^{\beta}\right]^{1-\alpha}\left(A_t^h\right)^{1-\alpha}\left(A_t^f\right)^{\alpha}\left[\left(l_{y,t}^{h,f}\right)^{1-\beta}\left(x_t^{h,f}\right)^{\beta}\right]^{\alpha}\left(G_t^h\right)^{\psi}}{(1-\alpha)\alpha}. \tag{26}$$

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Along the balanced-growth path, the government expenditure grows at the same rate as the final output, that is, $\frac{\dot{Y}^h}{Y^h} = \frac{\dot{G}^h}{G^h}$. Thus, by taking the log of equation (26) and differentiating with respect to time, we can obtain (see Appendix D):

$$\frac{\dot{Y}_t^h}{Y_t^h} = \frac{1-\beta}{1-\psi}\left[(1-\alpha)\frac{\dot{A}^h}{A^h} + \alpha\frac{\dot{A}_t^f}{A_t^f}\right]. \tag{27}$$

Defining $\gamma^h = \frac{\dot{Y}^h}{Y^h}$ as the balanced growth rate of final output in country h, equation (27) can be further rewritten as

$$\gamma^h = \frac{1-\beta}{1-\psi}\left[(1-\alpha)\varphi^h L_A^h + \alpha\varphi^f L_A^f\right], \tag{28}$$

where

$$L_A^k = 1 - \left[1 + \frac{\left(\mu^k - 1\right)(1-\beta)}{\Lambda^k}\right]\frac{\left(1+\rho/\varphi^k\right)}{\mu^k}, k = h, f.$$

The result reported in equation (28) leads us to establish the following proposition:

PROPOSITION 3. *Raising the tariff in one country depresses long-run growth in both countries.*

Proof. See Appendix D. ■

Proposition 1 delivers a similar insight as in Grossman and Helpman (1990), but the channel is rather different. Notice that in Grossman and Helpman (1990), because of the assumption of perfect international capital mobility that equalizes growth in both countries, a unilateral change in one country's tariff must lead to the same deteriorating effect on both countries' growth rates. This is not the case in our model. As implied by (28), raising the tariff in the home country can have negative impacts of different magnitudes on the domestic and foreign countries. We believe that this can be a more realistic way to characterize the tariff war.

As we have previously discussed with Lemma 2, raising the tariff depresses R&D activities in the home country. Moreover, as shown by (27), through trade in intermediate goods, the economic growth rate in one country is positively related to the R&D in both countries. Therefore, raising the tariff that hinders the home country's R&D will deteriorate growth in both countries. There is a vast literature that advocates a positive effect of trade on growth. The result of Proposition 1 joins this strand of the literature by highlighting the "innovation channel", that is, a trade barrier such as a tariff can reduce growth by suppressing R&D activities.

5. WELFARE IMPLICATIONS

This section conducts the welfare analysis. We first investigate the equilibrium tariffs chosen by noncooperative countries as we take the other country's actions as given (i.e. the Nash equilibrium). In Lemma 1, we show that the dynamic system

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has one positive characteristic root coupled with one jump variable. The steady-state equilibrium is thus characterized by local determinacy and there exists a unique growth path converging to it. Along the balanced-growth path, the representative household's consumption grows at a common rate γ^h. Let C_0^h denote the initial level of consumption. The time path of household's consumption can thus be written as $C_t^h = C_0^h e^{\gamma^h t}$. By substituting $C_t^h = C_0^h e^{\gamma^h t}$ into equation (1), we can derive the social welfare function (i.e. indirect lifetime utility of the household) under the regime of Nash equilibrium:

$$U^h = \frac{\ln C_0^h}{\rho} + \frac{\gamma^h}{\rho^2}. \tag{29}$$

Then, by differentiating U^h with respect to τ^h, we can obtain (see Appendix E)

$$\frac{\partial U^h}{\partial \tau^h} = \underbrace{\frac{1}{\rho^2}\frac{\partial \gamma^h}{\partial \tau^h}}_{-} + \underbrace{\frac{1}{\rho}\left[\frac{\alpha\beta - \psi}{(1-\psi)}\frac{\partial \Lambda^h}{\Lambda^h \partial \tau^h} + \frac{\psi}{1-\psi}\frac{1}{\tau^h(1+\tau^h)}\right]}_{?}. \tag{30}$$

The first term on the right-hand side (RHS) of (30) is the negative growth effect, as shown in Proposition 1. This term indicates that raising the tariff has a negative welfare effect because it reduces growth. The second term on the RHS of (30) is the effect of the tariff on the initial level of consumption. This term does not have a definite sign as it is composed of two opposite effects. On the one hand, as discussed previously, a higher tariff reduces the demand for domestic intermediate goods, which reduces final-goods production and thus consumption. On the other hand, raising the tariff increases the productive government spending, and therefore boosts the initial level of final-goods production and consumption. The overall effect of the tariff on the initial consumption level is then determined by the relative magnitudes of these two conflicting effects.

Notably, if the productivity of the government spending is disregarded, that is, $\psi \to 0$, we see that in (30), the initial consumption effect must be negative. This along with the negative growth effect indicates that raising the tariff always reduces welfare. In this case, hence, the Nash equilibrium tariff is zero, implying that there is no reason for noncooperative countries to set a positive tariff. However, when the government expenditure is sufficiently productive, that is, ψ is sufficiently high, a tariff will boost the initial level of consumption, and thus yields a positive effect on welfare. If this effect outweighs the negative growth effect, the Nash equilibrium tariff is no longer zero.

The ambiguous sign of equation (30) provides a theoretical explanation for why noncooperative countries have incentives to choose a positive tariff. As a result of the mathematical complexity of (30), however, we are unable to have an explicit condition under which the Nash equilibrium tariffs are greater than zero. In the next section, our numerical analysis based on the USA data will demonstrate the feasibility of positive Nash equilibrium tariffs.

We now proceed to analyze the optimal tariff in the interest of global welfare, and compare it with the levels of tariffs under the Nash equilibrium. The global

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welfare level is defined as a simple sum of two countries' welfare:

$$U^G = U^h + U^f = \frac{\ln \Lambda^h + \ln Y_0^h}{\rho} + \frac{\gamma^h}{\rho^2} + \frac{\ln \Lambda^f + \ln Y_0^f}{\rho} + \frac{\gamma^f}{\rho^2}. \tag{31}$$

Let τ_{NE}^h (τ_{NE}^f) denote the tariff chosen by the noncooperative country h (f) to maximize U^h (U^f), and let τ_G^h and τ_G^f be the tariffs that maximize the global welfare level U^G. Then, we differentiate U^G with respect to τ^h, which can be expressed as

$$\frac{\partial U^G}{\partial \tau^h} = \frac{\partial U^h}{\partial \tau^h} + \frac{1}{\rho}\frac{1}{Y_0^f}\frac{\partial Y_0^f}{\partial \tau^h} + \frac{1}{\rho^2}\frac{\partial \gamma^f}{\partial \tau^h}. \tag{32}$$

Evaluating (32) at the point $\tau^h = \tau_{NE}^h$ gives that the first term on the RHS of (32) equals zero. Moreover, the second term on the RHS is negative (see Appendix E) and the third term is also negative (Proposition 1). That is:

$$\frac{\partial U^G}{\partial \tau^h}\Big|_{\tau^h=\tau_{NE}^h} = \underbrace{\frac{\partial U^h}{\partial \tau^h}\Big|_{\tau^h=\tau_{NE}^h}}_{0} + \underbrace{\frac{1}{\rho}\frac{1}{Y_0^f}\frac{\partial Y_0^f}{\partial \tau^h}}_{-} + \underbrace{\frac{1}{\rho^2}\frac{\partial \gamma^f}{\partial \tau^h}}_{-} < 0. \tag{33}$$

Analogously, we can also infer that:

$$\frac{\partial U^G}{\partial \tau^f}\Big|_{\tau^f=\tau_{NE}^f} = \underbrace{\frac{\partial U^f}{\partial \tau^f}\Big|_{\tau^f=\tau_{NE}^f}}_{0} + \underbrace{\frac{1}{\rho}\frac{1}{Y_0^h}\frac{\partial Y_0^h}{\partial \tau^f}}_{-} + \underbrace{\frac{1}{\rho^2}\frac{\partial \gamma^h}{\partial \tau^f}}_{-} < 0. \tag{34}$$

Equations (33) and (34) imply that $\tau_G^h < \tau_{NE}^h$ and $\tau_G^f < \tau_{NE}^f$, which lead us to establish the following proposition:

PROPOSITION 4. *The globally optimal levels of tariffs are smaller than the levels of tariffs under the Nash equilibrium.*

Proof. Proven in the text. ■

The result of Proposition 2 is quite intuitive. A higher tariff in country h depresses the production and economic growth of country f. Under the setting of Nash equilibrium, this negative welfare externality is not taken into account by the home country. The same logic applies to the foreign country. As a result, both countries will sub-optimally choose too high a tariff from the perspective of global welfare. The welfare implication behind Proposition 2 is that introducing a supra-governmental authority (e.g. the World Trade Organization) that coordinates countries' tariff policies can be globally welfare improving.[15]

6. NUMERICAL RESULTS

In this section, we perform a numerical simulation to provide further insights. Instead of providing a comprehensive quantitative evaluation, our goals are

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TABLE 1. Baseline parameters

Parameter	Value	Source/Target
ρ	0.04	Kydland and Prescott (1991)
β	0.33	Jones (1995)
μ	1.1	Laitner and Stolyarov (2004)
τ	0.03	Data (World Bank)
ψ	0.05	Ercolani and Azevedo (2012)
α	0.42	Chu *et al.* (2015)
b	0.268	Transportation cost/GDP = 6%
φ	2.45	Output growth rate = 2%

to highlight the growth and welfare effects of the tariff, and to illustrate the possibility of positive optimal tariffs.

We consider two symmetric countries as our baseline case; thus, the superscripts h and f of parameters are suppressed. The model has eight parameters contained in the set: $\{\rho, \beta, \mu, \tau, \psi, \alpha, b, \varphi\}$. The parameters are chosen from commonly used values in the existing literature or calibrated to match the USA data. The discount rate is set to $\rho = 0.04$ [Kydland and Prescott (1991)], and the labor share is set to two-thirds [Jones (1995)], that is, $\beta = 0.33$. Based on the estimates in Laitner and Stolyarov (2004), the monopolistic markup is around 1.1; thus we set $\mu = 1.1$. According to the figure reported by the World Bank's World Development Indicators, the tariff rate in the US is around 3%, that is, $\tau = 0.03$.[16] The productivity of the government expenditure is set to $\psi = 0.05$ by following Ercolani and Azevedo (2012).[17] As for α, we use $\alpha = 0.42$ by following Chu *et al.* (2015), who analyze optimal monetary policies by developing a two-country R&D-based growth model similar to ours. The international transportation cost is calibrated such that the total transportation cost accounts for 6% of GDP [Balistreri and Hillberry (2006)]; thus we obtain $b = 0.268$. Finally, to generate a steady-state output growth rate of 2%, we derive the R&D productivity $\varphi = 2.45$. Table 1 reports our baseline parameter values.

Figure 1 depicts the growth effect and Figure 2 depicts the welfare effect of the tariff. In these two figures, we see that raising the tariff decreases economic growth, while it has a reverse U-shaped relation with individual country's welfare, implying that there exists a positive optimal tariff. The result is consistent with our analytical results reported in Proposition 1 and it also demonstrates the possibility that the Nash equilibrium tariffs can be greater than zero.

Figure 3 depicts how the Nash equilibrium tariff (τ_{NE}) and globally optimal tariff (τ_G) respond to the parameter reflecting the productivity of the government spending ψ. In association with a given value of ψ, the Nash equilibrium tariff is higher than the globally optimal tariff. In particular, Figure 3 shows that both the Nash equilibrium tariff and the globally optimal tariff are increasing in the

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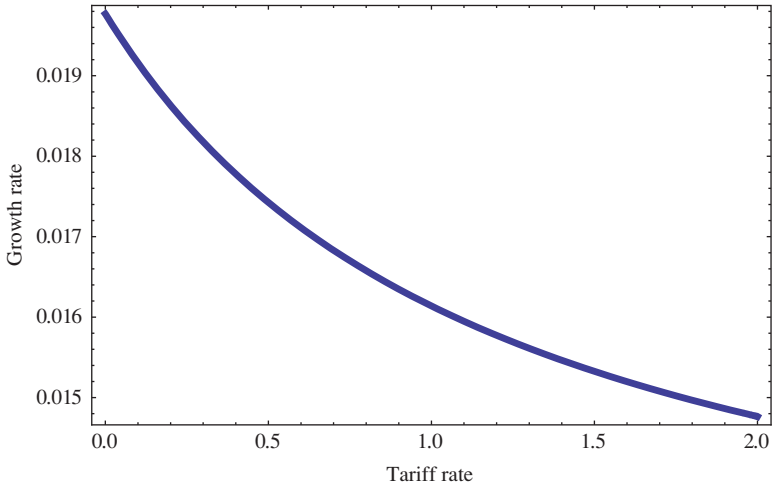


FIGURE 1. The growth effect of the tariff.

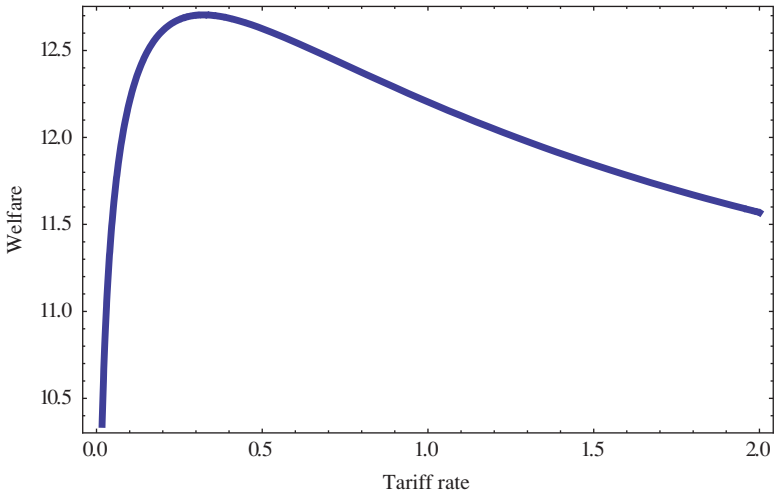


FIGURE 2. The welfare effect of the tariff.

government productivity parameter. The result exhibited in Figure 3 is quite intuitive as the benefit of the tariff comes from the productive government spending. When the government spending is more productive, the optimal tariffs should be higher.

Figure 4 depicts the relation between the R&D productivity and the Nash equilibrium tariff. The result shows that the Nash equilibrium tariff is decreasing in the parameter reflecting the R&D productivity. The intuition can be explained as follows. As shown in Proposition 1, raising the tariff reduces growth, and in turn

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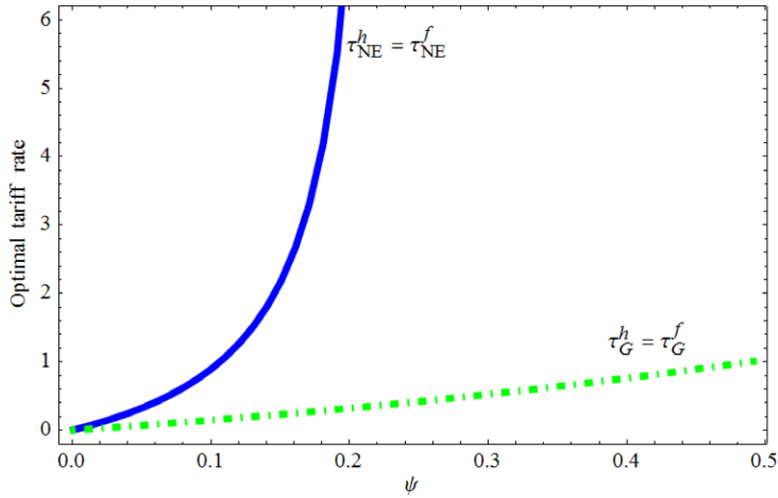


FIGURE 3. The Nash equilibrium tariff versus the globally optimal tariff.

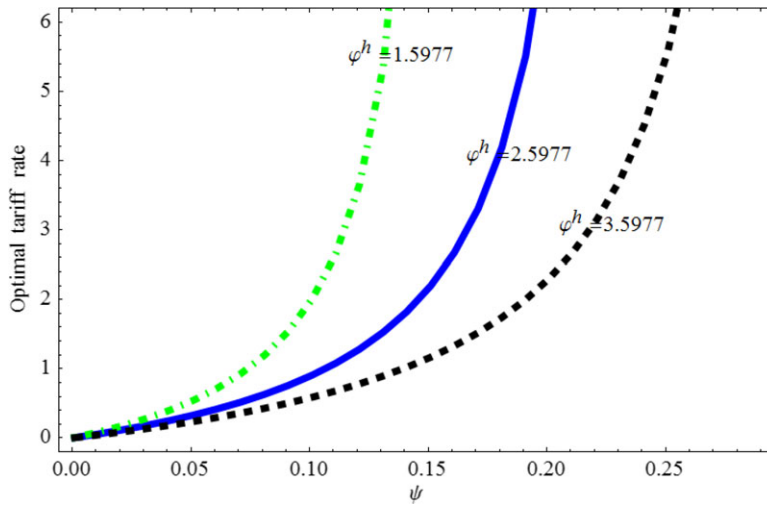


FIGURE 4. The Nash equilibrium tariff and R&D productivity.

has a negative welfare effect. This is the "innovation channel" through which the tariff affects growth and welfare that we highlight in this paper. When the R&D sector is more productive, the innovation channel is more important, making the negative growth effect of raising the tariff stronger. As a consequence, the country is motivated to set a lower tariff rate.

Figure 5 shows how the Nash equilibrium tariff and globally optimal tariff respond to the parameter representing the patent breadth, that is, the level of

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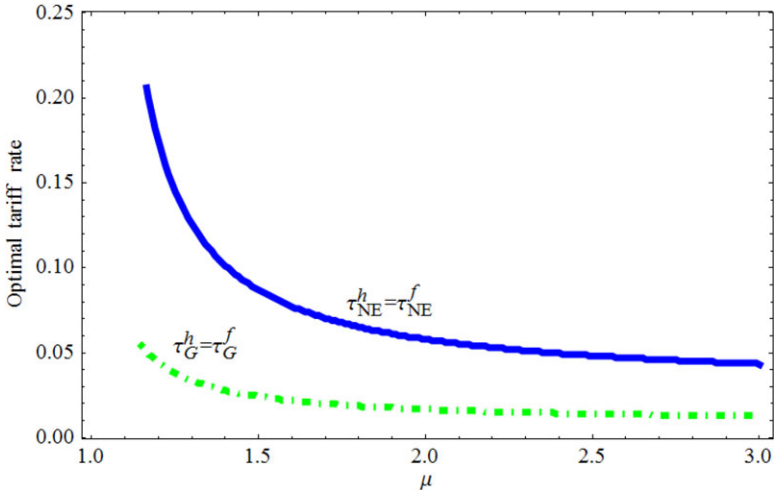


FIGURE 5. The optimal tariffs and patent breadth.

patent protection. In Figure 5, we see that a higher level of patent protection is associated with a lower optimal tariff, regardless of whether from the perspective of global welfare of noncooperative countries. The intuition is similar to what we have discussed in Figure 4. The negative welfare effect of the tariff results from its negative impact on growth. In an economy where the patent protection is stronger, the intermediate-good firms can charge a higher markup. This increases their profits, which then turn into the incentives for R&D. As this innovation channel is more important with a stronger patent protection, the negative effect of raising the tariff on growth is larger as well. Accordingly, the optimal tariffs should be smaller.

7. EXTENSIONS

In this section, we provide two extensions. In the first extension, we consider the case where two countries are asymmetric in terms of population size. In the second extension, we allow the positive effects of the productive government spending to differ across sectors. To save space, we only sketch out the main equation in the text and relegate the mathematical details to the appendix.

7.1. Asymmetric country size

Our baseline model considers two symmetric countries. However, a tariff war between countries with different population sizes can be more realistic, and thus examining this case could provide us with practical implications. Therefore, in this subsection, we relax this assumption by considering the asymmetric sizes of countries, and examine how relative country size affects our results. To do this,

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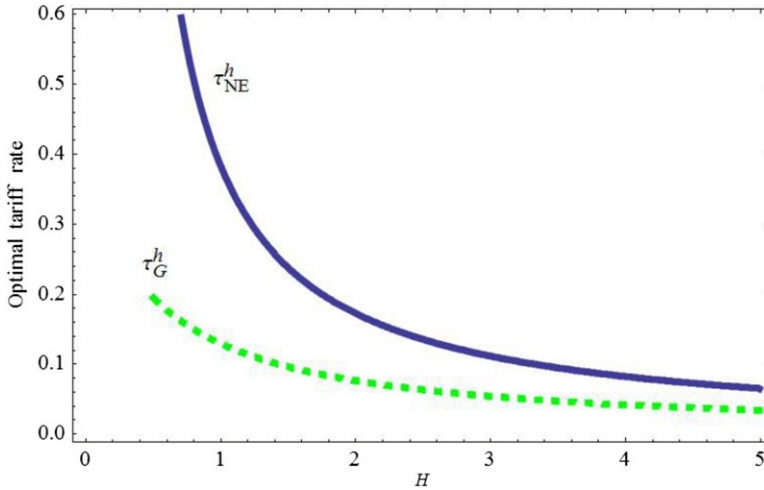


FIGURE 6. The optimal tariffs and relative size of countries.

we set the aggregate labor supply of the home country to be H:

$$L^h_{A,t} + L^h_{x,t} + L^h_{y,t} = H.$$

For simplicity, the aggregate labor supply of the foreign country is kept normalized to unity ($L^f_{A,t} + L^f_{x,t} + L^f_{y,t} = 1$) as in the baseline model. The parameter H, therefore, represents the relative population size of the two countries. Then, we vary H in the range of 0.5–5 to see how the Nash equilibrium tariff and the globally optimal tariff respond to relative country size. The results are shown in Figure 6.

We see from Figure 6 that the Nash equilibrium tariff is positive, and it is larger than the globally optimal tariffs. Hence, our main results are robust when countries have different population sizes. Moreover, the optimal tariffs are decreasing in H, meaning that a relatively large country tends to have a lower optimal tariff. This result echoes the findings in Naito (2019), who shows that a larger country sets a lower optimal tariff. This is also related to Naito (2020) since in his paper, a zero tariff is optimal for a larger country.[18] The intuition behind Figure 6 can be explained by the "scale effect" of the Romer (1990) model, that is, the growth rate is determined by the number of R&D workers. When a country has a larger population, due to the scale effect, the growth rate is higher. Accordingly, the negative growth effect of raising the tariff, as mentioned in Proposition 1, would be stronger, which discourages the country from setting a higher tariff.

7.2. Home bias in government spending

In this subsection, we relax the previous assumption that the government spending has an identical effect on different sectors. Specifically, we allow for different

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magnitudes of the government spending by replacing equations (8) and (9) with:

$$y_t^{h,h} = \left(G_t^h\right)^{\psi_1} \left(l_{y,t}^{h,h}\right)^{1-\beta} \int_0^{A_t^h} \left(x_t^{h,h}(i)\right)^{\beta} di, \tag{8}'$$

$$y_t^{h,f} = \left(G_t^h\right)^{\psi_2} \left(l_{y,t}^{h,f}\right)^{1-\beta} \int_0^{A_t^f} \left(x_t^{h,f}(j)\right)^{\beta} dj, \tag{9}'$$

in which ψ_1 reflects the productivities of the government spending in the sector of domestically produced intermediate goods and ψ_2 in the sector of the imported intermediate goods.

The bottom line of this generalization is that it will not change the qualitative nature of our results. To see this, we can insert (8)′ and (9)′ into (5) to obtain:

$$Y^h = \frac{\left[\left(l_y^{h,h}\right)^{(1-\beta)} \left(x^{h,h}\right)^{\beta}\right]^{1-\alpha} \left(A_t^h\right)^{1-\alpha} \left(A_t^f\right) \left[\left(l_y^{h,f}\right)^{(1-\beta)} \left(x^{h,f}\right)^{\beta}\right]^{\alpha} \left(G^h\right)^{\Psi}}{(1-\alpha)\alpha}. \tag{35}$$

where $\Psi \equiv (1-\alpha)\psi_1 + \alpha\psi_2$ is defined as the weighted average of the magnitudes of the government spending for the two sectors. By inspecting (35) and (26), we see that these two equations are identical except for the power term of G^h. This means that the analytical result that follows is still sustained, at least qualitatively.

To provide further insights, we perform a numerical exercise regarding this extension. In general, we expect the government spending to have a "home-bias" effect [e.g. Giovanni (2005)], in the sense that it possesses a higher productivity in the sector of domestically produced intermediate goods than in the sector of the imported intermediate goods, that is, $\psi_1 > \psi_2$. To capture this effect, we fix $\psi_1 = 0.05$ and use lower values so that $\psi_2 = 0$ and $\psi_2 = 0.03$. We also consider the case of a "foreign-bias" government spending by setting $\psi_2 = 0.07$. The growth and welfare effects of the tariff regarding these cases are shown in Figure 7, from which we see that our previous results are robust to this extension.

Finally, we examine how the Nash equilibrium tariff changes with ψ_1 and ψ_2. In Figure 8, the solid (dashed) line, namely the $\psi_2 = 0.05$ line (the $\psi_1 = 0.05$ line), means that we fix $\psi_2 = 0.05$ ($\psi_1 = 0.05$) while varying ψ_1 (ψ_2). As shown in this figure, the Nash equilibrium tariff is increasing in both parameters, since increasing each of them raises the weighted average productivity of the government spending. Figure 8 also reveals that the Nash equilibrium tariff is more sensitive to ψ_1. The implication we draw from this extension is that, given ψ_2, a stronger home bias for government spending results in a higher optimal tariff.

8. CONCLUDING REMARKS

This paper builds up a two-country R&D-based growth model to examine the interplay among the tariff, innovation, and growth. We find that when the home country raises its tariff, it will not only depress the growth of the foreign country, but also have a negative impact on the home country's innovation and growth.

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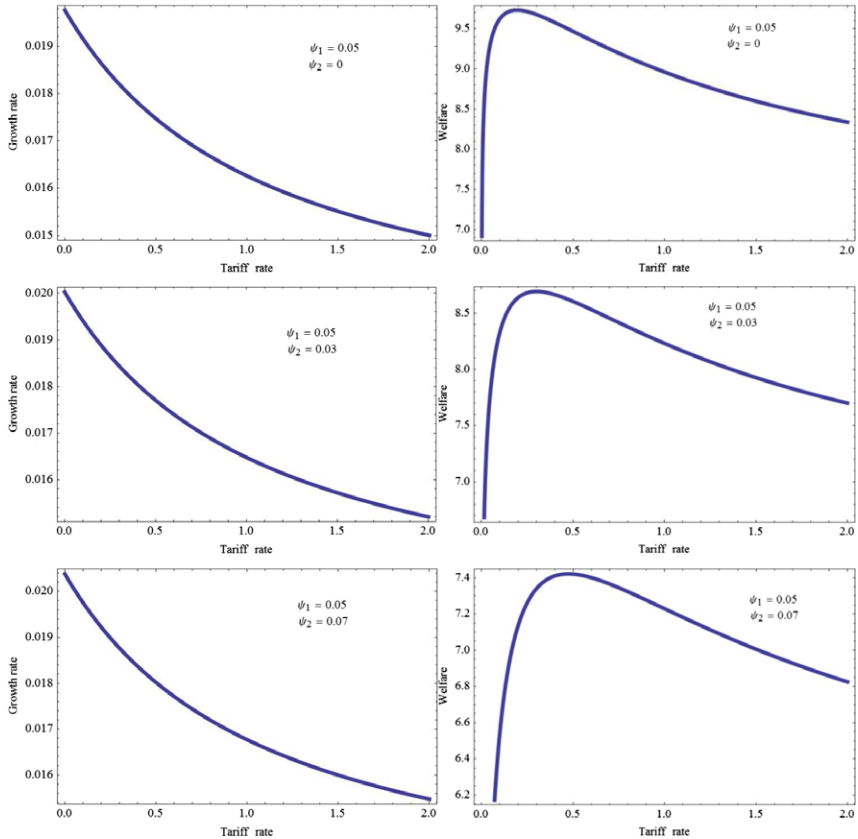


FIGURE 7. The growth and welfare effects of the tariff with different magnitudes of the productivity of government spending.

This result basically supports the view that freer trade is beneficial to economic growth. As for welfare, we show that, even though the tariff reduces growth, each country may still choose a positive tariff under the setting of Nash equilibrium in which each country only cares about its own residents' welfare. This provides a theoretical explanation for why tariffs are not zero in most countries. In addition, we also find that a zero tariff may be suboptimal in terms of maximizing global welfare.

This paper follows the literature on trade and growth in assuming a time-independent tariff [e.g. Grossman and Helpman (1990), Akcigit *et al.* (2018), and Naito (2020)]. This implies that our analysis of the tariff war is in the spirit of a one-shot game. Alternatively, one could consider analyzing a tariff war using a repeated game framework, but embedding a dynamic (time-dependent) tariff into our macroeconomic model will make it much less tractable.[19] According to game theory, cooperative outcomes may be sustained when the game is played

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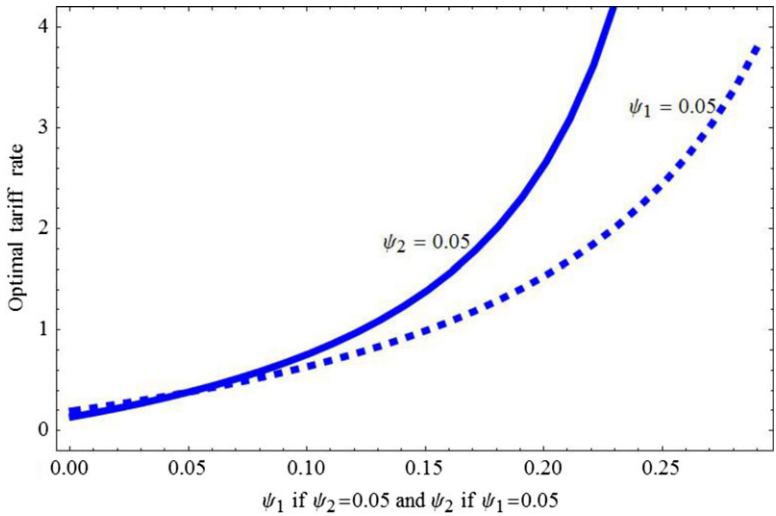


FIGURE 8. The Nash equilibrium tariff and the productivities of government spending.

repeatedly. Therefore, if we apply a repeated game to the current model, it is possible that the globally optimal tariffs will be achieved even if countries act non-cooperatively. Despite the cost of much greater complexity, we believe this is a valuable extension that deserves future exploration.

NOTES

1. Empirical studies suggest that trade has a positive effect on growth (or a higher tariff reduces growth) include, among others, Edwards (1992, 1998), Harrison (1996), Frankel and Romer (1999), Dollar and Kraay (2004) and Aghion *et al.* (2013). See also Billmeier and Nannicini (2009) for a brief review of the empirical literature. The theoretical literature will be reviewed in the next section.

2. Atkeson and Burstein (2010) specify two forms of innovation: product innovation, that is, innovation that increases the number of intermediate varieties, and process innovation, that is, innovation that increases the productivity (or reduces the production cost) of existing firms. Our model, in line with the literature on innovation-led growth models, only considers the product innovation.

3. Trade in intermediate goods plays the most important role in international trade as its share in total trade is about two-thirds [Bems *et al.* (2011)]. Moreover, as pointed out by Miroudot *et al.* (2009), imports of intermediates are more sensitive to trade costs. Thus, our analysis focuses on the effect of a tariff (a trade cost) on intermediate goods trade.

4. Barro (1990) and Futagami *et al.* (1993) first introduce the role of productive government expenditure into an endogenous growth model. Moreover, see, for example, Ercolani and Azevedo (2012) for the empirical evidence suggesting that public investment enhances the productivity of private factors.

5. Devereux and Lapham (1994) compare the world growth rate with and without trade, but they do not examine the issue of (optimal) tariffs, which is the main theme of our paper. Devereux (1997) analyzes the Nash equilibrium tariffs in a growth model, which is featured by learning-by-doing but is absent from innovation. Tang and Walde (2001) examine the welfare effect of openness to trade.

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These papers do not explore the optimal level of tariffs from the global perspective and compare them to the Nash equilibrium tariffs.

6. In the absence of the tariff policy, Grossman and Lai (2004) examine the intellectual property rights policy and Kondo (2013) examines the R&D subsidies in open-economy R&D-based endogenous growth models.

7. There is a vast literature on strategic trade policy that proposes a positive optimal tariff. See Brander (1995) for an earlier survey and, for example, Felbermayr *et al.* (2013) and Demidova (2017) for recent contributions. These studies do not consider growth models and thus do not explore the relationship between trade and growth. In addition, Azarnert (2018) presents a Ricardian model of trade, and shows that a tariff can be growth–enhancing. His study, however, is not an endogenous growth model and absent from innovation.

8. We follow most of the endogenous growth literature in assuming that government spending only increases the productivity of final outputs. However, more generally, government spending can also have a positive effect on the productivity of R&D. The inclusion of this effect in our model will enhance the benefits of raising the tariff, and as a result, a higher tariff may not necessarily harm economic growth. Nonetheless, our results of a positive optimal tariff will only be strengthened.

9. In the Peretto (2007) and Iwaisako (2013) models, the government spending benefits final goods production, which corresponds to the final inputs in our present model. Our analytical results will be unaffected, at least qualitatively, no matter the government spending affects the final goods sector or the final inputs sector. To be more specific, we can also assume that the government spending benefits the final goods sector by replacing $Y_t^h = \frac{(y_t^{h,h})^{1-\alpha}(y_t^{h,f})^{\alpha}}{(1-\alpha)\alpha}$ in (5) with $Y_t^h = \frac{(G_t^h)^{\psi}(y_t^{h,h})^{1-\alpha}(y_t^{h,f})^{\alpha}}{(1-\alpha)\alpha}$ and removing $(G_t^h)^{\psi}$ in (8) and (9), but this will not alter our analytical results.

10. The qualitative nature of our analytical results will not change in the absence of μ^h. However, introducing this parameter enables our model to deliver more realistic quantitative results.

11. In a previous version (available upon request) of this paper, we assume that, in addition to the tariff, the government can also use the (exogenous) labor income tax to finance the government spending. Our main results are robust to this generalization.

12. See Appendix A for the derivation of the relationship $C_t^h = \Lambda^h Y_t^h$ and the resource constraint.

13. When the domestic tariff is increased, the home country imports less from the foreign country. Under the balanced trade condition, the foreign country will also import less from the home country. This reduces the home country's aggregate income and thus its total consumption. We will elaborate on this intuition with Lemma 2 below.

14. As claimed in the literature, for example, Turnovsky (2000), in a dynamic rational expectation model where the number of unstable (positive) roots equals the number of jump variables in the dynamic system, the economy exists as a unique perfect foresight equilibrium solution.

15. Notably, Oladi and Beladi (2008) show that such trade coordination (i.e. the global trade bloc) is viable, while regional trade blocs are not viable.

16. See https://data.worldbank.org/indicator/TM.TAX.MANF.SM.AR.ZS?locations=US.

17. We choose the lowest value within the estimation by Ercolani and Azevedo (2012). The optimal tariff would be larger if we adopt a larger value of ψ.

18. In Naito (2020), a zero tariff can be supported as a Nash equilibrium. In our analysis, by contrast, the Nash equilibrium tariff is positive as long as the government expenditure is productive.

19. For example, Riezman (1991) and Saggi (2006) study dynamic optimal tariffs in a repeated game framework. However, these papers are static models and thus do not deal with economic growth.

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APPENDIX A: DERIVATION OF THE RESOURCE CONSTRAINT

This appendix provides the derivation of the relationship $C_t^h = \Lambda^h Y_t^h$ and the economy's resource constraint.

First, we combine (6)–(11) and apply the symmetry condition to obtain:

$$A_t^h p_{x,t}^{h,h} x_t^{h,h} = \beta(1-\alpha) Y_t^h, \tag{A1}$$

$$A_t^h (1+\tau^f) p_{x,t}^{f,h} x_t^{f,h} = \beta\alpha P_t^f Y_t^f. \tag{A2}$$

Then, multiplying both sides of (14) by A_t^h and inserting (A1) and (A2) into the resulting expression yield:

$$A_t^h \pi_{x,t}^h = \beta(1-\alpha) Y_t^h + \frac{(1-b)}{1+\tau^f} \beta\alpha P_t^f Y_t^f - w_t^h L_{x,t}^h. \tag{A3}$$

Equipped with (A3) and (18), we have:

$$w_t^h L_{x,t}^h = \frac{\beta}{\mu^h} \left((1-\alpha) + \frac{(1-b)}{1+\tau^f} \alpha \frac{P_t^f Y_t^f}{Y_t^h} \right) Y_t^h. \tag{A4}$$

The analogous expression of (A2) in country f is given as

$$A_t^f (1+\tau^h) p_{x,t}^{h,f} x_t^{h,f} = \beta\alpha Y_t^h. \tag{A5}$$

The condition of balanced trade is $\int_0^{A_t^f} p_{x,t}^{h,f}(j) x_t^{h,f}(j) dj = \int_0^{A_t^h} p_{x,t}^{f,h}(i) x_t^{f,h}(i) di$. Under symmetry, the condition can be expressed as $A_t^f p_{x,t}^{h,f} x_t^{h,f} = A_t^h p_{x,t}^{f,h} x_t^{f,h}$. When the tariff is raised, the relative price $p_{x,t}^{f,h}/p_{x,t}^{h,f}$ (terms of trade) will adjust so as to balance trade.

By substituting (A2) and (A5) into $A_t^f p_{x,t}^{h,f} x_t^{h,f} = A_t^h p_{x,t}^{f,h} x_t^{f,h}$, we can then obtain:

$$\frac{(1+\tau^f)}{(1+\tau^h)} = \frac{P_t^f Y_t^f}{Y_t^h}. \tag{A6}$$

Equation (A6) is a derivative condition of balanced trade that is expressed by the final goods of the two countries, in which terms of trade is replaced using the relative price of final goods P_t^f (given $P^h = 1$ at all times). Thus, as the tariff changes, the relative price P_t^f is the mechanism that adjusts to satisfy the relation described by (A6).

By inserting (A6) into (A4) to eliminate $P_t^f Y_t^f$, and after some calculations, we obtain:

$$w_t^h L_{x,t}^h = \frac{1}{\mu^h} \left(\Lambda^h - (1-\beta) \right) Y_t^h, \tag{A7}$$

where $\Lambda^h = 1 - \alpha\beta + \frac{\alpha\beta(1-b)}{1+\tau^h}$ is a composite parameter.

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Then, we substitute $a_t^h = A_t^h V_t^h$ into the household's constraint (2) to obtain:

$$\dot{A}_t^h V_t^h + A_t^h \dot{V}_t^h = r_t^h A_t^h V_t^h + w_t^h L_t^h - C_t^h. \tag{A8}$$

By substituting the labor market-clearing condition $L_{A,t}^h + L_{x,t}^h + L_{y,t}^h = L_t^h = 1$ into (A8), we have:

$$C_t^h = r_t^h A_t^h V_t^h + w_t^h (L_{A,t}^h + L_{x,t}^h + L_{y,t}^h) - \dot{A}_t^h V_t^h - A_t^h \dot{V}_t^h.$$

Next, we multiply both sides of (20) by A_t^h to obtain $A_t^h r_t^h V_t^h = A_t^h \pi_{x,t}^h + A_t^h \dot{V}_t^h$, and then multiply both sides of (19) by V_t^h to obtain $V_t^h \dot{A}_t^h = V_t^h \varphi^h A_t^h L_{A,t}^h$, which equals $w_t^h L_{A,t}^h$ by using (21). Finally, we insert the above expressions into (A8)', and then put (A7) and (18) into it; after some calculations, we can obtain:

$$C_t^h = \left[\Lambda^h - (1-\beta)\right] Y_t^h + w_t^h L_{y,t}^h. \tag{A9}$$

Using (6), (7), (12), (13), $L_{y,t}^h \equiv l_{y,t}^{h,h} + l_{y,t}^{h,f}$, and the zero-profit condition, we have:

$$w_t^h L_{y,t}^h = (1-\beta) Y_t^h. \tag{A10}$$

Inserting (A10) into (A9) yields the relationship between C_t^h and Y_t^h:

$$C_t^h = \Lambda^h Y_t^h. \tag{A11}$$

We now move to derive the resource constraint $Y_t^h = C_t^h + G_t^h + T_t^h$. First, by applying the symmetry condition to the government resource constraint (22) and inserting (A5) into the resulting expression, we obtain:

$$G_t^h = \left(\frac{\tau^h}{(1+\tau^h)} \alpha\beta \right) Y_t^h. \tag{A12}$$

Moreover, by applying the symmetry condition to the transportation cost T_t^h and using (A2), we can derive:

$$T_t^h = b A_t^h p_{x,t}^{f,h} x_t^{f,h} = \frac{b\alpha\beta}{(1+\tau^f)} P_t^f Y_t^f. \tag{A13}$$

We then eliminate $P_t^f Y_t^f$ in (A13) by using (A6), which gives:

$$T_t^h = \frac{b\alpha\beta}{(1+\tau^h)} Y_t^h. \tag{A14}$$

Finally, putting $C_t^h = \Lambda^h Y_t^h$, (A12), and (A14) together, after some algebraic manipulations, we can derive the resource constraint $Y_t^h = C_t^h + G_t^h + T_t^h$.

APPENDIX B: PROOF OF LEMMA 1

This appendix proves Lemma 1 that is associated with the dynamics of this model. Based on the expression $C_t^h = \Lambda^h Y_t^h$ reported in (A11), we have $\frac{\dot{C}_t^h}{C_t^h} = \frac{\dot{Y}_t^h}{Y_t^h}$. Combining it with (4) yields:

$$\frac{\dot{Y}_t^h}{Y_t^h} = r_t^h - \rho. \tag{B1}$$

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By taking log of both side of (A7) and differentiating the resulting expression with respect to time gives rise to

$$\frac{\dot{w}_t^h}{w_t^h} + \frac{\dot{L}_{x,t}^h}{L_{x,t}^h} = \frac{\dot{Y}_t^h}{Y_t^h} \tag{B2}$$

Combining (B1) and (B2) together yields:

$$\frac{\dot{w}_t^h}{w_t^h} = r_t^h - \rho - \frac{\dot{L}_{x,t}}{L_{x,t}}. \tag{B3}$$

The next step is to eliminate r_t^h in (B3). It follows from (18) to (21) along with some tedious calculations, we obtain:

$$r_t^h = \varphi^h \left\{ \left(\mu^h - 1\right) L_{x,t}^h - L_{A,t}^h \right\} + \frac{\dot{w}_t^h}{w_t^h}. \tag{B4}$$

Equipped with (B3) and (B4), we can then derive:

$$\frac{\dot{L}_{x,t}}{L_{x,t}} = \varphi^h \left[\left(\mu^h - 1\right) L_{x,t}^h - L_{A,t}^h \right] - \rho. \tag{B5}$$

We now turn to eliminate $L_{A,t}^h$ in (B5). Based on (6), (7), (12), and (13) coupled with the symmetry condition, we have:

$$w_t^h L_{y,t}^h / Y_t^h = (1 - \beta). \tag{B6}$$

Using (B6) along with (A7), we can express $L_{y,t}^h$ as a function of $L_{x,t}^h$, which is given by

$$L_{y,t}^h = \frac{(1-\beta)}{\Lambda^h - (1-\beta)} \mu^h L_{x,t}^h. \tag{B7}$$

Substituting (B7) into the labor market-clearing condition $L_{A,t}^h + L_{x,t}^h + L_{y,t}^h = 1$, we infer the relation between $L_{A,t}^h$ and $L_{x,t}^h$:

$$L_{A,t}^h = 1 - L_{x,t}^h - \frac{(1-\beta)}{\Lambda^h - (1-\beta)} \mu^h L_{x,t}^h. \tag{B8}$$

Finally, by inserting (B8) into (B5), we show that the dynamic system in this economy can be expressed by one differential equation:

$$\frac{\dot{L}_{x,t}}{L_{x,t}} = \frac{\Lambda^h}{\Lambda^h - (1-\beta)} \varphi^h \mu^h L_{x,t}^h - \varphi^h - \rho. \tag{B9}$$

The positive characteristic root of (B9) is $\frac{\Lambda^h}{\Lambda^h - (1-\beta)} \varphi^h \mu^h > 0$, which proves that the economy jumps immediately to a unique and stable balanced-growth path.

APPENDIX C: PROOF OF LEMMA 2

Along the BGP, the allocations of labor are stationary, implying that the result $\dot{L}_{x,t}^h = 0$ holds true. Putting $\dot{L}_{x,t}^h = 0$ into (B9) yields (23) in the main text. Then, we can substitute (23) into (B8) to obtain (24) and substitute (23) into (B7) to obtain (25). Thus, the proof of Lemma 2 is completed.

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APPENDIX D: DERIVATION OF (27) AND PROOF OF PROPOSITION 1

First, substituting (A12) into (26) and rearranging the resulting expression to eliminate G_t^h gives rise to:

$$Y_t^h = \left[\frac{\left[\left(l_{y,t}^{h,h} \right)^{1-\beta} \left(x_t^{h,h} \right)^{\beta} \right]^{1-\alpha} \left(A_t^h \right)^{1-\alpha} \left(A_t^f \right)^{\alpha} \left[\left(l_{y,t}^{h,f} \right)^{1-\beta} \left(x_t^{h,f} \right)^{\beta} \right]^{\alpha}}{(1-\alpha)\alpha} \right]^{1/(1-\psi)} \times \left(\frac{\tau^h}{(1+\tau^h)} \alpha\beta \right)^{\psi/(1-\psi)}. \tag{D1}$$

Next, we insert (17) and (A7) into (A1) to obtain

$$x_t^{h,h} = \frac{(1-\alpha)\beta}{A_t^h} \frac{L_{x,t}^h}{\Lambda^h - (1-\beta)}. \tag{D2}$$

Similarly, substituting the analogous expression of (17), that is, $p_{x,t}^{h,f}(j) = \mu^f \frac{w_t^f}{(1-b)}$, into (A5) gives rise to:

$$x_t^{h,f} = \frac{\beta\alpha Y_t^h}{(1+\tau^h) A_t^f \mu^f w_t^f} (1-b). \tag{D3}$$

Then, by inserting the analogous expression of (A7) in country f and (A6) into (D3), which yields:

$$x_t^{h,f} = \frac{\alpha\beta(1-b)}{A_t^f (1+\tau^f)} \frac{L_{x,t}^f}{\left(\Lambda^f - (1-\beta)\right)}. \tag{D4}$$

Finally, putting (D2), (D4), $l_{y,t}^{h,f} = \alpha L_{y,t}^h$, $l_{y,t}^{h,h} = (1-\alpha) L_{y,t}^h$, and (D1) together yields:

$$Y_t^h = \Omega \left\{ \left(A_t^h \right)^{(1-\alpha)(1-\beta)} \left(A_t^f \right)^{\alpha(1-\beta)} \left[\left(L_{y,t}^h \right)^{1-\beta} \left(L_{x,t}^h \right)^{\beta} \right]^{1-\alpha} \left[\left(L_{y,t}^f \right)^{1-\beta} \left(L_{x,t}^f \right)^{\beta} \right]^{\alpha} \right\}^{1/(1-\psi)}, \tag{D5}$$

where

$$\Omega \equiv \left(\frac{\tau^h}{(1+\tau^h)} \alpha\beta \right)^{\psi/(1-\psi)} \left(\frac{\Xi \left(\frac{(1-\alpha)\beta}{[\Lambda^h - (1-\beta)]} \right)^{\beta(1-\alpha)} \left(\frac{\alpha\beta(1-b)}{(1+\tau^f)[\Lambda^f - (1-\beta)]} \right)^{\alpha\beta}}{(1-\alpha)\alpha} \right)^{\frac{1}{1-\psi}}$$

and $\Xi = (1-\alpha)^{(1-\beta)(1-\alpha)} \alpha^{(1-\beta)\alpha}$.

Given that labor allocation is stationary along the BGP, we take log of (D5) and differentiate the resulting expression with respect to time, which gives us equation (27) in the main text.

Given that Lemma 2 implies that a higher tariff reduces R&D labor, that is, $\frac{\partial L_A^h}{\partial \tau^h} < 0$, and (28) indicates that the growth rate in one country is increasing in R&D labor for both countries, it is directly inferred that a higher tariff reduces growth in both countries, which proves Proposition 1.

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APPENDIX E: DERIVATION OF (30)

First, by inserting the condition $C_0^h = \Lambda^h Y_0^h$ into (29), the social welfare function can be alternatively expressed as

$$U^h = \frac{\ln\left(\Lambda^h\right) + \ln Y_0^h}{\rho} + \frac{\gamma^h}{\rho^2}. \tag{E1}$$

Initially, the economy is at a steady state. By substituting the above expression into (E1), and differentiating it with respect to τ^h, we have:

$$\frac{\partial U^h}{\partial \tau^h} = \frac{1}{\rho}\frac{\partial \Lambda^h}{\Lambda^h \partial \tau^h} + \frac{1}{\rho}\frac{1}{Y_0^h}\frac{\partial Y_0^h}{\partial \tau^h} + \frac{1}{\rho^2}\frac{\partial \gamma^h}{\partial \tau^h}. \tag{E2}$$

Without loss of generality, the initial levels of technology are normalized to unity, that is, $A_0^h = A_0^f = 1$. Putting $t = 0$ into (D5) with $A_0^h = A_0^f = 1$ gives:

$$Y_0^h = \Omega\left\{\left[\left(L_{y,0}^h\right)^{1-\beta}\left(L_{x,0}^h\right)^{\beta}\right]^{1-\alpha}\left[\left(L_{y,0}^f\right)^{1-\beta}\left(L_{x,0}^f\right)^{\beta}\right]^{\alpha}\right\}^{1/(1-\psi)}. \tag{E3}$$

Then, we insert (23) and (25) and their analogous expressions in country f into (E3), and differentiate the resulting expression with respect to τ^h. After tedious calculations, we can obtain:

$$\frac{\partial Y_0^h}{\partial \tau^h} = \left[-\frac{1-\alpha\beta}{(1-\psi)\Lambda^h}\frac{\partial \Lambda^h}{\partial \tau^h} + \frac{\psi}{1-\psi}\left(\frac{1}{\tau^h(1+\tau^h)}\right)\right] Y_0^h > 0, \tag{E4}$$

where

$$\frac{\partial \Lambda^h}{\partial \tau^h} = -\frac{\alpha\beta(1-b)}{(1+\tau^h)^2} < 0. \tag{E5}$$

Finally, by substituting (E4) and (E5) into (E2), we can derive (30) in the main text. Moreover, differentiating (E3) with respect to τ^f and putting (E5) into the resulting expression yield:

$$\frac{\partial Y_0^h}{\partial \tau^f} = -\frac{\alpha\beta\,(1-\alpha\beta)}{(1-\psi)\,(1+\tau^f)\left(1-\alpha\beta+\frac{\alpha\beta(1-b)}{(1+\tau^f)}\right)} Y_0^h < 0, \tag{E6}$$

which demonstrates that the sign of the second term on the RHS of (33) is negative.

APPENDIX F: MATHEMATICAL DERIVATIONS IN SECTION 7.1

This appendix derives the steady-state labor allocations in the case where the two countries have asymmetric population sizes. First, we make a restatement of equation (B7) in Appendix B:

$$L_{y,t}^h = \frac{(1-\beta)}{\Lambda^h - (1-\beta)}\mu^h L_{x,t}^h. \tag{B7}$$

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In this extension, the home country's labor market-clearing condition is given by $L_{A,t}^h + L_{x,t}^h + L_{y,t}^h = H$. By inserting (B7) into the above labor market-clearing condition yields:

$$L_{A,t}^h = H - L_{x,t}^h - \frac{(1-\beta)}{\Lambda^h - (1-\beta)}\mu^h L_{x,t}^h. \tag{F1}$$

Then, by inserting (F1) into (B5), we show that the dynamic system in this economy can be expressed by one differential equation:

$$\frac{\dot{L}_{x,t}^h}{L_{x,t}^h} = \frac{\Lambda^h}{\Lambda^h - (1-\beta)}\varphi^h \mu^h L_{x,t}^h - H\varphi^h - \rho. \tag{F2}$$

The characteristic root of the dynamic system reported in (F2) is positive, that is, $\frac{\Lambda^h}{\Lambda^h-(1-\beta)}\varphi^h\mu^h > 0$; this proves that the economy jumps immediately to a unique and stable balanced-growth path.

At the steady state, labor allocation is stationary, that is, $\dot{L}_{x,t}^h = 0$. Applying this to (F2), we have the steady-state value (the time index is dropped):

$$L_x^h = \frac{H + \rho/\varphi^h}{\mu^h}\frac{\left[\Lambda^h - (1-\beta)\right]}{\Lambda^h}. \tag{F3}$$

By substituting (F3) into (B7), we can obtain:

$$L_y^h = \left(H + \rho/\varphi^h\right)\frac{(1-\beta)}{\Lambda^h}. \tag{F4}$$

Lastly, by putting (F3), (F4), and the labor market-clearing condition together, we have:

$$L_A^h = H - \left(1 + \frac{(1-\beta)(\mu^h - 1)}{\Lambda^h}\right)\frac{\left(H + \rho/\varphi^h\right)}{\mu^h}. \tag{F5}$$

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