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Using Patent Capital to Estimate Tobin's *Q*

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Abstract

I construct a new proxy for Tobin's *Q* that incorporates the replacement cost of patent capital. This proxy, which I call patent *Q*, explains up to 62% more variation in investment than other proxies for *Q*. Furthermore, investment is more sensitive to patent *Q* than to other proxies for *Q*. Although investment is predicted more accurately by, and is more sensitive to, patent *Q*, controlling for patent *Q* leads to relatively larger, not smaller, cash flow coefficients. All results are stronger in subsamples with more patent capital. Overall, patent *Q* strengthens the historically weak investment–*Q* relation.

I. Introduction

Tobin's *Q*, which is the market value of capital divided by the replacement cost of capital, has been called the most commonly used regressor in studies of empirical corporate finance (Erickson and Whited (2012)). Despite the widespread use of *Q* as a proxy for investment opportunities, empirically, the investment–*Q* relation is surprisingly weak (Erickson and Whited (2000)). One reason for the weak investment–*Q* relation is that since Tobin's *Q* is not perfectly observable, proxies for *Q* likely contain substantial measurement error. Furthermore, in previous proxies for *Q*, estimates of the market value of capital include both physical capital and intangible capital, but estimates of the replacement cost of capital usually include only physical capital.[1] The exclusion of the replacement cost of intangible capital adds to measurement error in *Q* (Erickson and Whited (2006)), and as intangible capital has grown in recent decades, so has the associated measurement error in *Q*. In an effort to reduce this measurement error, I construct a new proxy for *Q* that includes an estimate of the replacement cost of intangible capital.

In the last 70 years, investment in intangible capital has nearly tripled to approximately 40% of total investment (Corrado and Hulten (2010), Gutiérrez

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[1]Recent examples include Alti (2003), Bolton, Chen, and Wang (2011), Chen and Chen (2012), and Erickson and Whited (2012). One notable exception is Peters and Taylor (PT) (2017); they develop a proxy for *Q* that includes an estimate of intangible capital in the denominator. However, I show that their proxy for *Q* actually explains *less* variation in all investment types than the standard proxy for *Q* that includes just physical capital in the denominator.

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and Philippon (2017)). Put in this perspective, the exclusion of the replacement cost of intangible capital in previous proxies for Q is striking. However, it is also understandable. Measuring intangible capital and determining which components should be included when estimating its replacement cost are difficult. For example, the value of one type of intangible capital, on-balance sheet intangible capital, appears on the balance sheet. On-balance sheet intangible capital includes many types of acquired intangible capital and any premium paid for physical capital. Since on-balance sheet intangible capital potentially includes physical capital, should it be included in the replacement cost of intangible capital? Most likely. Despite on-balance sheet intangible capital potentially including physical capital, on-balance sheet intangible capital includes many types of intangible capital and an observable fair market value, both of which make it an especially attractive candidate for inclusion in the replacement cost of intangible capital.

Conversely, the value of the other type of intangible capital, internally-created intangible capital, does not appear on the balance sheet. Thus, when measuring internally-created intangible capital, the first-best option would be to observe its market value on a secondary market for intangible capital, but such a market rarely exists (Corrado, Hulten, and Sichel (2009), Haskel and Westlake (2017)). Perhaps the second-best option is to focus on important components of internally-created intangible capital for which we can estimate the market value. One component that fulfills both criteria is patents. Patents are the most direct indicator of firm-level innovation (Pakes and Griliches (1980)); are important for firm value (Pakes (1985), Connolly and Hirschey (1988), and Schankerman (1998)); and can lead to higher growth in output, productivity, and profits (Kogan, Papanikolaou, Seru, and Stoffman (2017)). Their importance has also grown significantly in recent decades. Figure 1 shows that the estimated annual market value of patents issued to publicly traded firms has grown from fewer than $300 billion in the early 1990s to more than $2 trillion in almost every year since 1999. Furthermore, since patents are granted by the United States Patent and Trademark Office (USPTO) every week, patent values can be estimated using the market response to newly-granted patents (Kogan et al. (2017)).

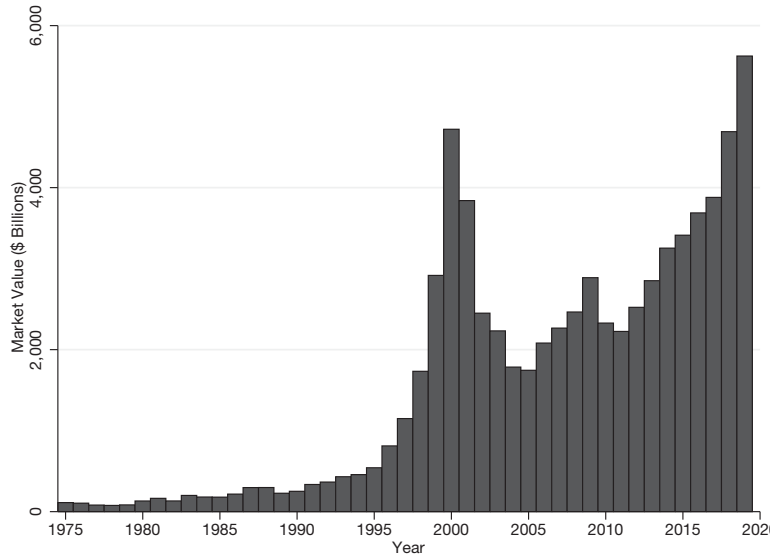
In conjunction with the marginal Q of new patents and the perpetual inventory method, I use the market value of patents to estimate the replacement cost of patent capital, which I include in my proxy for Q, patent Q. The numerator of patent Q is the market value of capital. The denominator of patent Q is physical capital plus my estimate of intangible capital, which includes on-balance sheet intangible capital and the replacement cost of patent capital. Thus, when estimating intangible capital, I include only elements that are a function of observable values.

Given Q's theoretical role as the sole determinant of new investment, I follow a large literature dating back over four decades (e.g., Ciccolo (1975), Abel (1980)) and empirically test the Q theory of investment by regressing investment on lagged Q. I find that patent Q explains 20% of the variation in physical investment, 36% of the variation in R&D investment, and 41% of the variation in total investment (i.e., physical investment plus R&D investment). These R^2 estimates are up to 31% (i.e., 9 percentage points) larger than those from regressions of investment on physical Q (Erickson and Whited (2012)), whose denominator includes only physical capital, and up to 62% (i.e., 14 percentage points) larger than those from

FIGURE 1

Market Value of Patents Issued to Publicly Traded Firms

Figure 1 presents the estimated annual market value of patents issued to publicly traded firms from 1975 to 2019.



regressions of investment on total Q (PT (2017)), whose denominator includes physical capital and an estimate of intangible capital whose value is a function of expenses from the income statement. To be clear, across all investment types, patent Q explains the most investment variation, physical Q explains the second most investment variation, and total Q explains the least investment variation.

Since Tobin's Q is not perfectly observable, any proxy for Q contains measurement error that leads to biased coefficient estimates on Q when using ordinary least squares (OLS; Erickson and Whited (2000)). To obtain bias-corrected coefficient estimates on Q, I use a cumulant estimator (Erickson, Jiang, and Whited (2014)). This methodology is important because when regressing investment on Q, the coefficient on Q can be interpreted as a determinant of the elasticity of investment with respect to Q (Whited (1994), Erickson and Whited (2000)), and previous work has found this coefficient to be lower than expected (Philippon (2009)). However, I find that coefficients on Q, and thus elasticities of investment with respect to Q, are almost always larger when using patent Q.

Patent Q's outperformance is driven in large part by the inclusion of patent capital, so I expect patent Q's outperformance to be stronger in subsamples with more patent capital, which is indeed what I find. In industries with the least patent capital, the explanatory power of patent Q is similar to that of other proxies for Q. However, in industries with the most patent capital, patent Q almost always explains the most investment variation. The same pattern is present in different time periods. In the earlier part of the sample, when patent capital is not as prevalent, the performance of patent Q is only slightly better than that of other proxies. As the importance

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of patent capital has grown over time, patent Q's relative outperformance over other proxies has also grown. In other words, patent Q performs as well as other proxies in subsamples with less patent capital and much better than other proxies in subsamples with more patent capital.

Although Q is the only determinant of new investment in Q theory, there exists a large literature showing that even after controlling for Q, investment is positively associated with cash flow (Hassett and Hubbard (1997), Caballero (1999)). The positive investment–cash flow relation has been interpreted as evidence of financing constraints (Fazzari, Hubbard, and Petersen (1988), Brown, Fazzari, and Petersen (2009)), the result of measurement error in Q (Erickson and Whited (2000), Gomes (2001)), and cash flow providing additional information about future investment opportunities (Gilchrist and Himmelberg (1995), Alti (2003)). Previous research suggests that controlling for a better proxy for Q should result in lower cash flow coefficients (e.g., Kaplan and Zingales (1997)). However, I find that although patent Q is a significantly better proxy for Q than both physical Q and total Q, controlling for patent Q leads to relatively larger, not smaller, cash flow coefficients.

This initially unexpected result is due to differences in investment–cash flow covariances between specifications. The Q theory of investment implies that when regressing investment on Q and cash flow, the denominators of all regression variables should be the same (Hayashi and Inoue (1991), Erickson and Whited (2012)). Thus, by changing the denominator of Q, I am also changing the denominators of both investment and cash flow. Changes in the denominators of investment and cash flow lead to changes in investment–cash flow covariances, and ultimately, larger cash flow coefficients when controlling for patent Q. This result shows that controlling for a better proxy for Q does not necessarily lead to lower cash flow coefficients, as suggested in previous work.

Extensive robustness tests show that my main results are i) insensitive to the depreciation rate used to estimate the replacement cost of patent capital, ii) insensitive to the marginal Q used to estimate the replacement cost of new patents, iii) robust to alternative econometric specifications, iv) driven by both the replacement cost of patent capital and on-balance sheet intangible capital, v) strongest when intangible capital is estimated using just the replacement cost of patent capital and on-balance sheet intangible capital, and vi) unlikely to be driven by a spurious correlation.

This article contributes to the vast literature associated with Tobin's Q proxy construction (e.g., Lewellen and Badrinath (1997), Tobin and Brainard (1977), Erickson and Whited (2006), and Philippon (2009)), but conceptually, this article is most closely related to PT (2017). Although PT (2017) were not the first to recognize the importance of including intangible capital in the denominator of Q, their attempt to do so is an important innovation.[2] In their construction of total Q, they estimate the replacement cost of intangible capital as on-balance sheet intangible capital plus proxies for knowledge capital and organization capital,

[2]See Griliches (1981), Cockburn and Griliches (1988), and Megna and Klock (1993) for some of the earliest calls to explicitly account for intangible capital in the denominator of Q.

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which they estimate by capitalizing R&D expenses (Bureau of Economic Analysis (BEA), Damodaran (1999)) and 30% of selling, general, and administrative (SG&A) expenses (Eisfeldt and Papanikolaou (2013), (2014)). However, as Ewens, Peters, and Wang (EPW) ((2020), p. 2) point out in *Measuring Intangible Capital with Market Prices*, "capitalizing in this way requires assumptions about the capital accumulation process." Indeed, there are significant challenges to capitalizing in this way.

First, intangible investment creates knowledge that is nonrival (i.e., spillovers), so the returns from intangible investment are not fully appropriable (Corrado, Haskel, Jona-Lasinio, and Iommi (2012), Hall, Helmers, Rogers, and Sena (2014)). Second, knowledge created by intangible investment is partly embodied in the firm's key employees (Eisfeldt and Papanikolaou (2013), (2014)). The level of employee embodiment varies between firms, and if employees leave the firm, so does some intangible capital. Both of these issues affect the intangible capital accumulation process in ways the researcher cannot measure. In stark contrast, patent capital is not subject to either problem; patents mitigate the spillover problem because of the associated legal protection (Haskel and Westlake (2017)), and patents stay with the firm even if their inventors leave. Furthermore, R&D is often missing for firms that engage in innovative activity and may actually be categorized as SG&A (McVay (2006), Koh and Reeb (2015)). Since organization capital is estimated as a fraction of SG&A, miscategorizing R&D as SG&A would not be a significant concern if we knew how much SG&A is investment. Unfortunately though, at the firm level, we have essentially no idea what fraction of SG&A is investment (Lev and Radhakrishnan (2005), Eisfeldt and Papanikolaou (2014), and EPW (2020)). These complications likely lead to especially noisy proxies for knowledge capital and organization capital.

It is difficult to determine what should be included in the replacement cost of intangible capital. Patent capital is a function of the market value of patents, but it is somewhat limited in scope. Proxies for knowledge capital and organization capital aim to be broader in scope, but their measurements are fraught with challenges. Although all three components are important, I argue that including components that are narrower in scope, but whose values are observable, should result in relatively less noisy estimates of the replacement cost of intangible capital, and thus, a better proxy for Tobin's Q. Indeed, my empirical results support my argument. Not only does patent Q perform better than total Q in all full sample tests and in almost all subsample tests, robustness tests show that patent Q performs *worse* when including proxies for knowledge capital, organization capital, or both. In fact, even when patent Q includes estimates of organization capital based on 10% or 20%, rather than the customary 30%, of SG&A as investment, patent Q explains less variation in all investment types.

The economic value of patent Q stems from its reliance on observable values for the replacement cost of intangible capital. The results in this article show that when estimating Q, including fewer types of intangible capital for which we have observable values, as in patent Q, leads to a stronger investment–Q relation.

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II. Intangible Capital and Tobin's *Q*

In this section, I discuss the difficulty involved in measuring intangible capital, the details of several types of intangible capital, and the replacement cost of intangible capital in proxies for *Q*.

A. Intangible Capital

The growing importance of intangible capital is hard to overstate. Intangible investment in the United States grew from 17% of total investment between 1948 and 1972 to 34% of total investment between 1995 and 2007 (Corrado and Hulten (2010)). Through the late 2000s and early 2010s, intangible investment in the United States grew to nearly 40% of total investment (Gutiérrez and Philippon (2017)). In some countries, intangible investment now outweighs tangible investment (Haskel and Westlake (2017)).

This growth in intangible investment has made it even more important for economists to accurately account for intangible capital in proxies for *Q*, which have usually excluded the replacement cost of intangible capital in the denominator despite always including the market value of intangible capital in the numerator. The difficulty involved in estimating the value of intangible capital is likely why many previous proxies for *Q* have excluded the replacement cost of intangible capital. One exception is the proxy for *Q* developed in PT (2017). They estimate the replacement cost of intangible capital as on-balance sheet intangible capital plus proxies for knowledge capital and organization capital. Another exception is the proxy for *Q* that I develop in this article. I estimate the replacement cost of intangible capital as on-balance sheet intangible capital plus patent capital. In the rest of this subsection, I discuss the details of on-balance sheet intangible capital, patent capital, knowledge capital, and organization capital.

1. On-Balance Sheet Intangible Capital

On-balance sheet intangible capital is the sum of other intangible capital and goodwill. Other intangible capital includes separately identifiable intangible capital acquired from another firm, such as copyrights, licenses, patents, trademarks, and software. Goodwill includes any premium paid for physical capital and intangible capital that is not separately identifiable. Intangible capital in goodwill is intellectual capital, which includes human capital and structural capital. Human capital includes employees' competence, interrelationship ability, and values. Structural capital includes customer capital (e.g., brand equity) and organization capital. Organization capital will be defined slightly differently below, but in the context of intellectual capital, organization capital includes innovation capital and process capital (Edvinsson and Malone (1997), Chen, Zhu, and Xie (2004)). In other words, on-balance sheet intangible capital contains many types of intangible capital whose value would be difficult to measure if it was not on the balance sheet.

2. Patent Capital

The value of patent capital is a function of the market value of patents. A patent is the grant of an exclusive property right to the inventor, and this grant is issued by the USPTO. The USPTO grants utility patents, design patents, and plant patents.

I focus on utility patents, which make up the bulk of all patent grants and are issued for the invention of a new and useful process or product. Generally speaking, utility patents granted before June 8, 1995 were enforceable for up to 17 years from the issue date, and patents filed on or after June 8, 1995 are enforceable for up to 20 years from the filing date.[3]

I estimate the market value of patents using the 3-day idiosyncratic return following the announcement of newly-granted patents.[4] As is the case with other capital stocks, the patent capital stock changes over time. New patents are constantly added, and the value of old patents depreciates over time (either through economic decay or discard). To account for these changes, I use the perpetual inventory method to estimate the replacement cost of patent capital:

$$K_{im}^{\text{PAT}} = (1 - \delta_P) K_{i,m-1}^{\text{PAT}} + P_{im}. \tag{1}$$

Subscripts i and m denote firms and months, δ_P is an industry-specific depreciation rate, and P_{im} is the replacement cost of new patents. Since the rest of my data (discussed below) are on an annual basis, I match the appropriate firm-month estimates of patent capital to firm-year data as of the same month.

R&D is closely related to subsequently issued patents, so beginning with the earliest possible data, I use industry-specific depreciation rates (δ_P) for R&D capital from Table 4 of Li (2012).[5,6] I cumulate and depreciate patent capital on a monthly basis, so I convert annual depreciation rates to monthly depreciation rates.[7]

I estimate the replacement cost of new patents (P_{im}) as the market value of new patents divided by the marginal Q of new patents, which I estimate using quarterly macro-level estimates of Q (Philippon (2009)).[8] These data span the second quarter of 1953 through the second quarter of 2007. The time-series average of quarterly Q estimates is 1.5, which I use as the marginal Q of new patents issued before the second quarter of 1953 and after the second quarter of 2007.[9]

3. Knowledge Capital and Organization Capital

Using the perpetual inventory method, knowledge capital and organization capital are estimated by capitalizing R&D and 30% of SG&A (PT (2017)). Knowledge capital is the estimated value of knowledge gained from R&D investments.

[3]See USPTO links https://www.uspto.gov/patents/basics and https://www.uspto.gov/web/offices/pac/mpep/s2701.html for more details concerning patents and changes in patent terms.

[4]I use estimated patent values from Stoffman, Woeppel, and Yavuz (2022). This methodology accounts for the probability of a successful patent application. See Appendix A for more details. For patents issued before 1976, I use matches from Kogan et al. (2017). For patents issued beginning in 1976, I use matches from Stoffman et al. (2022).

[5]Although the data in this article begin in 1975, estimated patent values begin in 1926.

[6]Depreciation rates range from 10% in pharmaceuticals to 40% in computers and peripheral equipment. Other notable industries include software (22%), semiconductor (25%), and communication equipment (27%). If the industry is not explicitly listed, I follow guidelines set by the BEA and use an annual rate of 15%.

[7]For example, if the annual accumulation rate is 0.85 (i.e., depreciation rate of 0.15), I use a monthly accumulation rate of 0.9865, which equals $0.85^{1/12}$.

[8]I obtain quarterly macro-level estimates of Q from Thomas Philippon's website.

[9]Of all quarterly estimates of Q, 56% of them are between 1.40 and 1.60, and 94% of them are between 1.30 and 1.70.

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Organization capital includes brand equity, customer relationships, distribution systems, and human capital.

R&D and a portion of SG&A qualify as investments because they reduce current cash flow to increase future cash flow (Corrado, Hulten, and Sichel (2005), (2009)). The estimated capital associated with these investments is important; firms with higher estimates of knowledge capital and organization capital earn higher stock returns and are more productive (Lev and Sougiannis (1996), Eisfeldt and Papanikolaou (2013)).

B. What Should Be Included in Tobin's *Q*?

Each of the previously discussed components of intangible capital is important, but which ones should be included in Tobin's *Q*? In this subsection, I weigh the advantages and disadvantages of each component and argue that the replacement cost of intangible capital should include patent capital and on-balance sheet intangible capital and exclude proxies for knowledge capital and organization capital.

1. On-Balance Sheet Intangible Capital

One advantage of including on-balance sheet intangible capital in estimates of the replacement cost of intangible capital is that it includes many different types of intangible capital: brand equity, copyrights, organization capital, patents, trademarks, and so on. Another advantage is that the value of on-balance sheet intangible capital is observable and recorded at fair value. After acquisition, both goodwill and other intangible capital are subject to annual impairment tests.[10] The primary disadvantage of including on-balance sheet intangible capital is that it potentially includes any premium paid for physical capital. However, since on-balance sheet intangible capital includes many different types of intangible capital and an observable fair value estimate, the advantages of including it in estimates of the replacement cost of intangible capital outweigh the disadvantages.

2. Patent Capital vs. Knowledge Capital and Organization Capital

When weighing which components to include in the replacement cost of intangible capital, estimates of patent capital, knowledge capital, and organization capital share advantages and disadvantages. On the one hand, each component is important and captures different parts of intangible capital (Megna and Klock (1993)). On the other hand, it is difficult to know how the value of each component should be depreciated (Pakes and Schankerman (1984), Lev and Radhakrishnan (2005)). More importantly though, many of the advantages of one component are direct disadvantages of another component.

The main advantage of patent capital is that its estimated value is a function of patent values, which are determined by the market response to newly-granted patents. As previously discussed, the value of any component of intangible capital is notoriously difficult to measure, so having observable values with which to back out replacement costs alleviates numerous sources of potential mismeasurement.

The estimation methodology of patent capital is not without its shortcomings. First, the market response to newly-granted patents may not perfectly measure

[10]For more details, see Statement of Financial Accounting Standards No. 142.

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patent values. One might wonder if this methodology overstates the value of patents, and thus, the effect of patent capital in estimates of patent Q. However, due to limited investor attention, estimating patent values using an event-study methodology likely underestimates actual patent values (Chemmanur, Li, Tseng, and Wang (2021)), so any positive effects of including patent capital in estimates of patent Q are likely understated, not overstated. Second, by using macro-level estimates of Q to estimate the marginal Q of new patents, I am assuming that average Q at the aggregate level is an appropriate estimate for the marginal Q of all new patents across all firms within a given quarter. Although this is a strong assumption, in robustness tests, I show that my results are almost identical when using alternative estimates for the marginal Q of new patents. Third, I use the same depreciation rate each time period for each patent in a particular industry. Although this is also a strong assumption, in robustness tests, I find that my results are unaffected by alternative depreciation rates. In summary, the estimation methodology of patent capital is not perfect, but its shortcomings do not outweigh its benefits.

The main disadvantage of patent capital is that it only captures the replacement cost of internally-created patents. Not all firms use patents to protect intellectual property (Hall et al. (2014)), and there are many other types of internally-created intangible capital. However, although patents are the product of successful R&D, patents rely on human capital and organizational development (Corrado and Hulten (2010)). Furthermore, patents are also the product of business process reengineering, mineral exploration, and software development (Haskel and Westlake (2017)). In other words, although patent capital focuses on patents, patents rely on investment from many different areas of the firm.

The main advantage of proxies for knowledge capital and organization capital is that they attempt to capture many types of intangible capital, such as brand equity, copyrights, customer relationships, distribution systems, human capital, patents, trademarks, and trade secrets. However, estimates of knowledge capital and organization capital share at least two serious disadvantages.

The first serious disadvantage is spillovers. Knowledge gained from intangible investment is nonrival, so returns from intangible investment are not fully appropriable. Thus, not all of the intangible investment made by a firm results in intangible capital. The second serious disadvantage is that intangible capital is partly embodied in the firm's key employees. The embodiment of intangible capital varies over time and across industries, and if key employees leave the firm, their share of intangible capital also leaves the firm. Accounting for these firm-level variables when estimating knowledge capital and organization capital is very difficult, if not impossible. Conversely, patents are not subject to either problem. The appropriability problem is considered to be the basic economic justification for patents, which give innovators a degree of protection (Corrado et al. (2012), Hall et al. (2014)). Furthermore, patents are issued to the firm, not the inventor, so even if an inventor leaves, the patent stays with the firm.

There are also idiosyncratic disadvantages of proxies for knowledge capital and organization capital. Knowledge capital is estimated by capitalizing R&D, but even for firms with patents, R&D is often missing. Put another way, estimated knowledge capital does not fully capture what it is intended to capture. Organization capital is estimated by capitalizing 30% of SG&A, and this is potentially hugely

problematic. Assuming that 30% of SG&A is intangible investment makes sense at the aggregate level (Corrado, Hulten, and Sichel (2009)), but at the firm level, there is no way to accurately measure what fraction of SG&A is investment (Haskel and Westlake (2017)).

In summary, there are advantages and disadvantages of including any of these components in the replacement cost of intangible capital. Patent capital is measured as a function of observed market values, but it is relatively narrow in scope. Knowledge capital and organization capital aim to be broader in scope, but their measurements are rife with serious problems. Since Tobin's Q is already so difficult to measure, I argue that the replacement cost of intangible capital should include only those components whose value can be observed, even if those components are comparatively narrow in scope. To that end, I argue that in estimates of Q, the replacement cost of intangible capital should include on-balance sheet intangible capital and patent capital and exclude proxies for knowledge capital and organization capital.

III. Sample

My sample includes all annual Compustat observations from fiscal years 1975 to 2019 except financials (SIC codes 6000–6799), utilities (SIC codes 4900–4999), and public administration and nonclassified firms (SIC codes 9000–9999). Following PT (2017), I exclude firms with nonpositive total assets; nonpositive sales; gross property, plant, and equipment of less than \$5 million (1990 dollars); and missing values of investment, Q, or cash flow.[11] Since I include firm fixed effects in each specification, I exclude singleton observations.[12] To mitigate the impact of outliers, I winsorize all regression variables at the 1% and 99% levels.[13]

A. Proxies for Tobin's Q

Tobin's Q is defined as the market value of capital divided by the replacement cost of capital. Using patent capital, I construct my proxy for Q, which I call patent Q (PATENT_Q):

$$\text{PATENT_Q}_{it} = \frac{V_{it}}{K_{it}^{\text{PHY}} + K_{it}^{\text{INT_PAT}}}. \tag{2}$$

Subscripts i and t denote firms and years. V_{it} is the market value of capital, which I estimate as outstanding common equity (CSHO × PRCC_F) plus the book value of short- and long-term debt (DLC + DLTT) minus the book value of current assets (ACT). I estimate the replacement cost of capital as physical capital (K_{it}^{PHY}) plus intangible capital ($K_{it}^{\text{INT_PAT}}$). I estimate physical capital as the book value of

[11] Some firms do not report R&D (Koh and Reeb (2015)), so specifications whose dependent variable is R&D investment are not able to capture all firms with actual R&D.

[12] Singleton observations can overstate statistical significance and lead to incorrect inferences (Correia (2015)).

[13] In unreported tests, I find that my results are qualitatively similar when winsorizing at the 5% and 95% levels.

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gross property, plant, and equipment (PPEGT). I estimate intangible capital as follows:

$$K_{it}^{\text{INT_PAT}} = K_{it}^{\text{PAT}} + K_{it}^{\text{ON_BS}}, \tag{3}$$

in which K_{it}^{PAT} is the replacement cost of patent capital, and $K_{it}^{\text{ON_BS}}$ is on-balance sheet intangible capital. The estimation procedure for patent capital is previously discussed in Section II.A.2, and on-balance sheet intangible capital is from Compustat (INTAN). I set all missing values of the replacement cost of patent capital and on-balance sheet intangible capital to 0.

In standard investment–Q regressions, I compare the performance of patent Q to that of two commonly used proxies for Q. The first proxy is from Erickson and Whited (2012), and since its denominator includes only physical capital, I refer to it as physical Q (PHYSICAL_Q):

$$\text{PHYSICAL_Q}_{it} = \frac{V_{it}}{K_{it}^{\text{PHY}}}. \tag{4}$$

The second proxy is from PT (2017), and it is called total Q (TOTAL_Q):

$$\text{TOTAL_Q}_{it} = \frac{V_{it}}{K_{it}^{\text{PHY}} + K_{it}^{\text{INT_TOT}}}. \tag{5}$$

The denominator of total Q includes an estimate of intangible capital that relies on expenses from the income statement:

$$K_{it}^{\text{INT_TOT}} = K_{it}^{\text{KNOW}} + K_{it}^{\text{ORG}} + K_{it}^{\text{ON_BS}}, \tag{6}$$

in which K_{it}^{KNOW} is estimated knowledge capital, and K_{it}^{ORG} is estimated organization capital.[14]

B. Summary Statistics

Table 1 presents summary statistics, which include estimates of different proxies for Q and estimates of intangible capital and patent capital for five different percentiles. Panel A presents estimates of Q for all firms. Since not all firms have patent capital, Panel B presents estimates of Q for firms with patent capital at any point during the sample period. Both panels show that the inclusion of my estimate of intangible capital in patent Q reduces many of the high estimates of physical Q.

Panel C of Table 1 presents different estimates of intangible capital for all firms, and Panel D presents different estimates of intangible capital for firms with patent capital. The first 2 rows in each panel show that my estimates of intangible capital (INTANGIBLE_CAPITAL) and patent capital (PATENT_CAPITAL) are both positively skewed. The last 2 rows in each panel present patent intensity (PATENT_INTENSITY), which is patent capital divided by total capital, and intangible intensity (INTANGIBLE_INTENSITY), which is intangible capital

[14]I obtain estimates of knowledge capital and organization capital from Wharton Research Data Services.

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TABLE 1

Summary Statistics

Table 1 presents estimates of different proxies for *Q* and estimates of intangible capital for five different percentiles. Panels A and B present estimates of different proxies for *Q*. The numerator of each proxy for *Q* is the market value of common equity plus the book value of short- and long-term debt minus the book value of current assets. The denominator of each proxy for *Q* contains physical capital (i.e., book value of gross property, plant, and equipment). The denominator of patent *Q* (PATENT_Q) also includes my estimate of intangible capital (i.e., patent capital plus on-balance sheet intangible capital). The denominator of total *Q* (TOTAL_Q) also includes intangible capital as estimated by Peters and Taylor (2017). Panels C and D present estimates of patent capital, intangible capital, patent intensity, and intangible intensity. Patent capital (PATENT_CAPITAL) and intangible capital (INTANGIBLE_CAPITAL) are in millions of dollars. Patent intensity (PATENT_INTENSITY) is patent capital divided by total capital, which is physical capital plus intangible capital. Intangible intensity (INTANGIBLE_INTENSITY) is my estimate of intangible capital divided by total capital. Panels A and C present estimates for all firms. Panels B and D present estimates for firms with patent capital at any point during the sample period.

	P10	P25	P50	P75	P90	No. of Obs.
Panel A. Estimates of Q, All Firms						
PATENT_Q	−0.09	0.30	0.76	1.67	4.04	161,564
PHYSICAL_Q	−0.11	0.35	0.99	2.87	8.41	161,564
TOTAL_Q	−0.05	0.21	0.59	1.23	2.56	161,564
Panel B. Estimates of Q, Firms with Patent Capital						
PATENT_Q	−0.09	0.28	0.75	1.74	4.26	85,422
PHYSICAL_Q	−0.11	0.35	1.13	3.61	10.36	85,422
TOTAL_Q	−0.05	0.19	0.57	1.25	2.62	85,422
Panel C. Estimates of Intangible Capital, All Firms						
PATENT_CAPITAL	0	0	0	5	180	161,564
INTANGIBLE_CAPITAL	0	0	7	120	1,046	161,564
PATENT_INTENSITY	0%	0%	0%	4%	28%	161,564
INTANGIBLE_INTENSITY	0%	0%	8%	36%	68%	161,564
Panel D. Estimates of Intangible Capital, Firms with Patent Capital						
PATENT_CAPITAL	0	0.2	4	71	908	85,422
INTANGIBLE_CAPITAL	0.1	2	24	320	2,359	85,422
PATENT_INTENSITY	0%	0%	3%	19%	50%	85,422
INTANGIBLE_INTENSITY	0%	3%	17%	48%	76%	85,422

divided by total capital. Importantly, Panel D shows that patent capital is often a significant portion of total intangible capital for firms that have patent capital.

Figure 2 presents firm-year observation totals and percentages in five Fama–French industries. Chart A shows that in the full sample, firms in high tech and healthcare comprise approximately 31% of all observations. However, Chart B shows that when focusing on firms with patent capital, firms in high tech and healthcare comprise 41% of the observations. These are the two industries one might expect a higher proportion of firms to have patent capital.

Since R&D investment is not as well populated in Compustat as physical investment, Charts C and D of Figure 2 present firm-year observation totals and percentages for firms with R&D investment. Chart C shows that firms in the same two industries, high tech and healthcare, comprise 53% of observations with R&D. A comparison of observation totals in Charts A and C reveals that high tech and healthcare are the only two industries in which more than half of all observations have R&D investment. Chart D shows that of all observations with R&D investment and patent capital at any point during the sample, 53% of them are in high tech and healthcare.

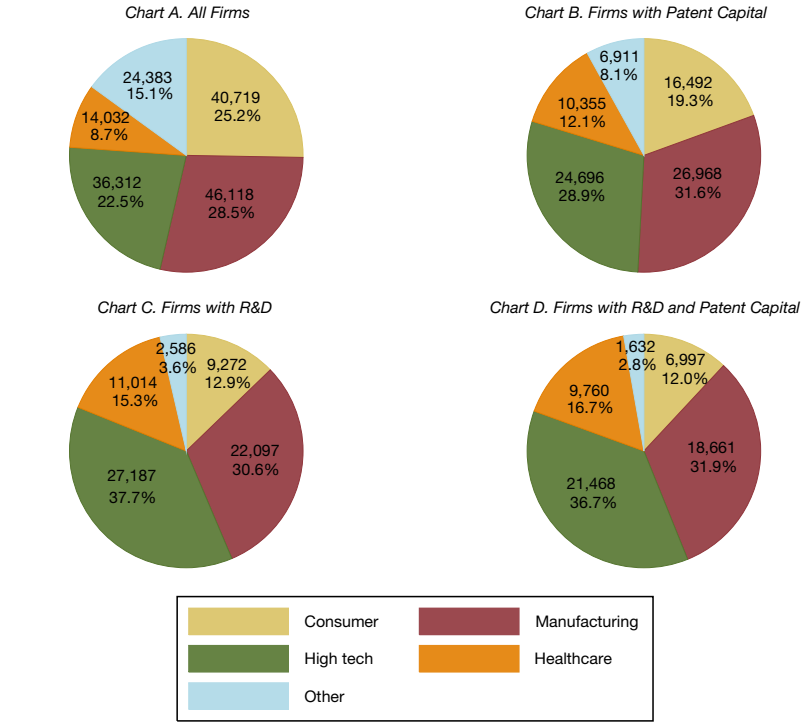
Overall, Figure 2 shows that there is significant inter-industry variation in the percentage of firms with patent capital. As one might expect, firms in high tech and healthcare are more likely to have patent capital than firms in other industries.

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FIGURE 2

Observations by Industry

Figure 2 presents firm-year observation totals and percentages in five Fama–French industries. Chart A presents totals and percentages for all firms. Chart B presents totals and percentages for firms with patent capital at any point during the sample period. Chart C presents totals and percentages for observations with R&D investment. Chart D presents totals and percentages for observations with R&D investment and patent capital at any point during the sample period.



Given this inter-industry variation, I expect the relative performance of patent Q to differ between industries, which I test for later by estimating regressions after sorting firms by industry.

IV. Investment–Q Relation

In this section, I test the Q theory of investment by regressing investment on each of the three proxies for Q. Since Q theory implies that the same divisor be used for all regression variables (Hayashi and Inoue (1991), Erickson and Whited (2012)), all investment variables are scaled by the denominator of lagged Q being regressed on by investment. Using different divisors for investment and Q breaks their natural positive correlation, which leads to lower coefficient estimates on Q and lower R^2 estimates.[15]

[15]Erickson and Whited (2012) state that using different divisors for investment and Q implies the identification assumption that the regression coefficient does not equal 0 (i.e., $\beta \neq 0$) is close to being violated, and no estimator provides reliable estimates when it is nearly unidentified. For the analysis of a large cross section, they advise using the same divisor for all regression variables.

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A. OLS Results

Table 2 presents results from OLS regressions of investment on lagged Q. Physical investment (PHYSICAL_INVEST) is scaled capital expenditures (CAPX). R&D investment (R&D_INVEST) is scaled R&D (XRD). Total investment (TOTAL_INVEST) is physical investment plus R&D investment. Following the investment–Q literature, I also include firm fixed effects and year fixed effects.[16] Since Q is measured with error, the coefficient on Q is biased toward 0 when using OLS (Erickson and Whited (2000)). For this reason, the discussion surrounding Table 2 focuses on within R^2 estimates, which are ordinary R^2 estimates from

TABLE 2

Investment and Q: Ordinary Least Squares

Table 2 presents results from ordinary least squares regressions of investment on lagged Q, firm fixed effects, and year fixed effects. The numerator of each proxy for Q is the market value of common equity plus the book value of short- and long-term debt minus the book value of current assets. The denominator of each proxy for Q contains physical capital (book value of gross property, plant, and equipment). The denominator of patent Q (PATENT_Q) also includes my estimate of intangible capital (i.e., patent capital plus on-balance sheet intangible capital). The denominator of total Q (TOTAL_Q) also includes intangible capital as estimated by Peters and Taylor (2017). Physical investment (PHYSICAL_INVEST) is scaled capital expenditures. R&D investment (R&D_INVEST) is scaled R&D expenses. Total investment (TOTAL_INVEST) is physical investment plus R&D investment. All investment variables are scaled by the denominator of lagged Q being regressed on by investment. Within R^2 estimates are ordinary R^2 estimates from estimating ordinary least squares on the transformed data. ΔR^2 estimates are R^2 estimates associated with patent Q minus R^2 estimates associated with either physical Q (columns 2, 5, and 8) or total Q (columns 3, 6, and 9) within each investment type. Panel A presents results for all firms. Panel B presents results for firms with patent capital at any point during the sample period. Standard errors in parentheses are clustered by firm. Standard errors below R^2 estimates and ΔR^2 estimates are estimated using influence functions. Bolded estimates are significant at the 5% level.

	PHYSICAL_INVEST			R&D_INVEST			TOTAL_INVEST		
	1	2	3	4	5	6	7	8	9
Panel A. All Firms									
PATENT_Q	**0.024** (0.0004)			**0.029** (0.0006)			**0.049** (0.0007)		
PHYSICAL_Q		**0.014** (0.0002)			**0.021** (0.0005)			**0.032** (0.0006)	
TOTAL_Q			**0.028** (0.0005)			**0.015** (0.0003)			**0.033** (0.0006)
Within R^2	0.203 (0.004)	0.168 (0.004)	0.146 (0.003)	0.358 (0.008)	0.274 (0.008)	0.220 (0.006)	0.412 (0.007)	0.327 (0.008)	0.285 (0.006)
ΔR^2	– –	**0.036** (0.004)	**0.058** (0.003)	– –	**0.084** (0.007)	**0.138** (0.006)	– –	**0.085** (0.007)	**0.126** (0.006)
No. of obs.	161,564	161,564	161,564	72,156	72,156	72,156	72,156	72,156	72,156
Panel B. Firms with Patent Capital									
PATENT_Q	**0.020** (0.0003)			**0.033** (0.0007)			**0.053** (0.0009)		
PHYSICAL_Q		**0.011** (0.0002)			**0.022** (0.0006)			**0.033** (0.0007)	
TOTAL_Q			**0.019** (0.0004)			**0.016** (0.0004)			**0.034** (0.0006)
Within R^2	0.263 (0.006)	0.199 (0.005)	0.178 (0.004)	0.369 (0.009)	0.277 (0.009)	0.225 (0.006)	0.425 (0.008)	0.328 (0.009)	0.293 (0.006)
ΔR^2	– –	**0.064** (0.005)	**0.085** (0.004)	– –	**0.093** (0.008)	**0.144** (0.007)	– –	**0.097** (0.008)	**0.132** (0.006)
No. of obs.	85,422	85,422	85,422	58,518	58,518	58,518	58,518	58,518	58,518

[16]When analyzing R&D investment and total investment, I remove firms whose R&D investment is 0 in each period because those firms have no within-firm variation when including firm fixed effects.

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estimating OLS on the transformed data. ΔR^2 estimates are within R^2 estimates associated with patent Q minus within R^2 estimates associated with either physical Q or total Q within each investment type. Measurement error in Q is addressed in the next subsection.

Panel A of Table 2 presents results for all firms. Columns 1–3 focus on physical investment, columns 4–6 focus on R&D investment, and columns 7–9 focus on total investment. For all investment types, R^2 estimates associated with patent Q are significantly larger than R^2 estimates associated with either physical Q or total Q. For example, patent Q explains 20% of the variation in physical investment, 36% of the variation in R&D investment, and 41% of the variation in total investment. These R^2 estimates are up to 31% (i.e., 9 percentage points) larger than those associated with physical Q and up to 62% (i.e., 14 percentage points) larger than those associated with total Q. Furthermore, differences in R^2 estimates are statistically significant; ΔR^2 estimates are between 10 and 23 times larger than their associated standard errors.[17,18]

Since patent Q's explanatory power is due in part to the inclusion of the replacement cost of patent capital, patent Q's outperformance should be stronger when focusing on firms with patent capital. Panel B of Table 2, which presents results for firms with patent capital at any point during the sample period, shows that this is indeed the case. R^2 estimates associated with patent Q in Panel B are between 3% and 30% larger than corresponding R^2 estimates in Panel A. Furthermore, Panel B shows that R^2 estimates associated with patent Q are between 30% and 64% (i.e., 6 and 14 percentage points) larger than R^2 estimates associated with other proxies for Q. ΔR^2 estimates show that these differences are again highly significant.

Overall, Table 2 shows that patent Q explains significantly more variation in all investment types than either physical Q or total Q. Patent Q's outperformance is stronger when focusing on firms with patent capital and is strongest when predicting R&D investment. The latter result is likely driven by the fact that R&D is the investment type most closely associated with the development of patent capital. Firms that engage in R&D are more likely to have patent capital than firms that do not engage in R&D. By including patent capital in these firms' estimates

[17]In Table 2 of PT (2017), most of the R^2 estimates associated with total Q are larger than those associated with physical Q (i.e., standard Q in their article). However, almost all of their specifications that include physical Q are misspecified. In 4 of the 5 columns of their Table 2, when regressing investment on physical Q, PT (2017) scale investment by the denominator of total Q, not physical Q. However, no estimator provides reliable estimates when using different divisors for investment and Q (see footnote 15 above). Thus, the comparisons made by PT (2017) in their Table 2 are mechanically set up for total Q to explain more investment variation than physical Q. In fact, in the only valid comparison of R^2 estimates in Table 2 of PT (2017), physical Q explains more variation in physical investment (23.3%; column 5 of Panel B) than does total Q (20.9%; column 1 of Panel A).

[18]Within R^2 estimates in my Table 2 are lower than R^2 estimates in Table 2 of PT (2017) because we account for fixed effects differently. We both account for firm fixed effects by demeaning regression variables by firm. However, to account for year fixed effects, I demean by year, and PT (2017) use year dummy variables. Importantly, this does not change the relative results. In unreported tests, I confirm that using dummy variables to account for year fixed effects still leads to R^2 estimates associated with patent Q (total Q) being the largest (smallest). I thank Ryan Peters for helping me resolve this inconsistency.

of Q, I am alleviating a potentially substantial source of measurement error in Q for these firms, which leads to a larger improvement when predicting R&D investment.[19]

B. Bias-Corrected Results

As previously discussed, coefficient estimates on Q are biased when using OLS. This bias stems from two sources of measurement error. First, Tobin's Q is not perfectly observable to the econometrician. Second, any estimate of Tobin's Q measures average Q, not marginal Q, which should be the true determinant of new investment in Q theory. Obtaining bias-corrected coefficients on Q is important because when regressing investment on Q, the coefficient on Q can be interpreted as a determinant of the elasticity of investment with respect to Q (Whited (1994), Erickson and Whited (2000)), and previous work has found this coefficient to be lower than expected (Philippon (2009)).

To obtain bias-corrected coefficient estimates on Q, Erickson and Whited (2000), (2002) develop a two-step generalized method of moments (GMM) estimator. Unfortunately, GMM results can be highly sensitive to starting value choices (Erickson and Whited (2012)), so Erickson et al. (2014) build upon the GMM framework and develop high-order cumulant estimators, which have closed-form solutions. Cumulant estimators provide unbiased estimates of β in the following errors-in-variables model:

$$y_{it} = q_{it}\beta + u_{it} \tag{7}$$

$$x_{it} = q_{it} + e_{it}, \tag{8}$$

in which y_{it} is investment, and x_{it} is an observable proxy for the true, unobservable q_{it}. The error terms, u_{it} and e_{it}, are independent of each other and of q_{it}.[20]

One might wonder why a new proxy for Q is necessary if the previously discussed procedure accounts for measurement error. Since investment and Q share a denominator, both variables have measurement error. The two measurement errors are correlated with each other, which violates the assumption that u_{it} and

[19]In addition to physical investment and R&D investment, PT (2017) also analyze intangible investment (INTANGIBLE_INVEST), which they define as scaled R&D expenses (set to 0 if missing) plus 30% of scaled SG&A expenses, and an alternative total investment (TOTAL_INVEST_PT), which they define as physical investment plus intangible investment. In Appendix B (Table B1), I show that when predicting intangible investment and the alternative total investment, patent Q explains the most variation, physical Q explains the second most variation, and total Q explains the least variation.

[20]To obtain identification for a third-order cumulant estimator, which is what I use for my main results and leads to an exactly identified system, take the following steps (dropping subscripts): first, square equation (7), multiply the results by equation (8), and take expectations of both sides; second, square equation (8), multiply the results by equation (7), and take expectations of both sides; third, assuming $\beta \neq 0$ and $\mathbb{E}(q^3) \neq 0$, divide the results from the first step by the results from the second step to obtain $\beta = \mathbb{E}(y^2x) / \mathbb{E}(yx^2)$. For additional details, see Erickson and Whited (2012). The treatment reviews identification for a third-order moment estimator, but as Erickson et al. (2014) point out, third-order cumulants equal third-order moments when the variables in question have a zero mean (as is the case in my sample).

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TABLE 3

Investment and *Q*: Cumulant Estimator

Table 3 presents results from cumulant estimator regressions of investment on lagged *Q*, firm fixed effects, and year fixed effects. The numerator of each proxy for *Q* is the market value of common equity plus the book value of short- and long-term debt minus the book value of current assets. The denominator of each proxy for *Q* contains physical capital (book value of gross property, plant, and equipment). The denominator of patent *Q* (PATENT_Q) also includes my estimate of intangible capital (i.e., patent capital plus on-balance sheet intangible capital). The denominator of total *Q* (TOTAL_Q) also includes intangible capital as estimated by Peters and Taylor (2017). Physical investment (PHYSICAL_INVEST) is scaled capital expenditures. R&D investment (R&D_INVEST) is scaled R&D expenses. Total investment (TOTAL_INVEST) is physical investment plus R&D investment. All investment variables are scaled by the denominator of lagged *Q* being regressed on by investment. ρ^2 is the within R^2 estimate from the hypothetical regression of investment on marginal *Q*. τ^2 is the within R^2 estimate from the hypothetical regression of estimated *Q* on marginal *Q*. Panel A presents results for all firms. Panel B presents results for firms with patent capital at any point during the sample period. Standard errors in parentheses are estimated using influence functions and are clustered by firm. Bolded coefficients are significant at the 5% level.

	PHYSICAL_INVEST			R&D_INVEST			TOTAL_INVEST		
	1	2	3	4	5	6	7	8	9
Panel A. All Firms									
PATENT_Q	**0.045** (0.0005)			**0.048** (0.0008)			**0.072** (0.0010)		
PHYSICAL_Q		**0.028** (0.0004)			**0.040** (0.0011)			**0.054** (0.0011)	
TOTAL_Q			**0.066** (0.0011)			**0.029** (0.0006)			**0.054** (0.0009)
ρ^2	0.385 (0.009)	0.340 (0.009)	0.347 (0.009)	0.586 (0.017)	0.526 (0.020)	0.429 (0.015)	0.605 (0.015)	0.542 (0.017)	0.463 (0.013)
τ^2	0.529 (0.011)	0.493 (0.012)	0.420 (0.011)	0.611 (0.018)	0.521 (0.022)	0.514 (0.017)	0.680 (0.016)	0.603 (0.020)	0.616 (0.017)
No. of obs.	161,564	161,564	161,564	72,156	72,156	72,156	72,156	72,156	72,156
Panel B. Firms with Patent Capital									
PATENT_Q	**0.034** (0.0005)			**0.052** (0.0010)			**0.077** (0.0011)		
PHYSICAL_Q		**0.021** (0.0004)			**0.042** (0.0013)			**0.056** (0.0013)	
TOTAL_Q			**0.040** (0.0007)			**0.030** (0.0007)			**0.053** (0.0010)
ρ^2	0.449 (0.013)	0.383 (0.014)	0.368 (0.012)	0.589 (0.018)	0.529 (0.023)	0.428 (0.016)	0.617 (0.016)	0.552 (0.020)	0.467 (0.015)
τ^2	0.587 (0.014)	0.519 (0.016)	0.484 (0.015)	0.627 (0.020)	0.522 (0.025)	0.526 (0.020)	0.689 (0.018)	0.595 (0.022)	0.628 (0.019)
No. of obs.	85,422	85,422	85,422	58,518	58,518	58,518	58,518	58,518	58,518

e_{it} are independent of each other.[21] To that end, an improved proxy for *Q* can still improve the observed relation between investment and *Q* in a bias-corrected setting.

Table 3 presents bias-corrected results from cumulant estimator regressions of investment on lagged *Q*, firm fixed effects, and year fixed effects.[22,23] Panel A presents results for all firms, and Panel B presents results for firms with patent capital. A comparison of *Q*-slopes within each investment type shows that with only

[21]For a detailed discussion, see footnote 10 in PT (2017).

[22]All regression variables have been demeaned to account for firm fixed effects and year fixed effects.

[23]Erickson et al. (2017) suggest that the order cumulant a researcher uses is an empirical choice. As mentioned in footnote 20, I use a third-order cumulant estimator. All Sargan–Hansen *J*-statistics (unreported) associated with fourth-order cumulants and above reject the null hypothesis that the overidentifying restrictions are valid. Regardless, robustness tests (Table 7) show that patent *Q* comfortably outperforms alternative proxies for *Q* in specifications that exploit higher-order cumulants.

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one exception in each panel, elasticities are larger when using patent Q than when using other proxies for Q. Furthermore, differences in coefficient estimates within investment types are generally larger when concentrating on firms with patent capital. Although all coefficient estimates on Q are smaller than the theorized coefficient of 1 (Hayashi and Inoue (1991)), coefficients on patent Q are indeed closer to 1 than coefficients on other proxies for Q.

Importantly, the relatively stronger investment–patent Q sensitivity does not come at the expense of explanatory power. The cumulant estimator provides two statistics for understanding how well a mismeasured proxy performs. The first is ρ^2, which is the within R^2 estimate from the hypothetical regression of investment on marginal Q (i.e., the within R^2 estimate from equation (7)). The second is τ^2, which is the within R^2 estimate from the hypothetical regression of estimated Q on marginal Q (i.e., the within R^2 estimate from equation (8)). If hypothetical marginal Q perfectly explains investment, ρ^2 equals 100%. Likewise, if any proxy is a perfect one for marginal Q, τ^2 equals 100%.

Panel A of Table 3 shows that within each investment type, ρ^2 and τ^2 estimates associated with patent Q are always larger than those associated with other proxies for Q. When using patent Q, marginal Q explains 39%, 59%, and 61% of the variation in physical investment, R&D investment, and total investment. These ρ^2 estimates are between 11% and 37% (i.e., 4 and 16 percentage points) larger than those associated with other proxies for Q. In addition, when using patent Q to predict physical investment, R&D investment, and total investment, τ^2 estimates are 53%, 61%, and 68%. These percentages are between 7% and 26% (i.e., 4 and 11 percentage points) larger than those associated with other proxies for Q.

Panel B of Table 3 shows that when focusing on firms with patent capital, patent Q's relative outperformance is again stronger than when focusing on all firms. All ρ^2 and τ^2 estimates associated with patent Q in Panel B are larger than corresponding estimates in Panel A. Furthermore, Panel B shows that when comparing ρ^2 and τ^2 estimates associated with patent Q to those associated with other proxies for Q, differences are generally larger than in Panel A.

The results in Table 3 show that bias-corrected coefficients are almost always larger when using patent Q. These results also show that for all investment types, patent Q is a better proxy for marginal Q than either physical Q or total Q. Patent Q's outperformance is again stronger when focusing on firms with patent capital and is strongest when focusing on R&D investment. Overall, Table 3 shows that even when using this bias-correcting methodology, results associated with patent Q are more consistent with Q theory than results associated with other proxies for Q.[24]

C. Subsample Analysis

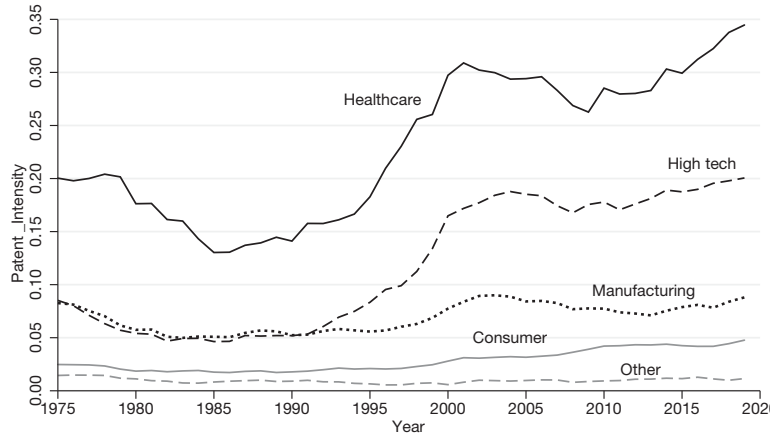
Figure 3 shows that average firm-level patent intensity varies significantly between industries and increases over time. Firms in healthcare and high tech have the highest average levels of patent intensity, which is consistent with observation totals and percentages in Figure 2. Since patent Q's outperformance in the full

[24] Table B2 shows that when examining intangible investment and the alternative total investment from PT (2017), both of which are described in footnote 19, bias-corrected coefficients, ρ^2 estimates, and τ^2 estimates associated with patent Q are always larger than those associated with other proxies for Q.

FIGURE 3

Patent Intensity Across Industries

Figure 3 presents average firm-level patent intensity in five Fama–French industries over time. Patent intensity (PATENT_INTENSITY) is plotted each fiscal year and is patent capital divided by total capital, which is the book value of gross property, plant, and equipment plus patent capital and on-balance sheet intangible capital.



sample is driven in part by the inclusion of the replacement cost of patent capital, the investment–patent Q relation should be relatively stronger in industries and time periods with more patent capital. In this subsection, I show that to be the case.[25]

1. Industry-Level Analysis

Figures 4–6 present estimates from regressions of investment on lagged Q, firm fixed effects, and year fixed effects after sorting firms by industry. Industries within each panel are sorted from lowest (top) to highest (bottom) with respect to their average firm-level patent intensity.

Figure 4 presents results from regressions of physical investment on different proxies for Q. Graphs A and B show that differences in R^2 and ρ^2 estimates are small in the three industries with the lowest levels of patent intensity (i.e., other, consumer, and manufacturing), but in the two industries with the highest levels of patent intensity (i.e., high tech and healthcare), R^2 and ρ^2 estimates associated with patent Q are up to 11 percentage points larger than those associated with other proxies. Graph C shows that patent Q is a better proxy for marginal Q than other proxies in all five industries, and differences in τ^2 estimates generally increase in patent intensity. Graph D shows that β estimates, or bias-corrected coefficient estimates on Q, are highest when using total Q and lowest when using physical Q. The results in Graph D are consistent with those in the full sample.

The results associated with patent Q in Figure 5, which focuses on R&D investment, are relatively stronger than those in Figure 4. Graphs A–C show that differences in R^2, ρ^2, and τ^2 estimates are mostly larger in industries with more

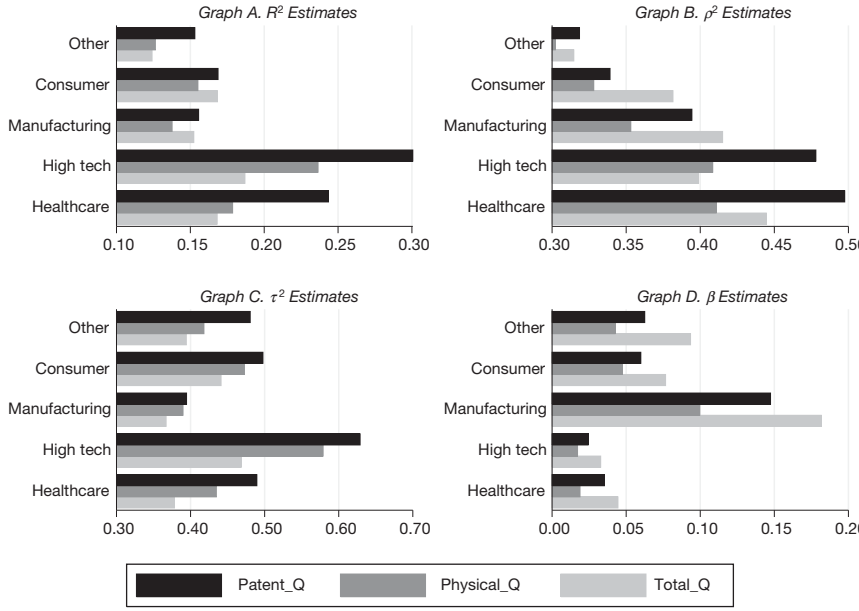
[25] All regression variables are winsorized either within industries or time periods. Winsorization over the full sample could still produce extreme outliers when regressions are estimated within industries or time periods.

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FIGURE 4

Physical Investment–*Q* Relation Across Industries

Figure 4 presents results from regressions of physical investment on lagged *Q*, firm fixed effects, and year fixed effects after sorting firms into five Fama–French industries. The numerator of each proxy for *Q* is the market value of common equity plus the book value of short- and long-term debt minus the book value of current assets. The denominator of each proxy for *Q* contains physical capital (book value of gross property, plant, and equipment). The denominator of patent *Q* (PATENT_Q) also includes my estimate of intangible capital (i.e., patent capital plus on-balance sheet intangible capital). The denominator of total *Q* (TOTAL_Q) also includes intangible capital as estimated by Peters and Taylor (2017). Physical investment is capital expenditures scaled by the denominator of lagged *Q* being regressed on by investment. Graph A presents within R^2 estimates from ordinary least squares (OLS) regressions, which are ordinary R^2 estimates from estimating OLS on the transformed data. Graph B presents ρ^2 estimates, which are within R^2 estimates from hypothetical regressions of physical investment on marginal *Q*. Graph C presents τ^2 estimates, which are within R^2 estimates from hypothetical regressions of *Q* on marginal *Q*. Graph D presents β estimates, which are bias-corrected coefficient estimates on *Q* from cumulant estimator regressions of physical investment on *Q*. Industries are ranked from lowest (top) to highest (bottom) with respect to their average firm-level patent intensity.



patent capital.[26] For example, R^2, ρ^2, and τ^2 estimates are relatively similar in consumer and manufacturing. In high tech, differences in estimates are up to 15 percentage points higher, and in healthcare, differences in estimates are up to more than 20 percentage points higher. Graph D shows that β estimates associated with patent *Q* are highest in all industries, and differences in β estimates generally increase in patent intensity.

Finally, Figure 6 presents results from regressions of total investment on different proxies for *Q*. Graph A shows that differences in R^2 estimates are more than 10 percentage points in high tech and reach more than 16 percentage points in healthcare. Graphs B and C show that ρ^2 and τ^2 estimates associated with patent *Q* are mostly similar to those associated with physical *Q* but are always larger than

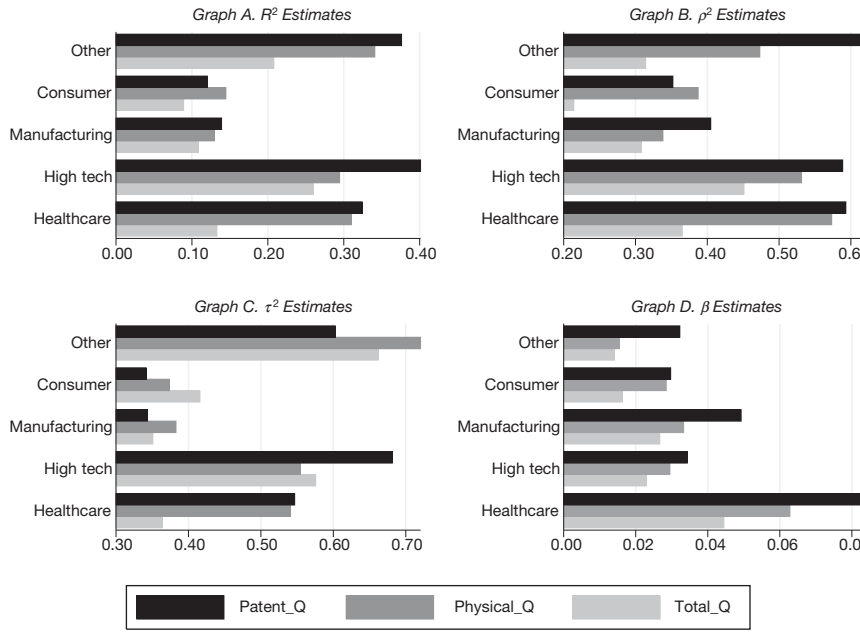
[26]As shown in Figure 2, there are only 2,586 firm-year observations in the "other" industry when analyzing R&D investment or total investment. Thus, the results associated with the other industry in Figures 5 and 6 are relatively noisy and should be interpreted with caution.

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FIGURE 5

R&D Investment–*Q* Relation Across Industries

Figure 5 presents results from regressions of R&D investment on lagged *Q*, firm fixed effects, and year fixed effects after sorting firms into five Fama–French industries. The numerator of each proxy for *Q* is the market value of common equity plus the book value of short- and long-term debt minus the book value of current assets. The denominator of each proxy for *Q* contains physical capital (book value of gross property, plant, and equipment). The denominator of patent *Q* (PATENT_Q) also includes my estimate of intangible capital (i.e., patent capital plus on-balance sheet intangible capital). The denominator of total *Q* (TOTAL_Q) also includes intangible capital as estimated by Peters and Taylor (2017). R&D investment is R&D expenses scaled by the denominator of lagged *Q* being regressed on by investment. Graph A presents within R^2 estimates from ordinary least squares (OLS) regressions, which are ordinary R^2 estimates from estimating OLS on the transformed data. Graph B presents ρ^2 estimates, which are within R^2 estimates from hypothetical regressions of R&D investment on marginal *Q*. Graph C presents τ^2 estimates, which are within R^2 estimates from hypothetical regressions of *Q* on marginal *Q*. Graph D presents β estimates, which are bias-corrected coefficient estimates on *Q* from cumulant estimator regressions of R&D investment on *Q*. Industries are ranked from lowest (top) to highest (bottom) with respect to their average firm-level patent intensity.



those associated with total *Q*. Graph D shows that once again, in all five industries, β estimates associated with patent *Q* are larger than β estimates associated with other proxies. Differences in β estimates are generally larger in industries with more patent capital.

The results in Figures 4–6 show that when analyzing industries with the least patent capital, R^2, ρ^2, τ^2, and β estimates associated with patent *Q* are mostly similar to those associated with physical *Q* and total *Q*. However, when analyzing industries with the most patent capital, estimates associated with patent *Q* are mostly larger than those associated with physical *Q* and almost always much larger than those associated with total *Q*. In other words, regardless of the industry, patent *Q* performs as well as, or better than, physical *Q* and total *Q*.

2. Time Period Analysis

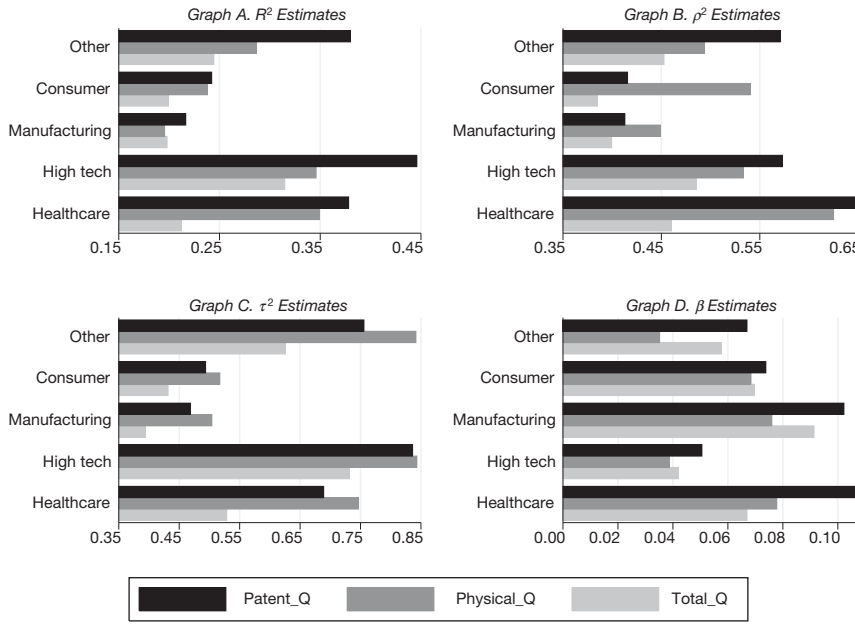
Given the growing importance of intangible capital in recent decades, I expect differences in explanatory power to change over time. I test this expectation by

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FIGURE 6

Total Investment–*Q* Relation Across Industries

Figure 6 presents results from regressions of total investment on lagged *Q*, firm fixed effects, and year fixed effects after sorting firms into five Fama–French industries. The numerator of each proxy for *Q* is the market value of common equity plus the book value of short- and long-term debt minus the book value of current assets. The denominator of each proxy for *Q* contains physical capital (book value of gross property, plant, and equipment). The denominator of patent *Q* (PATENT_Q) also includes my estimate of intangible capital (i.e., patent capital plus on-balance sheet intangible capital). The denominator of total *Q* (TOTAL_Q) also includes intangible capital as estimated by Peters and Taylor (2017). Total investment is capital expenditures plus R&D expenses scaled by the denominator of lagged *Q* being regressed on by investment. Graph A presents within R^2 estimates from ordinary least squares (OLS) regressions, which are ordinary R^2 estimates from estimating OLS on the transformed data. Graph B presents ρ^2 estimates, which are within R^2 estimates from hypothetical regressions of total investment on marginal *Q*. Graph C presents τ^2 estimates, which are within R^2 estimates from hypothetical regressions of *Q* on marginal *Q*. Graph D presents β estimates, which are bias-corrected coefficient estimates on *Q* from cumulant estimator regressions of total investment on *Q*. Industries are ranked from lowest (top) to highest (bottom) with respect to their average firm-level patent intensity.



regressing investment on lagged *Q*, firm fixed effects, and year fixed effects over rolling windows. In an effort to balance a reduction in noise and plot a sufficient number of estimates, I present results using 20-year rolling windows.[27]

Figure 7 plots estimates from regressions of physical investment on different proxies for *Q*. Each year in the plot represents the last fiscal year in the rolling window. Graphs A and B show that R^2 and ρ^2 estimates associated with patent *Q* were similar to those associated with other proxies in the first half of the sample. Over time though, differences in R^2 and ρ^2 estimates generally increase. Graph C shows that τ^2 estimates associated with patent *Q* are similar to those associated with physical *Q* but are more than 10 percentage points larger than those associated with total *Q*. Consistent with the main results, Graph D shows that bias-corrected coefficient estimates on patent *Q* are always larger than those on physical *Q* and

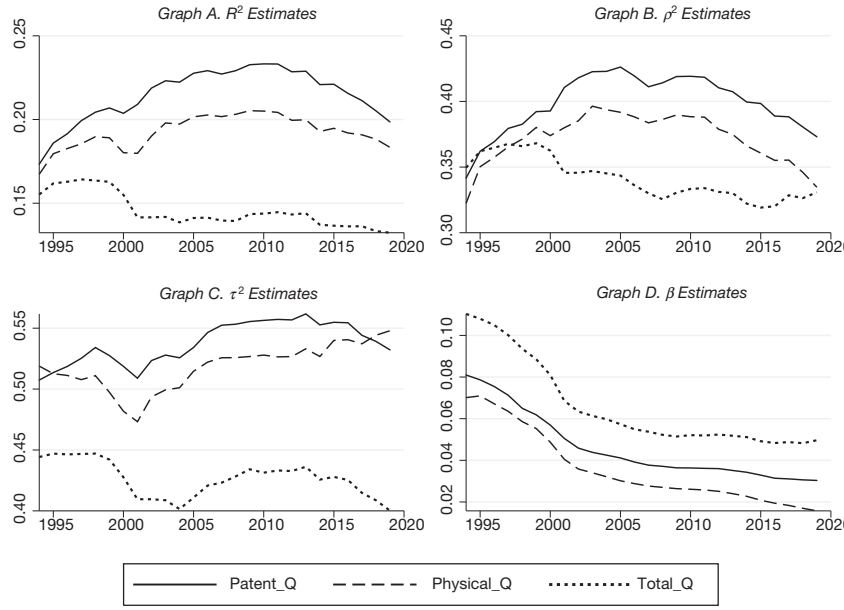
[27]In unreported tests, I find that my results are similar when using 15-year or 25-year rolling windows.

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FIGURE 7

Physical Investment–*Q* Relation over Time

Figure 7 presents results from regressions of physical investment on lagged *Q*, firm fixed effects, and year fixed effects over 20-year rolling windows. The numerator of each proxy for *Q* is the market value of common equity plus the book value of short- and long-term debt minus the book value of current assets. The denominator of each proxy for *Q* contains physical capital (book value of gross property, plant, and equipment). The denominator of patent *Q* (PATENT_Q) also includes my estimate of intangible capital (i.e., patent capital plus on-balance sheet intangible capital). The denominator of total *Q* (TOTAL_Q) also includes intangible capital as estimated by Peters and Taylor (2017). Physical investment is capital expenditures scaled by the denominator of lagged *Q* being regressed on by investment. Graph A presents within R^2 estimates from ordinary least squares (OLS) regressions, which are ordinary R^2 estimates from estimating OLS on the transformed data. Graph B presents ρ^2 estimates, which are within R^2 estimates from hypothetical regressions of physical investment on marginal *Q*. Graph C presents τ^2 estimates, which are within R^2 estimates from hypothetical regressions of *Q* on marginal *Q*. Graph D presents β estimates, which are bias-corrected coefficient estimates on *Q* from cumulant estimator regressions of physical investment on *Q*. Each year represents the last year in the 20-year window.



smaller than those on total *Q*. Declining coefficients on *Q* over time are well documented (e.g., Chen and Chen (2012)) and may be the result of changes in market competition and corporate governance (Gutiérrez and Philippon (2017)).

The results in Figure 8 show that the growing outperformance of patent *Q* is larger when analyzing R&D investment. Graphs A and C show that differences in R^2 and τ^2 estimates were small through the mid-1990s, but recent R^2 and τ^2 estimates associated with patent *Q* are approximately 10 percentage points larger than those associated with other proxies for *Q*. Graph B shows that ρ^2 estimates associated with patent *Q* are more recently approximately 6 and 15 percentage points larger than those associated with physical *Q* and total *Q*. Graph D shows that differences in β estimates have also grown over time.

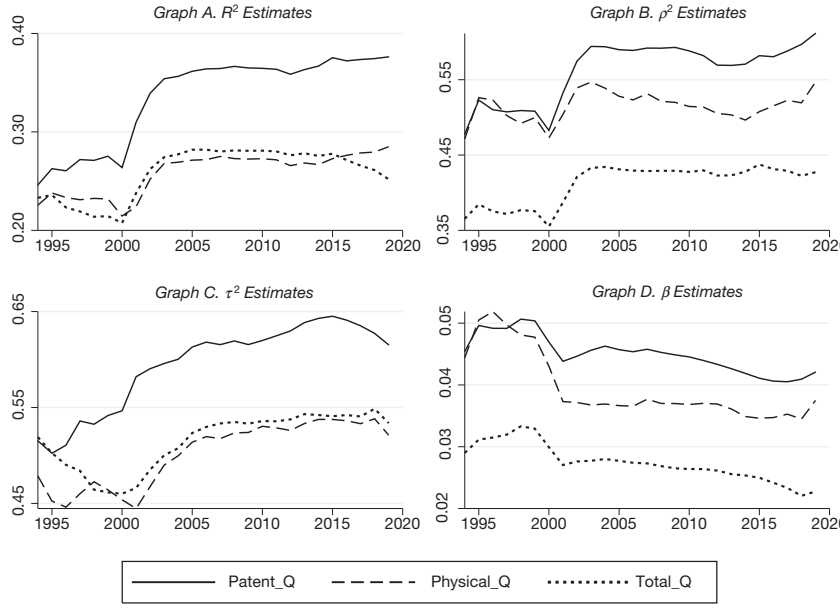
The growing outperformance of patent *Q* over time is especially salient in Figure 9, which focuses on total investment. Graph A shows that by the end of the sample, R^2 estimates associated with patent *Q* are 9 and 17 percentage points larger than those associated with physical *Q* and total *Q*. Graphs B and C show that ρ^2 and τ^2 estimates associated with patent *Q* generally grow at a faster rate than those

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FIGURE 8

R&D Investment–*Q* Relation over Time

Figure 8 presents results from regressions of R&D investment on lagged *Q*, firm fixed effects, and year fixed effects over 20-year rolling windows. The numerator of each proxy for *Q* is the market value of common equity plus the book value of short- and long-term debt minus the book value of current assets. The denominator of each proxy for *Q* contains physical capital (book value of gross property, plant, and equipment). The denominator of patent *Q* (PATENT_Q) also includes my estimate of intangible capital (i.e., patent capital plus on-balance sheet intangible capital). The denominator of total *Q* (TOTAL_Q) also includes intangible capital as estimated by Peters and Taylor (2017). R&D investment is R&D expenses scaled by the denominator of lagged *Q* being regressed on by investment. Graph A presents within R^2 estimates from ordinary least squares (OLS) regressions, which are ordinary R^2 estimates from estimating OLS on the transformed data. Graph B presents ρ^2 estimates, which are within R^2 estimates from hypothetical regressions of R&D investment on marginal *Q*. Graph C presents τ^2 estimates, which are within R^2 estimates from hypothetical regressions of *Q* on marginal *Q*. Graph D presents β estimates, which are bias-corrected coefficient estimates on *Q* from cumulant estimator regressions of R&D investment on *Q*. Each year represents the last year in the 20-year window.



associated with other proxies for *Q*. Graph D shows that β estimates associated with patent *Q* decline over time at a slower rate than those associated with either physical *Q* or total *Q*.

Figures 7–9 show that the outperformance of patent *Q* is present in nearly every 20-year window, is present across investment types, and has grown as the importance of patent capital has grown. These results are consistent with the implication in Andrei, Mann, and Moyen (2019) that as intangible capital has become more important over time, the explanatory power of different proxies for *Q* has been determined by their ability to estimate firm-level intangible capital.

V. Investment, *Q*, and Cash Flow

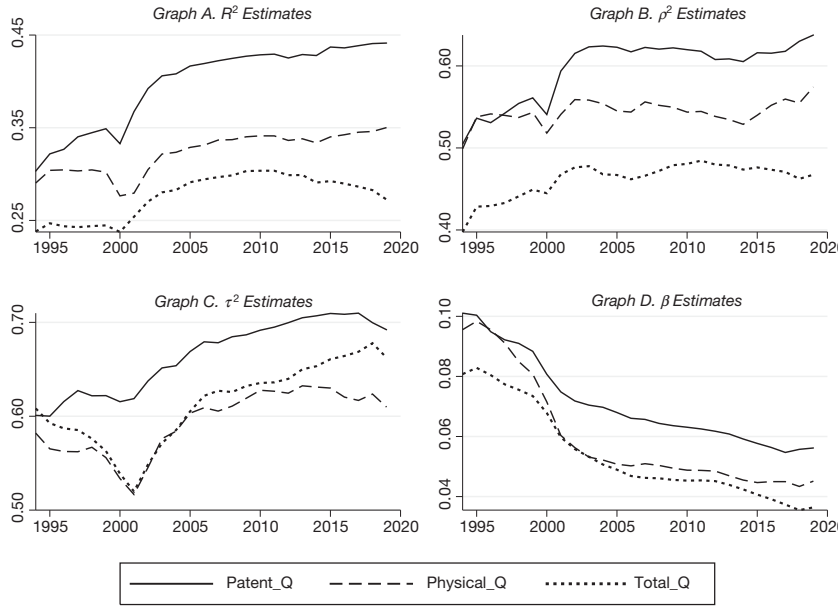
The *Q* theory of investment states that *Q* should be the sole determinant of new investment. However, many empirical tests show that when regressing investment on *Q* and cash flow, there is a positive coefficient on cash flow (Hassett and Hubbard (1997), Caballero (1999)).

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FIGURE 9

Total Investment–*Q* Relation over Time

Figure 9 presents results from regressions of total investment on lagged *Q*, firm fixed effects, and year fixed effects over 20-year rolling windows. The numerator of each proxy for *Q* is the market value of common equity plus the book value of short- and long-term debt minus the book value of current assets. The denominator of each proxy for *Q* contains physical capital (book value of gross property, plant, and equipment). The denominator of patent *Q* (PATENT_Q) also includes my estimate of intangible capital (i.e., patent capital plus on-balance sheet intangible capital). The denominator of total *Q* (TOTAL_Q) also includes intangible capital as estimated by Peters and Taylor (2017). Total investment is capital expenditures plus R&D expenses scaled by the denominator of lagged *Q* being regressed on by investment. Graph A presents within R^2 estimates from ordinary least squares (OLS) regressions, which are ordinary R^2 estimates from estimating OLS on the transformed data. Graph B presents ρ^2 estimates, which are within R^2 estimates from hypothetical regressions of total investment on marginal *Q*. Graph C presents τ^2 estimates, which are within R^2 estimates from hypothetical regressions of *Q* on marginal *Q*. Graph D presents β estimates, which are bias-corrected coefficient estimates on *Q* from cumulant estimator regressions of total investment on *Q*. Each year represents the last year in the 20-year window.



There are three main explanations for this positive cash flow coefficient. The first explanation is that a positive cash flow coefficient reflects the existence of financing constraints (Fazzari et al. (1988), Brown et al. (2009)). If *Q* is a sufficient proxy for investment opportunities, but firms cannot obtain external financing, investment will depend on internal cash flow. The second explanation is that measurement error in *Q* leads to a positive cash flow coefficient (Erickson and Whited (2000), Gomes (2001)). If *Q* is not a sufficient proxy for investment opportunities, a positive cash flow coefficient might be erroneously obtained even after controlling for *Q*. The third explanation is that cash flow provides additional information about future investment opportunities (Gilchrist and Himmelberg (1995), Alti (2003)). In all specifications, *Q* is lagged 1 period from investment, but since cash flow is concurrent with investment, cash flow might include information about future investment opportunities not contained in *Q*.

Given the importance of the extensive investment–cash flow literature, in this section, I investigate how patent *Q* affects the investment–cash flow relation.

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A. Investment–Cash Flow Relation

Since R&D investment is expensed before taxes, I adjust cash flow to reflect funds available before any investment is made:

$$\text{CASH_FLOW}_{it} = \frac{\text{IB}_{it} + \text{DP}_{it} + \text{R\&D}_{it}(1-\kappa)}{K^{\text{PHY}}_{i,t-1} + K^{\text{INT_PAT}}_{i,t-1}}, \tag{9}$$

in which IB is income before extraordinary items, DP is depreciation and amortization, and κ is the marginal tax rate. Following PT (2017), I estimate κ using simulated marginal tax rates from Graham (1996a), (1996b), and if not available, I use 30%. When regressing investment on other proxies for Q, the denominator of cash flow (CASH_FLOW) matches the denominator of those other proxies.

Table 4 presents results from cumulant estimator regressions of investment on Q, cash flow, firm fixed effects, and year fixed effects. Panel A presents results for all firms. Columns 1–3 show that although each proxy for Q is able to break the physical investment–cash flow relation, the only positive cash flow coefficient obtained is when controlling for patent Q. Columns 4–6 show that R&D investment is negatively associated with cash flow when controlling for each proxy for Q, but the cash flow coefficient is least negative when controlling for patent Q. Columns 7–9 show that when predicting total investment, the cash flow coefficient is largest, and highly significant, when controlling for patent Q. Although Tables 2 and 3 show that patent Q is a better proxy for Q, cash flow coefficients are always larger, albeit mostly insignificantly, when controlling for patent Q.[28]

Panel B of Table 4 shows that when focusing on firms with patent capital, differences in cash flow coefficients are larger. When predicting physical investment, controlling for patent Q leads to a cash flow coefficient that is highly significant and more than twice as large as that when controlling for physical Q or total Q. Columns 4–6 show that when regressing R&D investment on Q and cash flow, the cash flow coefficient is again least negative when controlling for patent Q. In columns 7–9, total investment is most sensitive to cash flow when controlling for patent Q.

The results in Table 4 show that when regressing investment on Q and cash flow, controlling for patent Q leads to larger cash flow coefficients. These results also show that differences in cash flow coefficients are larger when focusing on firms with patent capital. Overall, Table 4 shows that controlling for a better proxy for Q, which is patent Q in this case, does not necessarily lead to smaller cash flow coefficients, as conjectured in previous work (e.g., Kaplan and Zingales (1997)).

B. Why Does Controlling for Patent Q Lead to Larger Cash Flow Coefficients?

If patent Q is a better proxy for Tobin's Q, why are cash flow coefficients relatively larger when controlling for patent Q? A decomposition of the estimated

[28]Much of the literature obtains positive cash flow coefficients when regressing investment on Q and cash flow, but most of those coefficients are estimated using OLS. Negative cash flow coefficients are often found when using cumulant estimators (e.g., Erickson and Whited (2000), Erickson, et al. (2014), and PT (2017)).

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TABLE 4

Investment, *Q*, and Cash Flow

Table 4 presents results from cumulant estimator regressions of investment on lagged *Q*, cash flow, firm fixed effects, and year fixed effects. The numerator of each proxy for *Q* is the market value of common equity plus the book value of short- and long-term debt minus the book value of current assets. The denominator of each proxy for *Q* contains physical capital (book value of gross property, plant, and equipment). The denominator of patent *Q* (PATENT_Q) also includes my estimate of intangible capital (i.e., patent capital plus on-balance sheet intangible capital). The denominator of total *Q* (TOTAL_Q) also includes intangible capital as estimated by Peters and Taylor (2017). Physical investment (PHYSICAL_INVEST) is scaled capital expenditures. R&D investment (R&D_INVEST) is scaled R&D expenses. Total investment (TOTAL_INVEST) is physical investment plus R&D investment. Cash flow (CASH_FLOW) is income before extraordinary items plus depreciation and amortization plus tax-adjusted R&D expenses. All regression variables are scaled by the denominator of lagged *Q* being regressed on by investment. ρ^2 is the within R^2 estimate from the hypothetical regression of investment on marginal *Q*. τ^2 is the within R^2 estimate from the hypothetical regression of estimated *Q* on marginal *Q*. Panel A presents results for all firms. Panel B presents results for firms with patent capital at any point during the sample period. Standard errors in parentheses are estimated using influence functions and are clustered by firm. Bolded coefficients are significant at the 5% level.

	PHYSICAL_INVEST			R&D_INVEST			TOTAL_INVEST		
	1	2	3	4	5	6	7	8	9
Panel A. All Firms									
PATENT_Q	**0.045** (0.0006)			**0.050** (0.0009)			**0.073** (0.0011)		
PHYSICAL_Q		**0.028** (0.0004)			**0.040** (0.0011)			**0.054** (0.0011)	
TOTAL_Q			**0.066** (0.0012)			**0.031** (0.0006)			**0.055** (0.0010)
CASH_FLOW	0.002 (0.005)	−0.003 (0.004)	−0.010 (0.007)	**−0.023** (0.009)	**−0.032** (0.008)	**−0.033** (0.005)	**0.034** (0.011)	−0.004 (0.010)	**0.020** (0.009)
ρ^2	0.385 (0.009)	0.330 (0.009)	0.343 (0.009)	0.594 (0.017)	0.519 (0.019)	0.433 (0.015)	0.626 (0.015)	0.542 (0.017)	0.485 (0.013)
τ^2	0.528 (0.012)	0.507 (0.013)	0.426 (0.012)	0.605 (0.017)	0.530 (0.021)	0.515 (0.016)	0.659 (0.016)	0.603 (0.019)	0.589 (0.016)
No. of obs.	161,564	161,564	161,564	72,156	72,156	72,156	72,156	72,156	72,156
Panel B. Firms with Patent Capital									
PATENT_Q	**0.034** (0.0006)			**0.054** (0.0010)			**0.079** (0.0012)		
PHYSICAL_Q		**0.020** (0.0004)			**0.042** (0.0012)			**0.056** (0.0013)	
TOTAL_Q			**0.040** (0.0009)			**0.032** (0.0008)			**0.055** (0.0011)
CASH_FLOW	**0.013** (0.006)	0.003 (0.004)	0.006 (0.007)	**−0.027** (0.010)	**−0.037** (0.010)	**−0.036** (0.007)	**0.034** (0.013)	−0.007 (0.012)	**0.023** (0.010)
ρ^2	0.459 (0.012)	0.373 (0.013)	0.374 (0.012)	0.600 (0.018)	0.521 (0.021)	0.435 (0.016)	0.645 (0.015)	0.549 (0.019)	0.497 (0.015)
τ^2	0.574 (0.015)	0.534 (0.017)	0.476 (0.015)	0.618 (0.019)	0.535 (0.024)	0.524 (0.018)	0.660 (0.017)	0.597 (0.022)	0.591 (0.017)
No. of obs.	85,422	85,422	85,422	58,518	58,518	58,518	58,518	58,518	58,518

cash flow coefficient reveals that this result is due in large part to differences in investment–cash flow covariances between specifications.

When using the cumulant estimator to predict investment with both *Q* and cash flow, the errors-in-variables model is:

$$y_{it} = z_{it}\alpha + q_{it}\beta + u_{it}, \tag{10}$$

$$x_{it} = q_{it} + e_{it}, \tag{11}$$

in which y_{it} is investment, z_{it} is cash flow, and x_{it} is an observable proxy for the true, unobservable q_{it}. The cash flow coefficient can be estimated as follows:

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$$\alpha = \mu_y - \mu_x \beta, \tag{12}$$

in which μ_y = Cov(y, z)/Var(z) and μ_x = Cov(x, z)/Var(z).[29] Not only does the estimated cash flow coefficient depend on the measurement quality of a particular proxy for Q, as reflected in the size of β, but it also depends on μ_y and μ_z, whose estimates can be recovered from OLS regressions of investment on cash flow (i.e., y on z) and observable proxies for Q on cash flow (i.e., x on z).

To understand whether μ_y and μ_x estimates are driving differences in estimated cash flow coefficients, Table 5 presents μ_y and μ_x estimates alongside β and α estimates. Both panels show that there is indeed significant variation in μ_y and μ_x estimates. With only one exception in each panel, μ_y and μ_x estimates associated with patent Q are larger than those associated with other proxies for Q.

Since the cash flow coefficient (i.e., α) is increasing in μ_y, and decreasing in both μ_x and β, larger cash flow coefficients associated with patent Q must be the result of larger μ_y estimates, smaller μ_x estimates, smaller β estimates, or a combination thereof. Upon examination, we see that larger cash flow coefficients associated with patent Q are almost always driven by larger μ_y estimates.

For example, when focusing on physical investment in Panel A of Table 5, each estimate associated with patent Q is larger than that associated with physical Q. Since larger μ_x and β estimates lead to smaller α estimates, larger α estimates associated with patent Q must be driven by larger μ_y estimates. This pattern is

TABLE 5

Investment, *Q*, and Cash Flow: Determinants of the Cash Flow Coefficient

Table 5 presents estimates of the determinants of the estimated coefficient on cash flow (i.e., α) from cumulant estimator regressions of investment on lagged Q, cash flow, firm fixed effects, and year fixed effects. When using the cumulant estimator, $\alpha = \mu_y - \mu_x\beta$. μ_y = Cov(y, z)/Var(z), in which y is investment and z is cash flow. μ_x = Cov(x, z)/Var(z), in which x is an observable proxy for Q. β is the coefficient estimate on Q when using the cumulant estimator. The numerator of each proxy for Q is the market value of common equity plus the book value of short- and long-term debt minus the book value of current assets. The denominator of each proxy for Q contains physical capital (book value of gross property, plant, and equipment). The denominator of patent Q (PATENT_Q) also includes my estimate of intangible capital (i.e., patent capital plus on-balance sheet intangible capital). The denominator of total Q (TOTAL_Q) also includes intangible capital as estimated by Peters and Taylor (2017). Physical investment (PHYSICAL_INVEST) is scaled capital expenditures. R&D investment (R&D_INVEST) is scaled R&D expenses. Total investment (TOTAL_INVEST) is physical investment plus R&D investment. Cash flow is income before extraordinary items plus depreciation and amortization plus tax-adjusted R&D expenses. All regression variables are scaled by the denominator of lagged Q being regressed on by investment. Panel A presents results for all firms. Panel B presents results for firms with patent capital at any point during the sample period.

	PHYSICAL_INVEST				R&D_INVEST				TOTAL_INVEST			
	μ_y	μ_x	β	α	μ_y	μ_x	β	α	μ_y	μ_x	β	α
Panel A. All Firms												
PATENT_Q	0.174	3.81	0.045	0.002	0.191	4.29	0.050	−0.023	0.347	4.29	0.073	0.034
PHYSICAL_Q	0.091	3.39	0.028	−0.003	0.100	3.29	0.040	−0.032	0.172	3.29	0.054	−0.004
TOTAL_Q	0.193	3.06	0.066	−0.010	0.089	3.95	0.031	−0.033	0.239	3.95	0.055	0.020
Panel B. Firms with Patent Capital												
PATENT_Q	0.171	4.61	0.034	0.013	0.225	4.64	0.054	−0.027	0.398	4.64	0.079	0.034
PHYSICAL_Q	0.080	3.86	0.020	0.003	0.114	3.55	0.042	−0.037	0.190	3.55	0.056	−0.007
TOTAL_Q	0.175	4.21	0.040	0.006	0.105	4.38	0.032	−0.036	0.266	4.38	0.055	0.023

[29] See Section III of Erickson and Whited (2000) for details. The treatment reviews details for a GMM model, but as described in footnote 20, calculations are the same for third-order moments and third-order cumulants when the variables in question have a zero mean.

consistent across almost all comparisons that include patent Q. Since μ_y is investment–cash flow covariance divided by cash flow variance, larger cash flow coefficients associated with patent Q are mostly driven by larger investment–cash flow covariances.

The results in Table 5 show that estimated cash flow coefficients depend on much more than just the measurement quality of a particular proxy for Q. Developing a deeper understanding for how the denominator in different proxies for Q affects each of the three cash flow determinants (i.e., μ_y, μ_x, and β), and ultimately, the cash flow coefficient, is an interesting question for future research.

VI. Robustness

In this section, I show that my results are robust to using alternative patent depreciation rates, alternative estimates for the marginal Q of new patents, or higher-order cumulants. I also show that patent Q's performance is driven by both the replacement cost of patent capital and on-balance sheet intangible capital and is strongest when the replacement cost of intangible capital is estimated using just the replacement cost of patent capital and on-balance sheet intangible capital. Finally, I provide evidence suggesting that my results are not driven by a spurious correlation.

A. Changes to Patent Capital

In my main results, I depreciate the replacement cost of patent capital using industry-specific R&D depreciation rates. Although R&D investment is closely related to subsequently issued patents, the depreciation rates of the two may not be identical. For example, due to the property rights created by patents, patent depreciation rates might be slightly lower than R&D depreciation rates (Pakes and Schankerman (1984)). Panel A of Table 6 presents results when using alternative patent capital depreciation rates. The first row presents the main results. Rows 2–5 present results when using depreciation rates between 15% and 30%. The sixth row doubles all industry-specific depreciation rates. The results in the latter 5 rows are largely unchanged from the main results.

When estimating the replacement cost of patent capital, I estimate the marginal Q of new patents using quarterly macro-level estimates of Q that span the second quarter of 1953 through the second quarter of 2007 (Philippon (2009)). For marginal Q estimates outside of this time period, I assume a marginal Q of new patents of 1.5. To ensure that my results are not sensitive to alternative estimates of the marginal Q for new patents, Panel B of Table 6 presents results when making changes to this parameter. The first row presents the main results. In rows 2–6, I change the marginal Q of new patents to 1.0, 1.25, 1.5, 1.75, or 2.0. The results in the latter 5 rows are again largely unchanged from the main results.

Overall, Table 6 shows that patent Q's performance is not dependent on a particular patent capital depreciation rate or marginal Q estimate of new patents.

B. Higher-Order Cumulants

For all prior cumulant estimator results, I use a third-order cumulant, which results in an exactly identified system of equations. In Table 7, I present results from

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TABLE 6

Robustness: Changes to Patent Capital

Table 6 presents results from regressions of investment on lagged patent Q, firm fixed effects, and year fixed effects after making changes to how patent capital is estimated. The numerator of patent Q is the market value of common equity plus the book value of short- and long-term debt minus the book value of current assets. The denominator of patent Q is physical capital (book value of gross property, plant, and equipment) plus the replacement cost of patent capital and on-balance sheet intangible capital. Physical investment (PHYSICAL_INVEST) is scaled capital expenditures. R&D investment (R&D_INVEST) is scaled R&D expenses. Total investment (TOTAL_INVEST) is physical investment plus R&D investment. All investment variables are scaled by the denominator of patent Q. The first row of each panel presents the main results from Tables 2 and 3. Rows 2–6 of Panel A present results after making changes to the patent capital depreciation rate. Rows 2–6 of Panel B present results after making changes to the marginal Q of new patents. β is the bias-corrected coefficient on patent Q. R^2 estimates are ordinary R^2 estimates from estimating ordinary least squares on the transformed data. ρ^2 is the within R^2 estimate from the hypothetical regression of investment on marginal Q. τ^2 is the within R^2 estimate from the hypothetical regression of patent Q on marginal Q.

	PHYSICAL_INVEST				R&D_INVEST				TOTAL_INVEST			
	β	R^2	ρ^2	τ^2	β	R^2	ρ^2	τ^2	β	R^2	ρ^2	τ^2
Panel A. Changes to the Patent Capital Depreciation Rate												
1. Main results (industry delta)	0.045	0.203	0.385	0.529	0.048	0.358	0.586	0.611	0.072	0.412	0.605	0.680
2. δ_p of 15%	0.045	0.204	0.386	0.530	0.049	0.362	0.590	0.613	0.072	0.415	0.609	0.683
3. δ_p of 20%	0.045	0.203	0.385	0.528	0.048	0.358	0.586	0.611	0.072	0.412	0.606	0.680
4. δ_p of 25%	0.045	0.202	0.384	0.526	0.048	0.354	0.582	0.608	0.072	0.409	0.604	0.676
5. δ_p of 30%	0.045	0.201	0.383	0.525	0.048	0.350	0.579	0.605	0.072	0.405	0.601	0.673
6. Double industry δ_p	0.045	0.200	0.382	0.523	0.048	0.344	0.573	0.600	0.071	0.399	0.597	0.669
Panel B. Changes to the Marginal Q of New Patents												
1. Main results (macro Q)	0.045	0.203	0.385	0.529	0.048	0.358	0.586	0.611	0.072	0.412	0.605	0.680
2. Marginal Q of 1.00	0.046	0.207	0.387	0.535	0.048	0.371	0.595	0.623	0.073	0.424	0.614	0.692
3. Marginal Q of 1.25	0.046	0.205	0.386	0.531	0.048	0.364	0.590	0.617	0.073	0.418	0.609	0.685
4. Marginal Q of 1.50	0.045	0.204	0.385	0.529	0.048	0.359	0.587	0.611	0.072	0.412	0.606	0.680
5. Marginal Q of 1.75	0.045	0.202	0.384	0.527	0.048	0.355	0.582	0.609	0.072	0.409	0.603	0.677
6. Marginal Q of 2.00	0.044	0.201	0.382	0.527	0.048	0.351	0.580	0.606	0.071	0.405	0.600	0.675

the main investment specification using higher-order cumulants. Panel A presents results associated with patent Q, Panel B presents results associated with physical Q, and Panel C presents results associated with total Q. The first row in each panel presents the main results. Across panels and within investment types, estimates of β, ρ^2, and τ^2 associated with patent Q are almost always larger than those associated with either physical Q or total Q. Thus, regardless of the order cumulant used, results associated with patent Q are more consistent with Q theory than those associated with other proxies for Q.

C. Changes to Intangible Capital

My estimate of intangible capital includes both patent capital and on-balance sheet intangible capital, but since on-balance sheet intangible capital is also included in total Q, the novelty of patent Q comes from the inclusion of patent capital. To isolate the effect of each component of intangible capital on patent Q's performance, I regress investment on patent Q after excluding either patent capital or on-balance sheet intangible capital.

Panel A of Table 8 presents the results. The main results are in the first row. The results in the second and third rows of Panel A reflect the exclusion of patent capital or on-balance sheet intangible capital. In both rows, all estimates decline to similar levels regardless of whether patent capital or on-balance sheet intangible capital is excluded. These results show that each component of intangible capital is about equally important to patent Q's performance.

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TABLE 7

Robustness: Higher-Order Cumulants

Table 7 presents results from cumulant estimator regressions of investment on lagged Q, firm fixed effects, and year fixed effects using higher-order cumulants. The numerator of each proxy for Q is the market value of common equity plus the book value of short- and long-term debt minus the book value of current assets. The denominator of each proxy for Q contains physical capital (book value of gross property, plant, and equipment). The denominator of patent Q (PATENT_Q) also includes my estimate of intangible capital (i.e., patent capital plus on-balance sheet intangible capital). The denominator of total Q (TOTAL_Q) also includes intangible capital as estimated by Peters and Taylor (2017). Physical investment (PHYSICAL_INVEST) is scaled capital expenditures. R&D investment (R&D_INVEST) is scaled R&D expenses. Total investment (TOTAL_INVEST) is physical investment plus R&D investment. All investment variables are scaled by the denominator of lagged Q being regressed on by investment. β is the bias-corrected coefficient on patent Q. ρ^2 is the within R^2 estimate from the hypothetical regression of investment on marginal Q. τ^2 is the within R^2 estimate from the hypothetical regression of the proxy for Q on marginal Q.

	PHYSICAL_INVEST			R&D_INVEST			TOTAL_INVEST		
	β	ρ^2	τ^2	β	ρ^2	τ^2	β	ρ^2	τ^2
Panel A. Investment and Patent Q									
1. Main results (third-order)	0.045	0.385	0.529	0.048	0.586	0.611	0.072	0.605	0.680
2. Fourth-order	0.040	0.342	0.595	0.049	0.596	0.600	0.067	0.567	0.726
3. Fifth-order	0.041	0.348	0.584	0.049	0.595	0.601	0.068	0.574	0.717
4. Sixth-order	0.039	0.330	0.616	0.041	0.499	0.717	0.064	0.540	0.763
5. Seventh-order	0.041	0.345	0.590	0.048	0.587	0.610	0.068	0.575	0.716
6. Eighth-order	0.037	0.318	0.639	0.045	0.545	0.657	0.066	0.557	0.740
Panel B. Investment and Physical Q									
1. Main results (third-order)	0.028	0.340	0.493	0.040	0.526	0.521	0.054	0.542	0.603
2. Fourth-order	0.024	0.283	0.592	0.043	0.567	0.482	0.053	0.534	0.613
3. Fifth-order	0.024	0.286	0.585	0.040	0.530	0.517	0.053	0.533	0.614
4. Sixth-order	0.022	0.269	0.623	0.039	0.517	0.529	0.045	0.459	0.712
5. Seventh-order	0.024	0.284	0.590	0.037	0.490	0.558	0.053	0.534	0.613
6. Eighth-order	0.023	0.279	0.602	0.039	0.518	0.528	0.052	0.527	0.621
Panel C. Investment and Total Q									
1. Main results (third-order)	0.066	0.347	0.420	0.029	0.429	0.514	0.054	0.463	0.616
2. Fourth-order	0.082	0.231	0.341	0.031	0.452	0.488	0.050	0.432	0.660
3. Fifth-order	0.082	0.232	0.340	0.030	0.445	0.495	0.051	0.442	0.646
4. Sixth-order	0.181	0.512	0.154	0.028	0.418	0.527	0.047	0.403	0.708
5. Seventh-order	0.069	0.195	0.406	0.029	0.422	0.522	0.051	0.443	0.644
6. Eighth-order	0.158	0.446	0.177	0.027	0.392	0.562	0.055	0.471	0.605

Since the sum of patent capital and on-balance sheet intangible capital is unlikely to perfectly measure intangible capital, it is possible that the inclusion of other components of intangible capital improves patent Q's performance. In Panel B of Table 8, I present results from regressions of investment on patent Q after adding estimates of organization capital, knowledge capital, or both from PT (2017) or EPW (2020). The main results are in the first row. In the second and third rows, I include one third or two thirds of estimated organization capital, which imply that 10% or 20% of SG&A is investment, rather than the 30% used in PT (2017). For all investment types, the coefficient on patent Q increases slightly, but the explanatory power of patent Q declines. The pattern is the same in the fourth row, in which I include all estimated organization capital. In the fifth and sixth rows, I include proxies for knowledge capital or knowledge capital plus organization capital. In both rows, the coefficient increases when predicting physical investment, but declines when predicting R&D investment and total investment. Once again, the explanatory power of patent Q declines for all investment types. All of these patterns are nearly identical in the seventh through eleventh rows, in which I include components of intangible capital from EPW (2020).

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TABLE 8

Robustness: Changes to Intangible Capital

Table 8 presents results from regressions of investment on lagged patent *Q*, firm fixed effects, and year fixed effects after making changes to how intangible capital is estimated. The numerator of patent *Q* is the market value of common equity plus the book value of short- and long-term debt minus the book value of current assets. The denominator of patent *Q* is physical capital (book value of gross property, plant, and equipment) plus the replacement cost of patent capital and on-balance sheet intangible capital. Physical investment (PHYSICAL_INVEST) is scaled capital expenditures. R&D investment (R&D_INVEST) is scaled R&D expenses. Total investment (TOTAL_INVEST) is physical investment plus R&D investment. All investment variables are scaled by the denominator of patent *Q*. The first row of each panel presents the main results from Tables 2 and 3. Rows 2 and 3 of Panel A exclude the replacement cost of patent capital or on-balance sheet intangible capital. In Panel B, rows 2–6 add estimates of organization capital, knowledge capital, or knowledge capital plus organization capital from Peters and Taylor (PT) (2017). Rows 7–11 of Panel B include estimates of organization capital, knowledge capital, and knowledge capital plus organization capital from Ewens et al. (EPW) (2020). β is the bias-corrected coefficient on patent *Q*. R^2 estimates are ordinary R^2 estimates from estimating ordinary least squares on the transformed data. ρ^2 is the within R^2 estimate from the hypothetical regression of investment on marginal *Q*. τ^2 is the within R^2 estimate from the hypothetical regression of patent *Q* on marginal *Q*.

	PHYSICAL_INVEST				R&D_INVEST				TOTAL_INVEST			
	β	R^2	ρ^2	τ^2	β	R^2	ρ^2	τ^2	β	R^2	ρ^2	τ^2
Panel A. Changes to Patent Q												
1. Main results	0.045	0.203	0.385	0.529	0.048	0.358	0.586	0.611	0.072	0.412	0.605	0.680
2. Exclude patent capital	0.035	0.190	0.363	0.522	0.043	0.332	0.556	0.597	0.060	0.382	0.573	0.667
3. Exclude on-bs intangibles	0.036	0.187	0.365	0.513	0.043	0.317	0.557	0.569	0.062	0.372	0.571	0.651
Panel B. Changes to Intangible Capital												
1. Main results	0.045	0.203	0.385	0.529	0.048	0.358	0.586	0.611	0.072	0.412	0.605	0.680
2. Include 1/3 org. capital (PT)	0.050	0.186	0.375	0.497	0.049	0.333	0.576	0.577	0.073	0.390	0.593	0.657
3. Include 2/3 org. capital (PT)	0.054	0.178	0.371	0.479	0.051	0.320	0.574	0.556	0.075	0.380	0.594	0.639
4. Include all org. capital (PT)	0.056	0.172	0.366	0.471	0.053	0.312	0.577	0.541	0.077	0.374	0.594	0.630
5. Include know. capital (PT)	0.060	0.163	0.350	0.465	0.030	0.252	0.461	0.546	0.058	0.313	0.491	0.637
6. Include know. plus org. capital (PT)	0.072	0.151	0.355	0.426	0.033	0.237	0.460	0.515	0.061	0.301	0.487	0.619
7. Include 1/3 org. capital (EPW)	0.052	0.185	0.376	0.491	0.048	0.327	0.567	0.576	0.073	0.384	0.587	0.654
8. Include 2/3 org. capital (EPW)	0.057	0.175	0.371	0.472	0.050	0.312	0.567	0.551	0.075	0.373	0.584	0.638
9. Include all org. capital (EPW)	0.060	0.170	0.369	0.460	0.052	0.305	0.572	0.533	0.078	0.367	0.588	0.624
10. Include know. capital (EPW)	0.059	0.167	0.355	0.470	0.030	0.256	0.463	0.554	0.060	0.317	0.502	0.631
11. Include know. plus org. capital (EPW)	0.071	0.152	0.353	0.430	0.034	0.231	0.458	0.504	0.063	0.397	0.486	0.610

In summary, Table 8 shows that patent *Q*'s performance is dependent on both patent capital and on-balance sheet intangible capital. Table 8 also shows that the inclusion of proxies for organization capital, knowledge capital, or both always decreases the explanatory power of patent *Q*. Interestingly, the coefficient on patent *Q* increases when including different fractions of estimated organization capital. The latter result suggests that if one knew what fraction of SG&A is investment across firms and time, one might be able to construct a proxy for *Q* that results in both a larger investment–*Q* sensitivity and higher explanatory power. However, as of now, the replacement cost of intangible capital in the best proxy for *Q* includes only components for which we have observable values.

D. Are These Results Due to a Spurious Correlation?

Any adjustment to the capital stock affects both sides of the regression, so one concern might be that a spurious correlation between investment and patent *Q* is driving larger R^2 estimates. For example, R^2 estimates associated with patent *Q* might be artificially inflated because the inclusion of patent capital is creating excessive variability among regression variables within groups.

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TABLE 9

Investment and *Q*: Normalized Root-Mean-Square Error

Table 9 presents R^2 estimates and normalized root-mean-square error (NRMSE) estimates from ordinary least squares (OLS) regressions of investment on lagged *Q*, firm fixed effects, and year fixed effects. Root-mean-square error (RMSE) is the standard deviation of residuals from an OLS regression. The numerator of each proxy for *Q* is the market value of common equity plus the book value of short- and long-term debt minus the book value of current assets. The denominator of each proxy for *Q* contains physical capital (book value of gross property, plant, and equipment). The denominator of patent *Q* (PATENT_Q) also includes my estimate of intangible capital (i.e., patent capital plus on-balance sheet intangible capital). The denominator of total *Q* (TOTAL_Q) also includes intangible capital as estimated by Peters and Taylor (2017). Physical investment (PHYSICAL_INVEST) is scaled capital expenditures. R&D investment (R&D_INVEST) is scaled R&D expenses. Total investment (TOTAL_INVEST) is physical investment plus R&D investment. All investment variables are scaled by the denominator of lagged *Q* being regressed on by investment. For each investment type, I present three different NRMSE estimates. Std. Dev. is RMSE divided by the standard deviation of investment. IQR is RMSE divided by the interquartile range of investment. Range is RMSE divided by the range of investment. To ensure consistent comparisons between proxies for *Q*, investment has been demeaned to account for firm fixed effects and year fixed effects. Panel A presents results for all firms. Panel B presents results for firms with patent capital at any point during the sample period.

	PHYSICAL_INVEST				R&D_INVEST				TOTAL_INVEST			
	R^2	Std. Dev.	IQR	Range	R^2	Std. Dev.	IQR	Range	R^2	Std. Dev.	IQR	Range
Panel A. All Firms												
PATENT_Q	0.203	0.89	1.38	0.064	0.358	0.80	1.68	0.047	0.412	0.77	1.29	0.049
PHYSICAL_Q	0.168	0.91	1.36	0.068	0.274	0.85	2.74	0.041	0.327	0.82	1.71	0.044
TOTAL_Q	0.146	0.92	1.47	0.063	0.220	0.88	1.43	0.057	0.285	0.85	1.12	0.063
Panel B. Firms with Patent Capital												
PATENT_Q	0.263	0.86	1.19	0.068	0.369	0.79	1.51	0.049	0.425	0.76	1.20	0.049
PHYSICAL_Q	0.199	0.90	1.24	0.070	0.277	0.85	2.63	0.041	0.328	0.82	1.68	0.044
TOTAL_Q	0.178	0.91	1.21	0.066	0.225	0.88	1.37	0.057	0.293	0.84	1.06	0.067

To alleviate this concern, I use an additional statistic to compare model fits: normalized root-mean-square error (NRMSE). Scaling variables are different between proxies for *Q*, so root-mean-square error (RMSE) estimates do not allow for a valid comparison. By normalizing RMSE estimates, I am able to make comparisons across models with different scaling variables (Otto, Kadin, Casini, Torres, and Blenckner (2018)). Since no normalization method is obviously superior to another (Otto (2019)), I present three different NRMSE estimates in Table 9. Std. Dev. is RMSE divided by the standard deviation of investment. IQR is RMSE divided by the interquartile range of investment. Range is RMSE divided by the range of investment.

Both panels of Table 9 show that for each investment type, average NRMSE rankings associated with patent *Q* are lower than those associated with physical *Q* and total *Q*. For example, Panel A shows that when focusing on physical investment, the average NRMSE rankings of patent *Q*, physical *Q*, and total *Q* are 1.67, 2.00, and 2.33.[30] These rankings are similar across investment types and panels.[31]

It is possible that one normalization method is biased toward or against a particular proxy for *Q*. However, there is no clear relation between any particular RMSE normalization method and R^2 rankings. For example, although both panels of Table 9 show that physical *Q* explains the second most variation in physical

[30]When focusing on physical investment in Panel A, Std. Dev., IQR, and range estimates associated with patent *Q* are first lowest, second lowest, and second lowest. The average of 1, 2, and 2 is 1.67.

[31]In unreported tests, I find that R^2 rankings and NRMSE rankings are nearly identical when winsorizing at the 5% and 95% levels, which suggests that outliers are not driving my results.

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investment, the RMSE estimate normalized by the interquartile range is smallest for physical Q in Panel A but largest for physical Q in Panel B.

The results in Table 9 show that across investment types and normalization methods, the average rank of NRMSE estimates associated with patent Q is consistently lower than that associated with other proxies for Q. These results suggest that larger R^2 estimates associated with patent Q are not driven by a spurious correlation.

VII. Conclusion

In this article, I construct a new proxy for Tobin's Q, which I call patent Q, that includes the replacement cost of patent capital. The main result is that patent Q is a better proxy for Q than two commonly used proxies for Q. Specifically, I show that patent Q explains significantly more variation in firm-level investment and is more closely related to hypothetical marginal Q than both physical Q and total Q. I also show that bias-corrected coefficient estimates on Q are almost always larger when using patent Q. These results are stronger when focusing on subsamples with more patent capital.

Although investment is predicted more accurately by, and is more sensitive to, patent Q, I find that controlling for patent Q leads to relatively larger, not smaller, cash flow coefficients. Not only does this finding show that controlling for a better proxy for Q can lead to larger cash flow coefficients, but it suggests that relations between investment and other variables in the literature might depend on how those variables are scaled.

In summary, the results herein suggest that including components of intangible capital for which we have observable values is important for constructing a better proxy for Tobin's Q. Moving forward, it is important to understand whether the investment–Q relation can be further improved by including additional components of intangible capital for which we have observable values.

Appendix A. Estimated Patent Values

Appendix A describes how Stoffman et al. (2022) estimate patent values and how their methodology differs from that in Kogan et al. (2017). For complete details of the estimation procedure, see Section II.D of Kogan et al. (2017).

The idiosyncratic stock return R (i.e., firm's return minus the return on the CRSP value-weighted portfolio) for a given firm around the time that its patent j is issued is

$$R_j = v_j + \varepsilon_j, \tag{A-1}$$

in which v_j denotes the value of patent j, as a fraction of the firm's market capitalization, and ε_j denotes the component of the firm's stock return that is unrelated to the patent.

The economic value ξ of patent j is estimated as

$$\xi_j = M_j(1-\overline{\pi})^{-1}\frac{1}{N_j}E\left[v_j|R_j\right], \tag{A-2}$$

in which M_j is the market capitalization of the firm that is issued patent j on the day prior to the announcement of the patent issuance, and $\overline{\pi}$ is the unconditional probability of a

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successful patent application, which is approximately 56% of all progenitor applications (i.e., applications unrelated to any previously filed U.S. patent application) filed between 1996 and 2005 and examined before mid-2013 (Carley, Hedge, and Marco (2015). If multiple patents N_j are issued to the same firm on the same day as patent j, each patent is assigned fraction $1/N_j$ of the total value.

Since the market value of the patent v is assumed to be a positive random variable, v is distributed according to a normal distribution truncated at 0, $v_j \sim \mathcal{N}^+\left(0, \sigma^2_{vft}\right)$. The noise term is assumed to be normally distributed, $\varepsilon_j \sim \mathcal{N}\left(0, \sigma^2_{\varepsilon ft}\right)$. The filtered value of v_j, as a function of the idiosyncratic stock return R, is equal to

$$E\left[v_j | R_j\right] = \delta_{ft} R_j + \sqrt{\delta_{ft}}\sigma_{\varepsilon ft} \frac{\phi\left(-\sqrt{\delta_{ft}}\frac{R_j}{\sigma_{\varepsilon ft}}\right)}{1 - \Phi\left(-\sqrt{\delta_{ft}}\frac{R_j}{\sigma_{\varepsilon ft}}\right)}, \tag{A-3}$$

in which ϕ and Φ are the standard normal pdf and cdf, and δ is the signal-to-noise ratio,

$$\delta_{ft} = \frac{\sigma^2_{vft}}{\sigma^2_{vft} + \sigma^2_{\varepsilon ft}}. \tag{A-4}$$

Up until this point, the estimation procedure used in Stoffman et al. (2022) is the same as that in Kogan et al. (2017). The parameters σ^2_{vft} and $\sigma^2_{\varepsilon ft}$ need to be estimated. To do so, Kogan et al. (2017) specify that the signal-to-noise ratio is constant across firms and time (i.e., $\delta_{ft} = \delta$). To avoid using future information, Stoffman et al. (2022) allow the signal-to-noise ratio to vary over time but remain constant across firms (i.e., $\delta_{ft} = \delta_t$). To estimate δ_t, Stoffman et al. (2022) use γ_t to estimate the increase in the volatility of firm returns around patent announcement days from

$$\log\left(R_{fd}\right)^2 = \gamma_t I_{fd} + c_t Z_{fd} + u_{fdt}, \tag{A-5}$$

in which R_{fd} is the 3-day idiosyncratic return of firm f beginning on day d, and I_{fd} is a patent issue-day dummy. In each estimation, the sample is restricted to firms that have been granted at least one patent through month t, so every month, γ_t is updated to reflect all available information through that month. Controls Z_{fd} include day of the week and firm interacted with calendar month fixed effects (Kogan et al. (2017) use year fixed effects). The signal-to-noise estimate is recovered from the 1-month lagged estimated value of γ_t,

$$\widehat{\delta_t} = 1 - e^{-\widehat{\gamma_{t-1}}}. \tag{A-6}$$

The last step in estimating equation (A-3) involves estimating the variance of the measurement error $\sigma^2_{\varepsilon ft}$. This estimate is accomplished nonparametrically by using the sum of squared market-adjusted returns (Andersen and Teräsvirta (2009)), and although Kogan et al. (2017) allow this estimate to vary at an annual frequency (Stoffman et al. (2022) do so at a monthly frequency. Specifically, Stoffman et al. (2022) use a rolling 12-month period to provide a monthly update to the estimate of $\sigma^2_{\varepsilon ft}$. To obtain $\sigma^2_{\varepsilon ft}$, the realized mean idiosyncratic squared returns, σ^2_{ft}, are estimated using the squared residual from a rolling 12-month regression of daily firm returns on daily CRSP value-weighted returns (i.e., estimated idiosyncratic volatility is updated on a monthly basis). Since σ^2_{ft} is

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estimated over both announcement and nonannouncement days, it is comprised of both σ^2_{vft} and $\sigma^2_{\varepsilon ft}$. Given the estimate of σ^2_{ft}, the fraction of trading days that are announcement days, d_{ft}, and $\widehat{\gamma_t}$, the variance of the measurement error can be estimated, but once again, to ensure that no future information is used, Stoffman et al. (2022) use 1-month lagged parameters: $\sigma^2_{\varepsilon ft} = 3\sigma^2_{ft-1}\left(1 + 3d_{ft-1}\left(\widehat{e^{\gamma_{t-1}}} - 1\right)\right)^{-1}$.

Appendix B. Tables

TABLE B1

Investment and *Q*: Ordinary Least Squares

Table B1 presents results from ordinary least squares (OLS) regressions of investment on lagged *Q*, firm fixed effects, and year fixed effects. The numerator of each proxy for *Q* is the market value of common equity plus the book value of short- and long-term debt minus the book value of current assets. The denominator of each proxy for *Q* contains physical capital (book value of gross property, plant, and equipment). The denominator of patent *Q* (PATENT_Q) also includes my estimate of intangible capital (i.e., patent capital plus on-balance sheet intangible capital). The denominator of total *Q* (TOTAL_Q) also includes intangible capital as estimated by Peters and Taylor (2017). Intangible investment (INTANGIBLE_INVEST) is scaled R&D expenses (set to 0 if missing) plus 30% of scaled selling, general, and administrative expenses. Total investment from PT (2017) (TOTAL_INVEST_PT) is scaled capital expenditures (i.e., physical investment) plus intangible investment. Both investment variables are scaled by the denominator of lagged *Q* being regressed on by investment. Within R^2 estimates are ordinary R^2 estimates from estimating OLS on the transformed data. ΔR^2 estimates are R^2 estimates associated with patent *Q* minus R^2 estimates associated with either physical *Q* (columns 2 and 5) or total *Q* (columns 3 and 6) within each investment type. Panel A presents results for all firms. Panel B presents results for firms with patent capital at any point during the sample period. Standard errors in parentheses are clustered by firm. Standard errors below R^2 estimates and ΔR^2 estimates are estimated using influence functions. Bolded estimates are significant at the 5% level.

	INTANGIBLE_INVEST			TOTAL_INVEST_PT		
	1	2	3	4	5	6
Panel A. All Firms						
PATENT_Q	**0.039**			**0.067**		
	(0.0006)			(0.0008)		
PHYSICAL_Q		**0.030**			**0.046**	
		(0.0006)			(0.0007)	
TOTAL_Q			**0.019**			**0.050**
			(0.0003)			(0.0007)
WithinR^2	0.307	0.279	0.204	0.341	0.299	0.242
	(0.005)	(0.006)	(0.004)	(0.005)	(0.005)	(0.004)
ΔR^2	–	**0.029**	**0.104**	–	**0.042**	**0.099**
	–	(0.005)	(0.004)	–	(0.004)	(0.004)
No. of obs.	161,564	161,564	161,564	161,564	161,564	161,564
Panel B. Firms with Patent Capital						
PATENT_Q	**0.047**			**0.069**		
	(0.0009)			(0.0010)		
PHYSICAL_Q		**0.032**			**0.044**	
		(0.0007)			(0.0009)	
TOTAL_Q			**0.023**			**0.044**
			(0.0005)			(0.0007)
WithinR^2	0.373	0.311	0.249	0.410	0.339	0.292
	(0.007)	(0.008)	(0.005)	(0.007)	(0.007)	(0.005)
ΔR^2	–	**0.062**	**0.124**	–	**0.071**	**0.118**
	–	(0.007)	(0.006)	–	(0.007)	(0.005)
No. of obs.	85,422	85,422	85,422	85,422	85,422	85,422

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TABLE B2

Investment and *Q*: Cumulant Estimator

Table B2 presents results from cumulant estimator regressions of investment on lagged *Q*, firm fixed effects, and year fixed effects. The numerator of each proxy for *Q* is the market value of common equity plus the book value of short- and long-term debt minus the book value of current assets. The denominator of each proxy for *Q* contains physical capital (book value of gross property, plant, and equipment). The denominator of patent *Q* (PATENT_Q) also includes my estimate of intangible capital (i.e., patent capital plus on-balance sheet intangible capital). The denominator of total *Q* (TOTAL_Q) also includes intangible capital as estimated by Peters and Taylor (2017). Intangible investment (INTANGIBLE_INVEST) is scaled R&D expenses (set to 0 if missing) plus 30% of scaled selling, general, and administrative expenses. Total investment from PT (2017) (TOTAL_INVEST_PT) is scaled capital expenditures (i.e., physical investment) plus intangible investment. Both investment variables are scaled by the denominator of lagged *Q* being regressed on by investment. ρ^2 is the within R^2 estimate from the hypothetical regression of investment on marginal *Q*. τ^2 is the within R^2 estimate from the hypothetical regression of estimated *Q* on marginal *Q*. Panel A presents results for all firms. Panel B presents results for firms with patent capital at any point during the sample period. Standard errors in parentheses are estimated using influence functions and are clustered by firm. Bolded coefficients are significant at the 5% level.

	INTANGIBLE_INVEST			TOTAL_INVEST_PT		
	1	2	3	4	5	6
Panel A. All Firms						
PATENT_Q	**0.066** (0.0009)			**0.101** (0.0010)		
PHYSICAL_Q		**0.055** (0.0010)			**0.077** (0.0011)	
TOTAL_Q			**0.037** (0.0006)			**0.086** (0.0011)
ρ^2	0.515 (0.011)	0.506 (0.014)	0.389 (0.010)	0.517 (0.010)	0.497 (0.012)	0.415 (0.009)
τ^2	0.597 (0.014)	0.551 (0.016)	0.523 (0.013)	0.659 (0.013)	0.602 (0.014)	0.582 (0.012)
No. of obs.	161,564	161,564	161,564	161,564	161,564	161,564
Panel B. Firms with Patent Capital						
PATENT_Q	**0.072** (0.0011)			**0.097** (0.0012)		
PHYSICAL_Q		**0.056** (0.0014)			**0.078** (0.0014)	
TOTAL_Q			**0.039** (0.0007)			**0.067** (0.0010)
ρ^2	0.565 (0.014)	0.536 (0.019)	0.420 (0.012)	0.579 (0.013)	0.543 (0.017)	0.450 (0.012)
τ^2	0.661 (0.018)	0.580 (0.022)	0.594 (0.019)	0.709 (0.016)	0.625 (0.019)	0.649 (0.016)
No. of obs.	85,422	85,422	85,422	85,422	85,422	85,422

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