The 2016 Election: Like 1888 but not 1876 or 2000

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ABSTRACT Donald J. Trump won the 2016 US presidential election with fewer popular votes than Hillary R. Clinton. This is the fourth time this has happened, the others being 1876, 1888, and 2000. In earlier work, we analyzed these elections (and others) and showed how the electoral winner can often depend on the size of the US House of Representatives. This work was inspired by Neubauer and Zeitlin (2003, 721–5) in their paper, "Outcomes of Presidential Elections and the House Size." A sufficiently larger House would have given electoral victories to the popular vote winner in both 1876 and 2000. An exception is the election of 1888. We show that Trump's victory in 2016 is like Harrison's in 1888 and unlike Hayes's in 1876 and Bush's in 2000. This article updates our previous work to include the 2016 election. It also draws attention to some of the anomalous behavior that can arise under the Electoral College.

For the fourth time in American history, the winner of the popular vote in the presidential election is not the same as the winner of the electoral vote. Donald J. Trump obtained 46.09% of the popular vote (62,984,825 votes) to Hillary R. Clinton's 48.18% (65,853,516 votes).[1] Despite this, Clinton obtained only 233 votes in the Electoral College compared to Trump's 305.[2] The outcome was particularly ironic because Trump himself declared on Twitter in 2012, "The Electoral College is a disaster for a democracy."[3] As in 1876, 1888, and 2000, the winner of the electoral vote was a Republican. Following Nurmi (1998), we refer to these outcomes as examples of a "referendum paradox"—that is, a situation in which the electoral winner is not the same as the winner under a direct popular vote (i.e., a referendum).[4]

Barthélémy, Martin, and Piggins (2014) analyzed these presidential elections (and others) and noted how the electoral winner occasionally can depend on particular (and somewhat arbitrary) features of the Electoral College that they referred to as "architecture." More specifically, they showed (using a device called a representation graph) how variations in the size of the House of Representatives, *inter alia*, can change the electoral winner without anyone changing how they vote. This can occur because each state's electoral vote is based on its total congressional representation (i.e., its two Senators and House delegation), the relative size of which depends on its population. As House size increases (or decreases), the population-based component of electoral votes increases (or decreases); this implies that the relative weight of states in the Electoral College changes with House size. This in turn implies that a constant popular vote distribution may produce different shares of electoral votes—and perhaps different electoral vote winners—as House size changes.

This "House size effect" was first observed for the 2000 election by Neubauer and Zeitlin (2003) in an important paper published in this journal. A theoretical explanation was subsequently provided by Miller (2014). Neubauer and Zeitlin showed that in the 2000 election, a sufficiently large House would have given electoral victory to Al Gore instead of George W. Bush.[5] Barthélémy, Martin, and Piggins (2014) showed that a House size effect was also present in the 1876 election. In that election, a sufficiently large House would have given electoral victory to Samuel Tilden instead of Rutherford Hayes. This election is notable not only in that Tilden obtained more popular votes than Hayes but also in that he won an absolute majority (51%) of the popular vote. We can treat 1876 and 2000 as similar in that sufficiently larger House sizes would have guaranteed electoral vote majorities for the popular vote winner.[6]

An exception to this is the election of 1888. Grover Cleveland obtained more popular votes than Benjamin Harrison and lost the electoral vote. Barthélémy, Martin, and Piggins (2014) showed

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that no House size effect was present in that election; therefore, changing the size of the House would not have been to Cleveland's advantage. They called the referendum paradox of 1888 "entrenched" in that none of the architectural variations that they considered would have resulted in Cleveland's election.[7]

The purpose of this article is to update Barthélémy, Martin, and Piggins (2014) by considering the results of the 2016 presidential election, which was conspicuously subject to the referendum paradox. We show that Trump's victory was "entrenched" like Harrison's in 1888 but unlike Bush's in 2000 and Hayes's in 1876. None of the variations that we consider would have changed the electoral vote winner—despite the fact that 2016 was the most "severe" presidential election inversion in history in that Clinton had a substantial popular vote advantage whereas Trump had a substantial electoral vote advantage.[8] We believe that this quite strikingly demonstrates some of the anomalous behavior that can arise under the Electoral College.

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REPRESENTATION GRAPH

Our basic analytical device is a representation graph (Barthélémy, Martin, and Piggins 2014). This is a simple two-dimensional graph, with the number of electors measured on the horizontal axis and the proportion of electoral votes for the Democratic candidate measured on the vertical axis. Currently, the number of electors is 538, one for each of the 100 members of the Senate and 435 members of the House of Representatives, plus three for the District of Columbia.

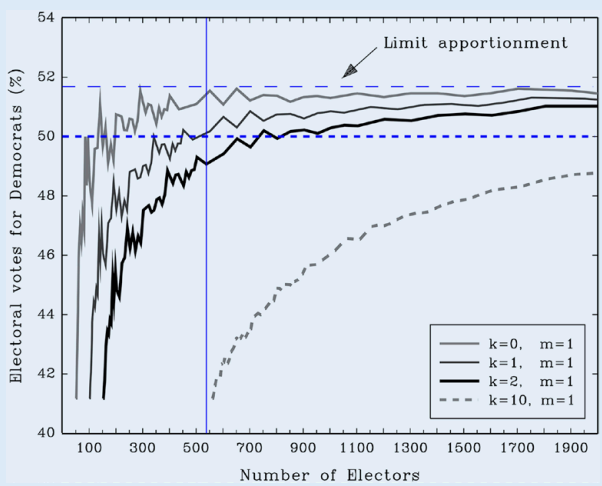
We used election data to graph how the proportion of electoral votes for the Democrat changes as the size of the House increases or decreases. For example, as House size increases (*ceteris paribus*), the number of electors will increase, and we move to the right along the horizontal axis in figure 1. For each additional seat in the House, we computed a new congressional apportionment and calculated the resulting proportion of electoral votes for the Democrat.[9] This allowed us to graph the effect of House size on election outcomes. Note that in all of our figures, the Democratic electoral vote percentage is displayed only in increments of five with respect to House size (and total electors), partly concealing what Miller (2014) called the "locally chaotic" relationship between House size and the electoral vote.

In our figures, if the plotted graph ever crosses 50%, then there is a House size effect.[10] The Electoral College, given that particular House size, will elect a different president. To aid the exposition, we first reproduce figure 1 in Barthélémy, Martin, and Piggins (2014), which refers to the 2000 presidential election.

A vertical line is drawn at 538 electors and a 50% dashed horizontal line is also plotted: k refers to the number of electoral votes each state has by virtue of its Senators and m is the lower bound on the number of electors a state has by virtue of its Representatives. The Constitution specifies that k=2 and m=1, which corresponds to the bold black line in figure 1. Four representation graphs are plotted, one for each of the possible parameter values listed in the legend. The referendum paradox of 2000 can be identified from the figure because the bold black line crosses the vertical line representing 538 electoral votes below the dashed 50% line. But while Gore loses at the existing House size, a sufficiently larger House would have ensured his victory. The representation graph lies above 50% as House size increases. The graph also exhibits the non-monotonicity identified by Neubauer and Zeitlin (2003); the winner oscillates back and forth for a certain interval of House sizes. Neubauer and Zeitlin (2003) showed that if the size of the House is smaller than 491, then Bush is always the winner; if it is greater than 597, then Gore is always the winner (with, somewhat surprisingly, a tie at 655). Between these two numbers, sometimes Bush narrowly wins, sometimes Gore narrowly wins, and sometimes there is a tie. The winner fluctuates chaotically.

However, in figure 1, as House size increases further the graph approaches a second horizontal dashed line labeled "limit apportionment." This horizontal line is drawn at the electoral vote percentage that the Democratic candidate would obtain if the size of the House was equal to the total apportionment population.[11] In this hypothetical situation, the size of the House is so large that each state, in effect, casts a number of electoral votes equal to its population.[12] As a matter of logic then, under the "winner-takes-all" rule, the limit apportionment electoral vote percentage must equal the percentage of the US apportionment population that

Figure 1
2000 Election



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resides in states carried by the Democratic candidate. Indeed, in the 2000 election, Gore carried states that accounted for 51.68% of the total apportionment population, which is the limit apportionment electoral vote percentage plotted in figure 1.[13] The corresponding representation graphs for the 1876 election are similar to figure 1.[14]

Figure 2 is the corresponding figure for the 2016 presidential election. Importantly, we can see in figure 2 that there is no House size effect. The four representation graphs plotted in figure 2 are safely below 50% on the vertical axis. Unlike 1876 and 2000, a larger House would not have been advantageous to the popular vote winner. Moreover, the limit apportionment is approximately 43.7%.[15]

Hillary Clinton's defeat in the Electoral College is "entrenched" and would have occurred under any of the variations considered. The only historical parallel is the election of 1888, which is shown in figure 4 in Barthélémy, Martin, and Piggins (2014) and reproduced here as figure 3. In 1888, Harrison received 233 electoral votes to Cleveland's 168; therefore, the vertical line in figure 3 is drawn for a total of 401 electors. The limit apportionment in figure 3 is 41.04%. Like Clinton, a larger House would not have been to Cleveland's advantage.

An immediate difference between figures 2 and 3 is that the four representation graphs plotted in figure 2 appear to be "inverses" of the corresponding graphs plotted in figure 3. For example, consider the k=10, m=1 graph. In figure 2, this graph begins beneath the limit apportionment electoral percentage and approaches it (from below) as House size increases. In figure 3, the corresponding graph begins above the limit apportionment electoral percentage and approaches it (from above) as House size increases. The fact that a representation graph approaches the limit apportionment electoral vote percentage as House size increases is a mathematical inevitability that was explained previously. However, the difference between the two k=10 graphs requires an explanation.[16] Trump carried 30 states to Clinton's 21 (including DC); therefore, if the number of electoral votes a state has by virtue of its Senators is increased from 2 to 10, then Trump's "Senate" electoral votes increase from 60 to 300 and Clinton's increase from 42 to 210. The smallest size that the Electoral College can be at this point is 561, with each state receiving one "House" electoral vote, with 11 electoral votes allocated to the District of Columbia. This is no different than a federal situation in which a majority of states (including DC) is sufficient to determine the electoral winner without any weight given to their relative populations. In this scenario, Clinton obtains 231/561≅41.18% of the electoral vote, which is why the graph begins below the limit apportionment electoral vote percentage of 43.7%.

As House size increases, the importance of these "Senate" electoral votes declines relative to "House" electoral votes and the graph approaches 43.7%. In 1888, Harrison carried 20 states to Cleveland's 18, and the states he carried accounted for a majority of the total apportionment population (i.e., approximately 58.96% to Cleveland's 41.04%).[17] A similar calculation shows that in the k=10, m=1 situation, Cleveland obtains 198/418≅47.37% of the electoral vote, which is why the graph begins above the limit

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Figure 2
2016 Election

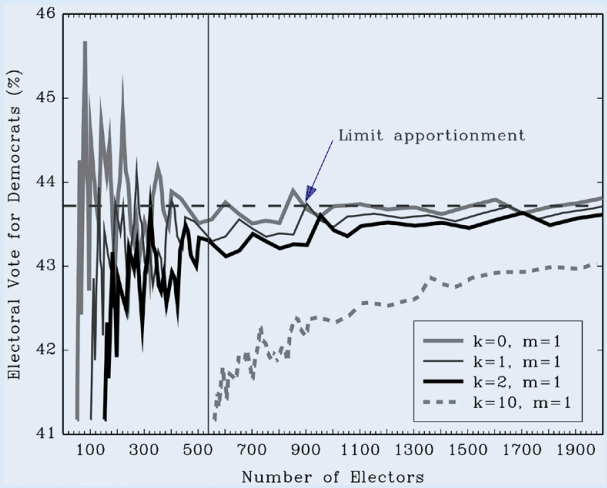
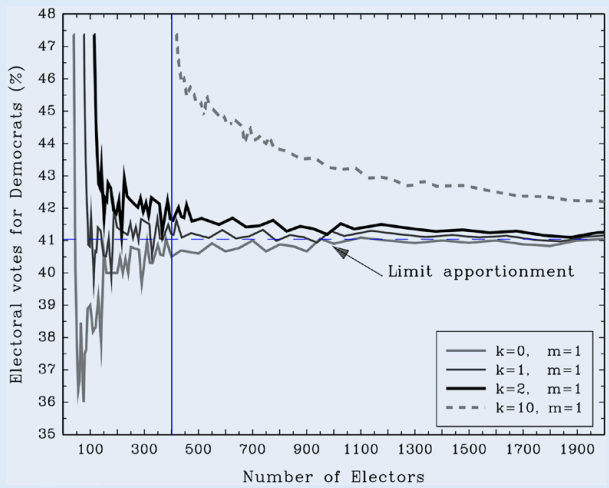


Figure 3
1888 Election



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apportionment electoral vote percentage of 41.04%. As before, the graph approaches 41.04% as House size increases. The explanation for the inverses, then, is that Clinton's loss under federalism was *more* severe than her loss under a weighted voting system in which each state casts a number of votes equal to its population. For Cleveland, this loss was *less* severe (recall that he carried slightly less than half of the states).

HOUSE SIZE EFFECT

When is there a House size effect? In an important paper, Miller (2014) identified the following sufficient condition.[18]

> **Proposition:** Given any method of apportioning seats in the House into whole numbers, a presidential election is subject to the House size effect if (1) one candidate (say A) carries a majority of states, and (2) the other candidate (say B) carries states that collectively hold a majority of the total apportionment population.

As a sufficient condition, a House size effect must arise as a matter of logic if conditions (1) and (2) are jointly satisfied. This sufficient condition was satisfied in both the 1876 and 2000 elections but not in 1888 and 2016. In 2000, Bush carried 30 states to Gore's 21 (including DC); therefore, condition (1) is satisfied. As noted previously, Gore's states accounted for approximately 51.68% of the total apportionment population; therefore, condition (2) is satisfied. In 1876, Hayes carried 21 states to Tilden's 17. However, Tilden's states accounted for 51.8% of the total apportionment population.[19] Again, conditions (1) and (2) are satisfied.

Therefore, Miller's sufficient condition shows that a House size effect must be present in both 1876 and 2000. Indeed, we verified through our simulations the existence of a House size effect for the 2000 election in figure 1, and the 1876 election is addressed in figure 5(a) in Barthélémy, Martin, and Piggins (2014).[20]

The intuition behind the Proposition was alluded to previously and we briefly repeat it here. When we are at the constitutional floor of one House seat per state, then the candidate who carries the majority of states (including DC) wins the electoral vote because each state casts three electoral votes (i.e., one "House" vote and two "Senate" votes). In the 2000 election, this candidate was Bush. As House size increases, the importance of these two "Senate" electoral votes decline relative to the "House" electoral votes. As House size increases, further and further, we approach the limit apportionment electoral vote percentage. At this point, the House has grown so large that, in effect, only "House" electoral votes count. The electoral vote percentage (and outcome) is determined by the fraction of the apportionment population in the states carried by the candidates. At some point in this process of making the House larger, the representation graph must cross over the 50% line and (for the 2000 election) Gore is elected President.[21]

Importantly, whereas conditions (1) and (2) are sufficient conditions for a House size effect, they are not necessary conditions. The reason for this is that House seats must be allocated as whole numbers, not fractional numbers. If House seats were allocated fractionally to states, each state could receive its exact "quota" of seats, which is the perfect proportion of the 435 seats in the House of Representatives it is due as a result of its share of the apportionment population. In these circumstances, Miller proved that conditions (1) and (2) are necessary and sufficient for a House size effect.[22]

This scenario would make our simulation in figure 2 unnecessary because the failure of condition (2) to be satisfied would be enough to know that there is no effect present for the 2016 election. However, given that seats are awarded in whole numbers, there is no way of knowing whether a House size effect is present in the 2016 election without running a simulation.

That said, Miller (2014, 266) provided another useful proposition (Proposition A in his paper) that has given historically accurate predictions of a House size effect, despite not being quite valid as a theoretical proposition (again, because House seats must be allocated as whole numbers). Proposition A states that a presidential election is subject to the House size effect if and only if: (1) one candidate (say A) wins a majority of the Senate electoral votes, and (2) the other candidate (say B) wins a majority of the House electoral votes. As discussed previously, Trump carried 30 states and therefore won 60 "Senate" electoral votes (a majority). However, he also won 305–60=245 "House" electoral votes (again, a majority). Therefore, condition (2) does not hold and Proposition A predicts that there will be no House size effect. Our simulation shows that this prediction is accurate.

In summary, figure 2 shows that Trump's election in 2016—like Harrison's in 1888 but unlike Hayes's in 1876 and Bush's in 2000—entailed an entrenched referendum paradox. It is tempting to think that one way to avoid the referendum paradox is to abandon "winner-takes-all" and allow states to allocate their electoral votes fractionally, in proportion to the popular vote percentages in each state.[23] However, such proportionality in general does not overcome the referendum paradox because it remains true that state electoral votes are not precisely proportional to state populations (and still less to state presidential popular votes). For example, in the 2000 election, such a procedure would generate approximately 259.17 electoral votes for Bush compared with 258.27 for Gore.[24] As this example demonstrates, the presence of third-party candidates means that a majority winner in the Electoral College is unlikely under a fractional system.[25] In the 2016 election, fractional voting would have produced approximately 255.43 electoral votes for Clinton compared with 249.07 for Trump. Unlike the election in 2000, Clinton (the popular vote winner) would emerge as the plurality winner in this case.

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NOTES

1. Data for the 2016 election are from "Official 2016 Presidential Election Results," Federal Election Commission, January 30, 2017. Available at www.fec.gov/pubrec/fe2016/2016presgeresults.pdf. Accessed April 20, 2017. Other election data used in this article are from David Leap's comprehensive *Atlas of US Presidential Elections*, available at http://uselectionatlas.org.
2. Seven of these electors were "faithless" and did not vote as pledged. This is an unusually high number. Throughout this article, we treat them as voting as pledged. For reasons given in footnote 7, we also assume that Clinton obtained all four of Maine's electoral votes.
3. Tweet by Donald J. Trump (@realDonaldTrump) on November 6, 2012, at 8:45 p.m. Available at https://goo.gl/KzaoIi. Accessed May 3, 2017.
4. Miller (2012) used the term "electoral inversion" rather than "referendum paradox"; we understand that this is the more common term in US political science. Note that the 1824 presidential election also could be viewed as paradoxical, in that the popular vote winner, Andrew Jackson, lost the election to John Quincy Adams.

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Jackson won a plurality of electoral votes but not a majority due to the presence of two additional candidates (William Crawford and Henry Clay), both of whom carried some states. Following the procedure described in the 12th Amendment, the House of Representatives subsequently elected Adams as president.

5. Neubauer and Zeitlin showed that the relationship between House size and electoral winner is not necessarily monotonic, which Miller (2014) referred to as "local chaos." Note that the size of the House was fixed at 435 in 1911. There was a temporary increase to 437 when Alaska and Hawaii were admitted as states in 1959. However, for the apportionment of seats in the 1960 census, which took effect for the election in 1962, the number of seats reverted to 435.
6. However, unlike Gore in 2000, certain smaller House sizes also would have guaranteed the election of Tilden in 1876 (see Miller 2014, 270). Interestingly, in 1876, the method of apportionment required by law was Hamilton's method. A total of 292 seats were apportioned by the 1870 census, with an extra three electoral votes for the new state of Colorado. Because there were 37 states (excluding Colorado), there were 74 Senators; therefore, the total number of electors was 369. Surprisingly, the apportionment does not correspond to Hamilton's method, but to Dean's method. This was particularly unfortunate for Tilden because he would have won under Hamilton's method. See Balinski and Young (2001, 37).
7. Barthélémy, Martin, and Piggins (2014) varied not only the size of the House in their election simulations but also the method of apportionment, the number of electors a state has by virtue of its Senators (currently two), and the lower bound (or floor) on the number of electors a state has by virtue of its representatives in the House (currently one). The Constitution determines these latter two values, whereas Congress determines House size and the method of apportionment. The method used to allocate a state's electoral votes among candidates is determined by the legislature of each state. At present, every state but two allocates all of its electoral votes to its popular plurality winner, whereas Maine and Nebraska each allocates one electoral vote to the popular plurality winner in each of its congressional districts and two electoral votes to its state-wide popular plurality winner. In Maine in 2016, this produced three electoral votes for Clinton and one for Trump. In the present analysis, we did not vary the method of apportioning House seats to states because this has no bearing on our observations pertaining to the 2016 election, and we must treat Maine and Nebraska as if they use the "winner-takes-all" allocation rule because there is no way to determine popular vote winners within the hypothetical districts implied by varying House size.
8. In percentage terms, Tilden had a greater popular vote advantage but suffered a minute (and bitterly contested) one-electoral-vote loss; similarly, Harrison won a somewhat greater electoral vote victory, but the popular vote was closer. Of course, the 2000 election was extremely close in both popular and electoral votes.
9. Consistent with the provisions of the 23rd Amendment, in our simulations we treat the District of Columbia as receiving the same number of electoral votes as the least populous state.
10. Our simulations were performed using the software GAUSS (see www.aptech.com).
11. The apportionment population includes the resident population for the 50 states, as ascertained by the decennial census, plus counts of overseas US military and federal civilian employees. The apportionment population excludes the population of the District of Columbia.
12. The two additional votes cast by each state by virtue of their Senators become insignificant when the total vote cast is so large.
13. Miller (2014, 266).
14. See figure 5(a) in Barthélémy, Martin, and Piggins (2014, 116).
15. Census data used for the 2010 apportionment is available at www.census.gov/data/tables/2010/dec/2010-apportionment-data.html.
16. A similar explanation accounts for the "inverse" nature of the k=0, k=1, and k=2 graphs in figures 2 and 3.
17. The states Trump carried accounted for approximately 56.3% of the apportionment population. Trump's situation and Harrison's are, therefore, similar in that both won a majority of states, and the states they won contained a majority of the total apportionment population.
18. This is Proposition 2 in Miller (2014), stated in its entirety.
19. Data are from table 1 in Miller (2014).
20. The House size effect being present for the 2000 election is, of course, Neubauer and Zeitlin's (2003) original finding.
21. As Neubauer and Zeitlin (2003) noted, there is no single crossover point.
22. This is Proposition 1 in Miller (2014). Note that fractional seats would eliminate the non-monotonicity observed by Neubauer and Zeitlin (2003).
23. For example, if Clinton obtained 34.36% of the popular vote in Alabama she would receive 3.0924 of Alabama's nine electoral votes.
24. Authors' calculations.
25. Miller (2009) considered fractional electoral votes (but proportional to popular vote percentages, not state populations) in the context of measuring individual voting power under different Electoral College variants.

REFERENCES

Balinski, Michel L., and H. Peyton Young. 2001. *Fair Representation: Meeting the Ideal of One Man, One Vote*. Washington, DC: Brookings Institution Press.

Barthélémy, Fabrice, Mathieu Martin, and Ashley Piggins. 2014. "The Architecture of the Electoral College, the House Size Effect, and the Referendum Paradox." *Electoral Studies* 34: 111–18.

Miller, Nicholas R. 2009. "A Priori Voting Power and the US Electoral College." *Homo Oeconomicus* 26: 341–80.

Miller, Nicholas R. 2012. "Electoral Inversions by the US Electoral College." In *Electoral Systems: Paradoxes, Assumptions, and Procedures*, eds. Dan S. Felsenthal and Moshé Machover, 93–127. Berlin: Springer.

Miller, Nicholas R. 2014. "The House Size Effect and the Referendum Paradox in US Presidential Elections." *Electoral Studies* 35: 265–71.

Neubauer, Michael G., and Joel Zeitlin. 2003. "Outcomes of Presidential Elections and the House Size." *PS: Political Science & Politics* 36: 721–5.

Nurmi, Hannu. 1998. "Voting Paradoxes and Referenda." *Social Choice and Welfare* 15: 333–50.

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