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Alfvén waves in shear flows: Driven wave formalism

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Abstract. The driven wave formalism, as it was later applied to the solar coronal plasma processes, was first developed in our earlier work (Kaghashvili, E. Kh. 2007 Alfvén wave-driven compressional fluctuations in shear flows. *Phys. Plasmas* **14**, 44502) that presented the analytical solutions for the plasma density fluctuations. In the driven-wave formalism, we look for the short-term changes in the initial waveform due to the linear interaction of the initial natural mode of the system and the flow inhomogeneity. This formalism allows us to obtain the analytical solutions for the driven waves that are excited in the system. While a full set of driven wave solutions for magnetohydrodynamic variables in the cold plasma case were presented earlier (Kaghashvili, E. 2012c Driven wave-generated electric field in the solar corona. *J. Geophys. Res.* **117**, A10103, doi:10.1029/2012JA018120), the purpose of this paper is to remove the cold-plasma restriction and to present the formal solutions for the initial linearly polarized Alfvén wave-driven fluctuations.

1. Introduction

All magnetic waves can transport energy in plasma, but one of the three magnetohydrodynamic (MHD) modes, the Alfvén wave, is the most efficient one for transporting energy and momentum over long distances. The Alfvén wave is a transverse magnetic wave that does not disturb the plasma density and drive no compressions, which makes it unique among the MHD waves. Soon after its discovery (Alfvén 1942), Alfvén waves were observed in the laboratory (Lundquist 1949), Earth's magnetosphere, and solar wind (e.g. Coleman et al. 1960). Today, it is firmly established that Alfvén waves play an important role in various laboratory (e.g. Gekelman et al. 1997), geophysical (e.g. Kletzing 1994; Génot et al. 1999, 2004; Chaston et al. 2002, 2007; Lysak and Song 2006, 2008), solar, and astrophysical plasma processes (e.g. Parker 1955; Hollweg 1974; Heyvaerts and Priest 1983; Markovskii et al. 2009; McClements and Fletcher 2009).

Alfvén waves have long been thought to play an important role in the solar atmosphere for several reasons, including (a) *in-situ* observations that found their presence in the solar wind (Coleman 1966, 1967, 1968; Unti and Neugebauer 1968; Belcher et al. 1969; Belcher and Davis 1971; Wang et al. 2012), (b) the non-compressive nature of Alfvén waves that allows them to transport available solar photospheric motion energy effectively without refraction and internal reflection (Hollweg 1978), (c) Alfvén waves can exert a force and influence the solar wind large-scale dynamics (Alazraki and Couturier 1971; Hollweg 1973a, b, 1978; Jacques 1978; Usmanov et al. 2011; Hansteen and Velli 2012), and (d) Alfvénic waves with periods of several minutes and higher have been reported recently in the chromosphere, transition region, and solar corona (De Pontieu et al. 2007; Tomczyk et al. 2010; Jess et al. 2009; Gupta et al. 2010; McIntosh et al. 2011). However, in the solar atmosphere, these same features, which make the Alfvén wave unique and special among MHD waves, also make it difficult for the Alfvén wave to dissipate where the energy deposition is needed most.

One of the ways to dissipate the Alfvén wave is to couple this mode into other MHD modes, namely into the slow or fast magnetosonic modes, both of which are compressive and, when the dissipative processes are considered, have the shorter dissipation times compared to the Alfvén wave (e.g. Barnes 1968; Hung and Barnes 1973, Porter et al. 1994). In our earlier work, we investigated the possible role of then newly discovered velocity shear-induced mode coupling (Chagelishvili et al. 1996, 1997) in the Alfvén wave transformation to the fast magnetosonic wave in the solar corona and the solar wind (Kaghashvili 1999a). Since then, a number of authors have also applied the same mathematical formalism to various solar wave phenomena occurring in the solar atmosphere (e.g. Poedts et al. 1998; Kaghashvili 1999b, Li et al. 2006; Shergelashvili et al. 2006; Gogoberidze et al. 2007; Camporeale et al. 2010).

The works cited above and the wave coupling studies (e.g. Melrose 1977a, b, Wentzel 1989; Chagelishvili et al. 1996, 1997; Poedts et al. 1998; Mahajan and Rogava 1999; Rogava et al. 2000; Gogoberidze et al. 2004; Rogava and Gogoberidze 2005; Webb et al. 2005a, b;

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Shergelashvili et al. 2006; Gogoberidze et al. 2007; Webb et al. 2007) describe a typical physical situation:

- The initial input into the system was specified to be a single natural frequency mode of the system.
- The objective was to partially or completely convert the initial mode into other MHD modes.
- The analytical solutions or the numerical simulations were analyzed using Wentzel–Kramers–Brillouin waves (WKB waves, which assume gradual changes in the wave phase) that were valid for long times.

However, it was shown (Kaghashvili 2002) that two separate wave modes need to begin almost in phase in order to transform efficiently from one MHD mode to another. Otherwise, for the typical Alfvénic spectrum of the lower solar atmosphere, the time-scale of the transformation appears to be large enough for waves to escape the solar corona before the coupling process can occur.

Recently, we considered another approach to the same problem. Instead of looking for how the initial natural frequency mode of the system partitions its energy into other MHD modes, we focused on how the initial wave evolves and propagates in the plasma with inhomogeneous flow. The goal was to understand and qualitatively describe fluctuations that are excited in the system by the velocity shear and that modify the initial waveform. Thus, in this driven wave formalism (Kaghashvili et al. 2006, 2009, Kaghashvili 2007, Hollweg et al. 2009; Hollweg and Kaghashvili 2012; Kaghashvili 2012a, b, c), we examined the short-term evolution of the system and changes in the initial waveform due to the direct driven non-WKB waves. These waves are excited by the linear interaction of a single mode with the background flow inhomogeneity even when the natural frequencies of the system are well separated. The general formalism for the driven waves, as presented in this paper, was first developed in Kaghashvili (2007).

In our previous work, we presented a full set of solutions for linear MHD waves in the cold plasma case (Kaghashvili 2012c). Hollweg and Kaghashvili (2012, see also Kaghashvili et al. 2009) presented the analytical solutions for the density and longitudinal magnetic field fluctuations without the cold plasma restriction. The goal of this work is to present the complete set of formal solutions for the initial linearly polarized Alfvén wave-driven fluctuations.

2. Linearly polarized Alfvén wave-driven fluctuations

Following Kaghashvili (2007), the background plasma is homogeneous and given by $\rho_0 = const.$, $\mathbf{B}_0 = (\mathrm{B}_0, 0, 0)$, and $\mathrm{P}_0 = const.$, where ρ, B, and P denote the density, magnetic field, and plasma pressure. Consider the simple case of the homogeneous magnetic field-aligned flow with a linear cross-field shear in one of the perpendicular directions:

$$\mathbf{V}_0 = (S_y y,\, 0,\, 0), \tag{1}$$

where $S_y = const.$ characterizes the shearing of the velocity field parallel to the y-axis. Due to the inhomogeneous background flow terms in the linearized MHD equations, no straightforward Fourier transformation is possible. A typical approach is to change the variables from laboratory to convecting Lagrangian coordinates (e.g. Goldreich and Lynden-Bell 1965; Koppel 1965; Phillips 1966; Craik and Criminale 1986; Cooper 1988; Balbus and Hawley 1992; Chagelishvili et al. 1996, 1997; Rogava et al. 1996, 2000, 2003; Mahajan and Rogava 1999; Bodo et al. 2001; Gogoberidze et al. 2004; Webb et al. 2005a, b; Li et al. 2006; Rogava and Gogoberidze 2007; Webb et al. 2007; Bakas 2009; Camporeale 2011), which converts the spatial inhomogeneity associated with the velocity shear into a temporal one. Alternatively, we can perform a spatial Fourier expansion of the fluctuating quantities, taking refraction into account, and study the temporal behavior of the spatial Fourier harmonics; thus, all fluctuating quantities, such as density, vary as $\rho(t) \times \exp(ik_x x + iK_y y + ik_z z)$, where k_x and k_z are constants and $K_y = k_y(t = 0) - k_x S_y t$, which changes according to geometric optics. (Since this is not a standard Fourier analysis in space and time, the approach is referred to as 'non-modal'; see Camporeale (2012) for a recent review.)

In our treatment, we are interested in the short-term evolution of the system (typically equal to several Alfvén time-scales), and the goal is to obtain analytical solutions for the flow-shear-driven fluctuations at early times before nonlinear processes take over. For the short-term evolution, the typical step is to take to $K_\mathrm{y} \cong k_{\mathrm{y}0} = const.$ A more accurate analytical solution for the driven wave evolution is obtained when the along-the-shear wave vector is 'frozen' in the system of the second-order ordinary differential equations. Then our short-term evolution assumption translates as an assumption that during the time-scale of interest no significant changes occur in the natural frequencies of the system, i.e. $\omega_i^2(t) \cong \omega_i^2(t = 0) = const.$, where i denotes the specific natural frequency that characterizes the plasma. Following this procedure, the system of equations governing the linear fluctuations now reads as follows:

$$\frac{\partial^2 \mathrm{D}}{\partial \mathrm{t}^2} + k^2 c_s^2 \mathrm{D} + k^2 v_a^2 \mathrm{b_x} = 2ik_x v_a S_y \mathrm{v_y} \tag{2}$$

$$\frac{\partial^2 \mathrm{b}_x}{\partial \mathrm{t}^2} + k^2 v_a^2 \mathrm{b_x} + k_\perp^2 c_s^2 \mathrm{D} = 2ik_x v_a S_y \mathrm{v_y}, \tag{3}$$

$$\frac{\partial^2 \mathrm{v_y}}{\partial \mathrm{t}^2} + k_x^2 v_a^2 \mathrm{v_y} = ic_s^2 \left(k_x S_y \mathrm{D} - K_y \frac{\partial \mathrm{D}}{\partial \mathrm{t}} \right) + iv_a^2 \left(k_x S_y \mathrm{b}_x - K_y \frac{\partial \mathrm{b_x}}{\partial \mathrm{t}} \right), \tag{4}$$

$$\frac{\partial^2 \mathrm{v_z}}{\partial \mathrm{t}^2} + k_x^2 v_\mathrm{a}^2 \mathrm{v_z} = -ic_s^2 k_z \frac{\partial \mathrm{D}}{\partial \mathrm{t}} - iv_a^2 k_z \frac{\partial \mathrm{b_x}}{\partial \mathrm{t}}, \tag{5}$$

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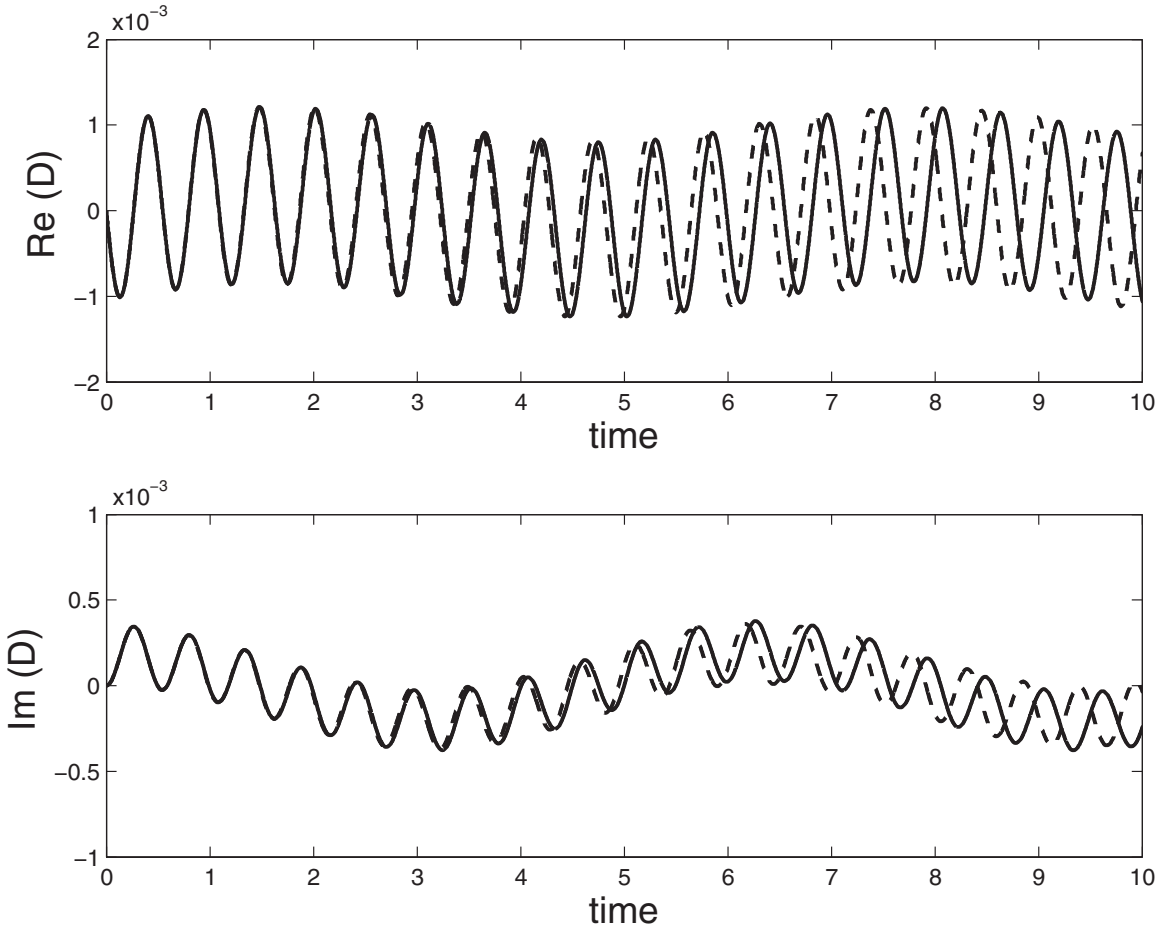


Figure 1. Comparison of analytical solutions for $\mathrm{D}(\boldsymbol{r},t)$ given by (12) at $\boldsymbol{r}=0$ with exact numerical solutions. The solid lines represent the numerical solutions and the dashed line indicates our analytical solutions, respectively. Time is normalized to 2×10^4 km. Plasma and wave parameters are taken from Hollweg and Kaghashvili (2012) as follows: $v_a = 800\,\mathrm{kms}^{-1}$, $c_s = 200\,\mathrm{kms}^{-1}$, $k_x = 2\pi/(1.6\times10^5\,\mathrm{km})$, $k_{y0}/k_x = k_z/k_x = 8$, $v_{y0} = 120\,\mathrm{kms}^{-1}$, and S_y, we take a flow difference of $50\,\mathrm{kms}^{-1}$ over 2×10^4 km. The exact and analytical solutions are in good agreement at early times and deviate from each other at later times.

$$\frac{\partial^2 \mathrm{b}_{\{\mathrm{y,z}\}}}{\partial \mathrm{t}^2} + k_x^2 v_a^2 \mathrm{b}_{\{\mathrm{y,z}\}} = k_x\{K_y, k_z\}\left(c_s^2\mathrm{D} + v_a^2\mathrm{b_x}\right), \quad (6)$$

$$\frac{\partial^2 \mathrm{v_x}}{\partial \mathrm{t}^2} + k_x^2 c_s^2 \mathrm{v_x} + k_x c_s^2 (K_y \mathrm{v_y} + k_z \mathrm{v_z}) = iS_y K_y\left(c_s^2\mathrm{D} + v_a^2\mathrm{b_x}\right) - iS_y k_x v_a^2 \mathrm{b_y}, \quad (7)$$

where $k_\perp^2 = K_y^2 + k_z^2$ and $k^2 = k_x^2 + k_\perp^2$ are the magnitudes of the perpendicular and total wave vectors, respectively. A closed MHD system consists of the normalized density, $\mathrm{D}\,(\equiv \rho/\rho_0)$, the plasma velocity, $\mathbf{v} = (\mathrm{v_x}, \mathrm{v_y}, \mathrm{v_z})$, and the normalized magnetic field, $\mathbf{b} = (\mathrm{b_x}, \mathrm{b_y}, \mathrm{b_z})$ fluctuations, respectively. The parameters c_s and v_a are the characteristic local sound and Alfvén speeds of the plasma, respectively. The above-mentioned ordinary differential equations are obtained from the linear first-order differential MHD wave equations describing the wave processes in the shear flows (e.g. Kaghashvili 2007).

In the absence of the background inhomogeneous flow, the above equations lead to the ordinary dispersion relation for MHD waves:

$$\omega_a^2 = k_x^2 v_a^2, \quad (8)$$

$$\omega_{s,f}^2 = \frac{k^2}{2}\left(v_a^2 + c_s^2\right)\left[1 \mp \sqrt{1 - \frac{4k_x^2 c_s^2 v_a^2}{k^2\left(v_a^2 + c_s^2\right)^2}}\right], \quad (9)$$

where subscripts a, s, and f denote the Alfvén, slow, and fast magnetosonic waves, respectively.

Following Kaghashvili (2007, see also Kaghashvili et al. 2009, Hollweg and Kaghashvili 2012), we take $Ky \cong k_{y0}$ in the above equations. The solutions for all variables in the given system of the differential equation can be written in the Laplace space. As an example, the solution for the density fluctuations in the Laplace space is given by

$$L[\mathrm{D}] = -\frac{s^2(s - i\omega_a)^2 S_y \mathrm{v_{y0}}}{v_a\left[\left(s^2+\omega_a^2\right)\left(s^2+\omega_a^2\right)\left(s^2+\omega_f^2\right) + 2\left(s^2 v_a^2 + c_s^2\left[\left(s^2+\omega_a^2\right)\right]\right)\left(k_x^2 S_y^2 - k_x k_y s S_y\right)\right]}, \quad (10)$$

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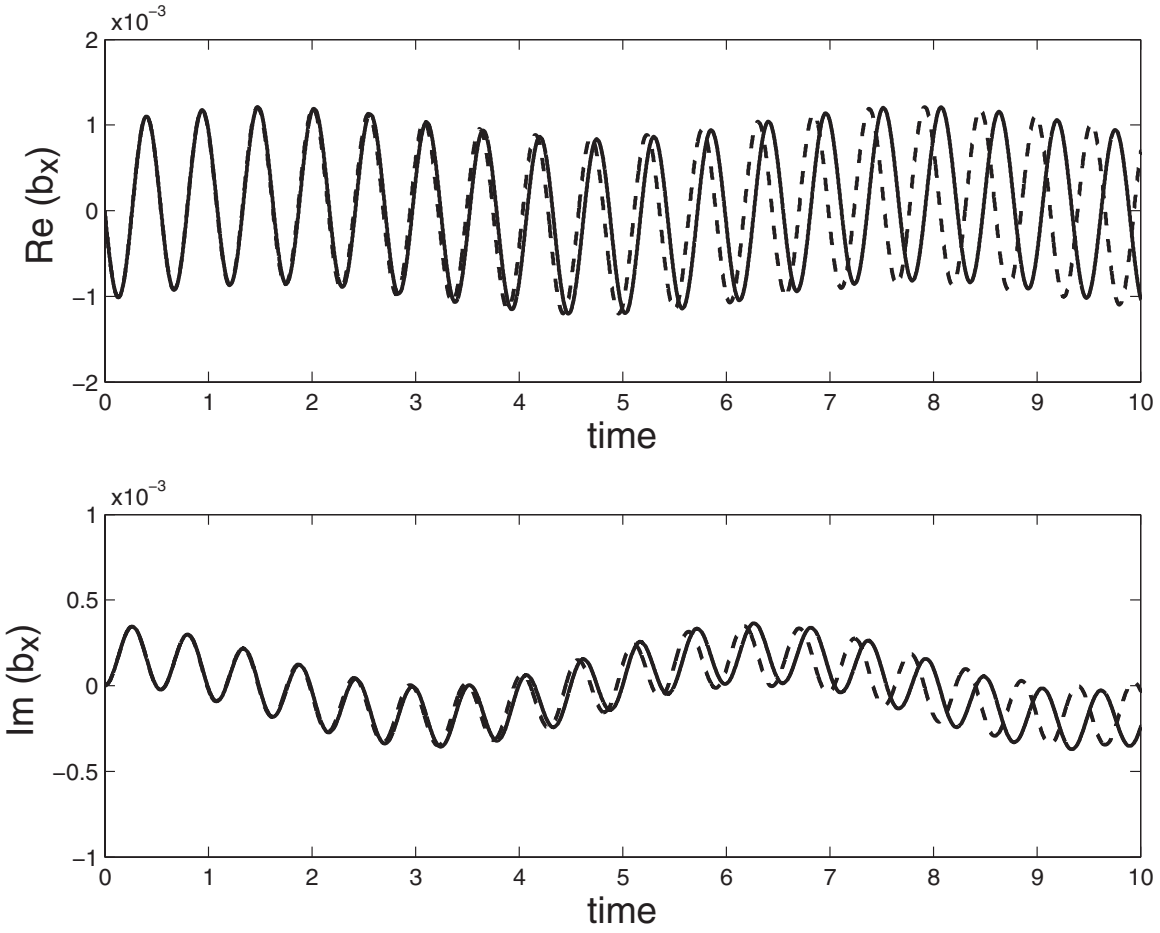


Figure 2. Same as Figure 1 but for the real and imaginary parts of $b_x(\boldsymbol{r}=0,t)$.

where v_{y0} is the initial linearly polarized Alfvén wave's velocity fluctuation along the velocity shear at $t=0$. Since we are taking the shear parameter S_y to be small, we expand the solution in Laplace space as a series in terms of this parameter:

$$L[\mathrm{D}] = -\frac{s^2(s-i\omega_a)S_y \mathrm{v}_{y0}}{v_a(s+i\omega_a)\left(s^2+\omega_s^2\right)\left(s^2+\omega_f^2\right)} + O\left(S_y^2\right). \tag{11}$$

In the final time-and-space-dependent solution, we keep only terms independent or linear to the shear parameter. We also apply one more restrictive assumption that the plasma conditions and the initial Alfvén wave characteristics are such that the three characteristic frequencies of the plasma are well separated. After applying these steps, the density fluctuations driven by the initial linearly polarized Alfvén wave is given by

$$\begin{aligned}\mathrm{D}(\boldsymbol{r},t) = &-\frac{4\omega_a^4 C_0}{\left(\omega_a^2-\omega_f^2\right)\left(\omega_a^2-\omega_s^2\right)}\exp(i\boldsymbol{kr}-i\omega_a t)\\ &-\frac{\omega_a\omega_f(\omega_a-\omega_f)C_0}{(\omega_a+\omega_f)\left(\omega_f^2-\omega_s^2\right)}\exp(i\boldsymbol{kr}+i\omega_f t)\\ &-\frac{\omega_a\omega_f(\omega_f+\omega_a)C_0}{(\omega_f-\omega_a)\left(\omega_f^2-\omega_s^2\right)}\exp(i\boldsymbol{kr}-i\omega_f t)\\ &-\frac{\omega_s\omega_a(\omega_a-\omega_s)C_0}{(\omega_a+\omega_s)\left(\omega_s^2-\omega_f^2\right)}\exp(i\boldsymbol{kr}+i\omega_s t)\\ &-\frac{\omega_s\omega_a(\omega_a+\omega_s)C_0}{(\omega_s-\omega_a)\left(\omega_s^2-\omega_f^2\right)}\exp(i\boldsymbol{kr}-i\omega_s t),\end{aligned} \tag{12}$$

where $C_0 = iS_y \mathrm{v}_{y0}/(2\omega_a v_a)$. This solution, with an initial condition of $\mathrm{v}_x(t=0) = -iS_y \mathrm{v}_{y0}/k_x v_a^2$, was first obtained in Hollweg and Kaghashvili (2012). Note that such an initial condition excludes the appearance of the linearly growing density fluctuations with time in $\beta=0$ plasma, where β is the ratio of the plasma pressure to the magnetic pressure. The possibility of such linearly growing solutions was shown in Falle and Hartquist (2002, see also, Webb et al. 2007; Kaghashvili et al. 2009).

Using the above-described procedure, we can calculate the solutions of all MHD variables described above. These solutions are given by (13)–(16):

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$$\mathrm{b_x}(\boldsymbol{r},t) = -\frac{4\omega_a^4(1-\beta)C_0}{\left(\omega_a^2-\omega_f^2\right)\left(\omega_a^2-\omega_s^2\right)}\exp(i\boldsymbol{kr}-i\omega_a t) + \frac{\omega_a(\omega_f+\omega_a)\left(\beta\omega_a^2-\omega_f^2\right)C_0}{\omega_f\left(\omega_f-\omega_a\right)\left(\omega_f^2-\omega_s^2\right)}\exp(i\boldsymbol{kr}-i\omega_f t)$$

$$+\frac{\omega_a(\omega_f-\omega_a)\left(\omega_f^2-\beta\omega_a^2\right)C_0}{\omega_f(\omega_f+\omega_a)\left(\omega_f^2-\omega_s^2\right)}\exp(i\boldsymbol{kr}+i\omega_f t) + \frac{\omega_a(\omega_s+\omega_a)\left(\beta\omega_a^2-\omega_s^2\right)C_0}{\omega_s(\omega_s-\omega_a)\left(\omega_s^2-\omega_f^2\right)}\exp(i\boldsymbol{kr}-i\omega_s t)$$

$$+\frac{\omega_a(\omega_s-\omega_a)\left(\omega_s^2-\beta\omega_a^2\right)C_0}{\omega_s(\omega_s+\omega_a)\left(\omega_s^2-\omega_f^2\right)}\exp(i\boldsymbol{kr}+i\omega_s t), \tag{13}$$

$$\mathrm{v_{\{y,z\}}}(\boldsymbol{r},t) = \mathrm{v_{\{y,z\}0}}\exp(i\boldsymbol{kr}-i\omega_a t) - \frac{k_{\{y,z\}}C_0}{k_x}v_a\left\{\frac{\omega_a^2\left(\omega_f^2+\beta\left[\omega_f^2-\omega_a^2\right]\right)}{\left(\omega_a-\omega_f\right)^2\left(\omega_f^2-\omega_s^2\right)}\exp(i\boldsymbol{kr}-i\omega_f t)\right.$$

$$+\frac{\omega_a^2\left(\omega_f^2+\beta\left[\omega_f^2-\omega_a^2\right]\right)}{(\omega_a+\omega_f)^2\left(\omega_f^2-\omega_s^2\right)}\exp(i\boldsymbol{kr}+i\omega_f t) + \frac{\omega_a^2\left(\omega_s^2+\beta\left[\omega_s^2-\omega_a^2\right]\right)}{(\omega_a-\omega_s)^2\left(\omega_s^2-\omega_f^2\right)}\exp(i\boldsymbol{kr}-i\omega_s t)$$

$$+\frac{\omega_a^2\left(\omega_s^2+\beta\left[\omega_s^2-\omega_a^2\right]\right)}{(\omega_a+\omega_s)^2\left(\omega_s^2-\omega_f^2\right)}\exp(i\boldsymbol{kr}+i\omega_s t) - \frac{2i\omega_a^5 t}{\left(\omega_a^2-\omega_f^2\right)\left(\omega_a^2-\omega_s^2\right)}\exp(i\boldsymbol{kr}-i\omega_a t)$$

$$\left.+\frac{2\omega_a^4\left(2\beta\left[\omega_a^2-\omega_f^2\right]\left[\omega_a^2-\omega_s^2\right]-\left(\omega_a^4-3\omega_f^2\omega_s^2+\omega_a^2\left[\omega_f^2+\omega_s^2\right]\right)\right)}{\left(\omega_a^2-\omega_f^2\right)^2\left(\omega_a^2-\omega_s^2\right)^2}\exp(i\boldsymbol{kr}-i\omega_a t)\right\}, \tag{14}$$

$$\mathrm{b_{\{y,z\}}}(\boldsymbol{r},t) = \mathrm{b_{\{y,z\}0}}\exp(i\boldsymbol{kr}-i\omega_a t) + \frac{k_{\{y,z\}}C_0}{k_x}\left\{\frac{\omega_a^3\left(\omega_f^2+\beta\left[\omega_f^2-\omega_a^2\right]\right)}{\omega_f(\omega_a-\omega_f)^2\left(\omega_f^2-\omega_s^2\right)}\exp(i\boldsymbol{kr}-i\omega_f t)\right.$$

$$-\frac{\omega_a^3\left(\omega_f^2+\beta\left[\omega_f^2-\omega_a^2\right]\right)}{\omega_f(\omega_a+\omega_f)^2\left(\omega_f^2-\omega_s^2\right)}\exp(i\boldsymbol{kr}+i\omega_f t) + \frac{\omega_a^3\left(\omega_s^2+\beta\left[\omega_s^2-\omega_a^2\right]\right)}{\omega_s(\omega_a-\omega_s)^2\left(\omega_s^2-\omega_f^2\right)}\exp(i\boldsymbol{kr}-i\omega_s t)$$

$$-\frac{\omega_a^3\left(\omega_s^2+\beta\left[\omega_s^2-\omega_a^2\right]\right)}{\omega_s(\omega_a+\omega_s)^2\left(\omega_s^2-\omega_f^2\right)}\exp(i\boldsymbol{kr}+i\omega_s t) - \frac{2i\omega_a^5 t}{\left(\omega_a^2-\omega_f^2\right)\left(\omega_a^2-\omega_s^2\right)}\exp(i\boldsymbol{kr}-i\omega_a t)$$

$$\left.+\frac{4\omega_a^4\left(\beta\left[\omega_a^2-\omega_f^2\right]\left[\omega_a^2-\omega_s^2\right]+\omega_f^2\omega_s^2-\omega_a^4\right)}{\left(\omega_a^2-\omega_f^2\right)^2\left(\omega_a^2-\omega_s^2\right)^2}\exp(i\boldsymbol{kr}-i\omega_a t)\right\}, \tag{15}$$

$$\mathrm{v_x}(\boldsymbol{r},t) = -C_0 v_a\left\{\frac{2\omega_a^2\left(2\beta\omega_a^2-k_\perp^2 v_a^2\right)}{\left(\omega_a^2-\omega_f^2\right)\left(\omega_a^2-\omega_s^2\right)}\exp(i\boldsymbol{kr}-i\omega_a t) + \frac{\omega_f^4-\left(k^2v_a^2+k_\perp^2c_s^2\right)\omega_f^2+\omega_a^2\left(k^2c_s^2+\beta\omega_a\omega_f\right)}{\omega_f(\omega_f-\omega_a)\left(\omega_f^2-\omega_s^2\right)}\exp(i\boldsymbol{kr}-i\omega_f t)\right.$$

$$+\frac{\omega_f^4-\left(k^2v_a^2+k_\perp^2c_s^2\right)\omega_f^2+\omega_a^2\left(k^2c_s^2-\beta\omega_a\omega_f\right)}{\omega_f(\omega_f+\omega_a)\left(\omega_f^2-\omega_s^2\right)}\exp(i\boldsymbol{kr}+i\omega_f t)$$

$$+\frac{\omega_s^4-\left(k^2v_a^2+k_\perp^2c_s^2\right)\omega_s^2+\omega_a^2\left(k^2c_s^2+\beta\omega_a\omega_s\right)}{\omega_s(\omega_s-\omega_a)\left(\omega_s^2-\omega_f^2\right)}\exp(i\boldsymbol{kr}-i\omega_s t)$$

$$\left.+\frac{\omega_s^4-\left(k^2v_a^2+k_\perp^2c_s^2\right)\omega_s^2+\omega_a^2\left(k^2c_s^2-\beta\omega_a\omega_s\right)}{\omega_s(\omega_s+\omega_a)\left(\omega_s^2-\omega_f^2\right)}\exp(i\boldsymbol{kr}+i\omega_s t)\right\}, \tag{16}$$

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where $\beta = c_s^2/v_a^2$. As seen, the general solutions are written as a sum of the initial Alfvén wave and driven wave components correspondingly. Fluctuations with '0' subscript denote the amplitudes of the initial Alfvén wave. For the longitudinal velocity and magnetic field fluctuations, the shear parameter-independent components are zero. Figures 1 and 2 compare our analytical solutions for $D(\boldsymbol{r} = 0, t)$ and $b_x(\boldsymbol{r} = 0, t)$ with numerical solutions. Both the real and imaginary parts are plotted and compared; the solid lines represent the numerical solutions and the dashed line indicates our analytical solutions, respectively. The exact and analytical solutions are in good agreement at early times and deviate from each other at later times.

3. Summary

Our straightforward way of solving fluctuating components involving an initial Alfvén wave does not eliminate secular terms, which limits the time interval over which the solutions are valid. These secular terms can be effectively removed by expanding the Laplace parameter 's' in terms of the shear parameter S_y, and then keeping the terms to the required order. This procedure was introduced for the cold plasma case recently by Hollweg et al. (accepted). This procedure significantly improves also the time interval for both $D(\boldsymbol{r} = 0, t)$ and $b_x(\boldsymbol{r} = 0, t)$ when the analytical solutions and numerical solutions are exact.

Note that all the driven wave solutions are shear parameter-dependent. Despite the fact that driven waves are implicitly present in the above linear equations, they were not elaborated in the earlier shear flow studies that used the commonly accepted wave coupling procedure. In the driven wave formalism, by formulating the linear wave and inhomogeneous background flow interaction problem differently, one is able to capture and qualitatively describe complex changes in the initial waveform.

To summarize, we have shown how the linearly polarized Alfvén wave propagating in the cross-field background shear flow can generate the driven waves and provided the solutions for all MHD fluctuating components. We argue that when the shear parameter is small, the obtained analytical solutions adequately describe driven waves for the short times before more powerful nonlinear processes take over.

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