J. Plasma Physics (2005), *vol.* 71, *part* 4, *pp.* 449–474. © 2005 Cambridge University Press
doi:10.1017/S0022377804003198 Printed in the United Kingdom

Studies of the particle diffusion coefficient in the fluctuating magnetic field by the direct-interaction approximation

M. TAGUCHI

College of Industrial Technology, Nihon University, 2-11-1 Shin-ei, Narashino, Chiba 275-0005, Japan
(taguchi@math.cit.nihon-u.ac.jp)

(Received 14 June 2004)

Abstract. The particle diffusion in a stochastic magnetic field is investigated by using the functional integral method. With the ensemble average of a functional representation to a kinetic equation over the fluctuating magnetic field and the random variables modeling the Coulomb collisions, a closed set of equations for an ensemble-averaged distribution function and a response function to an infinitesimal external perturbation is derived within the framework of the direct-interaction approximation (DIA). By using the Markovian approximation to the equation for the response function, the cross-field diffusion coefficients are found to be obtained from nonlinear ordinary differential equations. Explicitly calculated diffusion coefficients for various parameters are also shown in the shearless and sheared configurations.

1. Introduction

Understanding particle and heat transport is an important key issue in thermonuclear fusion research. Historically, one mechanism that has been considered in detail has been the effect of stochastic magnetic fields on cross-field transport (Rechester and Rosenbluth 1978; Krommes et al. 1983; Thyagaraja and Hass 1985; Isichenko 1991a, b; Rose 1992; Rax and White 1992). Interest in this topic persists today, not only because of its possible importance to fusion but also because its proper mathematical modeling raises fundamental issues of nonlinear statistical dynamics. A kinetic equation in the presence of a stochastic magnetic field is a reasonable starting point for the theoretical treatment of this problem. However, solving such a stochastic kinetic equation is extremely difficult, and so many different approaches and approximations are presented for this difficult problem. Until now, the various transport regimes caused by the stochastic magnetic field are proposed by intensive studies (Krommes et al. 1983).

Balescu et al. (1994) have presented an interesting analytically tractable model describing the particle transport in a fluctuating magnetic field. They have considered this transport in a slab configuration of the strong shearless magnetic field. The fluctuating components of the magnetic field are assumed to be a Gaussian random process, and the mutual collisions between particles are modeled by the random variation of their velocity. Even though the problem is quite simplified by using this

https://doi.org/10.1017/S0022377804003198 Published online by Cambridge University Press

model, the rigorous mathematical treatment is still impossible. In a subsequent paper, Wang et al. (1995) investigated the statistical properties of the fluctuating magnetic field in this model by using the Corrsin approximation. They combined this property with a random walk picture of a test particle, and derived the well-known Rechester–Rosenbluth diffusion coefficient (Rechester and Rosenbluth 1978) for a strongly collisional plasma. Eijnden and Balescu (1996b) proceeded to investigate this model further. Starting with a stochastic kinetic equation associating the equations of a test particle, they found by using the direct-interaction approximation (DIA) (Martin et al. 1973; Krommes 1984; Krommes 2002) and the Markovian approximation that the anomalous diffusion coefficient can be determined by solving a nonlinear second-order differential equation and, as a solution of this equation, the previously proposed various regimes of transport can be obtained.

The purpose of this paper is to more thoroughly study the particle-diffusion model of Eijnden and Balescu (1996b) within the framework of the DIA. We extend their theory to the form applicable to fast particles and also extend it to a finite shear configuration. In the Eijnden–Balescu theory, an ad hoc approximation, which is essential in deriving the Rechester–Rosenbluth diffusion coefficient, is implicitly used in addition to the DIA and the Markovian approximation. The validity of this additional approximation is also studied.

This paper is organized as follows. In Sec. 2, we first present our model, describing the particle transport in a fluctuating magnetic field. Our model agrees with that of Eijnden and Balescu (1996b) except for the inclusion of the particle velocity $v_\parallel$ parallel to the averaged magnetic field in a kinetic equation. We next introduce a functional integral representation for the kinetic equation. Ensemble-averaging this representation over the fluctuating magnetic field and random variables modeling the Coulomb collisions, and using the DIA, we obtain a closed set of equations for an ensemble-averaged distribution function and a response function to an infinitesimal external perturbation. In Sec. 3, the cross-field diffusion tensor is shown to be expressed in terms of the response function. Calculating this response function by using the Markovian approximation, we derive a second-order differential equation for the time-integral of the running diffusion coefficient. Explicitly solving this equation, we evaluate the diffusion coefficients for various parameters. Comparison of these explicitly evaluated diffusion coefficients with those of Eijnden and Balescu in the $v_\parallel = 0$ limit is also given in this section. In Sec. 4, the effect of magnetic shear on the diffusion tensor is considered. Extending the theory for the shearless configuration in Secs 2 and 3 to a finite shear configuration, we also derive a differential equation for the time-integral of the diffusion tensor in the collisionless and collisional regimes. Reduction of the diffusion coefficients by the effect of shear is shown by the explicit calculation. Concluding remarks are given in Sec. 5.

2. Basic equations

2.1. Kinetic equation

We consider a magnetic field given by

$$\mathbf{B}(\mathbf{r}) = B_0[\mathbf{e}_z + b_x(\mathbf{r})\mathbf{e}_x + b_y(\mathbf{r})\mathbf{e}_y], \tag{2.1}$$

where $\{\mathbf{e}_x, \mathbf{e}_y, \mathbf{e}_z\}$ are the unit vectors in the Cartesian coordinates (x, y, z), $B_0\mathbf{e}_z$ is the constant averaged magnetic field, and $B_0[b_x(\mathbf{r})\mathbf{e}_x + b_y(\mathbf{r})\mathbf{e}_y] \equiv B_0\mathbf{b}$ is the fluctuating magnetic field with the random variables $b_x(\mathbf{r})$ and $b_y(\mathbf{r})$ of the Gaussian

https://doi.org/10.1017/S0022377804003198 Published online by Cambridge University Press

process. We start with a kinetic equation for a distribution function in the magnetic field (2.1)

$$\partial_t f + v_\parallel \partial_z f + \sqrt{\epsilon}\, v_\parallel \mathbf{b}(\mathbf{r}) \cdot \nabla_\perp f - C(f) = 0, \tag{2.2}$$

where $\nabla_\perp = \mathbf{e}_x \partial/\partial x + \mathbf{e}_y \partial/\partial y$; the function f is normalized as $\int f(\mathbf{r}, t)\, d\mathbf{r} = 1$; and the subscripts $\parallel$ and $\perp$ refer to the parallel and perpendicular to the averaged magnetic field. In (2.2) we have introduced a formal ordering parameter ϵ, which physically means $\epsilon = \mathrm{O}(\mathbf{b}^2 l_\parallel^2 / l_\perp^2)$, where $l_\parallel$ and $l_\perp$ are the characteristic parallel and perpendicular lengths. The formal parameter ϵ is finally set to be one.

The effect of Coulomb collisions is modeled by using Gaussian random variation $\boldsymbol{\eta}(t)$ of the velocity as

$$C(f) = -\sqrt{\epsilon}\, \eta_\parallel(t) \partial_z f - \sqrt{\epsilon} [\boldsymbol{\eta}_\perp(t) + \sqrt{\epsilon}\, \eta_\parallel(t) \mathbf{b}(\mathbf{r})] \cdot \nabla_\perp f. \tag{2.3}$$

Let us now represent the ensemble averages of the random variables $\mathbf{b}(\mathbf{r})$, $\eta_\parallel(t)$ and $\boldsymbol{\eta}_\perp(t)$ by $\langle \cdot \rangle_b$, $\langle \cdot \rangle_\parallel$ and $\langle \cdot \rangle_\perp$, respectively. Then we assume the following forms of the correlation functions characterizing the statistical properties of Gaussian random variables $\mathbf{b}(\mathbf{r})$, $\eta_\parallel(t)$ and $\boldsymbol{\eta}_\perp(t)$:

$$\langle \eta_\parallel(t) \eta_\parallel(t') \rangle_\parallel = \chi_\parallel \nu R[\nu(t - t')], \tag{2.4}$$

$$\langle \boldsymbol{\eta}_\perp(t) \boldsymbol{\eta}_\perp(t') \rangle_\perp = \chi_\perp \nu R[\nu(t - t')] \mathbf{I}, \tag{2.5}$$

$$\langle \mathbf{b}(\mathbf{r}) \mathbf{b}(\mathbf{r}') \rangle_b = \langle \nabla A(\mathbf{r}) \times \mathbf{e}_z \nabla' A(\mathbf{r}') \times \mathbf{e}_z \rangle_b \equiv \mathbf{F}(\mathbf{r} - \mathbf{r}') \tag{2.6}$$

with $R[\nu(t - t')] = \exp(-\nu |t - t'|)$ and

$$\langle A(\mathbf{r}) A(\mathbf{0}) \rangle_b = \beta^2 \lambda_\perp^2 \exp\left[-\frac{z^2}{2\lambda_\parallel^2} - \frac{\mathbf{r}_\perp^2}{2\lambda_\perp^2} \right], \tag{2.7}$$

where $\mathbf{r}_\perp = x\mathbf{e}_x + y\mathbf{e}_y$; $\mathbf{I} = \mathbf{e}_x \mathbf{e}_x + \mathbf{e}_y \mathbf{e}_y$; $\chi_\parallel = v_\mathrm{T}^2 / 2\nu$ and $\chi_\perp = v_\mathrm{T}^2 \nu / 2\Omega^2$ are the parallel and perpendicular classical diffusion coefficients; v_T and Ω are the thermal velocity and the Larmor frequency; ν is the collision frequency; $\lambda_\parallel$ and $\lambda_\perp$ are the parallel and perpendicular correlation lengths; and β represents the amplitude of the fluctuating magnetic field $\mathbf{b}(\mathbf{r})$. We further assume that there is no correlation between $\mathbf{b}(\mathbf{r})$, $\eta_\parallel(t)$ and $\boldsymbol{\eta}_\perp(t)$.

2.2. Functional representation

Let us introduce a following functional representation for a functional $F[f]$ of the distribution function (Jensen 1981; Jouvet and Phythian 1979; Krommes 2002; Taguchi 2003):

$$F[f] = \mathcal{N} \int \mathscr{D}[f'] \mathscr{D}[\hat{f}] F[f'] e^{-\mathscr{L}} \tag{2.8}$$

with

$$\mathscr{L} = \hat{f}(1) \{ \partial_t + [v_\parallel + \sqrt{\epsilon}\, \eta_\parallel(t)] \partial_z + \sqrt{\epsilon} [(v_\parallel + \sqrt{\epsilon}\, \eta_\parallel(t)) \mathbf{b}(\mathbf{r}) + \boldsymbol{\eta}_\perp(t)] \cdot \nabla_\perp \} f'(1), \tag{2.9}$$

where the index 1 denotes $1 = (\mathbf{r}, t)$ and $\mathcal{N}$ is the multiplicative constant irrelevant to the following calculation. In this section, we assume the integration over repeated indices. Note that f' and $\hat{f}$ are merely integral variables in the functional integral representation. The ensemble average over the random variables $\mathbf{b}(\mathbf{r})$, $\eta_\parallel(t)$ and $\boldsymbol{\eta}_\perp(t)$, which is denoted by $\langle \cdot \rangle \equiv \langle \cdot \rangle_{b,\parallel,\perp}$, yields the ensemble-averaged functional

$$\langle F[f] \rangle = \mathcal{N} \int \mathscr{D}[f'] \mathscr{D}[\hat{f}] F[f'] e^{-L}, \tag{2.10}$$

https://doi.org/10.1017/S0022377804003198 Published online by Cambridge University Press

where

$$L = -\ln\langle e^{-\mathscr{L}}\rangle = \hat{f}(1)(\partial_t + v_\parallel\partial_z)f'(1) - \hat{L} \tag{2.11}$$

with

$$\hat{L} = \ln\langle\exp\{-\sqrt{\epsilon}\,\hat{f}(1)\eta_\parallel(t)\partial_z f'(1) - \sqrt{\epsilon}\,\hat{f}(1)[v_\parallel + \sqrt{\epsilon}\,\eta_\parallel(t)]\mathbf{b}(\mathbf{r})\cdot\nabla_\perp f'(1) - \sqrt{\epsilon}\,\hat{f}(1)\boldsymbol{\eta}_\perp(t)\cdot\nabla_\perp f'(1)\}\rangle. \tag{2.12}$$

Performing the ensemble average for $\hat{L}$, we have

$$\hat{L} = \frac{\epsilon}{2}\{v_\parallel^2\mathbf{F}(\mathbf{r}-\mathbf{r}_2) + \nu\chi_\perp R[\nu(t-t_2)]\mathbf{I}\} : \hat{f}(1)\hat{f}(2)\nabla_\perp f'(1)\nabla_{\perp 2}f'(2) + \ln\langle e^{-Q}\rangle_\parallel, \tag{2.13}$$

where

$$Q = \sqrt{\epsilon}\,\eta_\parallel(t)\hat{f}(1)\partial_z f'(1) - \frac{\epsilon^{3/2}}{2}[2v_\parallel + \sqrt{\epsilon}\,\eta_\parallel(t)]\eta_\parallel(t_2) \times \mathbf{F}(\mathbf{r}-\mathbf{r}_2) : \hat{f}(1)\hat{f}(2)\nabla_\perp f'(1)\nabla_{\perp 2}f'(2). \tag{2.14}$$

After some calculation given in Appendix A, we arrive at the following expression for $\langle e^{-Q}\rangle_\parallel$:

$$\begin{aligned}\langle e^{-Q}\rangle_\parallel = \exp\Bigg\{&\frac{\epsilon}{2}\nu\chi_\parallel R[\nu(t-t_2)]\hat{f}(1)\hat{f}(2)\partial_z f'(1)\partial_{z2}f'(2)\\ &+\frac{\epsilon^2}{2}\nu\chi_\parallel R[\nu(t-t_2)]\mathbf{F}(\mathbf{r}-\mathbf{r}_2) : \hat{f}(1)\hat{f}(2)\nabla_\perp f'(1)\nabla_{\perp 2}f'(2)\\ &+\frac{\epsilon^3}{2}(\nu\chi_\parallel)^2 R[\nu(t-t_2)]R[\nu(t_3-t_4)]\mathbf{F}(\mathbf{r}_2-\mathbf{r}_3)\\ &\quad : \hat{f}(1)\hat{f}(2)\hat{f}(3)\hat{f}(4)\partial_z f'(1)\nabla_{\perp 2}f'(2)\nabla_{\perp 3}f'(3)\partial_{z4}f'(4)\\ &-\epsilon^2 v_\parallel\nu\chi_\parallel R[\nu(t-t_3)]\mathbf{F}(\mathbf{r}-\mathbf{r}_2)\hat{f}(1)\hat{f}(2)\hat{f}(3)\nabla_\perp f'(1)\nabla_{\perp 2}f'(2)\partial_{z3}f'(3)\Bigg\}.\end{aligned} \tag{2.15}$$

In (2.13)–(2.15), $\partial_{zk} = \partial/\partial z_k$ and $\nabla_{\perp k} = \partial/\partial\mathbf{r}_{\perp k}$ for $k = 2, 3, 4$, and the index k in f' and $\hat{f}$ denotes $(\mathbf{r}_k, t_k)$.

Let us now introduce a generating functional

$$Z[\zeta,\xi] = \mathscr{N}\int\mathscr{D}[f']\mathscr{D}[\hat{f}]e^{-S/\epsilon} \equiv e^{W/\epsilon} \tag{2.16}$$

with $S = L - f'(1)\zeta(1) - \hat{f}(1)\xi(1)$, where we have made a change of integral variables in L as $f \to f/\sqrt{\epsilon}$ and $\hat{f} \to \hat{f}/\sqrt{\epsilon}$. Using this generating functional, we define the one-point function $g(1)$ and the response function $G(1-1')$ by

$$g(1) \equiv \epsilon\frac{\delta W}{\delta\zeta(1)}, \quad G(1-1') \equiv \epsilon^2\frac{\delta^2 W}{\delta\xi(1')\delta\zeta(1)}. \tag{2.17}$$

In the limit of $\zeta = \xi = 0$, the one-point function $g(1)$ becomes the ensemble-averaged distribution function, i.e. $g(1)|_{\zeta=\xi=0} = \langle f(1)\rangle$. We note here that the causality ensures the property $\langle\hat{f}(1)\cdots\rangle = 0$ in the limit of $\zeta = \xi = 0$ if t is the latest time in the ensemble average (Jouvet and Phythian 1979; Jensen 1981).

https://doi.org/10.1017/S0022377804003198 Published online by Cambridge University Press

The evolution equation for $g(1)$ is derived from the identity

$$\int \mathscr{D}[f']\mathscr{D}[\hat{f}]\frac{\delta}{\delta\hat{f}(1)}e^{-S/\epsilon} = 0 \tag{2.18}$$

as

$$\begin{aligned}
&\partial_t g(1) + v_\parallel \partial_z g(1) - \nu\chi_\parallel R[\nu(t-t_2)]\langle \hat{f}(2)\partial_z f(1)\partial_{z2} f(2)\rangle \\
&\quad - \{\nu\chi_\perp R[\nu(t-t_2)]\mathbf{I} + [\epsilon\nu\chi_\parallel R[\nu(t-t_2)] + v_\parallel^2]\mathbf{F}(\mathbf{r}-\mathbf{r}_2)\} \\
&\qquad : \langle \hat{f}(2)\nabla_\perp f(1)\nabla_{\perp 2} f(2)\rangle \\
&\quad - (\nu\chi_\parallel)^2 R[\nu(t-t_2)]R[\nu(t_3-t_4)]\mathbf{F}(\mathbf{r}-\mathbf{r}_3) \\
&\qquad : \langle \hat{f}(2)\hat{f}(3)\hat{f}(4)\nabla_\perp f(1)\partial_{z2} f(2)\nabla_{\perp 3} f(3)\partial_{z4} f(4)\rangle \\
&\quad - (\nu\chi_\parallel)^2 R[\nu(t-t_2)]R[\nu(t_3-t_4)]\mathbf{F}(\mathbf{r}_2-\mathbf{r}_4) \\
&\qquad : \langle \hat{f}(2)\hat{f}(3)\hat{f}(4)\partial_z f(1)\nabla_{\perp 2} f(2)\partial_{z3} f(3)\nabla_{\perp 4} f(4)\rangle \\
&\quad + v_\parallel\nu\chi_\parallel\{R[\nu(t-t_2)] + R[\nu(t_2-t_3)]\}\mathbf{F}(\mathbf{r}-\mathbf{r}_3) \\
&\qquad : \langle \hat{f}(2)\hat{f}(3)\nabla_\perp f(1)\partial_{z2} f(2)\nabla_{\perp 3} f(3)\rangle \\
&\quad + v_\parallel\nu\chi_\parallel R[\nu(t-t_2)]\mathbf{F}(\mathbf{r}_2-\mathbf{r}_3) \\
&\qquad : \langle \hat{f}(2)\hat{f}(3)\partial_z f(1)\nabla_{\perp 2} f(2)\nabla_{\perp 3} f(3)\rangle = \xi(1),
\end{aligned} \tag{2.19}$$

where the term of order ϵ is retained for comparison with the result of Eijnden and Balescu (1996b).

In the WKB approximation, the $(2m-1)$-point correlation function is approximated as

$$\begin{aligned}
\left\langle f(1)\prod_{i=2}^{m}\hat{f}(i)f(i)\right\rangle &= \epsilon^{2m-1}\frac{\delta}{\delta\zeta(1)}\prod_{i=2}^{m}\frac{\delta}{\delta\xi(i)}\frac{\delta}{\delta\zeta(i)}e^{W/\epsilon} \\
&\simeq \sum_{\sigma_m} G(i_1-2)G(i_2-3)\cdots G(i_{m-1}-m)g(i_m),
\end{aligned} \tag{2.20}$$

where $(i_1, i_2, \ldots, i_m)$ is the permutation of $(1, 2, \ldots, m)$, and $\sum_{\sigma_m}$ means the summation of all possible permutations satisfying the conditions $l \neq i_{l-1}$. Using this approximation, we have

$$\begin{aligned}
&\partial_t g(1) + v_\parallel \partial_z g(1) - \nu\chi_\parallel R[\nu(t-t_2)]\partial_z G(1-2)\partial_{z2} g(2) \\
&\quad - \{\nu\chi_\perp R[\nu(t-t_2)]\mathbf{I} + [\epsilon\nu\chi_\parallel R[\nu(t-t_2)] + v_\parallel^2]\mathbf{F}(\mathbf{r}-\mathbf{r}_2)\} \\
&\qquad : \nabla_\perp G(1-2)\nabla_{\perp 2} g(2) \\
&\quad - (\nu\chi_\parallel)^2 R[\nu(t-t_2)]R[\nu(t_3-t_4)]\mathbf{F}(\mathbf{r}-\mathbf{r}_3) \\
&\qquad : \sum_{\sigma_4}\hat{\nabla}_{i_l} G(i_1-2)\hat{\nabla}_{i_2} G(i_2-3)\hat{\nabla}_{i_3} G(i_3-4)\hat{\nabla}_{i_4} g(i_4) \\
&\quad - (\nu\chi_\parallel)^2 R[\nu(t-t_2)]R[\nu(t_3-t_4)]\mathbf{F}(\mathbf{r}_2-\mathbf{r}_4) \\
&\qquad : \sum_{\sigma_4}\tilde{\nabla}_{i_l} G(i_1-2)\tilde{\nabla}_{i_2} G(i_2-3)\tilde{\nabla}_{i_3} G(i_3-4)\tilde{\nabla}_{i_4} g(i_4)
\end{aligned}$$

https://doi.org/10.1017/S0022377804003198 Published online by Cambridge University Press

$$+ v_\parallel \nu\chi_\parallel \{R[\nu(t-t_2)] + R[\nu(t_2-t_3)]\}\mathbf{F}(\mathbf{r}-\mathbf{r}_3)$$
$$: \sum_{\sigma_3} \hat{\nabla}_{i_1} G(i_1-2)\hat{\nabla}_{i_2} G(i_2-3)\hat{\nabla}_{i_3} g(i_3)$$
$$+ v_\parallel \nu\chi_\parallel R[\nu(t-t_2)]\mathbf{F}(\mathbf{r}_2-\mathbf{r}_3) \sum_{\sigma_3} \tilde{\nabla}'_{i_1} G(i_1-2)\tilde{\nabla}'_{i_2} G(i_2-3)\tilde{\nabla}'_{i_3} g(i_3) = \xi(1), \tag{2.21}$$

where $\hat{\nabla}_{i_l} \equiv (\delta_{i_l 1}+\delta_{i_l 3})\nabla_{\perp i_l} + (1-\delta_{i_l 1}-\delta_{i_l 3})\partial_{z i_l}$, $\tilde{\nabla}_{i_l} \equiv (\delta_{i_l 2}+\delta_{i_l 4})\nabla_{\perp i_l} + (1-\delta_{i_l 2}-\delta_{i_l 4})\partial_{z i_l}$ and $\tilde{\nabla}'_{i_l} \equiv (\delta_{i_l 2}+\delta_{i_l 3})\nabla_{\perp i_l} + (1-\delta_{i_l 2})\partial_{z i_l}$. The evolution equation (2.21) includes the response function. Therefore, in order to obtain a closed set of equations for g and G, we need an equation for the response function $G(1-1')$. This equation is derived by taking the functional differentiation of (2.21) with respect to $\xi(1')$ and neglecting the three-point function $\delta G(1-2)/\delta\xi(1')$. The explicit form of this equation is given in (C 1). In this paper, the approximation leading to the closed set of equations for g and G is referred to as the DIA. Hereafter we set $\epsilon = 1$.

3. Diffusion coefficients

The cross-field running diffusion tensor $\mathbf{D}_\perp(t)$ is expressed in terms of the ensemble-averaged distribution function as

$$\mathbf{D}_\perp(t) = \frac{1}{2}\frac{d}{dt}\int d\mathbf{r}\,\mathbf{r}_\perp\mathbf{r}_\perp g(\mathbf{r},t) = \frac{1}{2}\int d\mathbf{r}\,\mathbf{r}_\perp\mathbf{r}_\perp \frac{\partial g}{\partial t}. \tag{3.1}$$

Substituting (2.21) into this expression and the cumbersome calculation given in Appendix B yields the following expressions:

$$\mathbf{D}_\perp(t) = \mathbf{D}_\perp^{\mathrm{cl}}(t) + \mathbf{D}_\perp^{\mathrm{an}}(t), \tag{3.2}$$

$$\mathbf{D}_\perp^{\mathrm{cl}}(t) = \nu\chi_\perp \int_0^t d\tau\, R(\nu\tau) G(\mathbf{k}=\mathbf{0},\tau)\mathbf{I} \equiv D_\perp^{\mathrm{cl}}(t)\mathbf{I}, \tag{3.3}$$

$$\begin{aligned}\mathbf{D}_\perp^{\mathrm{an}}(t) = {} & \nu\chi_\parallel \int_0^t d\tau\, R(\nu\tau) \int \frac{d\mathbf{k}}{(2\pi)^3}\mathbf{F}(\mathbf{k})G(\mathbf{k},\tau) \\ & - (\nu\chi_\parallel)^2 \int_0^t dt_2 \int_0^{t_2} dt_3 \int_0^{t_3} dt_4 \{R[\nu(t-t_2)]R[\nu(t_3-t_4)] \\ & + R[\nu(t-t_3)]R[\nu(t_2-t_4)]\} \\ & \times \int \frac{d\mathbf{k}}{(2\pi)^3} k_z^2 \mathbf{F}(\mathbf{k})G(\mathbf{k},t-t_2)G(\mathbf{k},t_2-t_3)G(\mathbf{k},t_3-t_4) \\ & - i v_\parallel \nu\chi_\parallel \int_0^t dt_2 \int_0^{t_2} dt_3 [R[\nu(t-t_2)] + R[\nu(t_2-t_3)]] \\ & \times \int \frac{d\mathbf{k}}{(2\pi)^3} k_z \mathbf{F}(\mathbf{k})G(\mathbf{k},t-t_2)G(\mathbf{k},t_2-t_3) \\ & + v_\parallel^2 \int_0^t d\tau \int \frac{d\mathbf{k}}{(2\pi)^3}\mathbf{F}(\mathbf{k})G(\mathbf{k},\tau),\end{aligned} \tag{3.4}$$

https://doi.org/10.1017/S0022377804003198 Published online by Cambridge University Press

where $G(\mathbf{k},t)$ is the Fourier transform of the response function, defined by

$$G(\mathbf{k},t)=\int d\mathbf{r}\,G(\mathbf{r},t)e^{-i\mathbf{k}\cdot\mathbf{r}}. \tag{3.5}$$

The Fourier transform of the correlation function $\mathbf{F}(\mathbf{r}-\mathbf{r}')$ is given by

$$\mathbf{F}(\mathbf{k})=(2\pi)^{3/2}\lambda_{\parallel}\lambda_{\perp}^{4}\beta^{2}e^{-\frac{1}{2}\lambda_{\parallel}^{2}k_{z}^{2}-\frac{1}{2}\lambda_{\perp}^{2}k_{\perp}^{2}}(k_{\perp}^{2}\mathbf{I}-\mathbf{k}_{\perp}\mathbf{k}_{\perp}). \tag{3.6}$$

In the decomposition (3.2), $\mathbf{D}_{\perp}^{\mathrm{cl}}(t)$ and $\mathbf{D}_{\perp}^{\mathrm{an}}(t)$ represent the classical part due to the collisions and the anomalous part due to the fluctuating magnetic field, respectively.

In order to obtain the response function $G(1-1')$, we need to solve the rather complex integro-differential equation (C 1). This equation for $G(1-1')$ is quite simplified by using the Markovian approximation. As is shown in Appendix C, this approximation gives us the following equation for the Fourier transform of the response function:

$$[\partial_t+ik_zv_{\parallel}+k_z^2D_{\parallel}(t-t')+\mathbf{k}_{\perp}\mathbf{k}_{\perp}:\mathbf{D}_{\perp}(t-t')]G(\mathbf{k},t-t')=\delta(t-t'), \tag{3.7}$$

where $D_{\parallel}$ is defined by (C 13). The solution to (3.7) is immediately obtained

$$G(\mathbf{k},t-t')=H(t-t')$$
$$\times\exp\left[-ik_zv_{\parallel}(t-t')-k_z^2\int_0^{t-t'}D_{\parallel}(\tau)\,d\tau-\mathbf{k}_{\perp}\mathbf{k}_{\perp}:\int_0^{t-t'}\mathbf{D}_{\perp}(\tau)\,d\tau\right]. \tag{3.8}$$

Substituting (3.8) into (3.4), we find that $\mathbf{D}_{\perp}^{\mathrm{an}}(t)$ is a diagonal tensor, i.e. $\mathbf{D}_{\perp}^{\mathrm{an}}(t)=D_{\perp}^{\mathrm{an}}(t)\mathbf{I}$, and the function μ defined through $D_{\perp}^{\mathrm{an}}(t)=(\lambda_{\perp}^2/4)\,d\mu/dt$ satisfies the equation

$$\frac{d^2\mu}{d\tau^2}=2\bar{\chi}_{\parallel}\alpha^2\frac{1}{[1+\bar{\chi}_{\perp}\psi(\tau)+\mu/2]^2}\frac{1}{[1+\bar{\chi}_{\parallel}\psi(\tau)]^{1/2}}$$
$$\times\left\{R(\tau)+2\bar{v}_{\parallel}^2-\frac{1}{2}\frac{\bar{\chi}_{\parallel}\varphi(\tau)}{1+\bar{\chi}_{\parallel}\psi(\tau)}\left[4\bar{v}_{\parallel}^2\tau+\varphi(\tau)\left(1-\frac{\bar{\chi}_{\parallel}\bar{v}_{\parallel}^2\tau^2}{1+\bar{\chi}_{\parallel}\psi(\tau)}\right)\right]\right\}$$
$$\times\exp\left[-\frac{1}{2}\frac{\bar{\chi}_{\parallel}\bar{v}_{\parallel}^2\tau^2}{1+\bar{\chi}_{\parallel}\psi(\tau)}\right] \tag{3.9}$$

with the initial conditions $\mu(0)=0$ and $d\mu(0)/d\tau=0$, where $\tau=\nu t$, $\alpha=\beta\lambda_{\parallel}/\lambda_{\perp}$, $\bar{v}_{\parallel}=v_{\parallel}/v_{\mathrm{T}}$, $\bar{\chi}_{\parallel}=2\chi_{\parallel}/\nu\lambda_{\parallel}^2$, $\bar{\chi}_{\perp}=2\chi_{\perp}/\nu\lambda_{\perp}^2$,

$$\varphi(\tau)=\int_0^{\tau}d\bar{\tau}\,R(\bar{\tau}),\quad \psi(\tau)=\int_0^{\tau}d\bar{\tau}\,\varphi(\bar{\tau}), \tag{3.10}$$

and we have used the following assumptions in the second and third terms on the right-hand side of (3.4):

$$\int_0^{t-t'}\mathbf{D}_{\perp}(\tau)\,d\tau\simeq\int_{t'}^{t}\mathbf{D}_{\perp}(\tau)\,d\tau\quad\text{and}\quad\int_0^{t-t'}D_{\parallel}(\tau)\,d\tau\simeq\int_{t'}^{t}D_{\parallel}(\tau)\,d\tau.$$

In the $\lambda_{\perp}=\infty$ limit, which is equivalent to neglecting the dependence of the fluctuating magnetic field on x and y, the differential equation (3.9) reduces to the exact one (D 9) obtained without using the DIA (Appendix D).

In the large $v_{\parallel}$ limit, the differential equation (3.9) is approximated as

$$\frac{d^2\mu}{dt^2}=\frac{4\alpha^2v_{\parallel}^2}{\lambda_{\parallel}^2}\frac{1}{(1+\mu/2)^2}\exp\left[-\frac{v_{\parallel}^2t^2}{2\lambda_{\parallel}^2}\right]. \tag{3.11}$$

https://doi.org/10.1017/S0022377804003198 Published online by Cambridge University Press

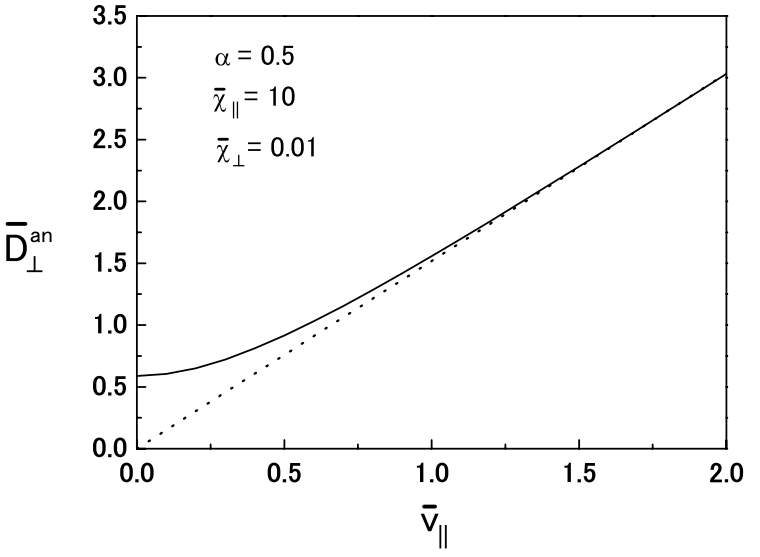


Figure 1. The normalized diffusion coefficient $\bar{D}_{\perp}^{\mathrm{an}}$ versus $\bar{v}_{\|}$ for $\alpha = 0.5$, $\bar{\chi}_{\|} = 10$ and $\bar{\chi}_{\perp} = 0.01$. The dotted curve shows the approximate diffusion coefficient obtained from (3.11).

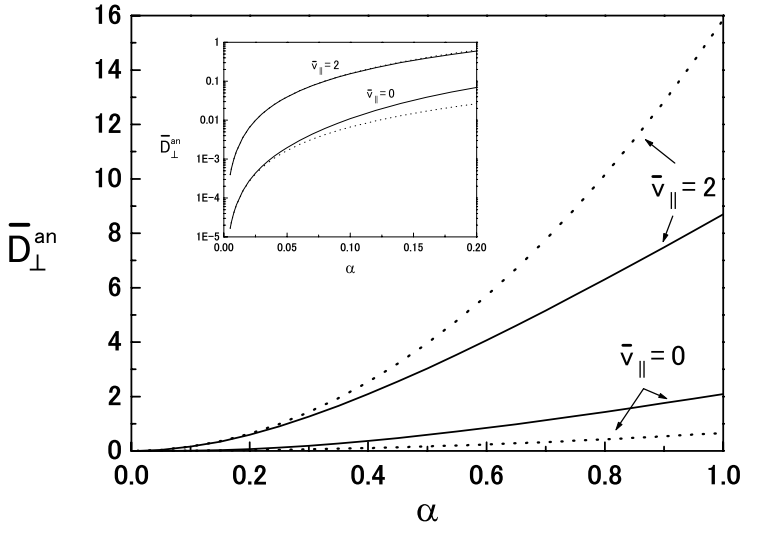


Figure 2. The normalized diffusion coefficient $\bar{D}_{\perp}^{\mathrm{an}}$ versus α for $\bar{\chi}_{\|} = 10$, $\bar{\chi}_{\perp} = 0.01$, and $\bar{v}_{\|} = 0$ and 2. The dotted curves represent the transport coefficients obtained by the quasi-linear approximation. The insert is the enlarged detail around $\alpha = 0$.

The explicit values of the diffusion coefficients calculated by solving (3.9) are shown in Figs 1–4. In Fig. 1, the normalized diffusion coefficients $\bar{D}_{\perp}^{\mathrm{an}} \equiv D_{\perp}^{\mathrm{an}}(\infty)/(\nu\lambda_{\perp}^2/2)$ are plotted as a function of $\bar{v}_{\|}$ for $\alpha = 0.5$, $\bar{\chi}_{\|} = 10.0$ and $\bar{\chi}_{\perp} = 0.01$. The dotted curve in this figure shows the approximate diffusion coefficient obtained from (3.11). We find from the calculation for various parameters that the approximation by (3.11) is fairly good for $\bar{v}_{\|} > 1$ in the region of $\bar{\chi}_{\|} > 1$ and $\bar{\chi}_{\perp} < 1$. In the small α expansion, the leading-order solution of μ is obtained by setting $\mu = 0$ on the right-hand side of (3.9). In Fig. 2, the normalized transport coefficients in this

https://doi.org/10.1017/S0022377804003198 Published online by Cambridge University Press

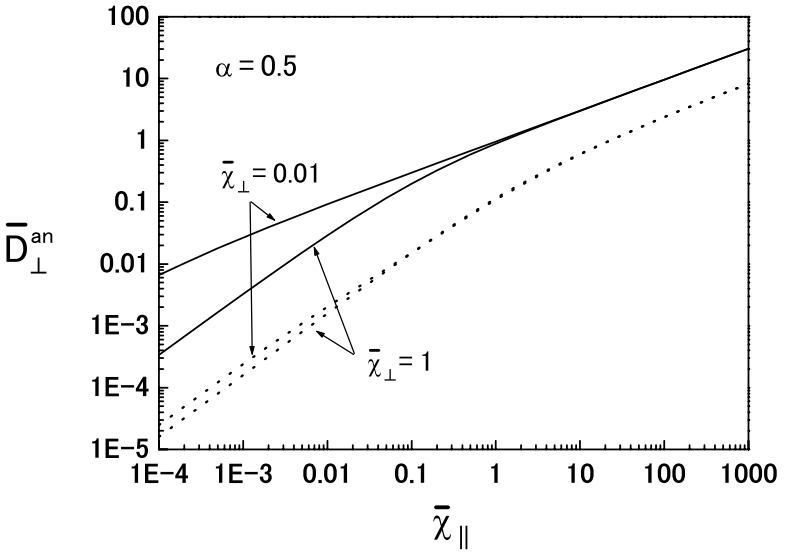


Figure 3. The normalized diffusion coefficient $\bar{D}_\perp^{an}$ versus $\bar{\chi}_\parallel$ for $\alpha = 0.5$, and $\bar{\chi}_\perp = 0.01$ and 1. The normalized parallel velocity is taken to be $\bar{v}_\parallel = 2$ (solid curves) and $\bar{v}_\parallel = 0$ (dotted curves).

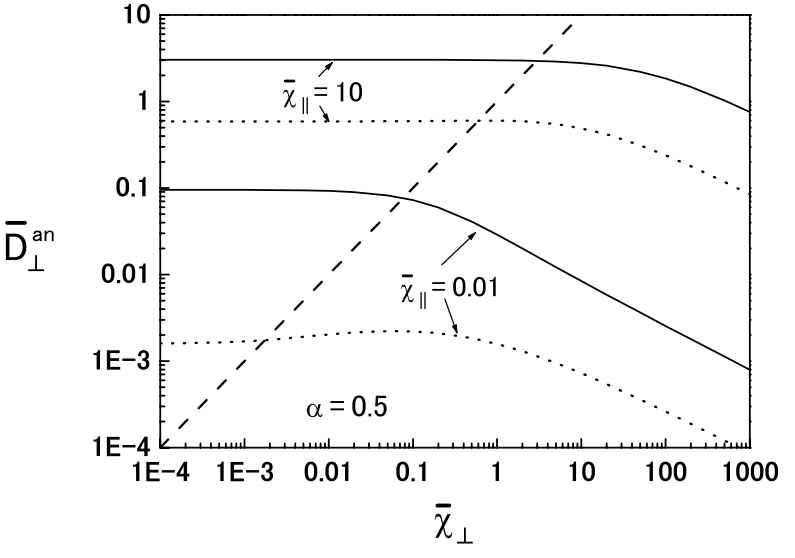


Figure 4. The normalized diffusion coefficient $\bar{D}_\perp^{an}$ versus $\bar{\chi}_\perp$ for $\alpha = 0.5$, and $\bar{\chi}_\parallel = 0.01$ and 10. The normalized parallel velocity is taken to be $\bar{v}_\parallel = 2.0$ (solid curves) and $\bar{v}_\parallel = 0$ (dotted curves). The dashed line shows the points satisfying $\bar{D}_\perp^{an} = \bar{\chi}_\perp$.

quasi-linear approximation, which are shown by the dotted curves, are compared with the exact ones (solid curves) for $\bar{\chi}_\parallel = 10$, $\bar{\chi}_\perp = 0.01$, and $\bar{v}_\parallel = 0$ and 2. The insert is the enlarged detail around $\alpha = 0$. The validity region of the quasi-linear approximation for α decreases as $\bar{v}_\parallel$ approaches zero, and the quasi-linear approximation for $\bar{v}_\parallel = 0$ is found to be good only for quite small values of α. The dependence of the normalized transport coefficients on the values of $\bar{\chi}_\parallel$ and $\bar{\chi}_\perp$ is shown in Figs 3 and 4 for $\alpha = 0.5$. We note here that the ratio of the anomalous

https://doi.org/10.1017/S0022377804003198 Published online by Cambridge University Press

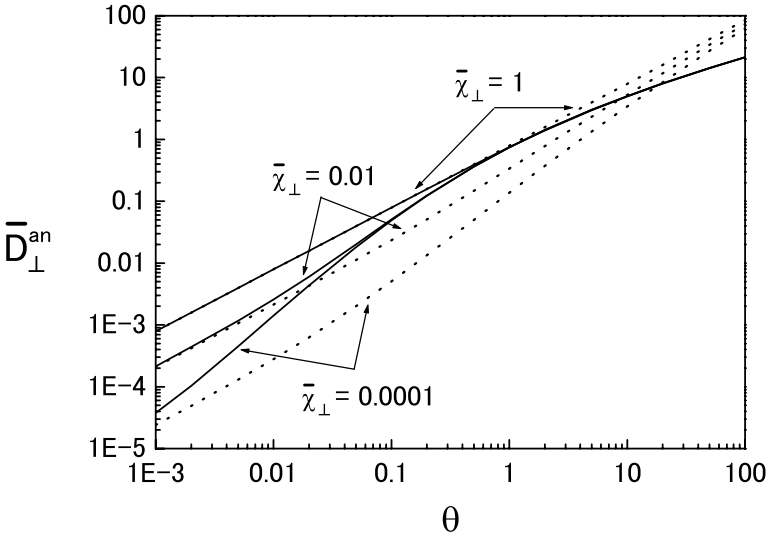


Figure 5. The normalized diffusion coefficient $\bar{D}_\perp^{\rm an}$ versus $\theta = \alpha^2 \bar{\chi}_\parallel^{1/2}$ for $\bar{\chi}_\parallel = 10$, and $\bar{\chi}_\perp = 0.0001$, 0.01 and 1. The dotted curves represent the transport coefficients obtained from the Eijnden–Balescu theory.

transport coefficient to the classical one is represented by $D_\perp^{\rm an}/D_\perp^{\rm cl} = \bar{D}_\perp^{\rm an}/\bar{\chi}_\perp$. In Fig. 4, the points satisfying $\bar{D}_\perp^{\rm an} = \bar{\chi}_\perp$ are depicted by the dashed line.

For comparison with the Eijnden–Balescu theory, we set $v_\parallel = 0$ in (3.9) and define $\hat{\mu} = \mu - \mu_0$, where

$$\mu_0(\tau) = 4\alpha^2\left(\sqrt{1 + \bar{\chi}_\parallel \psi(\tau)} - 1\right) \tag{3.12}$$

is the exact solution when $\boldsymbol{\eta}_\perp(t) = \mathbf{0}$ and the fluctuating magnetic field $\mathbf{b}$ depends only on the z coordinate. Note that $\hat{\mu}$ corresponds to μ in Eijnden and Balescu (1996b). The newly defined function $\hat{\mu}$ satisfies the equation

$$\frac{d^2\hat{\mu}}{d\tau^2} = \frac{2\bar{\chi}_\parallel \alpha^2}{(1 + \bar{\chi}_\parallel \psi(\tau))^{1/2}} \left[R(\tau) - \frac{1}{2} \frac{\bar{\chi}_\parallel \varphi(\tau)^2}{1 + \bar{\chi}_\parallel \psi(\tau)} \right] (\mathscr{F} - 1), \tag{3.13}$$

where

$$\mathscr{F} = \frac{1}{[1 + \bar{\chi}_\perp \psi(\tau) + (\hat{\mu} + \mu_0)/2]^2}. \tag{3.14}$$

The differential equation (3.13) agrees with that obtained by Eijnden and Balescu (1996b) when we neglect μ_0 in $\mathscr{F}$. This neglect of μ_0 is the additional assumption to the DIA and this assumption is implicitly used in Eijnden and Balescu (1996b).

In Fig. 5, we show the comparison of the normalized transport coefficients obtained from the full equation (3.9) and the equation (3.13), setting $\mu_0 = 0$ in $\mathscr{F}$. In this figure, the normalized diffusion coefficients are plotted as a function of $\theta \equiv \alpha^2 \bar{\chi}_\parallel^{1/2}$ for $\bar{\chi}_\parallel = 10$, and $\bar{\chi}_\perp = 0.0001$, 0.01 and 1. This figure shows that neglecting μ_0 in $\mathscr{F}$ is only valid in the region satisfying $\bar{\chi}_\perp > \theta$ for the chosen typical parameters $\bar{\chi}_\parallel$ and $\bar{\chi}_\perp$. Krommes et al. (1983) claimed that the well-known

https://doi.org/10.1017/S0022377804003198 Published online by Cambridge University Press

Rechester–Rosenbluth diffusion coefficient cannot be obtained within the framework of the DIA. This Rechester–Rosenbluth diffusion coefficient is derived in the Eijnden–Balescu theory. Their derivation is quite interesting but it may essentially depend on neglecting μ_0 in $\mathscr{F}$ in the region $\bar{\chi}_\perp < \theta$. Thus, we point out that the assumption beyond the DIA is used in their derivation of the Rechester–Rosenbluth diffusion coefficient.

The evolution of the ensemble-averaged distribution function is described by the quite complicated equation (2.21). However, we are often interested in the transport in some direction (for example, the x-direction) perpendicular to the averaged magnetic field. For this purpose, the distribution function averaged over y and z is only needed. The evolution equation for this distribution function $\bar{g}(x,t) = \int dy\, dz\, g(x,y,z,t)$ can be obtained by integrating (2.21) over y and z and using a similar calculation leading to (3.4):

$$\begin{aligned}
\partial_t \bar{g}(x,t) = \frac{\partial}{\partial x}\Bigg\{ & \nu\chi_\perp \int_0^t d\tau\, R(\nu\tau) \int d\mathbf{r}_2\, G(\mathbf{r}_2,\tau) \frac{\partial}{\partial x} \bar{g}(x-x_2, t-\tau) \\
& + \int_0^t d\tau [\nu\chi_\parallel R(\nu\tau) + v_\parallel^2] \int d\mathbf{r}_2\, F_{xx}(\mathbf{r}_2) G(\mathbf{r}_2,\tau) \frac{\partial}{\partial x} \bar{g}(x-x_2, t-\tau) \\
& + (\nu\chi_\parallel)^2 \int_0^t dt_2 \int_0^{t-t_2} dt_3 \int_0^{t-t_2-t_3} dt_4 \{ R(\nu t_2) R(\nu t_4) + R[\nu(t_2+t_3)] R[\nu(t_3+t_4)] \} \\
& \times \int d\mathbf{r}_2\, d\mathbf{r}_3\, d\mathbf{r}_4\, F_{xx}(\mathbf{r}_2+\mathbf{r}_3+\mathbf{r}_4) G(\mathbf{r}_2,t_2) \partial_{z3} G(\mathbf{r}_3,t_3) \partial_{z4} G(\mathbf{r}_4,t_4) \\
& \times \frac{\partial}{\partial x} \bar{g}(x-x_2-x_3-x_4, t-t_2-t_3-t_4) - \nu\chi_\parallel v_\parallel \int_0^t dt_2 \\
& \times \int_0^{t-t_2} dt_3 [R(\nu t_2) + R(\nu t_3)] \int d\mathbf{r}_2\, d\mathbf{r}_3\, F_{xx}(\mathbf{r}_2+\mathbf{r}_3) G(\mathbf{r}_2,t_2) \partial_{z3} G(\mathbf{r}_3,t_3) \\
& \times \frac{\partial}{\partial x} \bar{g}(x-x_2-x_3, t-t_2-t_3) \Bigg\},
\end{aligned} \tag{3.15}$$

where $F_{xx}(\mathbf{r}) = \mathbf{e}_x \cdot \mathbf{F}(\mathbf{r}) \cdot \mathbf{e}_x$. When we use the spatial and temporal Markovian approximation for $\bar{g}$, i.e.

$$\begin{aligned}
\bar{g}(x-x_2, t-t_2) \simeq \bar{g}(x,t), \quad \bar{g}(x-x_2-x_3, t-t_2-t_3) \simeq \bar{g}(x,t), \\
\bar{g}(x-x_2-x_3-x_4, t-t_2-t_3-t_4) \simeq \bar{g}(x,t),
\end{aligned} \tag{3.16}$$

the evolution equation (3.15) is simplified as

$$\frac{\partial}{\partial t} \bar{g}(x,t) = \frac{\partial}{\partial x} D_{xx}(t) \frac{\partial}{\partial x} \bar{g}(x,t), \tag{3.17}$$

where $D_{xx}(t) = \mathbf{e}_x \cdot \mathbf{D}_\perp(t) \cdot \mathbf{e}_x$.

4. Effect of finite shear

This section deals with the effect of finite shear on the transport coefficient. Restricting our consideration to only around a reference surface $x = 0$, we consider a

https://doi.org/10.1017/S0022377804003198 Published online by Cambridge University Press

sheared slab configuration

$$\mathbf{B}(\mathbf{r}) = B_0 \left[\mathbf{e}_z + b_x(\mathbf{r})\mathbf{e}_x + \left(b_y(\mathbf{r}) + \frac{x}{L_\mathrm{s}} \right) \mathbf{e}_y \right], \tag{4.1}$$

where L_s is the shear length and $x/L_\mathrm{s} \ll 1$ is assumed. Then the equation for the ensemble-averaged distribution function $g(1)$ is obtained from (2.21) by replacing ∂_z and ∂_{zl} $(l = 2, 3, 4)$ with $\partial_z + (x/L_\mathrm{s})\partial_y$ and $\partial_{zl} + (x_l/L_\mathrm{s})\partial_{yl}$. In order to go further analytically, we take the approximation x/L_s $(\simeq x_l/L_\mathrm{s})$ as a constant parameter in the derivation of the equation for the response function.

4.1. Collisionless regime

We first consider the collisionless regime by assuming $\eta_\parallel(t) = 0$ and $\boldsymbol{\eta}_\perp(t) = \mathbf{0}$. Now let us define the Fourier transform of the response function with respect to $\mathbf{r} - \mathbf{r}'$ by fixing x/L_s. Then this Fourier transform $G\left(\mathbf{k}, t - t'; x/L_\mathrm{s}\right)$ satisfies the equation

$$\left[\partial_t + i v_\parallel \left(k_z + \frac{x k_y}{L_\mathrm{s}} \right) + \mathbf{k}_\perp \mathbf{k}_\perp : \mathscr{D}_\perp(t - t', x) \right] G\left(\mathbf{k}, t - t'; \frac{x}{L_\mathrm{s}} \right) = \delta(t - t'), \tag{4.2}$$

where the diffusion tensor is defined as

$$\mathscr{D}_\perp(t, x) = v_\parallel^2 \int_0^t dt_2 \int d\mathbf{r}_2 \, \mathbf{F}(\mathbf{r} - \mathbf{r}_2) G(1, 2). \tag{4.3}$$

Solving this equation, we have

$$G\left(\mathbf{k}, \tau; \frac{x}{L_\mathrm{s}} \right) = H(\tau) \exp\left[-i v_\parallel \left(k_z + \frac{x k_y}{L_\mathrm{s}} \right) \tau - \mathbf{k}_\perp \mathbf{k}_\perp : \int_0^\tau \mathscr{D}_\perp\left(\bar{\tau}, x\right) d\bar{\tau} \right]. \tag{4.4}$$

Substitution of this solution into (4.3) yields the differential equations for $\mathscr{D}_\perp$. As is shown in Appendix E, the non-diagonal components of the diffusion tensor $\mathscr{D}_\perp$ become zero. Then the equations for the diagonal components of $\boldsymbol{\mu}(t; x) = (4/\lambda_\perp^2) \int_0^t \mathscr{D}_\perp(\tau, x) \, d\tau$ are given by

$$\frac{d^2 \mu_{xx}}{dt^2} = \frac{4\alpha^2 v_\parallel^2}{\lambda_\parallel^2} \frac{1}{(1 + \mu_{xx}/2)^{1/2} (1 + \mu_{yy}/2)^{3/2}} \left(1 - \frac{1}{1 + \mu_{yy}/2} \frac{v_\parallel^2 t^2}{\lambda_\parallel^2} \frac{x^2}{L_\mathrm{s}^2} \frac{\lambda_\parallel^2}{\lambda_\perp^2} \right) \times \exp\left[-\frac{1}{2} \frac{v_\parallel^2 t^2}{\lambda_\parallel^2} \left(1 + \frac{1}{1 + \mu_{yy}/2} \frac{x^2}{L_\mathrm{s}^2} \frac{\lambda_\parallel^2}{\lambda_\perp^2} \right) \right], \tag{4.5}$$

$$\frac{d^2 \mu_{yy}}{dt^2} = \frac{4\alpha^2 v_\parallel^2}{\lambda_\parallel^2} \frac{1}{(1 + \mu_{xx}/2)^{3/2} (1 + \mu_{yy}/2)^{1/2}} \times \exp\left[-\frac{1}{2} \frac{v_\parallel^2 t^2}{\lambda_\parallel^2} \left(1 + \frac{1}{1 + \mu_{yy}/2} \frac{x^2}{L_\mathrm{s}^2} \frac{\lambda_\parallel^2}{\lambda_\perp^2} \right) \right]. \tag{4.6}$$

In Fig. 6, we depict the transport coefficients normalized by $\lambda_\perp^2 |v_\parallel| / \lambda_\parallel$ as a function of α for $\Theta \equiv (\lambda_\parallel/\lambda_\perp)(x/L_\mathrm{s}) = 0$, 1 and 2. The solid and dotted curves in this figure represent (x, x) and (y, y) components of the normalized transport coefficients, respectively.

https://doi.org/10.1017/S0022377804003198 Published online by Cambridge University Press

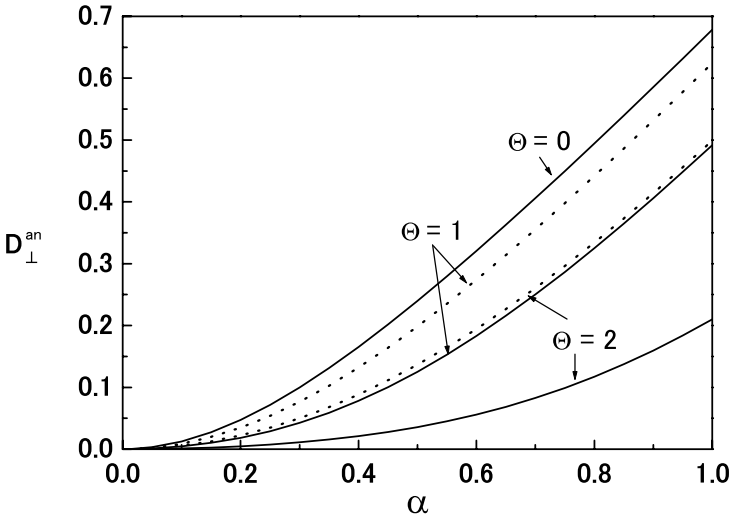


Figure 6. The transport coefficients normalized by $\lambda_\perp^2|v_\parallel|/\lambda_\parallel$ are plotted as a function of α in the collisionless regime for $\Theta = 0$, 1 and 2. The solid and dotted curves represent (x, x) and (y, y) components of the normalized transport coefficients.

The evolution equation for the ensemble-averaged distribution function in the sheared magnetic field can be written from (2.21) as

$$\partial_t g(1) + v_\parallel \left(\partial_z + \frac{x}{L_s}\partial_y\right) g(1) - v_\parallel^2 \nabla_\perp \cdot \int_0^t dt_2 \int d\mathbf{r}_2\, \mathbf{F}(\mathbf{r}-\mathbf{r}_2) G(1,2) \cdot \nabla_{\perp 2} g(2) = 0. \tag{4.7}$$

As is shown in Appendix F, the last term on the left-hand side of this equation is simplified by using the spatial and temporal Markovian approximation

$$\int d\mathbf{r}_2\, \mathbf{F}(\mathbf{r}-\mathbf{r}_2) G(1,2) \cdot \nabla_\perp g(2) \simeq \int d\mathbf{r}_2\, \mathbf{F}(\mathbf{r}-\mathbf{r}_2) G(1,2) \times \left[\mathbf{e}_x \frac{\partial}{\partial x} + \left(\mathbf{e}_y + \mathbf{e}_x \frac{z-z_2}{L_s}\right)\frac{\partial}{\partial y}\right] g(\mathbf{r}, t). \tag{4.8}$$

According to this approximation, (4.7) is expressed in the form

$$\partial_t g(1) + v_\parallel \left(\partial_z + \frac{x}{L_s}\partial_y\right) g(1) - \nabla_\perp \cdot \mathscr{D}_\perp^\star(t, x) \cdot \nabla_\perp g(1) = 0, \tag{4.9}$$

where $\mathscr{D}_{xx}^\star = \mathscr{D}_{xx}$, $\mathscr{D}_{yx}^\star = 0$, $\mathscr{D}_{yy}^\star = \mathscr{D}_{yy}$, and

$$\mathscr{D}_{xy}^\star(t, x) = v_\parallel^2 \int_0^t dt_2 \int d\mathbf{r}_2\, F_{xx}(\mathbf{r}-\mathbf{r}_2) \frac{z-z_1}{L_s} G(1,2). \tag{4.10}$$

https://doi.org/10.1017/S0022377804003198 Published online by Cambridge University Press

Substituting (4.4) into (4.10) provides the equation for the non-diagonal component $\mu^{\star}_{xy}(t;x) = (4/\lambda_{\perp}^2)\int_0^t \mathscr{D}^{\star}_{xy}(\tau,x)\,d\tau$:

$$\frac{d^2\mu^{\star}_{xy}}{dt^2} = \frac{4\alpha^2 v_{\parallel}^2}{\lambda_{\parallel}^2}\frac{v_{\parallel}t/L_{\mathrm{s}}}{(1+\mu_{xx}/2)^{1/2}(1+\mu_{yy}/2)^{3/2}}\left(1-\frac{1}{1+\mu_{yy}/2}\frac{v_{\parallel}^2t^2}{\lambda_{\parallel}^2}\frac{x^2}{L_{\mathrm{s}}^2}\frac{\lambda_{\parallel}^2}{\lambda_{\perp}^2}\right)$$
$$\times\exp\left[-\frac{1}{2}\frac{v_{\parallel}^2t^2}{\lambda_{\parallel}^2}\left(1+\frac{1}{1+\mu_{yy}/2}\frac{\lambda_{\parallel}^2}{\lambda_{\perp}^2}\frac{x^2}{L_{\mathrm{s}}^2}\right)\right]. \tag{4.11}$$

In the quasi-linear approximation, we set $\mu_{xx} = \mu_{yy} = 0$ on the right-hand sides of (4.5), (4.6) and (4.11). Then the diffusion coefficients in the limit of $t = \infty$ are given as

$$\mathscr{D}^{\star}_{xx} = \mathscr{D}_{xx} = \sqrt{\frac{\pi}{2}}\beta^2\lambda_{\parallel}|v_{\parallel}|\left(1+\frac{x^2}{L_{\mathrm{s}}^2}\frac{\lambda_{\parallel}^2}{\lambda_{\perp}^2}\right)^{-3/2}, \tag{4.12}$$

$$\mathscr{D}^{\star}_{yy} = \mathscr{D}_{yy} = \sqrt{\frac{\pi}{2}}\beta^2\lambda_{\parallel}|v_{\parallel}|\left(1+\frac{x^2}{L_{\mathrm{s}}^2}\frac{\lambda_{\parallel}^2}{\lambda_{\perp}^2}\right)^{-1/2}, \tag{4.13}$$

$$\mathscr{D}^{\star}_{xy} = \beta^2 v_{\parallel}\frac{\lambda_{\parallel}^2}{L_{\mathrm{s}}}\left(1-\frac{x^2}{L_{\mathrm{s}}^2}\frac{\lambda_{\parallel}^2}{\lambda_{\perp}^2}\right)\left(1+\frac{x^2}{L_{\mathrm{s}}^2}\frac{\lambda_{\parallel}^2}{\lambda_{\perp}^2}\right)^{-2}, \quad \mathscr{D}^{\star}_{yx} = 0. \tag{4.14}$$

The diffusion coefficients of the magnetic field lines in the configuration (4.1) were studied by Eijnden and Balescu (1996a) in the quasi-linear approximation. The expressions (4.12)–(4.14) divided by $v_{\parallel}(>0)$ agree with the diffusion coefficients that they obtained.

The diffusion tensor $\mathscr{D}_{\perp}$ defined in this section does not agree with the running diffusion tensor of (3.1) when $\mathscr{D}_{\perp}$ is space-dependent due to the effect of shear (Eijnden and Balescu 1996a). The (x,x) component of the running diffusion tensor is expressed in terms of $\mathscr{D}_{xx}$ and $\bar{g}(x,t)$ as

$$D_{xx}(t) = \int dx\,\frac{\partial}{\partial x}(x\mathscr{D}_{xx}(t,x))\bar{g}(x,t). \tag{4.15}$$

4.2. Effect of collisions

In this subsection, the collisional effect in the sheared magnetic field is considered by taking $v_{\parallel} = 0$. Then the Fourier transform of the response function becomes

$$G\left(\mathbf{k},\tau;\frac{x}{L_{\mathrm{s}}}\right) = H(\tau)\exp\left[-\left(k_z+\frac{k_yx}{L_{\mathrm{s}}}\right)^2\int_0^{\tau}D_{\parallel}(\bar{\tau})\,d\bar{\tau}\right.$$
$$\left.-k_{\perp}^2\int_0^{\tau}D_{\perp}^{\mathrm{cl}}(\bar{\tau})\,d\bar{\tau}-\mathbf{k}_{\perp}\mathbf{k}_{\perp}:\int_0^{\tau}\mathscr{D}_{\perp}^{\mathrm{an}}(\bar{\tau},x)\,d\bar{\tau}\right], \tag{4.16}$$

https://doi.org/10.1017/S0022377804003198 Published online by Cambridge University Press

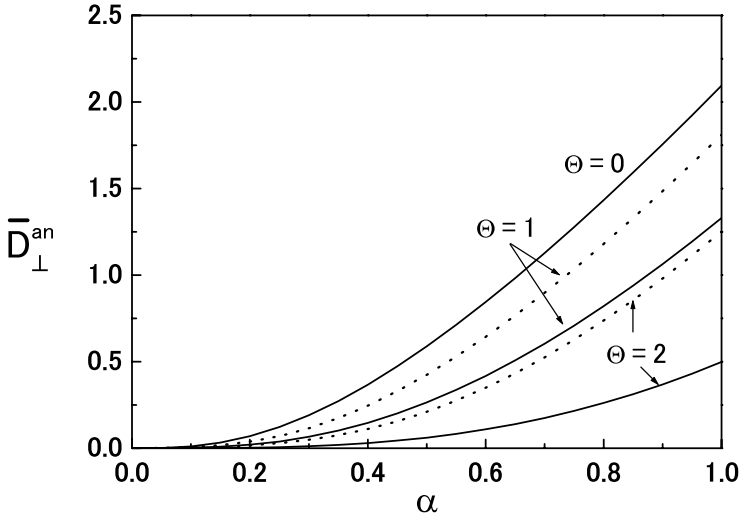


Figure 7. The normalized diffusion coefficients $\bar{\mathscr{D}}_{xx}^{\rm an}$ (solid curves) and $\bar{\mathscr{D}}_{yy}^{\rm an}$ (dotted curves) are plotted as a function of α for $\bar{v}_{\parallel}=0$, $\bar{\chi}_{\parallel}=10$, $\bar{\chi}_{\perp}=0.01$, and $\Theta=0$, 1 and 2.

where the anomalous diffusion tensor $\mathscr{D}_{\perp}^{\rm an}$ is expressed in terms of the Fourier transform of the response function as

$$\begin{aligned}\mathscr{D}_{\perp}^{\rm an}(t,x) &= \nu\chi_{\parallel}\int_0^t d\tau R(\nu\tau)\int\frac{d\mathbf{k}}{(2\pi)^3}\mathbf{F}(\mathbf{k})G\left(\mathbf{k},\tau;\frac{x}{L_{\rm s}}\right)\\ &\quad-(\nu\chi_{\parallel})^2\int_0^t dt_2\int_0^{t_2}dt_3\int_0^{t_3}dt_4\{R[\nu(t-t_2)]R[\nu(t_3-t_4)]\\ &\quad+R[\nu(t-t_3)]R[\nu(t_2-t_4)]\}\int\frac{d\mathbf{k}}{(2\pi)^3}\left(k_z+\frac{x}{L_{\rm s}}k_y\right)^2\mathbf{F}(\mathbf{k})\\ &\quad\times G\left(\mathbf{k},t-t_2;\frac{x}{L_{\rm s}}\right)G\left(\mathbf{k},t_2-t_3;\frac{x}{L_{\rm s}}\right)G\left(\mathbf{k},t_3-t_4;\frac{x}{L_{\rm s}}\right).\end{aligned}\tag{4.17}$$

Substituting (4.16) into (4.17) leads to the differential equations for $\boldsymbol{\mu}(t;x)=(4/\lambda_{\perp}^2)\int_0^t\mathscr{D}_{\perp}^{\rm an}(\tau,x)\,d\tau$:

$$\begin{aligned}\frac{d^2\mu_{xx}}{d\tau^2} &= \frac{2\bar{\chi}_{\parallel}\alpha^2}{(1+\bar{\chi}_{\perp}\psi(\tau)+\mu_{xx}/2)^{1/2}}\frac{1}{\mathscr{S}^{3/2}}\\ &\quad\times\left\{(1+\bar{\chi}_{\parallel}\psi(\tau))R(\tau)-\frac{1}{2}\bar{\chi}_{\parallel}\varphi(\tau)^2\left[1+\frac{3\left(\lambda_{\parallel}^2/\lambda_{\perp}^2\right)\left(x^2/L_{\rm s}^2\right)}{\mathscr{S}}\right]\right\},\end{aligned}\tag{4.18}$$

$$\begin{aligned}\frac{d^2\mu_{yy}}{d\tau^2} &= \frac{2\bar{\chi}_{\parallel}\alpha^2}{(1+\bar{\chi}_{\perp}\psi(\tau)+\mu_{xx}/2)^{3/2}(1+\bar{\chi}_{\parallel}\psi(\tau))}\frac{1}{\mathscr{S}^{1/2}}\\ &\quad\times\left\{(1+\bar{\chi}_{\parallel}\psi(\tau))R(\tau)-\frac{1}{2}\bar{\chi}_{\parallel}\varphi(\tau)^2\left[1+\frac{\left(\lambda_{\parallel}^2/\lambda_{\perp}^2\right)\left(x^2/L_{\rm s}^2\right)}{\mathscr{S}}\right]\right\},\end{aligned}\tag{4.19}$$

https://doi.org/10.1017/S0022377804003198 Published online by Cambridge University Press

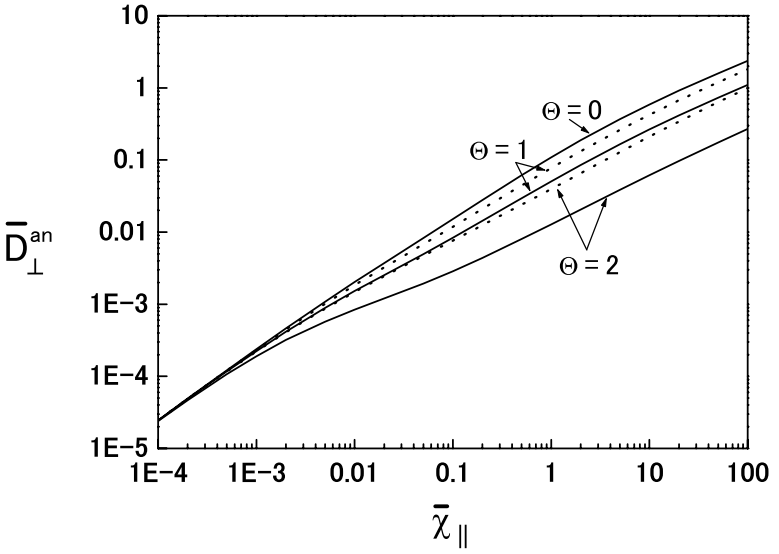


Figure 8. The normalized diffusion coefficients $\bar{\mathscr{D}}_{xx}^{an}$ (solid curves) and $\bar{\mathscr{D}}_{yy}^{an}$ (dotted curves) are plotted as a function of $\bar{\chi}_{\parallel}$ for $\bar{v}_{\parallel} = 0$, $\alpha = 0.5$, $\bar{\chi}_{\perp} = 0.01$, and $\Theta = 0$, 1 and 2.

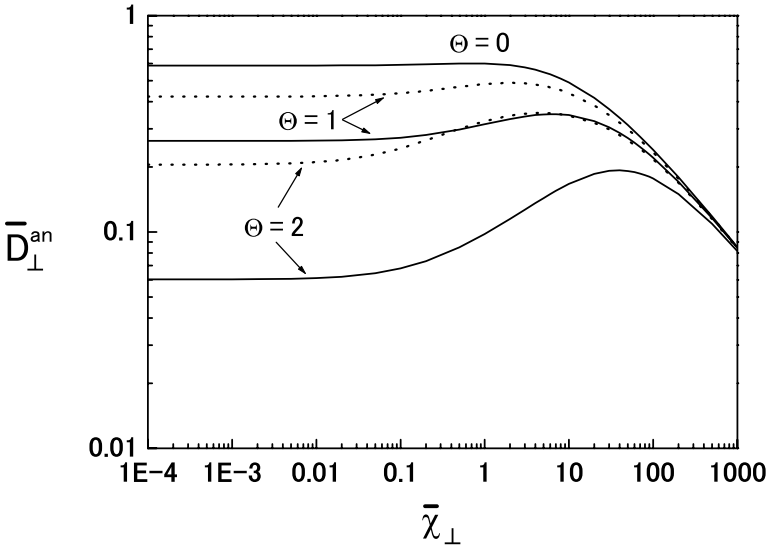


Figure 9. The normalized diffusion coefficients $\bar{\mathscr{D}}_{xx}^{an}$ (solid curves) and $\bar{\mathscr{D}}_{yy}^{an}$ (dotted curves) are plotted as a function of $\bar{\chi}_{\perp}$ for $\bar{v}_{\parallel} = 0$, $\alpha = 0.5$, $\bar{\chi}_{\parallel} = 10$, and $\Theta = 0$, 1 and 2.

where

$$\mathscr{S} = (1 + \bar{\chi}_{\parallel}\psi(\tau))(1 + \bar{\chi}_{\perp}\psi(\tau) + \mu_{yy}/2) + \left(\lambda_{\parallel}^2/\lambda_{\perp}^2\right)\left(x^2/L_s^2\right)\bar{\chi}_{\parallel}\psi(\tau). \tag{4.20}$$

The non-diagonal components are found to be zero from the explicit calculation.

Solving the coupled equations (4.18) and (4.19), we calculate the diffusion coefficients, and plot the normalized diffusion coefficients $\bar{\mathscr{D}}_{ll}^{an} = \mathscr{D}_{ll}^{an}(\infty, x)/(\nu\lambda_{\perp}^2/2)(l = x, y)$ for $\Theta \equiv (\lambda_{\parallel}/\lambda_{\perp})(x/L_s) = 0$, 1 and 2 in Figs 7–9. The solid and dotted

https://doi.org/10.1017/S0022377804003198 Published online by Cambridge University Press

curves in these figures represent the normalized transport coefficients $\bar{\mathscr{D}}_{xx}^{\rm an}$ and $\bar{\mathscr{D}}_{yy}^{\rm an}$, respectively.

Repeating the procedure leading to (3.17), we find the evolution equation for $\bar{g}$

$$\frac{\partial}{\partial t}\bar{g}(x,t) = \frac{\partial}{\partial x}\mathscr{D}_{xx}(t,x)\frac{\partial}{\partial x}\bar{g}(x,t), \tag{4.21}$$

where $\mathscr{D}_{xx}(t,x) = D_{xx}^{\rm cl}(t) + \mathscr{D}_{xx}^{\rm an}(t,x)$, and $\mathscr{D}_{xx}^{\rm an}(t,x)$ is obtained by solving the coupled equations (4.18) and (4.19). The diffusion coefficient $\mathscr{D}_{xx}$ is related to the running diffusion coefficient defined by (3.1) in the same way as (4.15).

5. Conclusions

The particle transport in the fluctuating magnetic field is investigated within the framework of the DIA. We have employed the model used by Balescu et al. (1994) for the fluctuating magnetic field and the Coulomb collisions. The differences in our starting point from the previous theories by Balescu et al. are: (1) the inclusion of the parallel particle velocity in the kinetic equation, by which the theory becomes applicable to fast particles; and (2) the consideration of the shear effect of the mean magnetic field.

First, we have studied the kinetic equation (2.2) in the shearless configuration (2.1). Applying the functional integral method to this kinetic equation and using the DIA, we have derived the closed set of equations (2.21) and (C 1) for the ensemble-averaged distribution function $g(1)$ and the response function $G(1{-}1')$. With the use of (2.21), the cross-field running diffusion tensor is shown to be expressed in terms of the response function as (3.4). The spatial and temporal Markovian approximation to (C 1) for the response function has quite simplified our problem, and the response function is then obtained as (3.8). Substituting this response function into the expression for the diffusion tensor, we found that the cross-field diffusion tensor has the diagonal form $\mathbf{D}_\perp(t) = [D_\perp^{\rm cl}(t) + D_\perp^{\rm an}(t)]\mathbf{I}$ and the anomalous diffusion coefficient $D_\perp^{\rm an}(t)$ is determined by the second-order differential equation (3.9) for $\mu(t)$. The explicit values of the anomalous diffusion coefficient are shown in Figs 1–4. In the large $v_\parallel$ limit, (3.9) is approximated by the more simplified equation (3.11). As is shown in Fig. 1, the anomalous diffusion coefficient calculated using the simplified equation is in fairly good agreement with the exact one obtained by (3.9) for $\bar{v}_\parallel > 1$ in the practically important region of $\bar{\chi}_\parallel > 1$ and $\bar{\chi}_\perp < 1$.

By setting $v_\parallel = 0$ in the shearless configuration, the starting point of our theory agrees with that of Eijnden and Balescu. However, our results differ from theirs. This difference is caused by their use of the additional approximation (neglecting μ_0 in $\mathscr{F}$) to the DIA and the Markovian approximation. In Fig. 5, this additional approximation is shown to be valid only in the region satisfying $\bar{\chi}_\perp > \theta$ for the typical values of parameters $\bar{\chi}_\parallel$ and $\bar{\chi}_\perp$. The derivation of the well-known Rechester–Rosenbluth diffusion coefficient in the Eijnden and Balescu theory may essentially depend on neglecting μ_0 in $\mathscr{F}$ in the region $\bar{\chi}_\perp < \theta$. Thus, the assumption that can not be validated within the framework of the DIA is used in their derivation.

The effect of finite shear on the transport coefficient is investigated in the collisionless and collisional regimes. Making use of a method similar to the shearless case, we found that the diffusion tensor is diagonal but not isotropic in the x–y plane. We also found that the diagonal components of the diffusion tensor can be calculated

https://doi.org/10.1017/S0022377804003198 Published online by Cambridge University Press

by solving the coupled equations (4.5) and (4.6) in the collisionless regime, and by solving (4.18) and (4.19) in the collisional regime. Explicitly calculated results shown in Figs 6–9 have shown the reduction of the diffusion coefficients due to the effect of shear.

Acknowledgement

This work was partially supported by a Grant-in-Aid for Scientific Research (C) from Japan Society for the Promotion of Science.

Appendix A

We rewrite $\langle e^{-Q}\rangle_\parallel$ in the form

$$\langle e^{-Q}\rangle_\parallel = \frac{e^{\frac{1}{2}\epsilon a(t)a(t')\tilde{A}^{-1}(t,t')}\int \mathscr{D}[\eta_\parallel]e^{-\frac{1}{2}\eta_\parallel(t)\eta_\parallel(t')\tilde{A}(t,t')}}{\int \mathscr{D}[\eta_\parallel]e^{-\frac{1}{2}\eta_\parallel(t)\eta_\parallel(t')A(t-t')}}, \tag{A 1}$$

where

$$a(t) = \int d\mathbf{r}\,\hat{f}(1)\partial_z f'(1) - \epsilon v_\parallel \int dt'\,\mathscr{P}(t,t'), \tag{A 2}$$

$$A^{-1}(t-t') = \chi_\parallel \nu R[\nu(t-t')], \tag{A 3}$$

$$\tilde{A}(t,t') = A(t-t') - \epsilon^2\mathscr{P}(t,t') \tag{A 4}$$

with

$$\mathscr{P}(t,t') = \int d\mathbf{r}\,d\mathbf{r}'\,\mathbf{F}(\mathbf{r}-\mathbf{r}') : \hat{f}(1)\hat{f}(1')\nabla_\perp f'(1)\nabla'_\perp f(1'). \tag{A 5}$$

Evaluating the Gaussian integrals (Zinn-Justin 1996), we have

$$\langle e^{-Q}\rangle_\parallel = e^{\frac{1}{2}\epsilon a(t)a(t')\tilde{A}^{-1}(t-t')}\frac{(\det A)^{1/2}}{(\det\tilde{A})^{1/2}}. \tag{A 6}$$

We neglect the terms of order $\mathrm{O}(\mathbf{F}^2)$. Then the inverse matrix $\tilde{A}^{-1}$ is then perturbatively calculated as

$$\tilde{A}^{-1}(t,t') \simeq A^{-1}(t-t') + \epsilon^2 A^{-1}(t-t_2)\mathscr{P}(t_2,t_3)A^{-1}(t_3-t'). \tag{A 7}$$

Using the identity

$$\det\tilde{A} = \exp(\mathrm{Tr}\ln\tilde{A}) = \exp\{\mathrm{Tr}[\ln A + \ln(1-\epsilon^2 A^{-1}\mathscr{P})]\}, \tag{A 8}$$

and expanding in powers of $A^{-1}\mathscr{P}$, we find

$$\det\tilde{A} \simeq \exp[\mathrm{Tr}(\ln A - \epsilon^2 A^{-1}\mathscr{P})] = \det A\exp[-\epsilon^2 A^{-1}(t-t')\mathscr{P}(t,t')]. \tag{A 9}$$

Substituting (A 7) and (A 9) into (A 6) yields the relation (2.15).

Appendix B

An awkward number of terms are obtained by substituting (2.21) into (3.1), but integration over $\mathbf{r}$ diminishes this number considerably.

https://doi.org/10.1017/S0022377804003198 Published online by Cambridge University Press

First we note that the following terms vanish by integrating over z:

$$\int dz \sum_{\sigma_4} \tilde{\nabla}_{i_l} G(i_1-2)\tilde{\nabla}_{i_2} G(i_2-3)\tilde{\nabla}_{i_3} G(i_3-4)\tilde{\nabla}_{i_4} g(i_4) = \int dz \frac{\partial}{\partial z}[\ \cdots\ ] = 0, \tag{B 1}$$

$$\int dz \sum_{\sigma_3} \tilde{\nabla}'_{i_l} G(i_1-2)\tilde{\nabla}'_{i_2} G(i_2-3)\tilde{\nabla}'_{i_3} g(i_3) = \int dz \frac{\partial}{\partial z}[\ \cdots\ ] = 0. \tag{B 2}$$

Integrating by parts, we immediately obtain

$$\int_0^t dt_2 \int d\mathbf{r} \int d\mathbf{r}_2\, \mathbf{r}_\perp \mathbf{r}_\perp R[\nu(t-t_2)] \nabla_\perp G(1-2)\cdot\nabla_{\perp 2} g(2)$$
$$= 2\int_0^t d\tau\, R(\nu\tau) \int d\mathbf{r}\, G(\mathbf{r},\tau)\mathbf{I}, \tag{B 3}$$

$$\int_0^t dt_2 \int d\mathbf{r} \int d\mathbf{r}_2\, \mathbf{r}_\perp \mathbf{r}_\perp \{\nu\chi_\parallel R[\nu(t-t_2)] + v_\parallel^2\}\mathbf{F}(\mathbf{r}-\mathbf{r}_2) : \nabla_\perp G(1-2)\nabla_{\perp 2} g(2)$$
$$= 2\int_0^t d\tau [\nu\chi_\parallel R(\nu\tau) + v_\parallel^2] \int d\mathbf{r}\, \mathbf{F}(\mathbf{r}) G(\mathbf{r},\tau), \tag{B 4}$$

$$\int_0^t dt_2 \int_0^t dt_3 \int d\mathbf{r} \int d\mathbf{r}_2 \int d\mathbf{r}_3\, \mathbf{r}_\perp \mathbf{r}_\perp \{R[\nu(t-t_2)] + R[\nu(t_2-t_3)]\}\mathbf{F}(\mathbf{r}-\mathbf{r}_3)$$
$$: \sum_{\sigma_3} \hat{\nabla}_{i_l} G(i_1-2)\hat{\nabla}_{i_2} G(i_2-3)\hat{\nabla}_{i_3} g(i_3)$$
$$= 2\int_0^t dt_2 \int_0^{t_2} dt_3 \{R[\nu(t-t_2)] + R[\nu(t_2-t_3)]\}$$
$$\times \int d\mathbf{r}_2 \int d\mathbf{r}_3\, \mathbf{F}(\mathbf{r}_2+\mathbf{r}_3) G(\mathbf{r}_2, t-t_2)\partial_{z3} G(\mathbf{r}_3, t_2-t_3). \tag{B 5}$$

The most tedious calculation is needed for the term

$$\prod_{k=2}^{4} \int_0^t dt_k \int d\mathbf{r}_k \int d\mathbf{r}\, \mathbf{r}_\perp \mathbf{r}_\perp R[\nu(t-t_2)] R[\nu(t_3-t_4)]\mathbf{F}(\mathbf{r}-\mathbf{r}_3) : (\mathbf{I}_1 + \mathbf{I}_2), \tag{B 6}$$

where

$$\mathbf{I}_1 + \mathbf{I}_2 = \sum_{\sigma_4} \hat{\nabla}_{i_l} G(i_1-2)\hat{\nabla}_{i_2} G(i_2-3)\hat{\nabla}_{i_3} G(i_3-4)\hat{\nabla}_{i_4} g(i_4) \tag{B 7}$$

with

$$\mathbf{I}_1 = \nabla_\perp G(1-2)\partial_{z2} G(2-4)\partial_{z4} G(4-3)\nabla_{\perp 3} g(3)$$
$$+ \nabla_\perp G(1-4)\partial_{z4} G(4-2)\partial_{z2} G(2-3)\nabla_{\perp 3} g(3) \tag{B 8}$$

https://doi.org/10.1017/S0022377804003198 Published online by Cambridge University Press

and

$$\begin{aligned}\mathbf{I}_2 = {} & \nabla_\perp G(1-2)\partial_{z2}G(2-3)\nabla_{\perp 3}G(3-4)\partial_{z4}g(4) \\ & + \nabla_\perp G(1-3)\nabla_{\perp 3}G(3-2)\partial_{z2}G(2-4)\partial_{z4}g(4) \\ & + \nabla_\perp G(1-4)\partial_{z4}G(4-3)\nabla_{\perp 3}G(3-2)\partial_{z2}g(2) \\ & + \nabla_\perp G(1-3)\nabla_{\perp 3}G(3-4)\partial_{z4}G(4-2)\partial_{z2}g(2). \end{aligned} \tag{B 9}$$

After this cumbersome calculation, we have

$$\begin{aligned}&\prod_{k=2}^{4}\int_0^t dt_k \int d\mathbf{r}_k \int d\mathbf{r}\, \mathbf{r}_\perp\mathbf{r}_\perp R[\nu(t-t_2)]R[\nu(t_3-t_4)]\mathbf{F}(\mathbf{r}-\mathbf{r}_3) : \mathbf{I}_1 \\ &\quad = 2\int_0^t dt_2 \int_0^{t_2} dt_3 \int_0^{t_3} dt_4 \{R[\nu(t-t_2)]R[\nu(t_3-t_4)] \\ &\qquad + R[\nu(t-t_3)]R[\nu(t_2-t_4)]\} \prod_{k=2}^{4}\int d\mathbf{r}_k\, \mathbf{F}(\mathbf{r}_2+\mathbf{r}_3+\mathbf{r}_4) \\ &\qquad \times G(\mathbf{r}_2, t-t_2)\partial_{z3}G(\mathbf{r}_3, t_2-t_3)\partial_{z4}G(\mathbf{r}_4, t_3-t_4), \end{aligned} \tag{B 10}$$

and

$$\prod_{k=2}^{4}\int d\mathbf{r}_k \int d\mathbf{r}\, \mathbf{r}_\perp\mathbf{r}_\perp R[\nu(t-t_2)]R[\nu(t_3-t_4)]\mathbf{F}(\mathbf{r}-\mathbf{r}_3) : \mathbf{I}_2 = \mathbf{0}. \tag{B 11}$$

Putting (B 3), (B 4), (B 5) and (B 10) together, we have

$$\mathbf{D}_\perp^{\mathrm{cl}}(t) = \nu\chi_\perp \int_0^t dt_2\, R[\nu(t-t_2)] \int d\mathbf{r}\, G(\mathbf{r}, t-t_2)\mathbf{I}, \tag{B 12}$$

$$\begin{aligned}\mathbf{D}_\perp^{\mathrm{an}}(t) = {} & \int_0^t dt_2 \{\nu\chi_\parallel R[\nu(t-t_2)] + v_\parallel^2\} \int d\mathbf{r}\, \mathbf{F}(\mathbf{r})G(\mathbf{r}, t-t_2) \\ & + (\nu\chi_\parallel)^2 \int_0^t dt_2 \int_0^{t_2} dt_3 \int_0^{t_3} dt_4 \{R[\nu(t-t_2)]R[\nu(t_3-t_4)] \\ & + R[\nu(t-t_3)]R[\nu(t_2-t_4)]\} \int d\mathbf{r}_2 \int d\mathbf{r}_3 \int d\mathbf{r}_4\, \mathbf{F}(\mathbf{r}_2+\mathbf{r}_3+\mathbf{r}_4) \\ & \times G(\mathbf{r}_2, t-t_2)\partial_{z3}G(\mathbf{r}_3, t_2-t_3)\partial_{z4}G(\mathbf{r}_4, t_3-t_4) \\ & - v_\parallel\nu\chi_\parallel \int_0^t dt_2 \int_0^{t_2} dt_3 \{R[\nu(t-t_2)] + R[\nu(t_2-t_3)]\} \\ & \times \int d\mathbf{r}_2 \int d\mathbf{r}_3\, \mathbf{F}(\mathbf{r}_2+\mathbf{r}_3)G(\mathbf{r}_2, t-t_2)\partial_{z3}G(\mathbf{r}_3, t_2-t_3). \end{aligned} \tag{B 13}$$

Rewriting (B 12) and (B 13) in terms of the Fourier transforms of the response and correlation functions, we arrive at (3.3) and (3.4).

https://doi.org/10.1017/S0022377804003198 Published online by Cambridge University Press

Appendix C

Functionally differentiating (2.21) with respect to $\xi(1')$ and neglecting the three point function $\delta G(1{-}2)/\delta\xi(1')$, we can derive the equation for the response function $G(1-1')$:

$$\partial_t G(1-1') + v_\parallel \partial_z G(1-1') - \sum_{k=1}^{7} R_k(1-1') = \delta(1-1'), \tag{C 1}$$

where

$$R_1(1-1') = \nu\chi_\parallel \partial_z \int_0^{t-t'} dt_2 R(\nu t_2) \int d\mathbf{r}_2 G(\mathbf{r}_2, t_2)\partial_z G(\mathbf{r}-\mathbf{r}'-\mathbf{r}_2, t-t'-t_2), \tag{C 2}$$

$$\begin{aligned} R_2(1-1') = \nu\chi_\perp \nabla_\perp \cdot \int_0^{t-t'} dt_2 R(\nu t_2) \\ \times \int d\mathbf{r}_2 G(\mathbf{r}_2, t_2)\nabla_\perp G(\mathbf{r}-\mathbf{r}'-\mathbf{r}_2, t-t'-t_2), \end{aligned} \tag{C 3}$$

$$\begin{aligned} R_3(1-1') = \nabla_\perp \cdot \int_0^{t-t'} dt_2 [\nu\chi_\parallel R(\nu t_2) + v_\parallel^2] \\ \times \int d\mathbf{r}_2\, \mathbf{F}(\mathbf{r}_2) G(\mathbf{r}_2, t_2) \cdot \nabla_\perp G(\mathbf{r}-\mathbf{r}'-\mathbf{r}_2, t-t'-t_2), \end{aligned} \tag{C 4}$$

$$\begin{aligned} R_4(1-1') &= (\nu\chi_\parallel)^2 \prod_{k=2}^{4} \int_0^t dt_k \int d\mathbf{r}_k\, R[\nu(t-t_2)]R[\nu(t_3-t_4)]\mathbf{F}(\mathbf{r}-\mathbf{r}_3) \\ &\quad : \sum_{\sigma_4} \hat{\nabla}_{i_1} G(i_1-2)\hat{\nabla}_{i_2} G(i_2-3)\hat{\nabla}_{i_3} G(i_3-4)\hat{\nabla}_{i_4} G(i_4-1') \\ &= (\nu\chi_\parallel)^2 \nabla \cdot \int_0^{t-t'} dt_2 \int_0^{t-t'-t_2} dt_3 \int_0^{t-t'-t_2-t_3} dt_4 \prod_{k=2}^{4} \int d\mathbf{r}_k \\ &\quad \times \{R[\nu(t_2+t_3+t_4)]R(\nu t_3)\mathbf{F}(\mathbf{r}_3+\mathbf{r}_4)\cdot G(2)\partial_{z3} G(3)\nabla_{\perp 4} G(4)\partial_z \\ &\quad + R[\nu(t_2+t_3+t_4)]R(\nu t_4)\mathbf{F}(\mathbf{r}_3)\cdot \partial_{z2} G(2)\nabla_{\perp 3} G(3)G(4)\partial_z \\ &\quad + R(\nu t_2)R(\nu t_3)\mathbf{F}(\mathbf{r}_2+\mathbf{r}_3+\mathbf{r}_4)\cdot G(2)\partial_{z3}(3)\partial_{z4} G(4)\nabla_\perp \\ &\quad + R[\nu(t_2+t_4)]R[\nu(t_2+t_3)]\mathbf{F}(\mathbf{r}_2+\mathbf{r}_3+\mathbf{r}_4)\cdot \partial_{z2} G(2)\partial_{z3} G(3)G(4)\nabla_\perp \\ &\quad + R(\nu t_2)R(\nu t_4)\mathbf{F}(\mathbf{r}_2+\mathbf{r}_3)\cdot \nabla_{\perp 2} G(2)\partial_{z3} G(3)G(4)\partial_z \\ &\quad + R[\nu(t_2+t_3)]R[\nu(t_2+t_4)]\mathbf{F}(\mathbf{r}_3)\cdot G(2)\nabla_{\perp 3} G(3)\partial_{z4} G(4)\partial_z\} \\ &\quad \times G(1-1'-2-3-4), \end{aligned} \tag{C 5}$$

https://doi.org/10.1017/S0022377804003198 Published online by Cambridge University Press

$$R_5(1-1') = (\nu\chi_\parallel)^2 \prod_{k=2}^{4} \int_0^t dt_k \int d\mathbf{r}_k \, R[\nu(t-t_2)] R[\nu(t_3-t_4)] \mathbf{F}(\mathbf{r}_2-\mathbf{r}_4)$$
$$: \sum_{\sigma_4} \tilde{\nabla}_{i_1} G(i_1-2) \tilde{\nabla}_{i_2} G(i_2-3) \tilde{\nabla}_{i_3} G(i_3-4) \tilde{\nabla}_{i_4} G(i_4-1')$$
$$= (\nu\chi_\parallel)^2 \partial_z \int_0^{t-t'} dt_2 \int_0^{t-t'-t_2} dt_3 \int_0^{t-t'-t_2-t_3} dt_4 \prod_{k=2}^{4} \int d\mathbf{r}_k$$
$$\times \{ R[\nu(t_2+t_3+t_4)] R(\nu t_3) \mathbf{F}(\mathbf{r}_2+\mathbf{r}_3) : \partial_{z2} G(2) \nabla_{\perp 3} G(3) G(4) \nabla_\perp$$
$$+ R[\nu(t_2+t_3+t_4)] R(\nu t_4) \mathbf{F}(\mathbf{r}_2) : \partial_{\perp 2} G(2) G(3) \partial_{z4} G(4) \nabla_\perp$$
$$+ R(\nu t_2) R(\nu t_3) \mathbf{F}(\mathbf{r}_4) : G(2) \nabla_{\perp 3} G(3) \nabla_{\perp 4} G(4) \partial_z$$
$$+ R[\nu(t_2+t_4)] R[\nu(t_2+t_3)] \mathbf{F}(\mathbf{r}_2) : \nabla_{\perp 2} G(2) \nabla_{\perp 3} G(3) G(4) \partial_z$$
$$+ R(\nu t_2) R(\nu t_4) \mathbf{F}(\mathbf{r}_3+\mathbf{r}_4) : G(2) \nabla_{\perp 3} G(3) \partial_{z4} G(4) \nabla_\perp$$
$$+ R[\nu(t_2+t_3)] R[\nu(t_2+t_4)] \mathbf{F}(\mathbf{r}_4) : \partial_{z2} G(2) G(3) \nabla_{\perp 4} G(4) \nabla_\perp \}$$
$$\times G(1-1'-2-3-4), \tag{C 6}$$

$$R_6(1-1') = -v_\parallel \nu\chi_\parallel \int_0^t dt_2 \int_0^t dt_3 \int d\mathbf{r}_2 \int d\mathbf{r}_3 \{ R[\nu(t-t_2)] + R[\nu(t_2-t_3)] \}$$
$$\times \mathbf{F}(\mathbf{r}-\mathbf{r}_3) : \sum_{\sigma_3} \hat{\nabla}_{i_1} G(i_1-2) \hat{\nabla}_{i_2} G(i_2-3) \hat{\nabla}_{i_3} G(i_3-1')$$
$$= -v_\parallel \nu\chi_\parallel \nabla \cdot \int_0^{t-t'} dt_2 \int_0^{t-t'-t_2} dt_3 \int d\mathbf{r}_2 \int d\mathbf{r}_3$$
$$\times \{ [R[\nu(t_2+t_3)] + R(\nu t_2)] \cdot \mathbf{F}(\mathbf{r}_3) \cdot G(2) \nabla_{\perp 3} G(3) \partial_z$$
$$+ [R(\nu t_2) + R(\nu t_3)] \cdot \mathbf{F}(\mathbf{r}_2+\mathbf{r}_3) \cdot G(2) \partial_{z3} G(3) \nabla_\perp \}$$
$$\times G(1-1'-2-3), \tag{C 7}$$

$$R_7(1-1') = -v_\parallel \nu\chi_\parallel \int_0^t dt_2 \int_0^t dt_3 \int d\mathbf{r}_2 \int d\mathbf{r}_3 \, R[\nu(t-t_2)] \mathbf{F}(\mathbf{r}_2-\mathbf{r}_3)$$
$$: \sum_{\sigma_3} \tilde{\nabla}'_{i_1} G(i_1-2) \tilde{\nabla}'_{i_2} G(i_2-3) \tilde{\nabla}'_{i_3} G(i_3-1')$$
$$= -v_\parallel \nu\chi_\parallel \partial_z \int_0^{t-t'} dt_2 \int_0^{t-t'-t_2} dt_3 \int d\mathbf{r}_2 \int d\mathbf{r}_3$$
$$\times \{ R[\nu(t_2+t_3)] \mathbf{F}(\mathbf{r}_2) : \nabla_{\perp 2} G(2) G(3) \nabla_\perp$$
$$+ R(\nu t_2) \mathbf{F}(\mathbf{r}_3) : G(2) \nabla_{\perp 3} G(3) \nabla_\perp \} G(1-1'-2-3). \tag{C 8}$$

https://doi.org/10.1017/S0022377804003198 Published online by Cambridge University Press

We use the ansatz of the spatial Markovian approximation, i.e.

$$G\left(\mathbf{r}-\mathbf{r}'-\sum_{k=2}^{n}\mathbf{r}_k, t-t'-\sum_{k=2}^{n}t_k\right) \simeq G\left(\mathbf{r}-\mathbf{r}', t-t'-\sum_{k=2}^{n}t_k\right). \tag{C 9}$$

Then we find $R_5 = R_7 = 0$ and

$$\begin{aligned} R_4(1-1') = (\nu\chi_\parallel)^2\nabla_\perp \cdot \int_0^{t-t'} dt_2 \int_0^{t-t'-t_2} dt_3 \int_0^{t-t'-t_2-t_3} dt_4 \\ \times \prod_{k=2}^{4} \int d\mathbf{r}_k\, \mathbf{F}(\mathbf{r}_2+\mathbf{r}_3+\mathbf{r}_4)\{R(\nu t_2)R(\nu t_4) + R[\nu(t_2+t_3)]R[\nu(t_3+t_4)]\} \\ \times G(2)\partial_{z3}G(3)\partial_{z4}G(4)\cdot\nabla_\perp G(\mathbf{r}-\mathbf{r}', t-t'-t_2-t_3-t_4), \end{aligned} \tag{C 10}$$

$$\begin{aligned} R_6(1-1') = -v_\parallel\nu\chi_\parallel\nabla_\perp \cdot \int_0^{t-t'} dt_2 \int_0^{t-t'-t_2} dt_3 [R(\nu t_2) + R(\nu t_3)] \\ \times \int d\mathbf{r}_2 \int d\mathbf{r}_3 \mathbf{F}(\mathbf{r}_2+\mathbf{r}_3) G(2)\partial_{z3}G(3)\cdot\nabla_\perp G(\mathbf{r}-\mathbf{r}', t-t'-t_2-t_3). \end{aligned} \tag{C 11}$$

Substituting R_k into (C 1) and further use of the temporal Markovian approximation, $G(\mathbf{r}-\mathbf{r}', t-t'-\sum_{k=2}^{n}t_k) \simeq G(\mathbf{r}-\mathbf{r}', t-t')$, we find the equation for the response function in the form

$$\begin{aligned} \partial_t G(1-1') + v_\parallel\partial_z G(1-1') - D_\parallel(t-t')\partial_z^2 G(1-1') \\ - \nabla_\perp\cdot\mathbf{D}_\perp(t-t')\cdot\nabla_\perp G(1-1') = \delta(1-1'), \end{aligned} \tag{C 12}$$

where

$$D_\parallel(t) = \chi_\parallel\nu\int_0^t d\tau\, R(\nu\tau). \tag{C 13}$$

From the equation (C 12), we have

$$\int G(\mathbf{r}, t-t')\, d\mathbf{r} = H(t-t'), \tag{C 14}$$

where H is the Heaviside step function.

Appendix D

When the fluctuating magnetic field is independent of the perpendicular coordinates x and y, the differential equation for the running diffusion coefficient $D_\perp^{\mathrm{an}}(t) = \mathbf{e}_x\cdot\mathbf{D}_\perp^{\mathrm{an}}(t)\cdot\mathbf{e}_x$ can be derived without using the DIA. Now let us consider the fluctuating magnetic field of the form $\mathbf{b} = b(z)\mathbf{e}_x$, and assume its correlation function as $\langle b(z_1)b(z_2)\rangle = \beta^2\exp[-(z_1-z_2)^2/(2\lambda_\parallel^2)]$. Then, using the similar method to Eijnden and Balescu (1996b), we can obtain the exact solution to (2.2) as

$$f(x, z, t; v_\parallel) = \delta(x - x_1(t) - x_2(z))\delta(z - z(t)), \tag{D 1}$$

where

$$x_1(t) = x_0 + \int_0^t \eta_x(\tau)\, d\tau \tag{D 2}$$

https://doi.org/10.1017/S0022377804003198 Published online by Cambridge University Press

with $\eta_x(t) = \boldsymbol{\eta}_\perp(t) \cdot \mathbf{e}_x$,

$$x_2(z) = \int_{z_0}^{z} b(z_1)\, dz_1, \tag{D 3}$$

and

$$z(t) = z_0 + v_\parallel t + \int_0^t \eta_\parallel(\tau)\, d\tau. \tag{D 4}$$

Performing the ensemble average on $b(z)$, $\eta_\parallel(t)$ and $\eta_x(t)$ yields the following expression for the ensemble-averaged distribution function $g(x, z, t; v_\parallel) = \langle f(x, z, t; v_\parallel)\rangle$:

$$g(x, z, t; v_\parallel) = \langle \delta(x - x_1(t) - x_2(z))\rangle_{b,\perp} \langle \delta(z - z(t))\rangle_\parallel \tag{D 5}$$

with

$$\langle \delta(z - z(t))\rangle_\parallel = \frac{1}{2\pi} \int dk_z \exp[ik_z(z - z_0 - v_\parallel t) - k_z^2 \chi_\parallel \psi(\nu t)/\nu] \tag{D 6}$$

and

$$\langle \delta(x - x_1(t) - x_2(z))\rangle_{b,\perp} = \frac{1}{2\pi} \int dk_x \exp\left[ik_x(x - x_0) - k_x^2 \chi_\perp \psi(\nu t)/\nu \right.$$
$$\left. - k_x^2 \int_{z_0}^{z} dz_1 \int_{z_0}^{z_1} dz_2\, \langle b(z_2) b(z_1)\rangle \right]. \tag{D 7}$$

Using (3.1) and the relation

$$\int x^2 g\, dz\, dx = 2\chi_\perp \frac{\psi(\nu t)}{\nu} + \frac{2\lambda_\parallel \beta^2}{\sqrt{2\pi}} \int \frac{dk}{k^2} e^{-\frac{1}{2}\lambda_\parallel^2 k^2} \left(1 - e^{-k^2 \chi_\parallel \psi(\nu t)/\nu + ikv_\parallel t}\right), \tag{D 8}$$

we arrive at the following differential equation for $D_\perp^{\mathrm{an}}(t)$:

$$\frac{dD_\perp^{\mathrm{an}}}{d\tau} = \frac{\chi_\parallel \beta^2}{[1 + \bar{\chi}_\parallel \psi(\tau)]^{1/2}} \left\{ R(\tau) + 2\bar{v}_\parallel^2 - \frac{1}{2} \frac{\bar{\chi}_\parallel \varphi(\tau)}{1 + \bar{\chi}_\parallel \psi(\tau)} \right.$$
$$\left. \times \left[4\bar{v}_\parallel^2 \tau + \varphi(\tau) \left(1 - \frac{\bar{\chi}_\parallel \bar{v}_\parallel^2 \tau^2}{1 + \bar{\chi}_\parallel \psi(\tau)} \right) \right] \right\} \exp\left[-\frac{1}{2} \frac{\bar{\chi}_\parallel \bar{v}_\parallel^2 \tau^2}{1 + \bar{\chi}_\parallel \psi(\tau)} \right], \tag{D 9}$$

where $\tau = \nu t$.

Appendix E

Using (4.3) and (4.4), we obtain the following equations for $\boldsymbol{\mu}$:

$$\frac{d^2 \mu_{xx}}{dt^2} = \frac{4\alpha^2 v_\parallel^2}{\lambda_\parallel^2} \frac{1}{(1 + \mu_{xx}/2)^{1/2}} \frac{1}{d^{3/2}} \left(1 - \frac{1}{d} \frac{v_\parallel^2 t^2}{\lambda_\parallel^2} \frac{x^2}{L_{\mathrm{s}}^2} \frac{\lambda_\parallel^2}{\lambda_\perp^2} \right)$$
$$\times \exp\left[-\frac{1}{2} \frac{v_\parallel^2 t^2}{\lambda_\parallel^2} \left(1 + \frac{1}{d} \frac{x^2}{L_{\mathrm{s}}^2} \frac{\lambda_\parallel^2}{\lambda_\perp^2} \right) \right], \tag{E 1}$$

https://doi.org/10.1017/S0022377804003198 Published online by Cambridge University Press

$$\frac{d^2\mu_{yy}}{dt^2} = \frac{4\alpha^2 v_\parallel^2}{\lambda_\parallel^2}\frac{1}{(1+\mu_{xx}/2)^{3/2}}\frac{1}{d^{1/2}}\left[1+\frac{1}{4d}\frac{\mu_{xy}^2}{1+\mu_{xx}/2}\left(1-\frac{1}{d}\frac{v_\parallel^2 t^2}{\lambda_\parallel^2}\frac{x^2}{L_s^2}\frac{\lambda_\parallel^2}{\lambda_\perp^2}\right)\right]$$
$$\times \exp\left[-\frac{1}{2}\frac{v_\parallel^2 t^2}{\lambda_\parallel^2}\left(1+\frac{1}{d}\frac{x^2}{L_s^2}\frac{\lambda_\parallel^2}{\lambda_\perp^2}\right)\right], \tag{E 2}$$

$$\frac{d^2\mu_{xy}}{dt^2} = \frac{2\alpha^2 v_\parallel^2}{\lambda_\parallel^2}\frac{\mu_{xy}}{(1+\mu_{xx}/2)^{3/2}}\frac{1}{d^{3/2}}\left(1-\frac{1}{d}\frac{v_\parallel^2 t^2}{\lambda_\parallel^2}\frac{x^2}{L_s^2}\frac{\lambda_\parallel^2}{\lambda_\perp^2}\right)$$
$$\times \exp\left[-\frac{1}{2}\frac{v_\parallel^2 t^2}{\lambda_\parallel^2}\left(1+\frac{1}{d}\frac{x^2}{L_s^2}\frac{\lambda_\parallel^2}{\lambda_\perp^2}\right)\right], \tag{E 3}$$

where we have used $\mu_{xy} = \mu_{yx}$ and

$$d = \frac{(1+\mu_{xx}/2)(1+\mu_{yy}/2) - \mu_{xy}^2/4}{1+\mu_{xx}/2}. \tag{E 4}$$

Imposing the initial conditions $\boldsymbol{\mu}(0) = \mathbf{0}$ and $d\boldsymbol{\mu}(0)/dt = \mathbf{0}$, we find $\mu_{xy} = \mu_{yx} = 0$ and thus $d = 1 + \mu_{yy}/2$. Therefore, the (4.5) and (4.6) for μ_{xx} and μ_{yy} are immediately obtained from (E 1) and (E 2).

Appendix F

In this section, we give a simplification of the following term

$$I = \int d\mathbf{r}_2\, \mathbf{F}(\mathbf{r}-\mathbf{r}_2) G(1,2)\cdot\nabla_{\perp 2} g(2). \tag{F 1}$$

From (4.4), the response function can be written as $G(1,2) = G(x-x_2, y-y_2-(z-z_2)x/L_s, z-z_2, t-t_2)$. Thus, introducing new variables $\hat{x} = x - x_2$, $\hat{y} = y - y_2 - (x/L_s)(z-z_2)$, $\hat{z} = z - z_2$, $\hat{t} = t - t_2$, we rewrite I in the form

$$I = \int d\hat{\mathbf{r}}_2\, \mathbf{F}\left(\hat{x}, \hat{y}+\frac{x}{L_s}\hat{z}, \hat{z}\right) G(\hat{x},\hat{y},\hat{z},\hat{t})$$
$$\times \left[\mathbf{e}_x\left(\frac{\partial}{\partial x}+\frac{\hat{z}}{L_s}\frac{\partial}{\partial y}\right)+\mathbf{e}_y\frac{\partial}{\partial y}\right] g\left(x-\hat{x}, y-\hat{y}-\frac{x}{L_s}\hat{z}, z-\hat{z}, t-\hat{t}\right). \tag{F 2}$$

We now adopt the spatial and temporal Markovian approximation for g, i.e. $g(x-\hat{x}, y-\hat{y}-(x/L_s)\hat{z}, z-\hat{z}, t-\hat{t}) \simeq g(x,y,z,t)$. Then we have the relation (4.8).

References

Balescu, R., Wang, H.-D. and Misguich, J. H. 1994 Langevin equation versus kinetic equation: Subdiffusive behavior of charged particles in a stochastic magnetic field. *Phys. Plasmas* **1**, 3826–3842.

Eijnden, E. V. and Balescu, R. 1996a Strongly anomalous diffusion in sheared magnetic configurations. *Phys. Plasmas* **3**, 815–823.

Eijnden, E. V. and Balescu, R. 1996b Statistical description and transport in stochastic magnetic fields. *Phys. Plasmas* **3**, 874–888.

Isichenko, M. B. 1991a Effective plasma heat conductivity in 'braided' magnetic field—I. Quasi-linear limit. *Plasma Phys. Control. Fusion* **33**, 795–807.

https://doi.org/10.1017/S0022377804003198 Published online by Cambridge University Press

Isichenko, M. B. 1991b Effective plasma heat conductivity in 'braided' magnetic field—II. Percolation limit. *Plasma Phys. Control. Fusion* **33**, 809–826.

Jensen, R. V. 1981 Functional integral approach to classical statistical dynamics. *J. Stat. Phys.* **25**, 183–210.

Jouvet, B. and Phythian, R. 1979 Quantum aspects of classical and statistical fields. *Phys. Rev.* A **19**, 1350–1355.

Krommes, J. A., Oberman, C. and Kleva, R. G. 1983 Plasma transport in stochastic magnetic fields. Part 3. Kinetics of test particle diffusion. *J. Plasma Phys.* **30**, 11–56.

Krommes, J. A. 1984 *Handbook of Plasma Physics II*. Amsterdam: North-Holland.

Krommes, J. A. 2002 Fundamental statistical theories of plasma turbulence in magnetic fields. *Phys. Reports* **360**, 1–351.

Lavel, G. 1993 Particle diffusion in stochastic magnetic fields. *Phys. Fluids* B **5**, 711–721.

Martin, P. C., Siggia, E. D. and Rose, H. A. 1973 Statistical dynamics of classical systems. *Phys. Rev.* A **8**, 423–437.

Rax, J. M. and White, R. B. 1992 Effective diffusion and nonlocal heat transport in a stochastic magnetic field. *Phys. Rev. Lett.* **68**, 1523–1526.

Rechester, A. B. and Rosenbluth, M. N. 1978 Electron heat transport in a Tokamak with destroyed magnetic surfaces. *Phys. Rev. Lett.* **40**, 38–41.

Rose, H. A. 1992 Test-particle transport in stochastic magnetic field: a fluid representation. *Phys. Rev. Lett.* **48**, 260–263.

Taguchi, M. 2003 Radio-frequency current drive theory in the presence of fluctuations. *Phys. Plasmas* **10**, 37–46.

Thyagaraja, A. and Haas, F. A. 1985 Fluid theory of anomalous transport due to a small-scale magnetic turbulence. *Physica Scripta* **31**, 83–86.

Wang, H.-D., Vlad, M., Eijnden, E. V., Spineanu, F., Misguich, J. H. and Balescu, R. 1995 Diffusive processes in a stochastic magnetic field. *Phys. Rev.* E **51**, 4844–4859.

Zinn-Justin, J. 1996 *Quantum Field Theory and Critical Phenomena*. Oxford: Clarendon.

https://doi.org/10.1017/S0022377804003198 Published online by Cambridge University Press