Feedback

On 100.02: Paul Stephenson writes: Pawel Jan Piskorz shows how to use properties of the familiar multiplication table to find an expression for the nth pyramidal number. The purpose of this note is to show how tetrahedral, pyramidal and octahedral numbers can be derived from the table by virtue of the relations (1) and (2) below. Let Tet_n be the sum of the first n triangular numbers, Pyr_n the sum of the first n square numbers, Oct_n the number represented by an octahedron whose principal section is the nth square. Then

$$Tet_{n-1} + Tet_n = Pyr_n, \tag{1}$$

$$Pyr_{n-1} + Pyr_n = Oct_n. \tag{2}$$

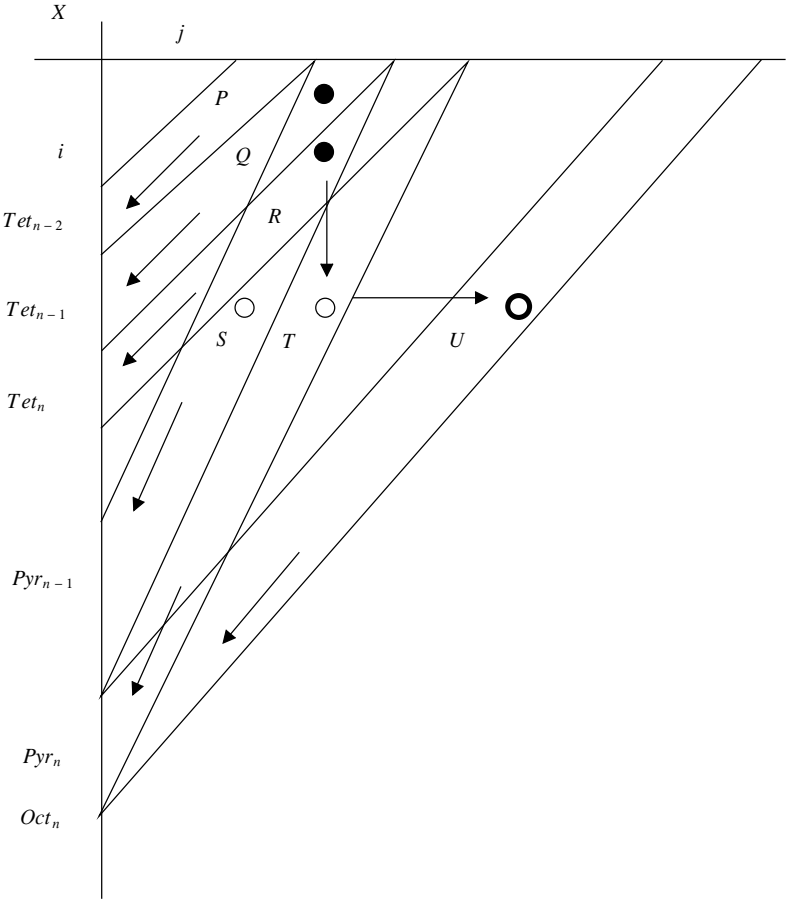


FIGURE 1

Figure 1 shows the multiplication table schematically. (As readers appreciate, the figure could be reflected in the main diagonal.)

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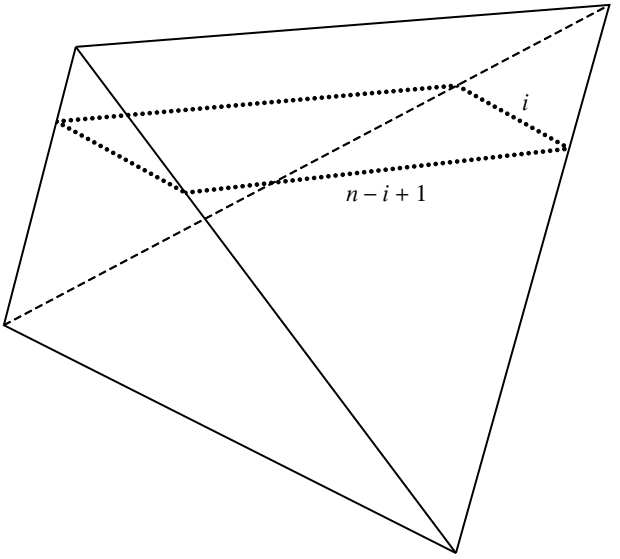


FIGURE 2

Dissecting a tetrahedron parallel to a pair of opposite edges as opposed to a face (Figure 2),

$$Tet_n = \frac{1}{2}\sum_{i=1}^{n} i\,(i+1) = \sum_{i=1}^{n} i\,(n-i+1),$$

we have products which fall on a 45° diagonal stripe. Adding the entries down stripes P, Q, R, we obtain the consecutive tetrahedral numbers shown on the left of the table.

Adding entries vertically, the sums form steeper diagonals. In Figure 1, sample entries on Q and R (filled circles) are shown to produce a sum on T (open circle). In accordance with (1), adding the entries down stripes S and T, we have sums which are consecutive pyramidal numbers.

We now add entries on S and T horizontally (open circles) to produce sums on a new 45° stripe, U (half-filled circle). In accordance with (2), adding the entries down this stripe produces an octahedral number. The entries on T occupy alternate cells so that we have the sequence of products corresponding to the terms in this sum:

$$Oct_n = \sum_{i=1}^{n} (2i-1)(2n-2i+1).$$

In figurate terms, this corresponds to lining up the corners of the squares in the octahedron and dissecting the solid into L-shaped prisms.

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