Neuro-fuzzy approach for performance optimisation of variable nozzle turbofan engine

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ABSTRACT

An algorithm employing adaptive neuro-fuzzy online identification and sequential quadratic programming optimisation techniques is developed to enhance aircraft engine performance. This algorithm is implemented and tested using digital simulation for two spool, mixed exhaust, variable geometry turbofan engine. Parametric study is conducted to select the proper measurable parameter that can represent the actual thrust during online optimisation. Subtractive clustering technique is applied to generate the minimum number of fuzzy rules that can model the engine performance at various input parameters and flight conditions. The resulting neuro-fuzzy system is then optimised through training algorithm to accurately represent the engine. This system can address engine variations by relearning the network using online measurements from the actual engine. Generating the optimum schedules and comparing them with those obtained from the complete non-linear engine model verify the algorithm. Benefits from this algorithm include fuel consumption savings, reductions in turbine inlet temperature, and preventing limit exceeding.

NOMENCLATURE

A_N	nozzle area
E	error
EPR	engine pressure ratio, ($P_{t4.5}/P_{t2}$)
FFR	fuel flow rate (kgs^{-1})
GF	gain factor
H_o	flight altitude
M_o	flight Mach number
N	spool speed (RPM)
P	pressure (Pa)
T	temperature (K)

Subscripts

2	fan inlet.
2·5	compressor inlet
3	compressor discharge
4	high pressure turbine inlet
4·5	high pressure turbine outlet
F	fuel
H	high pressure spool
L	low pressure spool
N	speed
rel	relative to design value
s	static
t	total
T	temperature

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Abbreviations

ANF	adaptive neuro-fuzzy
CSM	compressor surge margin
EPR	engine pressure ratio ($P_{t4.5}/P_{t2}$)
FPSC	fuzzy performance seeking control
FSM	fan surge margin
NLQP	non-linear quadratic programming
PLA	power lever angle.
PSC	performance seeking control
SLS	sea level static
TIT	turbine inlet temperature
TSFC	thrust specific fuel consumption

1.0 INTRODUCTION

Modern tactical aircrafts employ variable geometry turbofan engine with mixed exhaust and augmenter. Figure 1 gives a schematic representation for this engine. Such engines operate according to predetermined schedules for fuel flow and other engine control parameters. Typical control system for variable geometry turbofan engine is shown in Fig. 2. There are number of engine variations that produce sub optimal performance when operating on a fixed schedule. These sources include inaccuracy in the model at the design phase, shifts occurring during production phase, changes resulting from maintenance actions, conventional engine components deterioration, and variable engine operating environments. These differences tend to offset potential performance gains obtained by using specific model-based schedules.

An extensive effort in this field has been made jointly by NASA, McDonnell Aircraft Company and Pratt & Whitney between 1985 and 1995. They developed the so-called performance seeking control (PSC) algorithm with the objective of optimising the quasi-steady state performance of the F100 turbofan engine[1-10]. The algorithm flow diagram (Fig. 3) consists mainly of three activities; estimation, modelling, and optimisation. The estimation process is a Kalman-filter of five component deviation parameters. The second step formulates and uses the compact propulsion system model to estimate unmeasured engine outputs required to obtain an optimal solution.

A propulsion system matrix, which is a linear model for the engine, is derived from the non-linear compact model. A local optimal solution of the system matrix within the defined constraints is obtained using a linear programming algorithm. The global optimal engine operating point requires iterating on the compact model and linear programming optimisation for specified number of times. The F100 PSC algorithm as currently implemented, however, has some disadvantages that may reduce its efficiency and decrease its reliability[8]. These include the vital need of accurate models to predict actual flight hardware performance, the use of linear models in both the Kalman filter and propulsion system matrix modules, and inherent dependency on accurate measurements of the inputs and outputs of the system being optimised. For new developed engines, the model always has many inaccuracies, also using linear models limits the estimation capability to only few percent from the known model and require many iterations to reach global optimum.

It is intended to develop a new algorithm that can solve the above problems. The proposed algorithm uses adaptive neuro-fuzzy techniques due to Jang[11-13]. This approach constructs a fuzzy inference system for which the parameters can be updated to achieve the desired input-output mapping. The learning mechanism uses the gradient method for the non-linear parameters in the premise layer and least square estimation (Kalman filter) procedure for the linear parameters in the consequence layer. This combination is called hybrid-learning mechanism and it is able to fine-tune membership functions of the fuzzy rules as the training data set varies. The flow chart of the proposed fuzzy performance seeking control (FPSC) algorithm is plotted in Fig. 4. It covers modelling, scheduling, and

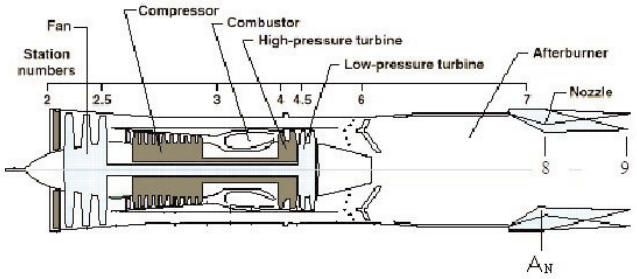


Figure 1. Typical variable geometry turbofan engine.

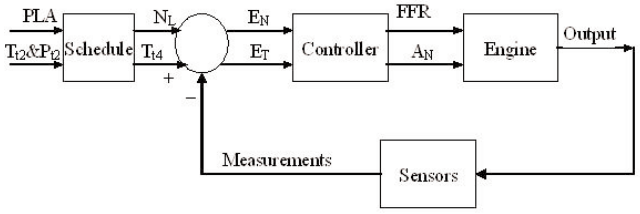


Figure 2. Variable geometry turbofan engine control system.

control of a two spool, mixed exhaust, variable geometry turbofan engine.

The FPSC system contains system identification using adaptive neuro-fuzzy (ANF) technique, and online optimisation via non-linear quadratic programming. Comparison of this system with PSC system highlights the following advantages of the proposed system[14]. No need for accurate models as the initial model represents first guess for the ANF to initiate learning. Also, since non-linear model (fuzzy model) is used, large excursions on the optimisation parameters can be tolerated allowing faster determination of the optimum set and large variations in engine health can be addressed. Third, small memory is required to store the fuzzy rules. Finally, it uses feedback between the true and identified engine, thus methods of sensor error diagnosis can be applied to exclude measurement bias.

2.0 THRUST OBSERVATION PARAMETER SELECTION

A measurable engine parameter has to be chosen to represent the thrust during online optimisation. The effects of variation in engine physical actuation parameters on actual thrust must be represented by such parameter. There are many candidates such as T_{t4}, $T_{t4.5}$, N_L, N_H, and engine pressure ratio (EPR). The most suitable parameter is that which varies with the engine actuation parameters in similar trend to that of actual thrust and thus can be employed online to realize the optimum thrust specific fuel consumption.

Figure 5(a) shows the turbine inlet temperature (TIT) and thrust variations with fuel flow rate and nozzle throat area. Increasing fuel flow increases actual thrust and TIT. On the other hand, the figure shows that the optimum nozzle area producing maximum thrust at constant fuel flow, i.e., minimum TSFC, corresponds to the minimum TIT at constant fuel flow. This contradicts the previous trend of the thrust-TIT relationship. Hence if the TIT parameter, and similarly $T_{t4.5}$, is employed online to minimise thrust specific fuel consumption instead of actual thrust, distorted results will be obtained. Similar conclusions can be extracted from Fig. 5(b) for the case of using N_H as a thrust observation parameter. Figure 5(c) indicates that the resulting optimum nozzle area if N_L is used to

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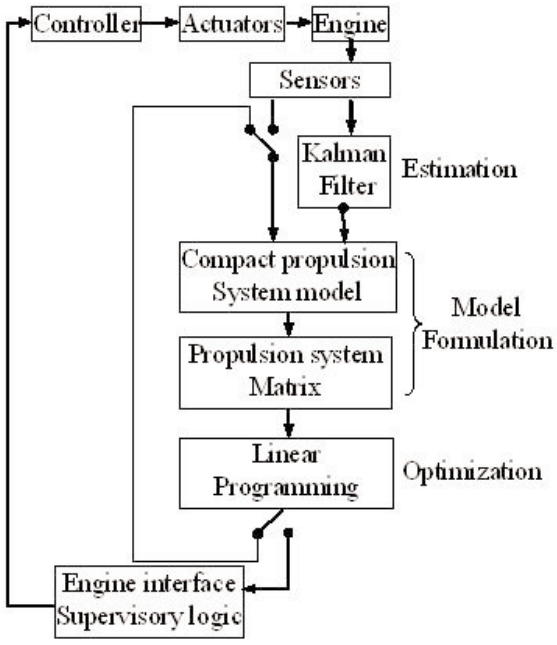


Figure 3. Performance seeking control algorithm.

observe thrust will be different from the actual optimum nozzle area. The case of using EPR as a thrust observation parameter is very indicative as the optimum nozzle area obtained from the use of this parameter is almost the same as that obtained from the application of actual thrust as implied by Fig. 5(d). This is due to positive sensitivity of EPR with thrust for changes in fuel flow and area nozzle in the whole operating range. This sensitivity for the above parameters changes its sign with variations in nozzle area leading to unfavourable effect on the online optimisation process. Measuring the EPR is simple and conventional for many engines. Moreover, it is required to sense N_L, N_H, T_{t2}, P_{t2}, $P_{t2\cdot5}$, $T_{t2\cdot5}$, *FFR*, $T_{t4.5}$ and P_{S3} to determine the status of limits and use them in updating the fuzzy models related to these parameters[14].

(a) Variations in thrust and *TIT* with fuel flow and nozzle area.
(b) Variations in thrust and N_H with fuel flow and nozzle area.
(c) Variations in thrust and N_L with fuel flow and nozzle area.
(d) Variations in thrust and EPR with fuel flow and nozzle area.

3.0 FUZZY CLUSTERING

Fuzzy modelling and identification is used in many fields, however, its application in the field of aircraft engine dynamics and control is still limited. Here, fuzzy logic is used as an identification algorithm along with neural networks to increase the efficiency of fuzzy systems in the area of adaptation. This algorithm consists mainly of two steps: fuzzy clustering and adaptive neuro-fuzzy learning. The cluster estimation method is applied to determine the number of clusters and their initial fuzzy rules for starting iterative optimisation-based clustering algorithm such as adaptive neuro-fuzzy inference systems. The first guess for the fuzzy model is obtained through the application of the Chiu cluster estimation method[15].

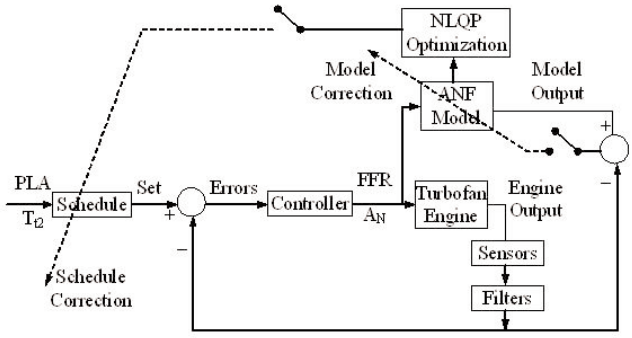


Figure 4. Fuzzy performance seeking control (FPSC) algorithm flow chart.

This method is based on evaluating a measure of potential for each data point and iteratively reducing the potential of data points near new cluster centres. The computation grows linearly with the dimension of the problem and as the square of the number of data points. Another method is presented by Yager and Filev[16], which uses a grid to determine the cluster centre. The cluster estimation method and linear least-squares estimation procedure provide fast and robust algorithm for identifying fuzzy models from numerical data. Consider a collection of *n*-data points (x_1, x_2… x_n) in an m-dimensional space, normalised in each dimension. Define measure of the potential of each data point x_i as:

$$P_i = \sum_{j=1}^{n} e^{-\alpha\|x_i - x_j\|^2} \text{ where } \alpha = \frac{4}{r_a^2} \text{ and } r_a \text{ is a positive constant} \quad \ldots (1)$$

Thus, the measure of potential for a data point is a function of its distance from all other data points. A data point with many neighbouring data points has a high potential value. The constant r_a is effectively the radius defining a neighbourhood status. Data points outside this radius have little influence on the potential. A large r_a generally results in fewer clusters and hence a coarser model, while a small r_a can produce excessive number of clusters and a model that does not generalise well. The data point with the highest potential is selected as the first cluster centre. Let x_1^* be the location of the first cluster centre and P_1^* be its potential value. The potential of each data point x_i is revised by the formula:

$$P_i = P_i - P_1^* e^{-\beta\|x_i - x_1^*\|^2} \text{ where } \beta = \frac{4}{r_b^2} \text{ and } r_b \text{ is a positive constant} \quad \ldots (2)$$

Subtracting an increment of potential from each data point as function of its distance from the first cluster centre, the data points near the first cluster centre will have greatly reduced potential, and therefore are unlikely to be selected as the next cluster centre. The constant is effectively the radius defining the neighbourhood that will have measurable reductions in potential. Revising the potential of all data points according to Equation (2), we select the data point with the highest remaining potential as the second cluster centre, and so on. The cluster centres are accepted or rejected according to a criterion that provides a good trade-off between having a sufficient potential for a point and being far enough from existing cluster centres. When applying the cluster estimation method to a collection of input/output data, each cluster centre is basically an ideal point that demonstrates a characteristic behaviour of the system[16].

Next, a set of c-cluster centres (x_1^*, x_2^*, ... x_c^*) in *m*-dimensional space is considered. Let the first *n*-dimensions correspond to input variables and the last (*m*-*n*) dimensions correspond to output variables. Decompose each vector $\mathbf{x_i}^*$ into two component vectors $\mathbf{y_i}^*$ and $\mathbf{z_i}^*$, where $\mathbf{y_i}^*$ contains the first *n*-elements of $\mathbf{x_i}^*$ and $\mathbf{z_i}^*$ contains the last (*m*-*n*) elements. Each cluster centre $\mathbf{x_i}^*$ locates a rule that describes the system behaviour. Given an input vector $\mathbf{y}$, the

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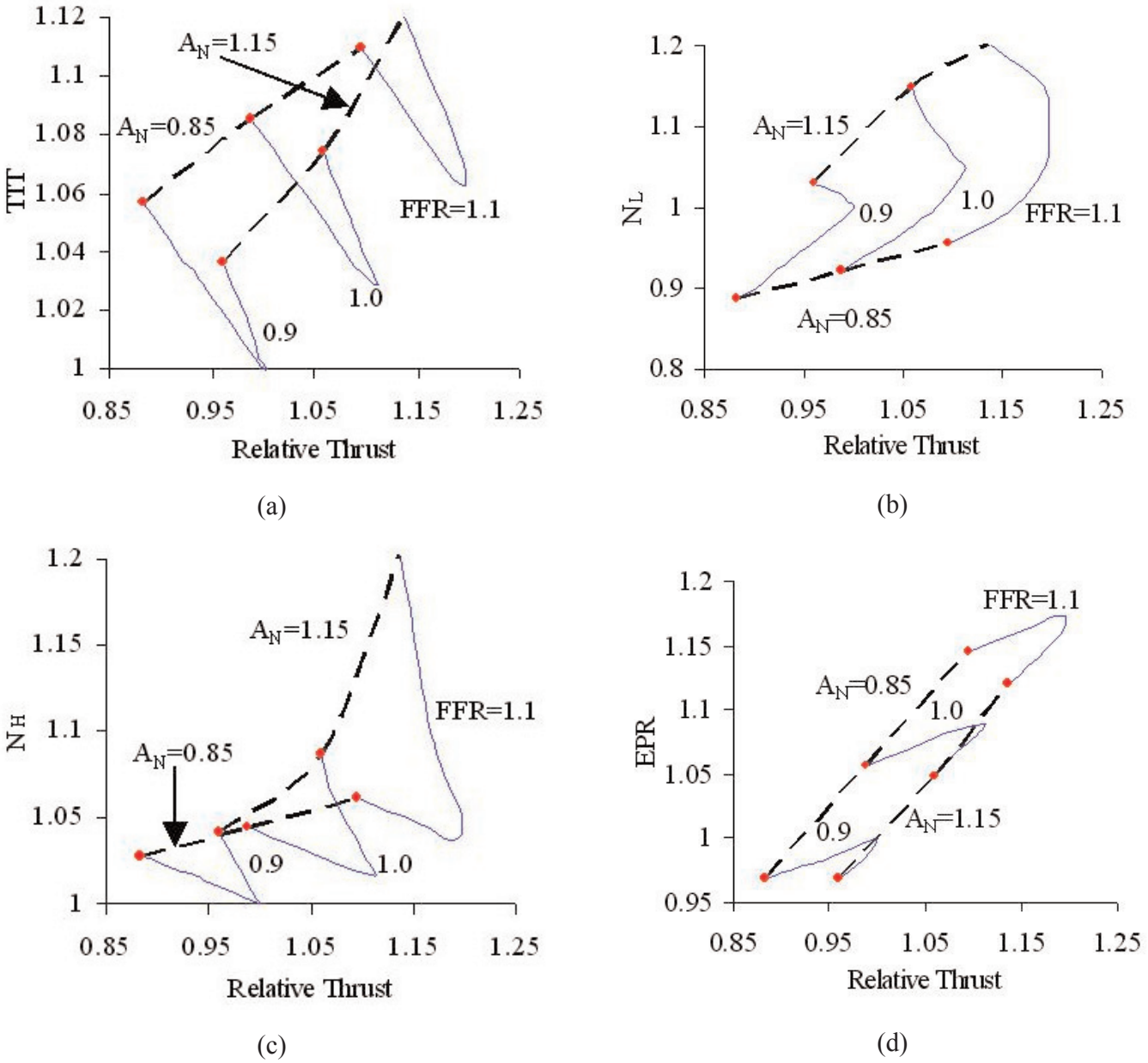


Figure 5. Engine parameters correlation with thrust at SLS conditions.

degree to which rule (*i*) is fulfilled is defined by a membership function such as the Gaussian function expressed by

$$w_i = e^{-\alpha\left\|y-y_i^*\right\|^2} \quad \ldots (3)$$

In this equation, α is the constant defined by Equation (1) and is determined by the cluster radius. The output vector **z** is computed via:

$$z = \frac{\sum_{i=1}^{c} w_i z_i^*}{\sum_{i=1}^{c} w_i} \quad \ldots (4)$$

This computational model can be viewed as a fuzzy inference system employing traditional fuzzy if-then rules and modelled as Sugeno zero-order fuzzy system[13]. In this fuzzy system, the output from each rule is a constant value at the centre of the rule (cluster centre) in the output space. The firing strength for this output value is obtained from Equation (3). The overall output is the weighted average of the outputs from all rules. Each rule has the following form:

If y_1 is A_1 and y_2 is A_2 and ... Then z_1 is B_1 and z_2 is B_2 $\ldots (5)$

Where y_j is the *j*th input variable and z_j is the *j*th output variable; A_j is an exponential membership function and B_j is a singleton.

For the *i*th rule that is represented by cluster centre x_i^*, A_j and B_j are given by:

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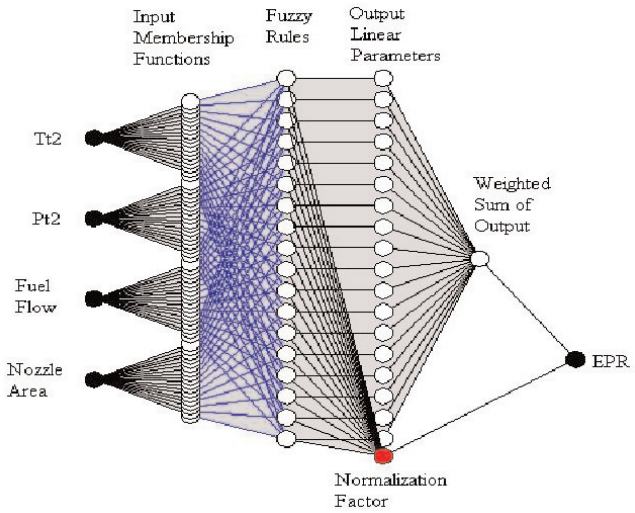


Figure 6. Adaptive neuro-fuzzy model structure.

$$A_j(q) = e^{-\alpha_j (q - y_{ij}^*)^2} \quad \ldots (6)$$

$$B_j = z_{ij}^* \quad \ldots (7)$$

Where y_{ij}^* is the jth element of $\mathbf{y_i}^*$, z_{ij}^* is the jth element of $\mathbf{z_i}^*$ and α_j is the width of the membership function related to the jth input variable. Equations (3) and (4) provide a simple and direct way to translate a set of cluster centres into a fuzzy model.

4.0 TRAINING SCHEMES

One approach to optimize the rules of a fuzzy model such that it accurately represents a specific system is to adjust the y_{ij}^*, z_{ij}^* and individual α_j parameters in Equations (6) and (7). Another different approach is to allow $\mathbf{z_i}^*$ in Equation (4) to be a linear function of the input variables instead of a simple constant. That is

$$\mathbf{z}_i^* = \mathbf{G}_i \cdot y + \mathbf{h}_i \quad \ldots (8)$$

In this equation, $\mathbf{G_i}$ is an $(m\text{-}n) \times n$ constant matrix, and $\mathbf{h_i}$ is a constant column vector with $m\text{-}n$ elements. The equivalent if-then rules then become the Sugeno first order type. Given a set of rules with fixed premises (y_1^* and α_j), optimising the parameters in the consequent equations ($\mathbf{G_i}$ and $\mathbf{h_i}$) with respect to training data reduces to a linear least-squares estimation problem[13]. Such problem can be easily solved and the solution is always globally optimal. Then, another algorithm is used to optimise the parameters in the premises (membership function centres and widths) using back propagation gradient descent method. This hybrid learning rule decreases the dimension of the search space in the gradient method and cut down substantially the convergence time[12,13].

5.0 TURBOFAN ENGINE CASE STUDY

In actual case, a set of measurements is obtained from the flight engine at various flight conditions. This set is applied to the fuzzy performance seeking control as a training data. Here, for the considered turbofan engine, General Electric F110[17]; a training data set is collected using the non-linear engine model[18,19] at various flight conditions to cover the expected range of T_{t2} and P_{t2}. Also, the training data covers the range of operation of both fuel flow rate and the nozzle throat area. These are the four independent inputs for this fuzzy system and the thrust measured by *EPR* is considered as the output. Initially, these input/output pairs are clustered with a radius of influence of 0·25. This results in fuzzy system with eighteen rules. This model is a coarse representation for the engine, but this step is very important to decrease the number of rules and start as close as possible from the accurate fuzzy model. The network structure for this model is outlined in Fig. 6.

This model leads to adaptive neuro-fuzzy model with the following information: 18 fuzzy rules, 90 linear parameters [(No of inputs + 1) × No of rules], 144 non-linear parameters [No of defining parameters for used membership function × No. of rules × No of inputs], and 234 total number of defining parameters.

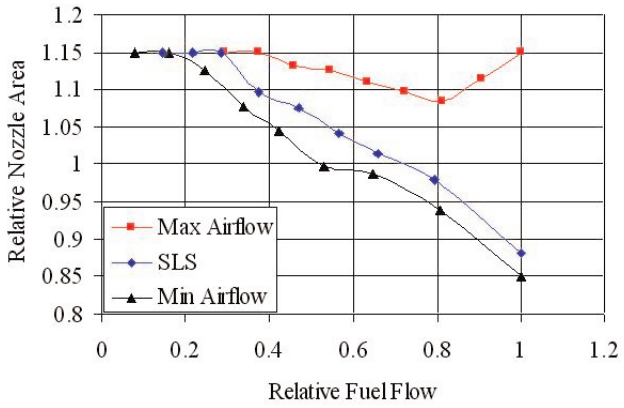


Figure 7. Optimum schedule for minimum, maximum airflow and SLS conditions.

5.1 Fuzzy model verification

The non-linear quadratic programming optimisation algorithm[20,21] is applied using the resulting fuzzy model for engine output and engine limits to reproduce the schedule at various flight conditions. The problem may be stated as having an objective function expressed as

$$\min \; TSFC(\mathbf{x}); \quad \mathbf{x} \in \mathrm{R}^2 \quad \ldots (9)$$

In this equation $\mathbf{x} = [FFR, A_N]$, and the optimality problem is solved subject to the steady state operating limits and with the thrust equal to or more than the demanded level. The solution is also bounded by the geometrical extremes. The resulting nozzle area schedules for SLS, maximum and minimum airflow flight conditions are given in Fig. 7 from idle to maximum available thrust for each case. Lower thrust is obtained by opening the nozzle to preserve fan surge margin limit. Thus the nozzle operates at its maximum area for low thrust levels and as the demand for thrust increases it closes. For the maximum airflow conditions, the nozzle operates near its maximum opening for most of the power range. This may be explained by high levels of engine pressures encountered at these flight conditions.

Figures 8 and 9 give the variations of thrust-TSFC and EPR-TIT, respectively. These two figures present the operating schedule at sea level static conditions for both the complete engine model and the fuzzy model. It is clear from these figures that the fuzzy model gives close results to that obtained from the non-linear model of the engine from which data points are collected producing a very representative fuzzy model for the engine. The time needed to evaluate the objective function and constraints from the fuzzy model is very

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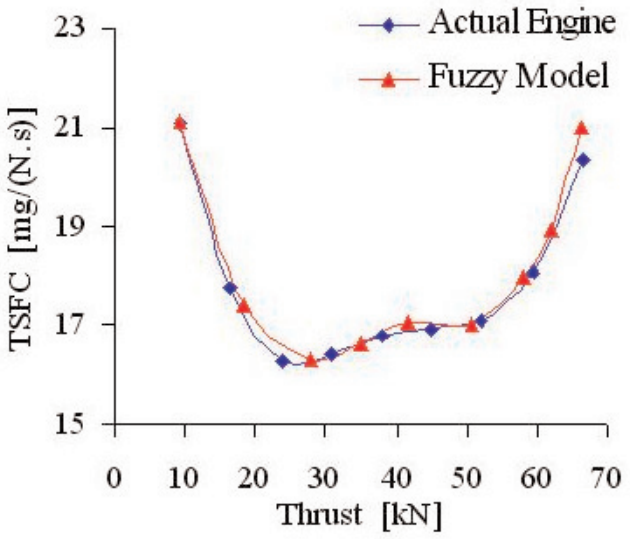


Figure 8. Thrust-TSFC variations for both actual engine and fuzzy model.

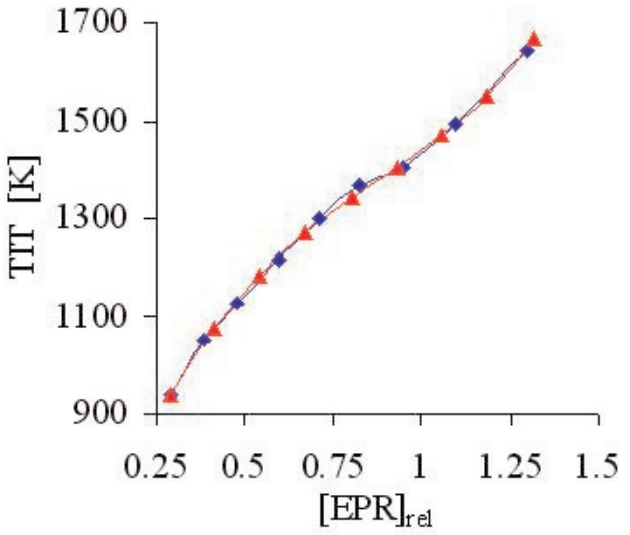


Figure 9. EPR-TIT variations for both actual engine and fuzzy model.

small compared with that needed for the complete non-linear model of the engine. This guarantees the real time requirements for online schedule optimisation. Moreover, the memory size for the fuzzy model is very small compared with that of the non-linear model.

5.2 Deteriorations in the compression system

The strong potential of the fuzzy performance seeking control (FPSC) algorithm lies on its adaptive nature, i.e. the engine fuzzy model can be changed to accurately represent variations caused by component deteriorations or other deviations from the ideal model by which the fuel and nozzle area schedules are built. It is assumed that the fan and high pressure compressor have deteriorated by 5% efficiencies. This level of deterioration is common for military turbofan engines and corresponds to moderate to severe degradation of the engine health[22]. Also, applying such high level of deterioration validates the high capabilities of the recommended fuzzy system in engine model identification and online optimisation of the schedule.

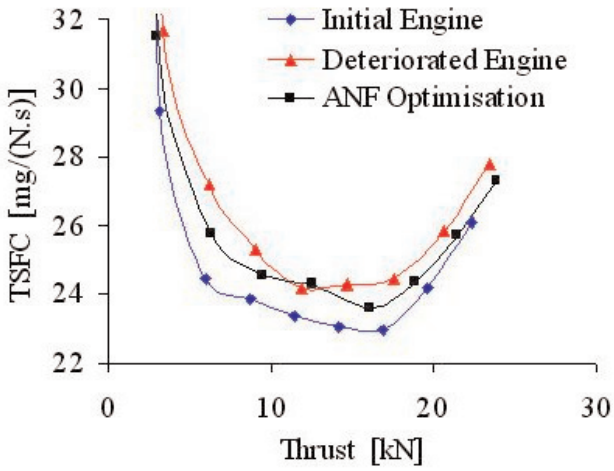


Figure 10. Thrust-TSFC variations at cruise flight conditions.

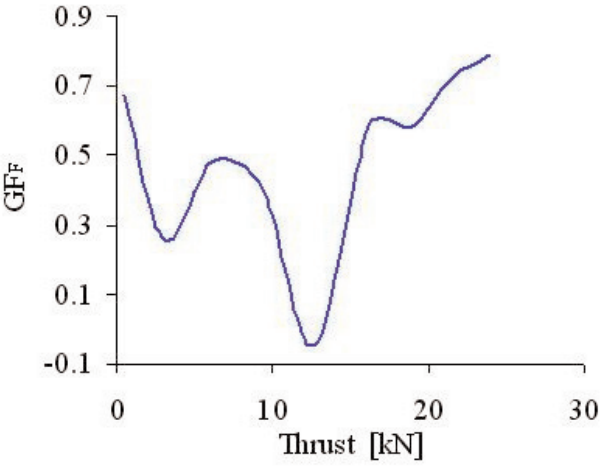


Figure 11. TSFC gain factor variation at cruise flight conditions.

Figure 10 gives a comparison between the thrust-TSFC variation at cruise flight conditions (M_O = 0·85 and H_O = 10km) for the initial engine, deteriorated engine operating with the initial schedule and with the adaptive neuro-fuzzy optimisation output. The curve resulting from the application of neuro-fuzzy model lies below that resulting from the application of the initial schedule on a deteriorated engine. To realise the gain of employing the recommended approach, a TSFC gain factor (TSFC *GF*) is defined as:

$$GF_F = \frac{TSFC_{\text{Deteriorated Engine}} - TSFC_{\text{ANF Optimisation}}}{TSFC_{\text{Deteriorated Engine}} - TSFC_{\text{Initial Engine}}} \quad \ldots (10)$$

The TSFC gain factor at various thrust levels for cruise flight conditions is plotted in Fig. 11. The gain fluctuates between –8% to 80% with an average of about 50 % for the entire thrust range at these flight conditions, i.e. the FPSC approach can save about 50% from the losses attained due to the use of fixed schedule on a deteriorated engine. The thrust-TIT variations are plotted in Fig. 12 for the above three cases at SLS conditions. It is obvious from this figure that the application of the initial schedule on the deteriorated engine results

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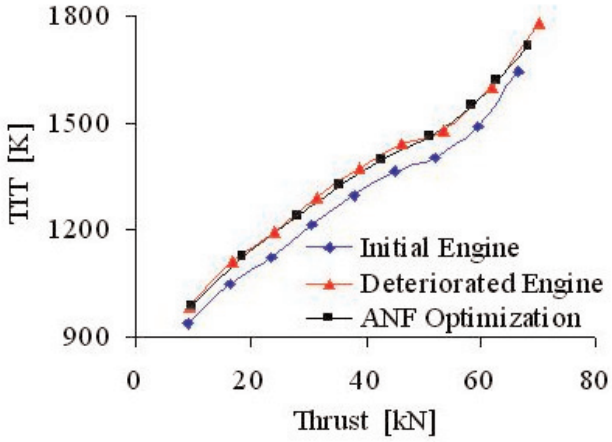


Figure 12. Thrust-TIT variations with at SLS conditions.

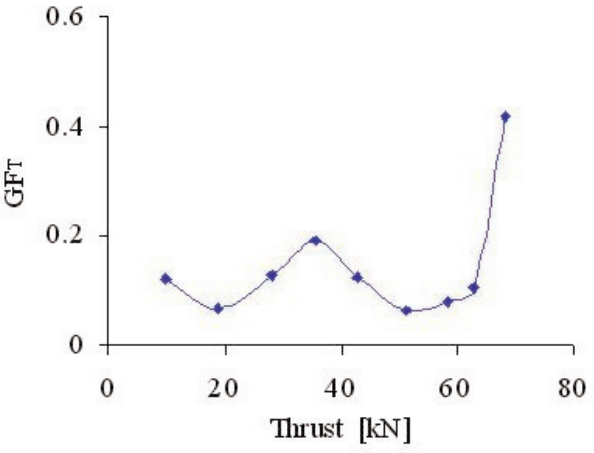


Figure 13. TIT gain factor variations at SLS conditions.

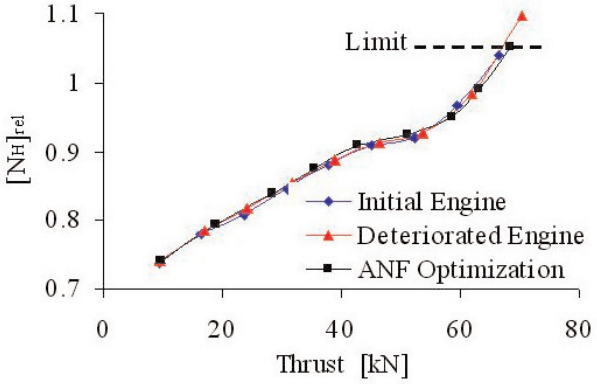


Figure 14. Thrust-N_H variations at SLS.

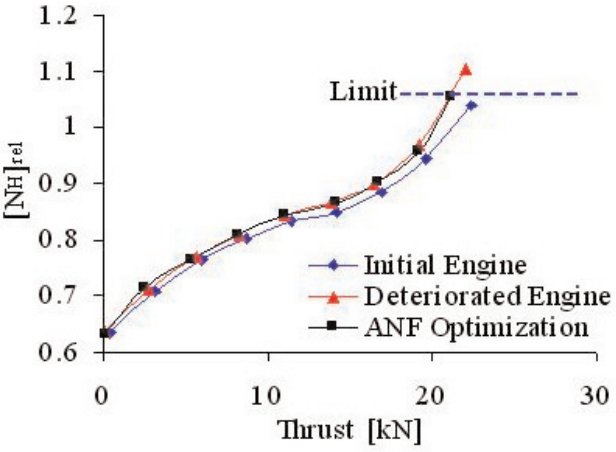


Figure 15. N_H limit exceeding at cruise flight conditions.

in higher temperature level. The optimisation using the updated fuzzy model reduces this temperature. Defining a TIT gain factor (GF_T) similar to that for TSFC:

$$GF_T = \frac{TIT_{\text{Deteriorated Engine}} - TIT_{\text{ANF Optimisation}}}{TIT_{\text{Deteriorated Engine}} - TIT_{\text{Initial Engine}}} \quad (11)$$

The resulting gain in temperature at SLS conditions is plotted in Fig. 13. This figure shows that reductions up to 40% from the temperature increase can be obtained especially at maximum thrust. At this thrust level, the reductions in TIT can reach more than 70°F. Based solely on temperature effects, Refs 5 and 9 claim that every 70°F (21°C) reduction in TIT doubles the turbine life if it was operating at maximum temperature for long periods.

Figure 14 presents the variations of the high pressure spool speed (N_H) with thrust for the same three cases at SLS conditions. The figure indicates that the application of the initial schedule on the deteriorated engine leads to limit exceeding for the high pressure spool speed parameter. A new safe schedule results from updating the fuzzy model and optimising the schedule at these conditions using real data from the deteriorated engine. Although, the fan surge margin doesn't suffer from limit exceeding, it is improved slightly when applying the fuzzy performance seeking control. The above benefits in both TSFC and TIT can be enhanced by equipping the engine with more variable geometry such as fan inlet guide vanes, and compressor stator variable vanes besides the nozzle area. These variations in engine geometry can result in wide range for the optimisation operation and consequently lead to more benefits in the optimised parameters.

5.3 Deteriorations in the turbines

Performance degradation resulting in drop of 5% in the two turbines efficiency is considered. The initial fuzzy model is updated using training data from the degraded engine. The updated fuzzy model is used to optimize the TSFC at cruise flight conditions. Benefits in TSFC and TIT such as those obtained with compression system performance deterioration are obtained. However, the application of the initial schedule on a deteriorated engine has very important effect on limit exceeding and can cause engine damage at some flight conditions especially at power lever angle extremes (idle and maximum dry). Figure 15 presents this effect on N_H at cruise flight conditions, and Fig. 16 declares it on fan surge margin limit at SLS idle conditions. It is obvious from these figures that the fuzzy performance seeking algorithm is capable of preserving the engine limits at various flight conditions.

However, the compressor surge margin is not affected by the deteriorations as it is clear from Fig. 17. It is evident that simultaneous deterioration of both the compression system and turbines results in the fuzzy performance seeking control algorithm becoming more valuable and vital for engine safety.

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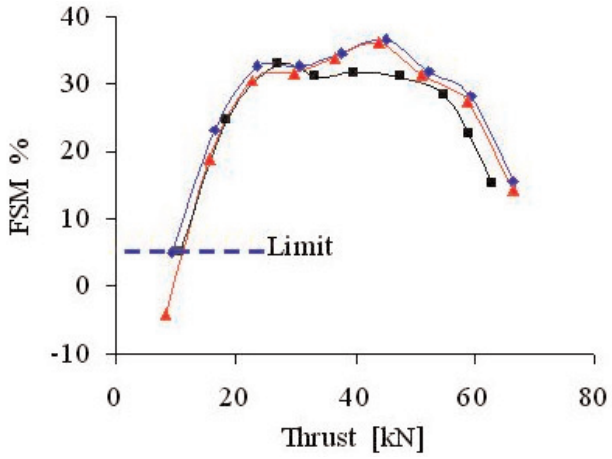


Figure 16. FSM limit exceeding at SLS conditions.

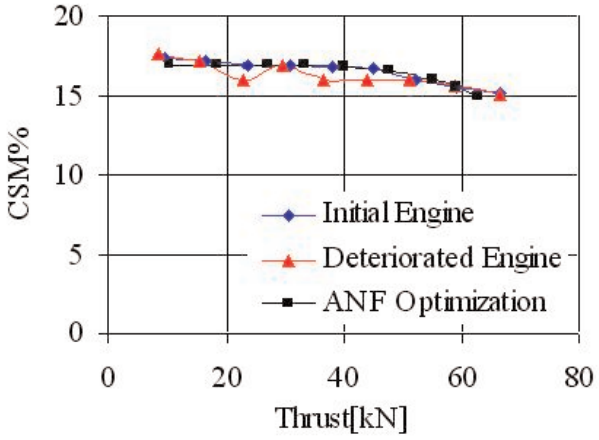


Figure 17. CSM limit at SLS conditions.

6.0 CONCLUSIONS

- A new algorithm is developed to update the steady state engine operating schedule online to enhance the performance and prevent limit exceeding. The neuro-fuzzy modelling technique is used through a comprehensive algorithm to accomplish this task. This algorithm doesn't need accurate engine model and can precisely represent the actual engine after variations in engine components.
- For a variable geometry turbofan engine used in military applications, the engine pressure ratio ($P_{t4.5}/P_{t2}$) is the most suitable measurable engine parameter to represent actual thrust during on line optimisation.
- Subtractive clustering technique is a simple and reliable method for generating the minimum number of fuzzy if-then rules that model the variations in turbofan engine parameters as function of the actuation parameters and flight conditions. The resulting fuzzy model is optimised through learning algorithms to accurately imitate the engine performance. Changes in the engine can be addressed through retraining the fuzzy model using true measurements.
- The implementation of online optimisation with the neuro-fuzzy model results in some benefits in both thrust specific fuel consumption and turbine inlet temperature. Also it avoids limit exceeding due to unrealistic modelling of the engine either because of model inaccuracy or engine deterioration.

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