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Kinematic coupling complexity of heavy-payload forging manipulator

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SUMMARY
There exist several kinds of definitions for complexity in different areas. In mechanical engineering area, some formulations capable of measuring the complexity of robotic architectures have been proposed at the conceptual-design stage. This paper presents the definition of Kinematic Coupling Complexity to measure the kinematic coupling degree of a manipulator based on the input/output velocity matrix. In order to distinguish the differences among many matrixes, three factors will be considered including the proportion of nonzero elements in matrix J, the difference of nonzero elements' numbers in every row among many matrixes, and the difference of nonzero elements' positions among many matrixes. To illustrate the methods introduced in this paper, four types of heavy-payload forging manipulators are analyzed and the type of forging manipulator with the smallest Kinematic Coupling Complexity is chosen as our prototype.

KEYWORDS: Kinematic coupling complexity; Forging manipulator; Input/output velocity relationship.

1. Introduction

The definition of Complexity has been used in many different areas. Shannon[1] equated Complexity with the amount of information to describe an object or a message. Cover and Thomas[2] defined the Complexity of process to measure how much computing it takes to solve a problem. Suh[3] associated Complexity with how much effort it takes to manufacture a product. In axiomatic approach,[4,5] design is modeled as a mapping process between a set of Functional Requirements (FRs) in the functional domain and a set of Design Parameters (DPs) in the physical domain, and then, Complexity is defined as a measure of uncertainty in achieving the specified FRs. In the mechanical engineering field, Khan *et al.*[6] proposed a formulation capable of measuring Complexity of robotic architectures at the conceptual-design stage with joint-number complexity, loop complexity, joint-type complexity, link diversity, actuator-type complexity, and actuator diversity considered. Caro *et al.*[7] used the Complexity proposed by Khan to organize the topologies of serial Schönflies-motion generators in an ascending order.

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In this paper, the definition of Kinematic Coupling Complexity (*KCC*) will be proposed to evaluate the kinematic coupling degree of a manipulator. Jin and Yang[8] classified the kinematic decoupling into two types, i.e., complete decoupling (each independent output is controlled by only one input so the matrix is diagonal) and partial decoupling (neither complete decoupling nor complete coupling). However, the definition of partial decoupling is too broad and almost all parallel manipulators can be classified in this category. Then, Jin *et al.*[9] proposed a new classification method and added Group decoupling, furthermore, they showed some different types of Group decoupling. Jin and Yang[10] put forward the concept of Complexity for coupling degree by considering every single-opened-chain (SOC).

This paper will give the definition of *KCC* to measure the kinematic coupling degree based on the input–output velocity matrix J. *KCC* is different from other indices mentioned in many papers because it is a specific evaluation for the kinematic coupling degree. In order to distinguish the difference among many matrixes, the following three aspects will be considered:

- KCC_s is the proportion of nonzero elements in matrix J.
- KCC_e is the difference of nonzero elements' numbers in every row among many matrixes.
- KCC_d is the difference of nonzero elements' positions among many matrixes.

The rest of this paper is organized as follows: In Section 2, the definition of *KCC* is described in general terms. Next, Section 3 illustrates the definition by using a type of forging manipulator as an example and calculates its *KCC*. Then, in Section 4, another three types of forging manipulators are introduced and their *KCC* are calculated, respectively. According to these analyses, the best candidate out of the four types is chosen as our prototype. Finally, Section 5 presents some conclusions.

2. The Definition of *KCC*

KCC will be defined as a specific value to measure the kinematic coupling degree based on the input–output velocity matrix J. In order to include all details about the difference among many matrixes, this section will take four steps to define *KCC*.

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2.1. KCC_s: The proportion of nonzero elements in matrix J

There are two input–output velocity matrixes J_a and J_b in Eq. (1):

$$\begin{bmatrix}\dot{M}_1\\ \dot{M}_2\\ \dot{M}_3\\ \dot{M}_4\end{bmatrix} = J_a \begin{bmatrix}\dot{S}_1\\ \dot{S}_2\\ \dot{S}_3\\ \dot{S}_4\end{bmatrix} = \begin{bmatrix}* & 0 & 0 & 0\\ 0 & * & 0 & 0\\ 0 & 0 & * & 0\\ 0 & 0 & 0 & *\end{bmatrix}\begin{bmatrix}\dot{S}_1\\ \dot{S}_2\\ \dot{S}_3\\ \dot{S}_4\end{bmatrix}$$
$$\begin{bmatrix}\dot{M}_1\\ \dot{M}_2\\ \dot{M}_3\\ \dot{M}_4\end{bmatrix} = J_b \begin{bmatrix}\dot{S}_1\\ \dot{S}_2\\ \dot{S}_3\\ \dot{S}_4\end{bmatrix} = \begin{bmatrix}* & 0 & 0 & 0\\ 0 & * & * & *\\ 0 & 0 & * & 0\\ 0 & 0 & 0 & *\end{bmatrix}\begin{bmatrix}\dot{S}_1\\ \dot{S}_2\\ \dot{S}_3\\ \dot{S}_4\end{bmatrix}, \quad (1)$$

where

$\dot{M}_i (i = 1 \sim 4)$: The velocities of output motions,
$\dot{S}_i (i = 1 \sim 4)$: The velocities of input motions,
* : nonzero element.

It is obvious that J_a is better than J_b because there are fewer nonzero elements in J_a than J_b. According to this reason, KCC_s can be defined:

$$KCC_s = \frac{N}{N_{\max}} = \frac{N}{n \times n}, \quad (2)$$

where

N: The number of nonzero elements in matrix J,
n: The degree of the square matrix J to be considered.

In the example in Eq. (1), $KCC_s(J_a) = 4/16$, $KCC_s(J_b) = 6/16$. This index is important for a manipulator's control and reliability: (1) The inverse kinematics is relevant to the kinematic coupling degree of the output motions, therefore, the control procedures deduced by the inverse kinematics will be easier if the index KCC_s is smaller. (2) The smaller KCC_s, the less number of actuators will be required to execute the desired task. Therefore, the manipulator's reliability will be better. For example, if KCC_s of a manipulator is equal to 1, all actuators must be required to effect each output motion, and this does the cylinders more harm. It is an unfavorable factor to the manipulator's reliability.

2.2. KCC_e: The difference of nonzero elements' numbers in every row among many matrix

This subsection will focus on the difference of nonzero elements' numbers in every row among many different matrixes

$$\begin{bmatrix}\dot{M}_1\\ \dot{M}_2\\ \dot{M}_3\\ \dot{M}_4\end{bmatrix} = J_b \begin{bmatrix}\dot{S}_1\\ \dot{S}_2\\ \dot{S}_3\\ \dot{S}_4\end{bmatrix} = \begin{bmatrix}* & 0 & 0 & 0\\ 0 & * & * & *\\ 0 & 0 & * & 0\\ 0 & 0 & 0 & *\end{bmatrix}\begin{bmatrix}\dot{S}_1\\ \dot{S}_2\\ \dot{S}_3\\ \dot{S}_4\end{bmatrix}$$
$$\begin{bmatrix}\dot{M}_1\\ \dot{M}_2\\ \dot{M}_3\\ \dot{M}_4\end{bmatrix} = J_c \begin{bmatrix}\dot{S}_1\\ \dot{S}_2\\ \dot{S}_3\\ \dot{S}_4\end{bmatrix} = \begin{bmatrix}* & 0 & 0 & 0\\ 0 & * & * & 0\\ 0 & 0 & * & *\\ 0 & 0 & 0 & *\end{bmatrix}\begin{bmatrix}\dot{S}_1\\ \dot{S}_2\\ \dot{S}_3\\ \dot{S}_4\end{bmatrix}. \quad (3)$$

For example in Eq. (3), $KCC_s(J_b) = KCC_s(J_c)$. However, J_b is still different from J_c. In order to distinguish the difference of nonzero elements' numbers in every row between J_b and J_c, $N_e^i (i = 2 \sim n-1)$ will be defined firstly:

$$N_e^i = \sum_{j=1}^{n} (N_{ej} - i) \quad (i = 2 \sim n-1), \quad (4)$$

where N_{ej} is the number of nonzero elements in the jth row of matrix J.

It must be stressed that the above expression $(N_{ej} - i)$ is a nonnegative value. When $(N_{ej} - i)$ becomes negative, it should be set to zero.

Then, the definition of KCC_e will be proposed to include all the information in every row of matrix J:

$$KCC_e = \frac{\sum_{i=2}^{n-1} N_e^i}{\max\left(\sum_{i=2}^{n-1} N_e^i\right)} = \frac{\sum_{i=2}^{n-1} N_e^i}{0.5 \times n(n-1)(n-2)}. \quad (5)$$

For the matrix J_g whose every element is nonzero,

$$N_e^2(J_g) = n(n-2),\ N_e^3(J_g) = n(n-3),\ N_e^{n-1}(J_g) = n,$$
$$\sum_{i=2}^{n-1} N_e^i(J_g) = 0.5 \times n(n-1)(n-2), \ \text{so,}$$
$$\max\left(\sum_{i=2}^{n-1} N_e^i\right) = 0.5 \times n(n-1)(n-2).$$

In the example in Eq. (3), $KCC_e(J_b) = 1/12$, $KCC_e(J_c) = 0$. This index is proposed to express the whole distribution of the nonzero elements in the matrix J. Its value will be smaller when the nonzero elements are more equally distributed in every row. The manipulator with small KCC_e will be easy to operate because there will be fewer output motions, which must be fulfilled by many actuators.

2.3. KCC_d: The difference of nonzero elements' positions among many matrixes

In addition to the above factors KCC_s and KCC_e, there is another factor KCC_d that has impact on the kinematic coupling degree of a manipulator. For example, in Eq. (6), $KCC_s(J_c) = KCC_s(J_d) = 6/16$, $KCC_e(J_c) = KCC_e(J_d) = 0$, but J_c is still different from J_d. The matrix J_d belongs to the classification of Group decoupling and J_c does not

$$\begin{bmatrix}\dot{M}_1\\ \dot{M}_2\\ \dot{M}_3\\ \dot{M}_4\end{bmatrix} = J_c \begin{bmatrix}\dot{S}_1\\ \dot{S}_2\\ \dot{S}_3\\ \dot{S}_4\end{bmatrix} = \begin{bmatrix}* & 0 & 0 & 0\\ 0 & * & * & 0\\ 0 & 0 & * & *\\ 0 & 0 & 0 & *\end{bmatrix}\begin{bmatrix}\dot{S}_1\\ \dot{S}_2\\ \dot{S}_3\\ \dot{S}_4\end{bmatrix}$$
$$\begin{bmatrix}\dot{M}_1\\ \dot{M}_2\\ \dot{M}_3\\ \dot{M}_4\end{bmatrix} = J_d \begin{bmatrix}\dot{S}_1\\ \dot{S}_2\\ \dot{S}_3\\ \dot{S}_4\end{bmatrix} = \begin{bmatrix}* & 0 & 0 & 0\\ 0 & * & * & 0\\ 0 & * & * & 0\\ 0 & 0 & 0 & *\end{bmatrix}\begin{bmatrix}\dot{S}_1\\ \dot{S}_2\\ \dot{S}_3\\ \dot{S}_4\end{bmatrix}. \quad (6)$$

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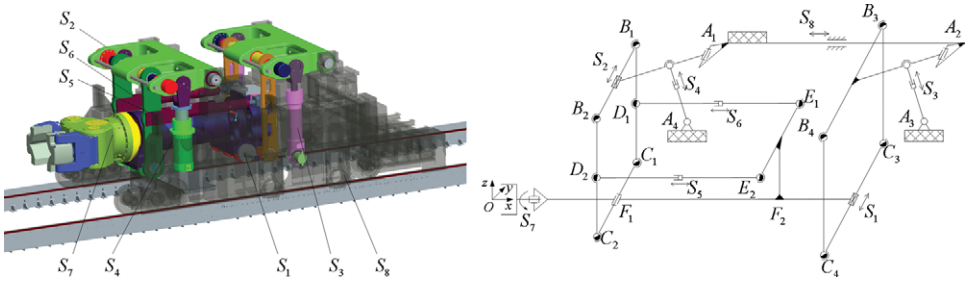


Fig. 1. (Colour online) CAD model of the heavy-payload forging manipulator (FM I).

In order to distinguish the difference between J_c and J_d, the index KCC_d is defined:

$$KCC_d = \frac{\sum_{i=1}^{n} N_d^i}{\max\left(\sum_{i=1}^{n} N_d^i\right)} = \frac{\sum_{i=1}^{n} N_d^i}{0.5 \times (n+1)n}, \tag{7}$$

where

N_d^i: The number of the columns that are distributed by some particular nonzero elements and these particular nonzero elements belong to the rows where the number of nonzero elements is i.

For the upper triangular matrix J_h whose every element in the upper triangular part is nonzero,

$$N_d^1(J_h) = 1, N_d^2(J_h) = 2, N_d^n(J_h) = n, \sum_{i=1}^{n} N_d^i(J_h) = 0.5 \times (n+1)n, \text{ so,}$$
$$\max\left(\sum_{i=1}^{n} N_d^i\right) = 0.5 \times (n+1)n.$$

In the example in Eq. (6), $KCC_d(J_c) = 5/10$, $KCC_d(J_d) = 4/10$. Out of those partially decoupled manipulators, the one with Group decoupling is preferred by many researchers. This index KCC_d is designed to measure the Group decoupling degree of a manipulator.

2.4. The definition of KCC

With the above three factors considered, KCC can be defined:

$$KCC = KCC_s + KCC_e + KCC_d. \tag{8}$$

Briefly speaking, KCC_s indicates the proportion of nonzero elements in the matrix, KCC_e expresses the degree of distribution of nonzero elements, and KCC_d measures the Group decoupling degree of a matrix. Among these three factors, KCC_s is the first one to be considered and is more important than the other two. Therefore, when a designer wants to compare different types of robotic architectures from the perspective of kinematic coupling:

(1) If KCC_s of J_u is much greater than KCC_s of J_v, choose J_v.
(2) If there is a little difference between KCC_s of J_u and KCC_s of J_v, the designer should employ the overall KCC to make a decision.

It is necessary to mention that J_e is different from J_f in Eq. (9), but they have the same kinematic coupling degree, so there is no need to distinguish the difference between them and $KCC(J_e) = KCC(J_f)$:

$$\begin{bmatrix} \dot{M}_1 \\ \dot{M}_2 \\ \dot{M}_3 \\ \dot{M}_4 \end{bmatrix} = J_e \begin{bmatrix} \dot{S}_1 \\ \dot{S}_2 \\ \dot{S}_3 \\ \dot{S}_4 \end{bmatrix} = \begin{bmatrix} * & 0 & 0 & 0 \\ 0 & * & * & 0 \\ 0 & 0 & * & 0 \\ 0 & 0 & 0 & * \end{bmatrix} \begin{bmatrix} \dot{S}_1 \\ \dot{S}_2 \\ \dot{S}_3 \\ \dot{S}_4 \end{bmatrix}$$
$$\begin{bmatrix} \dot{M}_1 \\ \dot{M}_2 \\ \dot{M}_3 \\ \dot{M}_4 \end{bmatrix} = J_f \begin{bmatrix} \dot{S}_1 \\ \dot{S}_2 \\ \dot{S}_3 \\ \dot{S}_4 \end{bmatrix} = \begin{bmatrix} * & 0 & 0 & 0 \\ 0 & * & 0 & * \\ 0 & 0 & * & 0 \\ 0 & 0 & 0 & * \end{bmatrix} \begin{bmatrix} \dot{S}_1 \\ \dot{S}_2 \\ \dot{S}_3 \\ \dot{S}_4 \end{bmatrix}. \tag{9}$$

Some important results are as follows:

(1) $KCC_s > 0 \quad KCC_d > 0$. The reason is that there is at least one nonzero element in every row of matrix J.
(2) The maximum values of KCC_s and KCC_e will be reached when the matrix J is full of nonzero elements.
(3) The maximum value of KCC_d will be reached when the matrix J is a triangular matrix and has $0.5 \times n(n+1)$ nonzero elements.
(4) $0 < KCC < 3$.

3. *KCC* of Heavy-Payload Forging Manipulators

In this section, one type of heavy-payload forging manipulator (as shown in Fig. 1) will be taken as an example to illustrate the definition of *KCC*. Yan *et al.*[11] introduced a type of forging manipulator where $B_i(i = 1 \sim 4)$, $C_i(i = 1 \sim 4)$, $D_i(i = 1 \sim 2)$, and $E_i(i = 1 \sim 2)$ are Hooke joints, while these joints are changed to spherical joints in this paper. This manipulator consists of five function parts:

(a) Frame part, formed by $A_1 - A_2 - A_3 - A_4$, includes the gear actuator S_8;
(b) Front lifting part, formed by $A_1 - B_1 - B_2 - C_2 - C_1$, includes the front lifting cylinder S_4 and the front side shifting cylinder S_2;
(c) Rear lifting part, formed by $A_2 - B_3 - B_4 - C_4 - C_3$, includes the rear lifting cylinder S_3 and the rear side shifting cylinder S_1;

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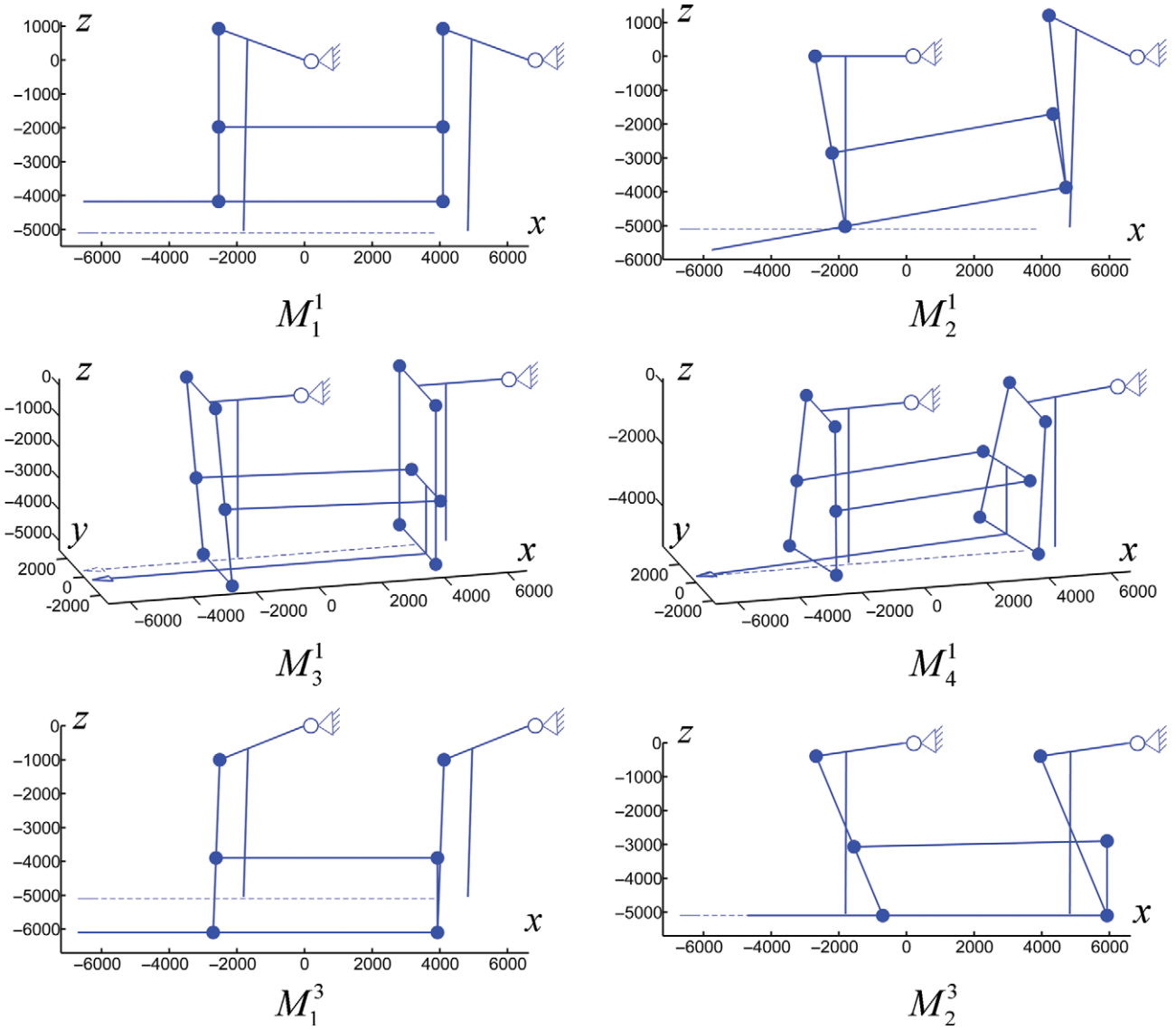


Fig. 2. (Colour online) Some basic output motions of the heavy-payload forging manipulator (the dashed line is the initial position of the gripper).

(d) Buffer mechanism, formed by $D_1 - D_2 - E_2 - E_1$, includes the hydraulic damping cylinders S_5 and S_6;
(e) Gripper part, formed by $F_1 - F_2$, includes the rotation motor S_7.

3.1. All the basic output motions

One of the primary issues is to represent the pose of a manipulator through a set of parameters. The most classical way is to use three translation coordinates for its position, and three rotation coordinates for its orientation. However, for a manipulator in an actual workshop, there are some special output motions:

(1) Some complex manipulators work in different stages related to different functions. Six parameters are not enough to represent the pose of a manipulator, because there are more than six types of output motions in these stages.
(2) The output motions used in the actual workshop are not the standard single translation or rotation coordinate. They may be combined by some of the six standard coordinates and can be called basic output motions. These basic output motions are easy to control, and sometimes only one actuator is needed to complete one basic output motion. Moreover, these basic motions can be employed to fulfill all the feasible motions.

The whole working process of the heavy-payload forging manipulator in the workshop can be divided into three stages related to the different functions: grasping stage M^1, transporting stage M^2, and forging stage M^3. Then, all the basic output motions (as shown in Fig. 2) will be figured out in each stage. For convenience of presentation, a moving reference frame $O - xyz$ is placed in the center of the gripper in Fig. 1, its x-axis is parallel to the gripper's axis F_1F_2, its y-axis is parallel to C_2C_1, and its z-axis can be decided by the right-hand rule.

In the grasping stage M^1, these following basic motions are chosen for the simplicity of its control:

M_1^1: Combined by the translations along x- and z-axes and can be completed by the same movements from the cylinders S_3 and S_4.

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Table I. Actuator status.

Status	Output motions	Locked actuators	Activated actuators
1	$M_1^1\ M_2^1$	$S_1\ S_2\ S_5\ S_6\ S_7\ S_8$	$S_3\ S_4$
2	M_3^1	$S_3\ S_4\ S_5\ S_6\ S_7\ S_8$	$S_1\ S_2$
3	M_4^1	$S_7\ S_8$	$S_1\ S_2\ S_3\ S_4\ S_5\ S_6$
4	$M_1^2\ M_2^2$	$S_1\ S_2\ S_3\ S_4\ S_5\ S_6$	$S_7\ S_8$
5	$M_1^3\ M_2^3$	$S_1\ S_2\ S_7\ S_8$	$S_3\ S_4\ S_5\ S_6$

M_2^1: Combined by the translations along x- and z-axes and the rotation about y-axis and can be completed by the cylinder S_3 when the cylinder S_4 is locked.
M_3^1: The translation along y-axis.
M_4^1: The rotation about z-axis.

In the transporting stage M^2, these basic motions are chosen:

M_1^2: The rotation about x-axis and can be completed by the only rotation motor S_7.
M_2^2: The translation along x-axis and can be completed by the gear actuator S_8.

In the forging stage M^3, these basic motions are chosen:

M_1^3: The translation along x-axis.
M_2^3: The translation along z-axis.

3.2. Input/output velocity matrix of the heavy-payload forging manipulator

The different basic output motions will be completed by different actuators, so those activated actuators must be identified according to each basic output motion (as shown in Table I), and then, it is possible to calculate the matrix representing the relationship between the actuated joint velocities and the output motion velocities. This matrix is different from Jacobian matrix[12–15] because its degree may be more than six.

Status 1: The constrained mechanism is equivalent to a 2-DOF (i.e., degree of freedom) planar mechanism, so the relationship between the actuated joint velocities and the output motion velocities can be obtained easily:

$$\begin{bmatrix} \dot{M}_1^1 \\ \dot{M}_2^1 \end{bmatrix} = \begin{bmatrix} J_{13} & J_{14} \\ J_{23} & 0 \end{bmatrix} \begin{bmatrix} \dot{S}_3 \\ \dot{S}_4 \end{bmatrix}. \tag{10}$$

Status 2: The special mechanism is equivalent to a 1-DOF mechanism. The output motion M_3^1 can be fulfilled by the same movements from the cylinders S_1 and S_2:

$$\dot{M}_3^1 = [J_{31} \quad J_{32}][\dot{S}_1 \quad \dot{S}_2]^{\mathrm{T}}. \tag{11}$$

Status 3: This is the most complex output motion. To complete it, six cylinders $S_i (i = 1 \sim 6)$ must be actuated simultaneously. From the geometry of the mechanism, one can write

$$\begin{cases} (X_{B1} - X_{C1})^2 + (Y_{B1} - Y_{C1})^2 + (Z_{B1} - Z_{C1})^2 = c_1^2, \\ (X_{B2} - X_{C2})^2 + (Y_{B2} - Y_{C2})^2 + (Z_{B2} - Z_{C2})^2 = c_2^2, \\ (X_{B3} - X_{C3})^2 + (Y_{B3} - Y_{C3})^2 + (Z_{B3} - Z_{C3})^2 = c_3^2, \\ (X_{B4} - X_{C4})^2 + (Y_{B4} - Y_{C4})^2 + (Z_{B4} - Z_{C4})^2 = c_4^2, \\ (X_{D1} - X_{E1})^2 + (Y_{D1} - Y_{E1})^2 + (Z_{D1} - Z_{E1})^2 = S_6^2, \\ (X_{D2} - X_{E2})^2 + (Y_{D2} - Y_{E2})^2 + (Z_{D2} - Z_{E2})^2 = S_5^2, \end{cases} \tag{12}$$

where $c_i (i = 1 \sim 4)$ is the length of the link $B_i C_i (i = 1 \sim 4)$.

Then, differentiating Eq. (11) with respect to time, one obtains

$$\dot{M}_4^1 = [J_{41} \quad J_{42} \quad J_{43} \quad J_{44} \quad J_{45} \quad J_{46}] \times [\dot{S}_1 \quad \dot{S}_2 \quad \dot{S}_3 \quad \dot{S}_4 \quad \dot{S}_5 \quad \dot{S}_6]^{\mathrm{T}}. \tag{13}$$

Status 4: The output motions M_1^2 and M_2^2 can be fulfilled by S_7 and S_8, respectively,

$$\begin{bmatrix} \dot{M}_1^2 \\ \dot{M}_2^2 \end{bmatrix} = \begin{bmatrix} J_{57} & 0 \\ 0 & J_{68} \end{bmatrix} \begin{bmatrix} \dot{S}_7 \\ \dot{S}_8 \end{bmatrix}. \tag{14}$$

Status 5: The output motions M_1^3 and M_2^3 are used in the forging stage. The cylinders S_3, S_4, S_5, and S_6 must be actuated simultaneously in order to forge different parts

$$\begin{bmatrix} \dot{M}_1^3 \\ \dot{M}_2^3 \end{bmatrix} = \begin{bmatrix} J_{73} & J_{74} & J_{75} & J_{76} \\ J_{83} & J_{84} & J_{85} & J_{86} \end{bmatrix} \begin{bmatrix} \dot{S}_3 \\ \dot{S}_4 \\ \dot{S}_5 \\ \dot{S}_6 \end{bmatrix}. \tag{15}$$

Moreover, the Eqs. (10), (11), (13)–(15) can be written in matrix form, which leads to

$$\begin{bmatrix} \dot{M}_1^1 \\ \dot{M}_2^1 \\ \dot{M}_3^1 \\ \dot{M}_4^1 \\ \dot{M}_1^2 \\ \dot{M}_2^2 \\ \dot{M}_1^3 \\ \dot{M}_2^3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & J_{13} & J_{14} & 0 & 0 & 0 & 0 \\ 0 & 0 & J_{23} & 0 & 0 & 0 & 0 & 0 \\ J_{31} & J_{32} & 0 & 0 & 0 & 0 & 0 & 0 \\ J_{41} & J_{42} & J_{43} & J_{44} & J_{45} & J_{46} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & J_{57} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & J_{68} \\ 0 & 0 & J_{73} & J_{74} & J_{75} & J_{76} & 0 & 0 \\ 0 & 0 & J_{83} & J_{84} & J_{85} & J_{86} & 0 & 0 \end{bmatrix} \times \begin{bmatrix} \dot{S}_1 \\ \dot{S}_2 \\ \dot{S}_3 \\ \dot{S}_4 \\ \dot{S}_5 \\ \dot{S}_6 \\ \dot{S}_7 \\ \dot{S}_8 \end{bmatrix} = \mathbf{J}_1 \begin{bmatrix} \dot{S}_1 \\ \dot{S}_2 \\ \dot{S}_3 \\ \dot{S}_4 \\ \dot{S}_5 \\ \dot{S}_6 \\ \dot{S}_7 \\ \dot{S}_8 \end{bmatrix}. \tag{16}$$

Now, the matrix $\mathbf{J}_1$, which relates the velocities of actuated joints to the velocities of all the basic output motions, is obtained.

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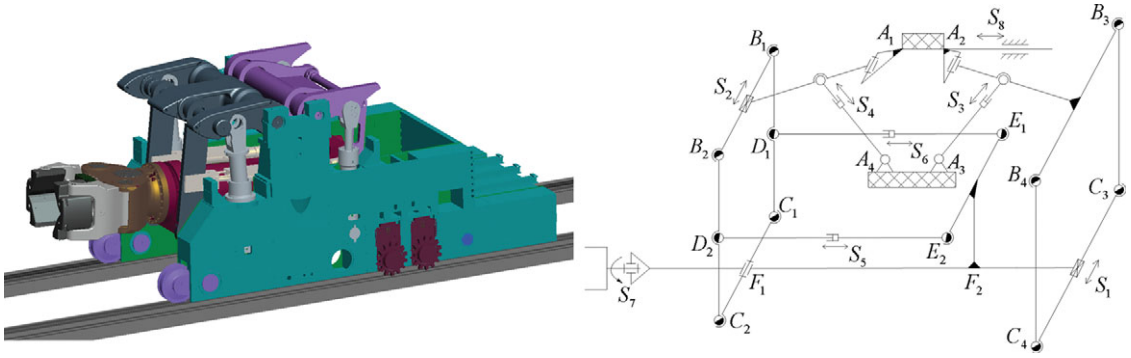


Fig. 3. (Colour online) CAD model of the heavy-payload forging manipulator (FM II).

Note that, if the tasks of a robotic system can not be readily decomposed into a set of basic motions, the designer could use the general Jacobian matrix although its Jacobian matrix may be a little complicated. The general Jacobian matrix represents the relationship between the velocities of actuated joints and the velocities of the general output motions $x\ y\ z\ \alpha\ \beta\ \gamma$, therefore, KCC of the general Jacobian matrix also allows the designer to compare other types of robotic architectures from the perspective of kinematic coupling.

3.3. *KCC of FM I*

After getting the velocity matrix J_1, the indexes $KCC_s(J_1)$, $KCC_e(J_1)$, and $KCC_d(J_1)$ can be calculated by Eqs. (2), (5), and (7):

$$KCC_s(J_1) = 21/64 \quad KCC_e(J_1) = 16/168$$
$$KCC_d(J_1) = 17/36. \tag{17}$$

Using Eq. (8), the KCC of FM I can be obtained:

$$KCC(J_1) = KCC_s(J_1) + KCC_e(J_1) + KCC_d(J_1) = 0.8956. \tag{18}$$

4. *KCC* of Other Types of Heavy-Payload Forging Manipulators

4.1. *KCC of FM II*

The second type of heavy-payload forging manipulator is shown in Fig. 3. Using the same methods as above, the matrix $\mathbf{J}_2$ can be obtained

$$\mathbf{J}_2 = \begin{bmatrix} 0 & 0 & J_{13} & J_{14} & J_{15} & J_{16} & 0 & 0 \\ 0 & 0 & J_{23} & 0 & 0 & 0 & 0 & 0 \\ J_{31} & J_{32} & 0 & 0 & 0 & 0 & 0 & 0 \\ J_{41} & J_{42} & J_{43} & J_{44} & J_{45} & J_{46} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & J_{57} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & J_{68} \\ 0 & 0 & J_{73} & J_{74} & J_{75} & J_{76} & 0 & 0 \\ 0 & 0 & J_{83} & J_{84} & J_{85} & J_{86} & 0 & 0 \end{bmatrix}. \tag{19}$$

After getting the velocity matrix J_2, the indexes $KCC_s(J_2)$, $KCC_e(J_2)$, and $KCC_d(J_2)$ can be calculated by Eqs. (2), (5), and (7):

$$KCC_s(J_2) = 23/64, \quad KCC_e(J_2) = 19/168,$$
$$KCC_d(J_2) = 15/36. \tag{20}$$

Using Eq. (8), the KCC of FM II can be obtained:

$$KCC(J_2) = KCC_s(J_2) + KCC_e(J_2) + KCC_d(J_2) = 0.8891 \tag{21}$$

4.2. *KCC of FM III*

The third type of heavy-payload forging manipulator is shown in Fig. 4. Using the same methods as above, the matrix

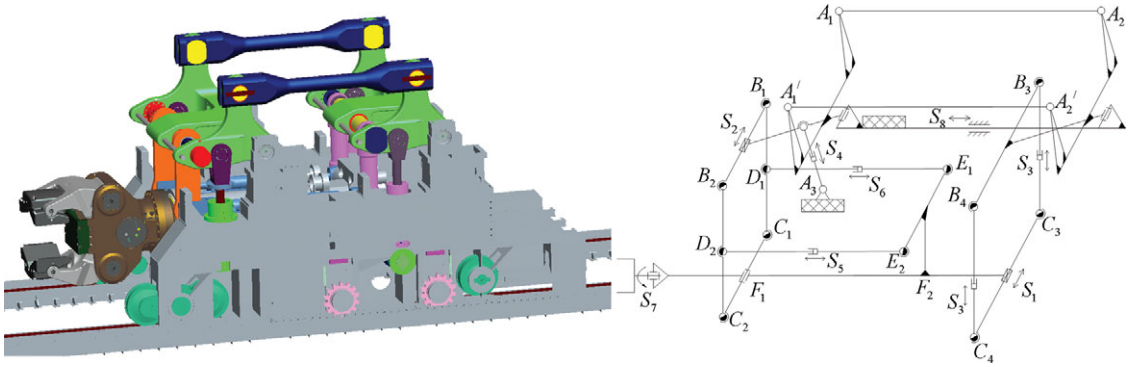


Fig. 4. (Colour online) CAD model of the heavy-payload forging manipulator (FM III).

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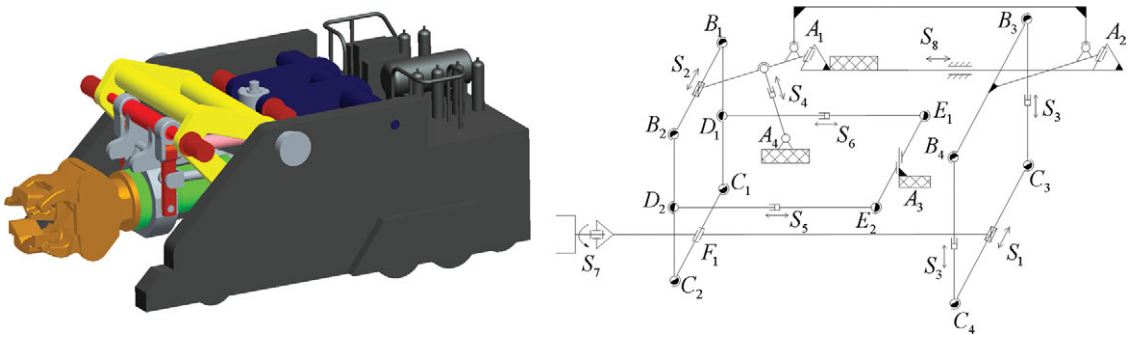


Fig. 5. (Colour online) CAD model of the heavy-payload forging manipulator (FM IV).

$\mathbf{J}_3$ can be obtained

$$\mathbf{J}_3 = \begin{bmatrix} 0 & 0 & 0 & J_{14} & 0 & 0 & 0 & 0 \\ 0 & 0 & J_{23} & 0 & 0 & 0 & 0 & 0 \\ J_{31} & J_{32} & 0 & 0 & 0 & 0 & 0 & 0 \\ J_{41} & J_{42} & J_{43} & J_{44} & J_{45} & J_{46} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & J_{57} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & J_{68} \\ 0 & 0 & 0 & J_{74} & J_{75} & J_{76} & 0 & 0 \\ 0 & 0 & 0 & J_{84} & J_{85} & J_{86} & 0 & 0 \end{bmatrix}. \tag{22}$$

After getting the velocity matrix J_3, the indexes $KCC_s(J_3)$, $KCC_e(J_3)$, and $KCC_d(J_3)$ can be calculated by Eqs. (2), (5), and (7):

$$KCC_s(J_3) = 18/64, \quad KCC_e(J_3) = 12/168, \\ KCC_d(J_3) = 15/36. \tag{23}$$

Using Eq. (8), the *KCC* of FM III can be obtained:

$$KCC(J_3) = KCC_s(J_3) + KCC_e(J_3) \\ + KCC_d(J_3) = 0.7693. \tag{24}$$

4.3. KCC of FM IV

The fourth type of heavy-payload forging manipulator is shown in Fig. 5. Using the same methods as above, the matrix $\mathbf{J}_4$ can be obtained

$$\mathbf{J}_4 = \begin{bmatrix} 0 & 0 & 0 & J_{14} & J_{15} & J_{16} & 0 & 0 \\ 0 & 0 & J_{23} & 0 & J_{25} & J_{26} & 0 & 0 \\ J_{31} & J_{32} & 0 & 0 & J_{35} & J_{36} & 0 & 0 \\ J_{41} & J_{42} & J_{43} & J_{44} & J_{45} & J_{46} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & J_{57} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & J_{68} \\ 0 & 0 & 0 & J_{74} & J_{75} & J_{76} & 0 & 0 \\ 0 & 0 & 0 & J_{84} & J_{85} & J_{86} & 0 & 0 \end{bmatrix}. \tag{25}$$

After getting the velocity matrix J_4, the indexes $KCC_s(J_4)$, $KCC_e(J_4)$, and $KCC_d(J_4)$ can be calculated by Eqs. (2), (5), and (7):

$$KCC_s(J_4) = 24/64, \quad KCC_e(J_4) = 17/168, \\ KCC_d(J_4) = 16/36. \tag{26}$$

Using Eq. (8), the *KCC* of FM IV can be obtained:

$$KCC(J_4) = KCC_s(J_4) + KCC_e(J_4) \\ + KCC_d(J_4) = 0.9206. \tag{27}$$

Table II. KC of all FMs.

Type	FM I	FM II	FM III	FM IV
KC	0.8956	0.8891	0.7693	0.9206

5. Conclusions

The input/output velocity matrix contains a lot of kinematic information for a manipulator. For a heavy-payload forging manipulator in the workshop, it is necessary to obtain its velocity matrix relating the velocities of actuated joints to the velocities of all basic output motions. Based on the above matrix, this paper proposed the definition of *KCC* to measure the kinematic coupling degree for the manipulator. In order to distinguish the differences among many velocity matrixes, three factors will be considered: KCC_s is the proportion of nonzero elements in matrix J, KCC_e is the difference of nonzero elements' numbers in every row among many matrixes, and KCC_d is the difference of nonzero elements' positions among many matrixes. As an example, four different types of heavy-payload forging manipulators were analyzed by using the concept of *KCC*, respectively. There is no much difference among the KCC_s of four types of manipulator. According to Table II, the third type of heavy-payload forging manipulator with smallest *KCC* was chosen as our prototype (as shown in Fig. 6).

It is necessary to mention that before the final decision was made, other criteria must be considered firstly, including DOF, workspace, force, weight, and cost. Due to the little differences of other criteria among four types of manipulators, *KCC* was used to make the final decision because the kinematic coupling degree is an important issue especially for the heavy-payload manipulators.

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Fig. 6. (Colour online) The prototype of FM III.

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