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The Term Structure of Expected Recovery Rates

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Abstract

There is widespread agreement that corporate debts' recovery rates are time varying, but empirical work in this area is limited. We show that the joint information from the term structure of senior and subordinate credit default swaps can identify the level and the dynamics of recovery rates. We estimate a reduced-form no-arbitrage model on 46 firms across different industries. We find that the term structure of expected recovery rates is, on average, downward sloping. However, an inversion occurs during the 2008 crisis, suggesting the market expects higher recoveries conditional on short-term survival. The inversion is more pronounced for firms in distressed industries.

I. Introduction

Credit spreads consist of two main components: the default probability and the loss given default (LGD). The LGD is often expressed in terms of the debt's recovery rate, which is defined as 1 minus the LGD. Although much is known about the modeling of default probabilities, the same cannot be said for risk-neutral recovery rates. The standard practice in the literature is to assume a constant recovery rate.[1] Unfortunately, this assumption is not realistic given the mounting

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[1]For examples, see Pan and Singleton (2008), Chen, Cheng, Fabozzi, and Liu (2008), Schneider, Sögner, and Veža (2010), Doshi, Ericsson, Jacobs, and Turnbull (2013), and Elkamhi, Jacobs, and Pan (2014).

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evidence in support of time-varying ex post realized recovery rates in the credit-risk literature.[2]

In this article, we use information from both senior and subordinate credit default swaps (CDSs) to isolate the stochastic recovery-rate dynamic of credit spreads. The senior CDSs are insurance contracts for the senior unsecured bonds, whereas the subordinate CDSs are insurance contracts for the subordinate or lower tier 2 bonds. In the event of default, the expected loss is larger for the subordinate debt. Consequently, the spread of subordinate CDS contracts is larger than the spread of their corresponding senior contracts despite having the same default probability. We make use of the information embedded in the seniority-driven gap between senior and subordinate CDS spreads to identify their recovery rates' dynamic, which in turn enables us to study the *term structure* of expected recovery.

The firm-by-firm estimation results show that both the default intensity and the recovery components vary significantly over time. The average expected risk-neutral recovery rate across firms and default horizons is approximately 34% for senior contracts and 20% for subordinate contracts. Our estimates of recovery rates are economically plausible and consistent with realized recovery rates reported in the literature.

Our empirical framework consists of modeling the senior and subordinate CDS contracts in a 5-factor reduced-form no-arbitrage model. The model allows for stochastic interest rates, stochastic default intensity, and stochastic LGD. We model the short-rate dynamic, the default intensity, and the natural log of the LGD rate using a quadratic specification. This approach ensures that the short rate and the default intensity are always positive and that the LGD is bounded between 0 and 1. We estimate the model on 46 firms individually. Firms in our sample are from different industries, with the sample period spanning from Jan. 1, 2001 to May 31, 2012.[3]

We obtain a number of important findings about the recovery-rate dynamic. First, we document a sharp decline in recovery rates during the financial crisis. Specifically, we show that the average recovery rate falls dramatically from mid-2007 onward, which marks the onset of the financial crisis. Second, we find that the term structure of expected recovery implied by CDS contracts significantly changes shape during the 2008 financial crisis. On average, the term structure of expected recovery is downward sloping; however, it becomes upward sloping during the 2008 financial crisis. This inversion suggests that the market expects firms to recover less if they were to default in the midst of the recession than if they were to survive and default at future dates when the economy improves. Our finding that the term structure of expected recovery is upward sloping during bad economic times is in line with Zhang (2009), who shows that realized recovery rates are negatively correlated with lagged macroeconomic conditions.

[2]For instance, Altman, Brady, Resti, and Sironi (2005) examine recovery rates for corporate bond defaults between 1982 and 2002 and find that they vary significantly over time and are negatively correlated with default rates. Using an extensive set of firms, Acharya, Bharath, and Srinivasan (2007) also document large cross-sectional and time-series variations in realized recovery rates.

[3]Our sample consists of firms in the Markit database that have CDS data on both senior and subordinate debts for at least 1 year before the financial crisis.

Third, we find that increases in CDS spreads during the financial crisis are mainly due to the increase in default probabilities. Although the increase in LGD is small relative to the change in default probabilities, its magnitude is economically significant. Importantly, we find that the increase in LGD during the financial crisis is much larger at the short end of the term-structure horizon. The relatively larger increase in LGD at shorter default horizons explains why the term structure of recovery flattens or inverts during the financial crisis period.

Fourth, we find that industry characteristics are an important determinant of the CDS implied recovery rates, consistent with the theoretical argument of Shleifer and Vishny (1992) and the findings of Acharya et al. (2007), who study realized recovery rates on an extensive set of firms. The inversion of the expected recovery term structure during the crisis is more pronounced for firms in distressed industries (e.g., financial firms). We find that recovery rates decrease most dramatically for financial firms relative to the other industries during the crisis period. Although industry characteristics impact firms' recovery rates, we do not find that firms' credit ratings monotonically explain the cross-sectional differences in recovery rates. This finding is consistent with Altman and Kishore (1996), who show that bond ratings have no impact on the recovery rates once the seniority of the bonds is taken into consideration. Credit ratings, on the other hand, are solely assigned based on the firm's default probability, and we find a clear monotonic relationship that firms' risk-neutral default probabilities increase with their credit ratings.

We verify that our main conclusions are robust to the relative difference in market liquidity between senior and subordinate CDS spreads by including daily bid–ask spreads in the model's estimations. We show that the price discovery mechanism between senior and subordinate CDS contracts was not significantly affected during the crisis, suggesting that our main conclusions are not driven by structural changes in their relative liquidity. We verify that our main conclusions are robust to an alternative estimation approach, and that parameter estimates are stable using subperiod tests.

We provide an extensive discussion of how our estimation method using multiple-seniority CDS term structures facilitates the identification of the recovery-rate dynamic. We perform simulation exercises showing that the parameters in our model are well identified. Further, using a static analysis, we show that the identification of senior and subordinate recovery levels comes from their relative CDS term structures at a given point in time. Therefore, by using panel data of senior and subordinate CDS term structures, we can identify the stochastic dynamics of their recovery rates.[4]

To our knowledge, this study is the first that uses information from CDS contracts with multiple seniorities to estimate the *stochastic dynamics* of risk-neutral recovery. Few other studies exist that consider a stochastic recovery-rate model. Bakshi, Madan, and Zhang (2006) examine a stochastic recovery-rate model

[4] Pan and Singleton (2008) show that the constant recovery rate is identified using the term structure of CDS spreads. Our identification builds on their strategy and shows that the term structure of CDS spreads with more than one seniority allows for the identification of the stochastic dynamic of recovery rates.

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using single-seniority BBB-rated bonds. Their approach differs from ours in that their recovery-rate and default-intensity dynamics are driven by factors describing the risk-free rate. In our approach, we model recovery-rate and default-intensity dynamics using latent firm-specific factors in addition to the default-free term structure. This method allows firm-specific factors to influence the time-varying recovery rate. Christensen (2007) estimates a stochastic recovery model using CDS data for the Ford Motor Corporation. Our study differs from this article in at least two ways. First, the scale of our empirical exercise is much larger. We estimate 46 firms (instead of 1). This allows us to investigate differences in the term structure of expected recovery across industries and rating categories. Second, we use the term structure of CDS contracts with multiple seniorities, resulting in an improved identification of the recovery-rate dynamic.

A study that is closely related to ours is that of Schläfer and Uhrig-Homburg (2014), which estimates a time-varying recovery-rate model that is a function of firm-specific and macro variables. Their study identifies risk-neutral recovery rates using the ratio of 5-year senior-to-subordinate CDS spreads together with firm-level capital structure information, and they find that risk-neutral recovery rates are systematically lower than historical realizations. The focus of Schläfer and Uhrig-Homburg (2014) is on documenting the recovery risk premium, which is an important topic. Our study differs from theirs, and from much of the extant literature, in that we study time variations in the *term structure* of risk-neutral recovery rates. We also fit the model to a large panel of senior and subordinate CDS contracts across various maturities, and we jointly estimate the dynamics of risk-neutral recovery and default intensity. This, in turn, provides us with new insights on the shape of the term structure of expected recovery rates, which becomes inverted during the subprime crisis for financial firms.[5]

The remainder of the article is organized as follows: Section II discusses other related literature and our contributions. Section III introduces the model. Section IV discusses the data and the estimation method. Section V presents the empirical results. Section VI shows that our findings are robust to liquidity-related concerns in the CDS market. Section VII illustrates the model's identification of the recovery-rate dynamic. Finally, Section VIII concludes.

II. Other Related Literature

Jarrow (2001), Madan, Guntay, and Unal (2003), Madan and Unal (1998), Carr and Wu (2010), Das and Hanouna (2009), and Le (2015), among others, show that information from more than one type of security can be used to infer risk-neutral recovery rates. Building on the insight of these articles, we use information embedded in multiple-seniority CDS spreads to infer the term structure of expected recovery rates.

Pan and Singleton (2008) show that the term structure of single-seniority CDS spreads allows for separate identification of the default intensity dynamic and the constant recovery rate. They estimate a reduced-form model using the

[5]The sample period and the type of firms that we study also differ from Schläfer and Uhrig-Homburg (2014). We focus on financial firms, and our sample spans the pre-crisis to the post-crisis period.

term structure of sovereign CDS spreads over a sample period of 5.5 years. Their setup allows for the identification of a constant recovery parameter but not the time-varying recovery-rate dynamic. Schneider et al. (2010) estimate a jump model with a constant recovery-rate parameter across a large cross section of firms. The mean implied risk-neutral recovery rate reported in their study is approximately 79%, which is larger than the historical realized recovery that is found to be between 40% and 50% (see Altman and Kishore (1996), Emery, Ou, and Tennant (2008)). Le (2015) uses information from option prices to estimate the dynamics of the risk-neutral default intensity.[6] The estimated default intensity is then used together with the 5-year maturity CDS spreads to compute the implied recovery rates. This method assumes that the CDS and equity options markets are integrated. Relatedly, Carr and Wu (2010) model CDSs and equity options jointly and estimate a constant recovery-rate model. They note that their estimates should be treated cautiously because of the possible segmentation between the equity options and the CDS market.

Das and Hanouna (2009) use the term structure of CDS spreads together with stock price and volatility to extract the term structure of default probabilities and recovery rates. The recovery rate and default probabilities in their model are, however, driven by only one state variable, which is the stock price. In addition, their model requires calibration at each point in time (i.e., they estimate different parameters every month), whereas we estimate our model using the time-series information for the term structure of CDS spreads with multiple seniorities. Song (2008), in a related article, shows that the recovery rate can be separated from the default probabilities using a series of cross-sectional no-arbitrage relationships between the spot and forward CDSs. However, as with most other articles in the literature, he estimates a static recovery-rate model for sovereign CDSs.[7]

In addition to the aforementioned literature, there is a large body of literature that studies the relationship between default rates and *realized* recovery rates ex post of defaults. Jokivuolle and Peura (2003) find a negative relationship between the default rates and the recovery rates in a modeling framework similar to that of Merton (1974). Carey and Gordy (2004) examine the relationship between realized recovery rates and default rates using a large cross section of data and find that their correlation is, on average, close to 0 except during bad economic conditions, when it becomes significantly negative. Altman et al. (2005) and Hu and Perraudin (2002) also find a negative relationship between the default probability and the realized recovery rate using different empirical approaches. Finally, Chava, Stefanescu, and Turnbull (2011) jointly model the physical default probabilities and the realized recovery rates using observable covariates and find a negative relationship between them, with their magnitude varying with the credit cycle. Motivated by the stream of literature that has documented the cyclical relationship between the default probability and the realized recovery rate, we

[6]Similarly, Conrad, Dittmar, and Hameed (2013) use information from option prices to infer recovery rates.

[7]Berd (2005) derives a no-arbitrage relationship between digital default swaps and conventional CDS contracts to extract risk-neutral recovery rates.

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allow their risk-neutral dynamics to depend on the common factors driving the default-free term structure.

Our article differs from the aforementioned studies described in this section in two important ways. First, we study the stochastic dynamics of risk-neutral recovery rates. Second, we use CDS contracts with multiple seniorities to facilitate the identification of the recovery rates' dynamics.

III. Model

We model default-free securities and CDSs using the reduced-form no-arbitrage framework. We assume that the model is described by five independent latent factors. We use a latent-variable approach because our objective is to extract the time-varying dynamics of recovery rates directly from CDS spreads in a flexible way without imposing additional assumptions on observable covariates that could affect the dynamics. Further, previous studies (e.g., Pan and Singleton (2008), Schneider et al. (2010)) show that a latent-variable approach is able to price CDS spreads across different maturities reasonably well with relatively few variables.[8]

In this section, we start by stating the five latent-state variables that fully describe our model under the risk-neutral measure. We then describe the short-rate model and the pricing of default-free bonds. Subsequently, we describe the default intensity and LGD dynamics in the CDS valuation model. Finally, we present our specification for the market price of risk and describe the dynamic of latent-state variables under the physical measure.

A. State Variables

The model is fully described by five latent factors denoted by $X_{i,t}$ for $i = 1,\ldots,5$. We assume that the latent factors are independent and follow an autoregressive process with Gaussian innovation. Their dynamic under the risk-neutral measure is described by

$$X_{i,t+1} = \mu_i + \rho_i X_{i,t} + \sigma_i \varepsilon_{i,t}, \quad \text{for } i = 1,\ldots,5. \tag{1}$$

The parameters μ_i, ρ_i, and σ_i in equation (1) correspond to the mean, autoregressive term, and standard deviation of each factor, respectively. We assume the residual shock in each factor dynamic follows a standard normal distribution, $\varepsilon_{i,t+1} \sim \mathrm{N}(0,1)$.

The dynamic of the state variables can be written compactly using matrix notation as

$$X_{t+1} = \mu + \rho X_t + \Sigma \varepsilon_{t+1}, \tag{2}$$

where X_t is a 5×1 vector containing the five latent factors describing the model, and μ is a 5×1 vector of factor drift, with the ith element denoted by μ_i. In equation (2), ρ and Σ are diagonal 5×5 matrices containing the standard deviation

[8] An alternative approach in the reduced-form framework is to model default-intensity and recovery-rate dynamics as functions of observable covariates. This approach is suited for studying the determinants of credit spreads and for quantifying the credit-risk premium associated with a specific observable covariate, which are not the objectives of this article.

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and autoregressive terms of each factor, with the ith diagonal element denoted by ρ_i and σ_i, respectively. We assume the 5×1 vector of residual shocks in equation (2) is generated by $\varepsilon_{t+1} \sim \mathrm{N}(0, I)$, where I is the identity matrix. Appendix A provides a matrix representation of the full model.

B. Default-Free Model

Let r_t denote the instantaneous default-free interest rate (i.e., risk-free rate). We follow Duffee (1999) and model the interest-rate dynamic using two latent factors, $X_{1,t}$ and $X_{2,t}$. We assume that r_t follows a quadratic specification.[9] Its dynamic is given by

$$r_t = (\delta_0 + \delta_1 X_{1,t} + \delta_2 X_{2,t})^2, \tag{3}$$

where δ_0, δ_1, and δ_2 are loading coefficients in the short-rate dynamic. The dynamics of $X_{1,t}$ and $X_{2,t}$ are identified using the term structure of default-free securities.

The price of a zero-coupon bond at time t that matures in h periods is given by $B(t, t+h) = \mathrm{E}_t^Q[\exp(-\sum_{j=0}^{h-1} r_{t+j})]$, where E_t^Q indicates the expectation under the risk-neutral measure. Given the dynamic of the short rate in equation (3), the price of a default-free zero-coupon bond is available in closed form and shown in Appendix B.

C. Default Intensity and Loss Given Default

Following the framework set forth by Jarrow and Turnbull (1995), Duffie and Singleton (1997), (1999), and Lando (1998), we model default as a surprise event driven by a Poisson process. The risk-neutral intensity for the Poisson process at time t is defined as λ_t. The risk-neutral probability of surviving at least h periods conditional on no default up until time t is

$$\mathrm{E}_t^Q[1_{\tau > t+h}] = \mathrm{E}_t^Q\left[\exp\left(-\sum_{j=0}^{h-1} \lambda_{t+j}\right)\right]. \tag{4}$$

where $1_{\tau > t+h}$ is an indicator function that is equal to 1 if default happens after time $t+h$, and 0 otherwise.

Similar to the short-rate dynamic, we model the default intensity using a quadratic specification.[10] This functional form ensures that the default intensity is nonnegative. We let the default-intensity dynamic depend on four latent factors, as follows:

$$\lambda_t = (\alpha_0 + \alpha_1 X_{1,t} + \alpha_2 X_{2,t} + \alpha_3 X_{3,t} + \alpha_4 X_{4,t})^2, \tag{5}$$

where α_i for $i = 0, \ldots, 4$ are loading coefficients. Factors $X_{1,t}$ and $X_{2,t}$ drive the short-rate dynamic (see equation (3)) and are identified using the term structure of default-free securities. Factors $X_{3,t}$ and $X_{4,t}$ are firm-specific latent factors driving

[9] See, for example, Longstaff (1989), Constantinides (1992), Ahn, Dittmar, and Gallant (2002), Leippold and Wu (2002), Li and Zhao (2006), Ang, Boivin, Dong, and Loo-Kung (2011), and Doshi et al. (2013) for quadratic term-structure models. See Gourieroux and Monfort (2008) for an application of a discrete time quadratic factor model to mortality intensity modeling.

[10] See also Doshi et al. (2013) for a similar specification of the default intensity.

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the default-intensity dynamic. They are identified using the term structure of CDS spreads.

We assume that when the firm defaults, its loss rate at time t (i.e., LGD) for senior and subordinate CDS contracts is given by

$$\text{LGD}_t^s = \exp(-(\beta_0^s + \beta_1^s X_{1,t} + \beta_2^s X_{2,t} + \beta_5^s X_{5,t})^2), \tag{6}$$

for $s \in \{\text{sen}, \text{sub}\}$. We assume the loading coefficients in the LGD dynamic are different between senior and subordinate CDS contracts. We let β_i^{sen} and β_i^{sub}, for $i = 0, 1, 2, 5$, denote factor loadings in the senior and subordinate LGD dynamics, respectively. Equation (6) shows that the LGD dynamic depends on the latent factors describing the short-rate dynamic $X_{1,t}$ and $X_{2,t}$, and a latent factor $X_{5,t}$ that is specific to the LGD. For each firm, the dynamic of $X_{5,t}$ is identified using the term structure of CDS spreads written on the firm's senior and subordinate bonds. The modeling approaches in equations (5) and (6) imply that the default intensity and the recovery-rate dynamic are related through their dependence on the default-free latent factors $X_{1,t}$ and $X_{2,t}$.

D. Credit Default Swap Valuation

Let S denote the annualized CDS spread. The protection buyer (PB) promises to make payments $S\Delta$ on each coupon date, conditional on no default, where Δ is the time between successive payment dates in years. For simplicity, we assume that the payments are equally spaced. If a credit event occurs, the protection buyer receives a payment from the protection seller (PS), and the contract terminates. The present value of the payment by the protection buyer on a payment date h periods ahead is

$$\text{PB}(t, t+h) = \text{E}_t^Q \left[S\Delta \times \exp\left[-\sum_{j=0}^{h-1} r_{t+j} \right] \times 1_{\tau > t+h} \right], \tag{7}$$

where $1_{\tau > t+h}$ is an indicator function that is equal to 1 if default happens after time $t+h$ for $h > 0$, and 0 otherwise.

Next, we consider the cash flow of the protection seller. When default occurs between time interval $t+h-1$ and $t+h$, the protection seller makes a payment of LGD_{t+h-1}. The expected value of this cash flow from the perspective of protection seller is

$$\begin{aligned} \text{PS}(t, t+h) = {} & \text{E}_t^Q \left[\text{LGD}_{t+h-1} \times \exp\left(-\sum_{j=0}^{h-1} r_{t+j} \right) \times 1_{\tau > t+h-1} \right] \\ & - \text{E}_t^Q \left[\text{LGD}_{t+h-1} \times \exp\left(-\sum_{j=0}^{h-1} r_{t+j} \right) \times 1_{\tau > t+h} \right]. \end{aligned} \tag{8}$$

The expectations in equations (7) and (8) can be solved analytically using iterated expectations. To save space, we provide their solutions in Appendix C.

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The CDS spread level is set such that the present value of the payments by the protection buyer is equal to the present value of the payments by the protection seller:

$$\sum_{h=1}^{T/\Delta} \mathrm{PB}(t, t+h\Delta N) \quad = \quad \sum_{k=1}^{N\times T} \mathrm{PS}(t, t+k), \tag{9}$$

where N refers to the number of trading days in a year, Δ refers to the time period between two successive premium payments, and T refers to the maturity in years of the CDS contract. The time period for payments between two successive premium payments is $\Delta = 0.25$ years. This corresponds to the payment frequency of the U.S. CDSs used in our study.

E. Market Prices of Risk

So far, we have discussed the model under the risk-neutral measure Q. This section specifies the market prices of risk for the model. To change from the risk-neutral measure to the physical measure, we specify the Radon–Nikodym derivative to take the form

$$\frac{\Delta P}{\Delta Q} \quad = \quad \frac{\exp\left(-\sum_{i=1}^{5} \Lambda_{i,t}\varepsilon_{i,t+1}\right)}{\mathrm{E}_t\left[\exp\left(-\sum_{i=1}^{5} \Lambda_{i,t}\varepsilon_{i,t+1}\right)\right]}, \tag{10}$$

where $\Lambda_{i,t}$ is the price-of-risk parameter corresponding to shock $\varepsilon_{i,t+1}$. We follow the term-structure literature and assume time-varying prices of risk that depend on the state variables, that is, $\Lambda_{i,t} = \lambda_{0,i} + \lambda_{1,i} X_{i,t}$ for $i = 1, \ldots, 5$, where $\lambda_{0,i}$ and $\lambda_{1,i}$ are scalar coefficients. Given the assumption of the Radon–Nikodym derivative in equation (10) and the risk-neutral dynamic in equation (2), the dynamic of each state variable $X_{i,t}$, for $i = 1, \ldots, 5$, under the physical measure is

$$X_{i,t+1} \quad = \quad \mu_i^P + \rho_i^P X_{i,t} + \sigma_i \varepsilon_{i,t+1}, \tag{11}$$

where $\mu_i^P = \mu_i - \sigma_i \lambda_{0,i}$, and $\rho_i^P = \rho_i - \sigma_i \lambda_{1,i}$. The measure change affects only the mean and the autoregressive term of each state variable. Their physical-measure parameters are denoted with the superscript P.

IV. Data and Estimation

A. Data

We obtain data used for estimating the default-free term-structure model from Bloomberg. We use the daily 6-month London Interbank Offered Rate (LIBOR) and the daily swap rates with maturities of 1, 2, 3, 4, 5, 7, and 10 years. Daily CDS data are obtained from Markit. Our sample consists of single-name CDS contracts that are denominated in U.S. dollars between Jan. 1, 2001, and May 31, 2012. We keep daily CDS spreads on senior and subordinate debts with maturities of 1, 3, 5, 7, and 10 years. We eliminate firms that have less than 1 year

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of data for both senior and subordinate contracts before Dec. 2007, which is the start of the recession. This filter ensures that firms in our sample have a sufficiently long time series of CDS data prior to the financial crisis, thereby enabling comparison of their recovery rates before and during the financial crisis. The final sample consists of 46 firms.

Table 1 describes the firms in our sample. We report the sample averages and standard deviations of daily CDS spreads for senior and subordinate contracts. Figure 1 plots the time-series averages of CDS spreads across firms. To save space, we only report summary statistics for CDS contracts with 1-, 5-, and 10-year maturities. The columns next to each firm's name report the firm's ticker, its overall entity ratings, and its start and end dates of CDS data. The overall entity rating corresponds to the firm's sample monthly average of Standard & Poor's (S&P) long-term entity credit ratings from Compustat. For each firm-month, we assign an integer value to each rating class from 1 (AAA) to 6 (B). The monthly ratings are averaged over the sample and rounded off to the nearest integer, then translated back to a character rating.

Table 1 shows that firms in our sample are distributed across the rating categories of AAA to B, with a relatively higher concentration in the A and BB categories. Freddie Mac and Fannie Mae have the lowest average spreads for senior and subordinate contracts in our sample, reflecting their higher creditworthiness relative to the other firms. In most cases, we find that average CDS spreads increase with maturity, indicating an upward-sloping term structure of credit spreads for both senior and subordinate contracts. The exceptions are Bear Stearns, Countrywide Financial, and Lehman Brothers, which have data starting only after Dec. 2006. We find that these firms have exceptionally large spreads at short maturities starting in mid-2007, suggesting that they were in distress. The start and end periods of CDS data vary across firms as dictated by the availability of both senior and subordinate contracts.

B. Estimation Strategy and Model Parameters

This section provides an overview of our estimation strategy and summarizes the model parameters that we estimate. We leave discussions about the filtering and estimation method to the next subsection.

We estimate the model using a panel of default-free securities, senior CDS spreads, and subordinate CDS spreads. The estimation is executed in two steps. In the first step, we estimate the latent factors X_1 and X_2 from the term structure of LIBOR swap rates. Because the risk-free-rate dynamic is common to all firms, X_1 and X_2 are estimated only once and assumed to be the same across all firms. We refer to them as the default-free factors because these latent state variables are filtered from panel data of default-free securities. Panel A of Table A1 in Appendix C shows the parameters that are identified using the LIBOR swap rates. Overall, the first-stage estimation produces 13 parameter estimates and yields daily filtered time series of X_1 and X_2.

In the second step, we estimate the physical and risk-neutral credit-risk parameters for each of the 46 firms. Following Duffee (1999), the first-stage parameters and the filtered state variables X_1 and X_2 are taken as observables and applied to fit the term structure of CDS spreads. The second-stage estimation is

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TABLE 1

Summary Statistics: Individual Firms

Table 1 describes the 46 firms in our sample. We obtain data on the daily credit default swap (CDS) spreads from Markit. The column labeled *Data Availability* indicates the start and end dates of the CDS-spread data in our sample in month/year format. For each firm, we report the sample mean and standard deviation (Std. Dev.) of the daily CDS spreads (in basis points (bps)) on senior and subordinate debts for three maturities: 1 year, 5 years, and 10 years. An asterisk next to the firm's name indicates that it is a financial firm. We define financial firms as those categorized under the finance, insurance, and real estate sectors following the classification on Kenneth French's Web site. We report, next to each firm, its official ticker and its corresponding overall credit-rating level. We obtain monthly Standard & Poor's (S&P) long-term entity credit ratings from Compustat.

				Senior, Market Spreads (bps)						Subordinate, Market Spreads (bps)					
				1 Yr		5 Yr		10 Yr		1 Yr		5 Yr		10 Yr	
Firm Name	Ticker	Entity Ratings	Data Availability	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.
Amkor Tech Inc	AMKR	B	05/03–08/10	408	277	641	250	689	193	548	325	808	302	866	278
Bear Stearns Co Inc*	BSC	A	12/06–07/08	135	175	113	101	109	84	152	187	133	112	130	94
Best Buy Co Inc	BBY	BBB	04/03–05/12	66	79	144	137	168	141	79	83	160	137	184	146
Boyd Gaming Corp	BYD	BB	12/05–05/12	567	666	863	617	813	485	590	725	874	633	834	521
Cap One Bk*	COF	BBB	04/04–05/12	88	123	101	91	101	72	110	154	121	108	121	83
Citigroup Inc*	C	AA	02/01–05/12	83	141	102	116	111	104	121	212	141	172	151	153
Cox Comms Inc	COX	BBB	03/01–05/12	62	93	96	76	115	68	100	165	136	147	151	156
Ctrywde Finl Corp*	CFC	BBB	12/06–11/08	523	590	299	266	252	206	576	636	322	275	280	208
D R Horton Inc	DHI	BB	03/06–01/08	157	180	190	134	232	102	194	200	229	145	273	111
Fed Home Ln Mtg Corp*	FRE	AAA	12/02–09/08	10	9	21	15	28	14	29	57	48	46	55	40
Fed Natl Mtg Assn*	FNM	AAA	09/02–09/08	10	9	22	14	29	13	30	57	51	44	60	39
Goldman Sachs Gp Inc*	GS	A	12/06–05/12	150	130	158	97	158	88	189	147	203	123	200	114
Harrahs Entmt Inc	HET	BB	07/04–11/10	672	1,515	1,051	1,560	965	1,298	705	1,541	1,061	1,494	1,002	1,308
Health Mgmt Assoc Inc	HMA	BB	01/05–05/12	177	204	331	250	353	229	210	236	371	292	388	258
Iron Mtn Inc	IRM	BB	05/05–05/12	131	70	334	94	374	90	133	73	344	98	387	95
JPMorgan Chase & Co*	JPM	A	01/01–05/12	38	39	64	42	74	39	58	58	85	60	97	55
KB Home	KBH	BB	06/03–05/12	224	181	356	212	371	203	265	196	399	212	415	202
Kerr-McGee Corp	KMG	BB	03/06–05/12	23	20	51	49	67	44	31	25	63	58	81	51

(continued on next page)

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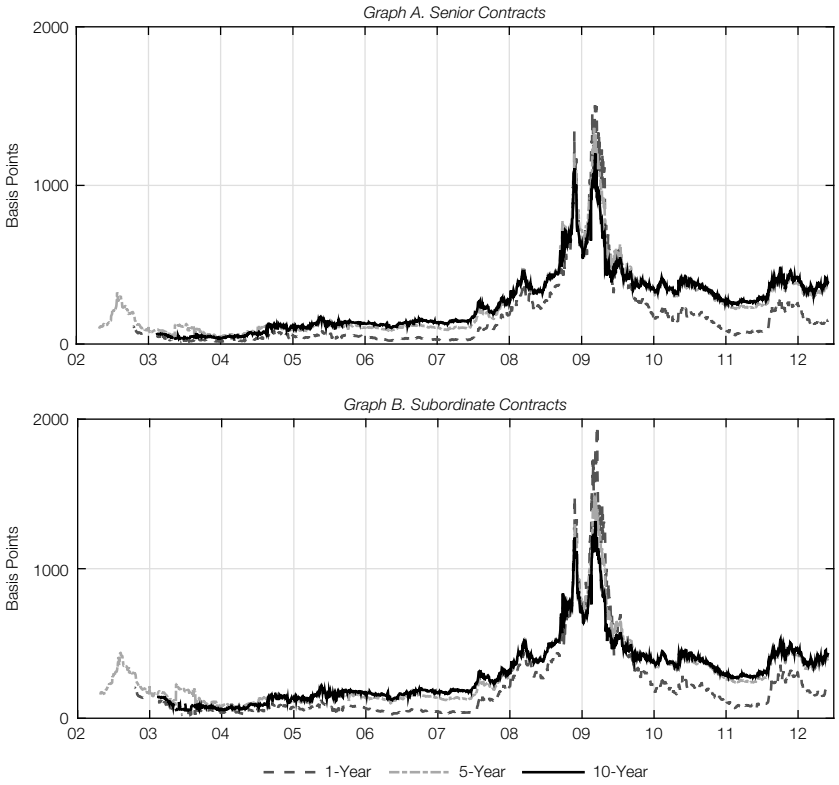
TABLE 1 (continued)
Summary Statistics: Individual Firms

Firm Name	Ticker	Entity Ratings	Data Availability	Senior, Market Spreads (bps) 1 Yr Mean	1 Yr Std. Dev.	5 Yr Mean	5 Yr Std. Dev.	10 Yr Mean	10 Yr Std. Dev.	Subordinate, Market Spreads (bps) 1 Yr Mean	1 Yr Std. Dev.	5 Yr Mean	5 Yr Std. Dev.	10 Yr Mean	10 Yr Std. Dev.
Kohls Corp	KSS	BBB	01/04–05/12	41	52	86	65	104	59	47	58	87	64	105	60
L 3 Comms Corp	LLL	BBB	08/05–05/12	61	33	146	44	176	43	63	32	152	44	179	43
Lehman Bros Hldgs Inc*	LEH	A	01/07–09/08	198	206	143	104	129	81	216	217	164	122	147	96
Loews Corp*	L	A	08/03–05/12	29	22	57	32	72	31	37	35	69	41	86	40
Merrill Lynch & Co Inc*	MER	A	03/06–05/12	174	166	180	125	175	112	233	230	239	182	232	165
MGM Mirage	MGM	BB	06/04–06/10	662	1586	705	1048	688	898	807	1896	799	1080	763	855
Morgan Stanley*	MS	A	06/04–05/12	162	246	156	155	154	131	194	260	193	188	190	163
Navistar Intl Corp	NAV	BB	03/05–08/10	305	284	465	258	502	215	310	290	483	248	519	210
Neiman Marcus Gp Inc	NMG	B	01/06–05/12	363	532	606	468	603	369	421	589	659	448	657	343
Office Depot Inc	ODP	BB	12/03–05/12	351	671	388	469	402	375	358	666	391	444	404	378
Omnicare Inc	OCR	BB	08/04–05/12	117	82	269	115	305	107	120	78	282	116	315	105
Sanmina SCI Corp	SANM	B	01/05–05/12	217	284	472	357	490	309	486	912	730	616	728	485
Sinclair Broadcast Gp Inc	SBGI	BB	10/04–05/12	479	850	554	501	547	362	483	845	564	465	550	321
Solectron Corp	SLR	BB	04/06–10/07	39	18	157	56	211	56	69	25	278	81	349	84
Sungard Systems	SNDT/SDS	B	08/06–08/10	229	187	462	201	508	158	334	255	571	219	636	187
SunTrust Bks Inc*	STI	A	09/04–03/11	81	92	101	97	101	92	151	169	157	157	150	149
Tesoro Corp	TSO	BB	11/04–05/12	127	121	278	175	306	163	131	117	295	163	319	149
Time Warner Inc	TWX	A	04/02–05/12	24	22	103	107	91	29	37	32	149	151	121	49
TJX Cos Inc	TJX	A	08/03–07/11	21	24	48	30	63	30	31	34	59	32	79	33
Toll Bros Inc	TOL	BBB	04/04–05/12	105	100	158	73	179	63	127	113	183	77	204	62
Triad Hosps Inc	TRI	BB	09/04–03/08	71	37	161	61	220	62	119	62	228	81	296	68
Tribune Co	TRB	BBB	10/03–12/08	827	1,787	908	1,670	895	1,540	832	1,723	917	1,667	899	1,554
TRW Automotive Inc	TRW	BBB	10/05–08/10	452	873	537	570	529	439	459	801	552	512	556	403
U S Bancorp*	USB	AA	11/04–03/12	36	39	65	56	78	66	66	64	92	76	99	80
Utd Rents Inc	URI	BB	07/04–04/11	393	306	567	278	587	232	422	246	627	243	662	224
WA Mut Inc*	WM	A	05/04–09/08	284	935	188	540	169	409	295	908	223	575	214	522
Wachovia Corp*	WB	A	03/02–12/08	39	94	52	86	61	83	53	125	67	107	79	107
Wells Fargo & Co*	WFC	AA	10/03–05/12	45	59	68	57	75	51	69	89	94	86	102	78

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FIGURE 1
Average Market-Observed Spreads

In Figure 1 we plot the daily average credit default swap (CDS) spreads (in basis points) across firms in our sample (see Table 1). Graphs A and B plot daily time-series averages of the market spread for senior and subordinate CDS contracts, respectively, with maturities of 1, 5, and 10 years. We obtain daily CDS spreads data from Markit. The data availability of CDS spreads differs across firms. Thus, the number of firms used to calculate the time-series averages varies over time. It increases over time as more firms have CDSs trading on their debts in the later part of the sample period. The start of the time-series averages differs between different maturities and seniorities. We require that CDS data are available on at least five firms to calculate the daily average.



conducted firm by firm. This helps us identify the dynamic of X_3, X_4, and X_5. We refer to them as the credit-risk factors because these latent state variables are filtered from a panel of CDS data for each firm. Panel B of Table A1 in Appendix C summarizes the parameters that are estimated firm-by-firm using the CDS data.

We facilitate the firm-by-firm estimation step by normalizing parameters that drive the dynamics of default intensity and LGD rate. This method reduces redundant parameters from the model's estimation and does not impact our conclusions. Equation (5) shows that X_3 and X_4 are latent factors driving the dynamic of default intensity. The default-intensity dynamic of senior and subordinate spreads written on the same entity are identical. Therefore, the dynamics of X_3 and X_4 are identified by the term structures of both senior *and* subordinate CDS spreads. For parsimony, we set $\alpha_3 = \alpha_4 = 1$ in equation (5). This implies that the level of

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latent factors X_3 and X_4 will be scaled to reflect the normalization of α_3 and α_4, respectively.

Similarly, we can reduce the number of parameters to be estimated for the LGD dynamic shown in equation (6) through a simple re-parametrization. Here, the dynamic of latent factor X_5 is identified through the *relative* term structure of senior and subordinate CDS spreads (i.e., their CDS spread ratios across various maturities).[11] We normalize $\beta_5^{\text{sen}} = 1$ in equation (6), which is the loading coefficient on the senior LGD dynamic on X_5, and leave the loading coefficient β_5^{sub} on the subordinate LGD dynamic as a free parameter to be estimated. As a result, the latent factor X_5 and the coefficient β_5^{sub} will be scaled to reflect the normalization of β_5^{sen}.

Finally, we further reduce the number of parameters to be estimated by normalizing the physical-measure drift, that is, μ_i^P, of the credit-risk state variables X_3, X_4, and X_5. Here, we set $\mu_3^P = \mu_4^P = \mu_5^P = 0$. Panel B of Table A1 in Appendix C summarizes the parameter normalization. Overall, the re-parametrization of model parameters reduces the number of risk-neutral and physical-measure parameters to be estimated in the firm-by-firm estimation from 28 down to 22.

C. Estimation Method

We use the unscented Kalman filter to extract latent-state variables on a daily basis. The state-space system in the first and second estimation steps can be written in a similar form using matrix notation as follows:

$$X_t = \mu^P + \rho^P X_{t-1} + \Sigma \varepsilon_t, \tag{12}$$

$$Y_t = G(X_t) + u_t, \tag{13}$$

where X_t is an N-dimensional vector of latent factors, and Y_t is a D-dimensional vector of observables.[12] The values for N and D differ between the first and second estimation steps, as detailed in the following discussion.

The state propagation equation (12) is a linear Gaussian process that describes the dynamic of latent variables under the physical measure. Equation (13) is the measurement equation specified under the risk-neutral measure. It describes how latent variables affect the pricing of observables (i.e., default-free securities or CDS spreads) through the nonlinear pricing function G. Model parameters in the pricing function G correspond to risk-neutral parameters. We use the unscented Kalman filter because it can accommodate nonlinearity in the measurement equation.[13] We assume an additive D-dimensional normally distributed error term, u_t, with an unconditional covariance matrix (i.e., $\mathrm{E}[u_t^T u_t] = \Omega$) that is diagonal.

[11] Section VII.B provides more details on the identification of senior and subordinate LGD dynamics using a static analysis. We show that the ratios of senior-to-subordinate CDS spreads at various maturities help identify the LGD dynamics.

[12] Here, μ^P is an N-dimensional vector containing the physical-measure drift parameters μ_i^P, for $i = 1 \ldots N$, and ρ^P is an $N \times N$ diagonal matrix whose elements are ρ_i^P, for $i = 1 \ldots N$.

[13] We use the square-root unscented Kalman filter proposed by Van der Merwe and Wan (2001). Other studies that apply this unscented Kalman filter to estimate the default-free term-structure and credit-risk models include Carr and Wu (2010), Chen et al. (2008), and Christoffersen, Dorion, Jacobs, and Karoui (2014).

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The first estimation step extracts daily default-free latent factors and their parameters from the term structure of default-free securities. In this case, X_t in the propagation equation (12) is a 2×1 vector of default-free latent factors $[X_{1,t}\ X_{2,t}]'$, and Y_t is an 8×1 vector of observables consisting of a 6-month LIBOR and LIBOR swap rates at seven maturities. In the second firm-by-firm estimation step, we estimate credit-risk latent factors and the model's parameters that describe the firm's default-intensity and LGD dynamics. Here, X_t in equation (12) is a 3×1 vector represented by $[X_{3,t}\ X_{4,t}\ X_{5,t}]'$, and Y_t is a 10×1 vector of observables consisting of senior and subordinate CDS spreads at five maturities.

In both estimation steps, we construct the log-likelihood value at time t assuming normally distributed forecasting errors, as follows:

$$l_t(\Theta) = -\frac{1}{2}\ln|\overline{\Omega}_t| - \frac{1}{2}(Y_t - \overline{Y}_t)^T(\overline{\Omega}_t)^{-1}(Y_t - \overline{Y}_t), \tag{14}$$

where $\overline{Y}_t$ represents the ex ante forecasts conditional on time $t-1$ of Y_t, and $\overline{\Omega}_t$ is the conditional covariance matrix obtained from the unscented Kalman filter. Model parameters are chosen to maximize the log likelihood of the data series. For more details of the algorithm for unscented Kalman filter, we refer the reader to Christoffersen et al. (2014).

V. Empirical Results

A. Default-Free Term Structure

Table 2 presents estimation results for the default-free term structure. Panel A reports the parameter estimates describing the dynamics of latent term-structure factors, X_1 and X_2, as well as their loadings on the instantaneous risk-free rate (see equation (3)). All estimates in Table 2 are statistically significant. The first factor is closely related to the long-term interest rate. The correlation between the 10-year zero rate and the first factor is approximately 90%, whereas the correlation between changes in the 10-year zero rate and changes in the first factor is approximately 83.16%. The second factor is closely related to the difference between the long-term zero rates and the short-term zero rates. The correlation between the difference in 10-year zero rates and 6-month zero rates with the second factor is -91%. Overall, our estimates suggest the first factor is closely related to the level of the yield curve, whereas the second factor is closely related to the slope of the yield curve, which is consistent with Duffee (1999). Panel B of Table 2 reports root-mean-squared errors (RMSEs) for different maturity yields. The RMSEs range from 7.15 to 21.03 basis points (bps) across different maturity contracts.

B. Firm-by-Firm Parameter Estimates and Model Fit

Table 3 presents the distribution of the parameter estimates across the 46 firms in the sample. As mentioned, the CDS valuation model consists of five latent factors. The first two factors (i.e., X_1 and X_2) are obtained from the default-free term-structure estimation in the first stage and are identically applied to all firms. The remaining three factors (i.e., X_3, X_4, and X_5) are estimated independently for each firm as part of the second-stage estimation. See Panel B of Table A1 in

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TABLE 2

Default-Free Term Structure's Estimates and Properties

Table 2 reports the estimation results of the default-free term structure. We apply the unscented Kalman filter to estimate a 2-factor latent model on the term structure of the default-free rate. The factor dynamics and short-rate loadings are estimated using the 6-month London Interbank Offered Rate (LIBOR), together with swap rates with maturities of 1, 2, 3, 4, 5, 7, and 10 years. Panel A reports parameter estimates of the default-free term-structure factors. The standard error is reported (in parentheses) beneath each estimate. Panel B reports the root-mean-squared errors (RMSEs) and the standard deviation of the measurement error equation (ME SD) for the default-free term structure.

Panel A. Default-Free Term-Structure Factor Loadings and Dynamics

	X_1	X_2
δ_0	0.043 (9.94E-04)	
δ_1	0.345 (1.89E-03)	
δ_2		1.005 (3.71E-03)
ρ	0.999 (2.25E-04)	0.998 (5.57E-04)
ρ^P	0.997 (3.22E-03)	0.997 (8.33E-03)
$\mu \times 100$	−0.002 (1.81E-04)	−0.004 (1.43E-05)
$\mu^P \times 100$	−0.012 (2.44E-03)	−0.006 (2.06E-03)
$\sigma \times 100$	0.035 (3.17E-04)	0.009 (9.85E-05)

Panel B. RMSE and Measurement Error Standard Deviation (bps)

	6 Months	1 Year	2 Years	3 Years	4 Years	5 Years	7 Years	10 Years
RMSE	21.03	7.51	7.43	6.17	4.68	4.67	7.56	12.52
ME SD	22.28	7.77	9.12	8.88	7.94	8.78	10.12	13.93

Appendix C for a summary of the parameters estimated in the second stage. Firm-by-firm estimation produces 46 sets of parameter estimates, one for each firm. For conciseness, Table 3 reports the cross-sectional distribution of the parameter estimates.

Panel A of Table 3 reports the distribution of factor loadings for the default intensity, the LGD of the senior contracts, and the LGD of the subordinate contracts. For each factor loading, we report the cross-firm mean, standard deviations, and distributional percentiles at the 25th, 50th, and 75th levels, respectively. Recall that we normalize factor the loadings of the intensity dynamic on X_3 and X_4 to 1 (i.e., $\alpha_3 = \alpha_4 = 1$ in equation (5)), and the factor loading of the senior LGD is normalized to 1 (i.e., $\beta_5^{\text{sen}} = 1$ in equation (6)). Therefore, their distributions are not reported. The last row in Panel A reports the percentage of cross-firm estimates that are statistically significant at the 5% level.

Panel A of Table 3 shows a significant variation in the factor loadings of the latent factors X_1 and X_2 (i.e., α_1 and α_2 in equation (5)). This is partly because we have a large cross section of firms, and it is the cross-sectional difference in spreads that drives the variation in their loadings. The intensity dynamic loads significantly on the default-free latent variables X_1 and X_2, suggesting a strong dependence of default risk on the default-free term structure. Similarly, the LGD dynamics significantly load on X_1 and X_2 for most of the firms in our sample. Looking at the estimates of the constant term for LGDs (i.e., β_0 in equation (6)), we find that they are substantially larger for senior contracts relative

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TABLE 3
Parameter Estimates and Model Fit

Table 3 reports the distribution of parameter estimates obtained from firm-by-firm estimations on 46 individual firms. For each firm, we estimate the credit default swap (CDS) valuation model using the time-series data of senior and subordinate CDS contracts with maturities of 1, 3, 5, 7, and 10 years. The estimation exercises yield 46 sets of parameter estimates. We report the distribution of parameter estimates. See Table A1 in Appendix C for a summary of the model parameters. Panel A reports the factor loadings of the default intensity, the senior loss given default (LGD), and the subordinate LGD on the latent factors. In Panel B, we report the distribution of parameter estimates that drive the factor dynamics of X_3, X_4, and X_5. The dynamics of factors X_1 and X_2 are estimated in the first stage using the 6-month London Interbank Offered Rate (LIBOR) and swap rates (see Table 2). For each estimate, we report the cross-firm mean, standard deviations, and distributional percentiles at the 25%, 50%, and 75% levels. The last row in Panels A and B reports the percentage of cross-firm estimates that are statistically significant at the 5% level. Panel C reports the distribution of the measurement errors from fitting the senior and subordinate CDS contracts across different maturities.

Panel A. Distribution of Factor-Loading Parameters

	Default Intensity (λ_t)			Senior LGD			Subordinate LGD			
Statistics	Constant $\alpha_0 \times 100$	X_1 $\alpha_1 \times 100$	X_2 $\alpha_2 \times 100$	Constant β_0^{sen}	X_1 β_1^{sen}	X_2 β_2^{sen}	Constant β_0^{sub}	X_1 β_1^{sub}	X_2 β_2^{sub}	X_5 β_5^{sub}
Mean	0.590	−1.656	−6.105	0.979	−0.429	2.991	0.549	−0.437	1.156	0.648
25%	0.076	−5.128	−13.18	0.496	−0.997	−5.602	0.072	−2.558	−6.181	0.235
50%	0.436	−1.048	−1.639	0.879	−0.206	−1.580	0.433	−0.101	0.091	0.683
75%	1.417	4.329	12.587	1.126	0.527	4.242	0.806	1.905	1.712	0.925
Std. dev.	1.311	19.669	48.198	0.824	3.750	44.795	0.625	4.254	19.588	0.789
Percent significant	89%	98%	90%	100%	80%	93%	87%	87%	96%	91%

Panel B. Distribution of Parameters in the Factor Dynamics of X_3, X_4, and X_5

	Intensity Factor X_3				Intensity Factor X_4				Recovery Factor X_5			
Statistics	ρ_3	ρ_3^P	$\mu_3 \times 100$	$\sigma_3 \times 100$	ρ_4	ρ_4^P	$\mu_4 \times 100$	$\sigma_4 \times 100$	ρ_5	ρ_5^P	$\mu_5 \times 100$	$\sigma_5 \times 100$
Mean	0.983	0.967	−0.005	0.038	0.963	0.974	−0.001	0.016	0.995	0.965	−0.044	2.042
25%	0.994	0.984	−0.001	0.010	0.970	0.983	0.000	0.008	0.997	0.986	−0.040	0.841
50%	0.998	0.995	0.000	0.020	0.998	0.993	0.000	0.011	0.999	0.998	−0.013	1.611
75%	0.999	1.000	0.000	0.029	1.000	1.000	0.000	0.021	1.000	1.000	−0.001	2.497
Std. dev.	0.032	0.035	0.018	0.073	0.040	0.032	0.003	0.014	0.011	0.036	0.100	1.633
Percent significant	100%	100%	90%	100%	98%	100%	72%	100%	100%	100%	80%	100%

Panel C. Distribution of Measurement Error Standard Deviations (bps)

	Senior CDS					Subordinate CDS				
Statistics	1 Yr	3 Yr	5 Yr	7 Yr	10 Yr	1 Yr	3 Yr	5 Yr	7 Yr	10 Yr
Mean	26.59	17.63	16.41	15.94	15.37	25.77	17.84	15.45	14.69	14.86
25%	3.92	3.22	3.21	3.67	3.69	4.57	4.82	4.40	4.57	5.11
50%	8.25	7.26	6.55	6.22	6.82	11.41	8.25	7.18	7.79	8.62
75%	31.72	17.71	16.38	15.02	12.63	30.93	20.84	19.32	17.92	22.07
Std. dev.	14.74	7.43	6.98	5.98	4.43	13.41	8.47	7.60	6.77	8.95

to subordinate contracts. This finding is in line with our expectation of larger expected recovery for senior contracts. The larger constant in the LGD dynamic for senior contracts implies that it has a lower LGD rate relative to subordinate contracts.

In Panel B of Table 3, we report the distribution of parameter estimates that drive the dynamic of factors X_3, X_4, and X_5. As mentioned, the parameters driving the dynamic of factors X_1 and X_2 are estimated using the default-free term structure and are reported in Table 2. We report parameter estimates under the physical and risk-neutral measures. Recall that the physical-measure drifts, μ^P, of latent factors X_3, X_4, and X_5 are normalized to 0, and hence they are not reported. Panel B shows that the LGD-specific latent factor, X_5, is highly persistent, with most estimates of ρ being close to 1. Panel C reports the distribution of standard deviations of the measurement error. The measurement error's standard deviation is the highest for 1-year CDS contracts, whereas it is relatively comparable for the

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remaining maturities. Nevertheless, the measurement error's standard deviation is generally low.

We examine the CDS model's performance in fitting CDS data using two metrics: the relative RMSE and the absolute percentage error (APE). Table 4 reports average values of the relative RMSE and APE grouped by various categories. To save space, only results for senior and subordinate CDS spreads with 1-, 5-, and 10-year maturity are reported. The average relative RMSE across firms is between 10% and 19%. The average APE across firms is between 7% and 15%. Importantly, the model fit measured using these two metrics is similar for both seniorities. This finding suggests our CDS model is able to fit senior and subordinate contracts equally well.

We find that 1-year CDS contracts have the largest RMSE and APE, whereas for CDS contracts at longer default horizons, the RMSE values are comparable. Table 4 shows that, on average, the mean relative RMSE and the mean APE are lower for firms in higher rating categories. The breakdown based on industry categories shows that the model performs equally well in fitting the term structure of CDS spreads across industries.

TABLE 4
Model Fit

Table 4 reports the average in-sample errors from the firm-by-firm estimation of the credit default swap (CDS) valuation model. We report two measures of average in-sample fit: the relative root-mean-squared error (RMSE) and the mean absolute percentage error (APE). The relative RMSE for maturity h contracts is calculated as

$$\text{Relative RMSE} = \sqrt{\frac{1}{G}\sum_{j=1}^{G}\left(\frac{\text{CDS}(j,h)-\text{CDS}^{M}(j,h)}{\text{CDS}(j,h)}\right)^{2}},$$

where G is the number of observations used, and $\text{CDS}(j,h)$ and $\text{CDS}^{M}(j,h)$ are the market-observed and model-implied spreads, respectively. Similarly, the APE is calculated as

$$\text{APE} = \frac{1}{G}\sum_{j=1}^{G}\frac{|\text{CDS}(j,h)-\text{CDS}^{M}(j,h)|}{\text{CDS}(j,h)}.$$

We report the average values of relative RMSE and mean APE in percentage terms for three maturities: 1 year, 5 years, and 10 years. The results are reported for the overall sample, as well as for subsamples grouped by credit rating and industry sector. The last column reports the number of firms in each category. We obtain monthly Standard & Poor's (S&P) long-term entity credit ratings from Compustat. Industry classifications are obtained from Kenneth French's Web site. *FIR* refers to firms in the finance, insurance, and real estate sectors. *Mining/Constr.* includes firms in the mining and construction industries.

	Relative RMSE (%)						APE (%)						
	Senior			Subordinate			Senior			Subordinate			
Category	1 Yr	5 Yr	10 Yr	1 Yr	5 Yr	10 Yr	1 Yr	5 Yr	10 Yr	1 Yr	5 Yr	10 Yr	No. of Firms
Overall	18.9	9.9	11.2	19.1	10.3	11.3	14.4	7.6	8.5	14.8	7.9	8.6	46
AAA/AA	18.6	8.8	11.1	19.0	8.0	7.8	14.4	6.9	8.3	14.8	6.4	5.8	5
A	18.5	11.5	12.6	19.6	12.0	14.4	13.8	8.9	9.6	15.0	9.2	11.0	12
BBB	18.9	10.6	12.0	19.4	10.9	12.1	14.5	8.0	9.2	15.0	8.3	9.1	9
BB	19.0	9.1	10.5	18.4	9.3	9.9	14.5	6.8	7.7	14.3	7.1	7.5	16
B	19.7	8.6	8.9	19.4	10.1	10.9	15.6	6.8	6.9	15.5	8.0	8.8	4
Mining/Constr.	15.9	9.0	10.7	18.7	9.9	9.8	11.4	7.0	8.1	14.3	8.0	7.5	4
Manufacturing	19.7	9.9	11.4	18.4	11.5	10.4	15.0	7.4	8.5	14.2	8.8	8.2	8
Communications	19.6	11.0	10.3	19.7	10.0	10.8	15.5	9.0	7.7	15.6	7.6	7.9	3
Retail trade	19.4	8.1	10.7	19.2	10.4	11.5	15.0	6.4	7.9	15.1	8.2	8.6	6
FIR	18.4	11.1	12.0	19.4	10.0	12.0	13.8	8.4	9.1	14.9	7.5	9.1	17
Services	20.0	9.0	10.6	18.8	9.8	11.8	15.8	6.8	8.0	14.7	7.4	8.9	8

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C. Term Structure of Recovery Rates

We first examine cross-sectional differences in expected recovery rates. Then, we examine the time-varying dynamic of expected recovery rates, with a particular focus on financial firms.

1. Expected Recovery Rates across Firms

Using the parameters and the factor dynamics estimated from the CDS valuation model for each firm, we calculate the time series of *expected recovery*, which is defined as

$$R^{\text{EXP}}(t,h) \quad = \quad 1-\text{E}_t^{\mathcal{Q}}[\text{LGD}_{t+h-1}], \tag{15}$$

where $\text{E}_t^{\mathcal{Q}}[\text{LGD}_{t+h-1}]$ follows from equation (6). The expected recovery, $R^{\text{EXP}}(t,h)$, represents the market's expectation of the recovery rate at time t, if the firm were to default at time $t+h$ for $h \geq 0$. We calculate expected recovery rates for various horizons. However, to save space, we report results for three horizons: 1-, 5-, and 10-year, which together comprehensively represent the term structure of expected recovery.

Table 5 reports the time-series averages of expected recovery and default probability for the overall sample and for firms grouped by different ratings and industries. The results are reported separately for senior and subordinate contracts. The table also reports the time-series averages of the 1-year *default probabilities* implied by the model.[14] The 1-year default probability at time t is calculated as $1-\text{E}_t^{\mathcal{Q}}[1_{\tau>t+h}]$, where $\text{E}_t^{\mathcal{Q}}[1_{\tau>t+h}]$ is the survival probability given in equation (4) and evaluated at $h=1$ year. To see how the default probability is related to the expected recovery rates, we report the time-series correlations between the 1-year expected recovery and the 1-year default probability for senior and subordinate contracts.

We find that the expected recovery for the senior contracts is between 33.25% and 39.54%. As for the subordinate CDS contracts, the overall expected recovery falls between 20.64% and 24.65%, which is slightly more than half of the levels implied by senior contracts. It is useful to compare our estimates of expected recovery that are evaluated under the risk-neutral measure to the realized estimates found in the literature. Altman and Kishore (1996) back out the realized recovery from a sample of 696 defaulted bonds between 1971 and 1995. They find an average realized recovery of 47.65% for the senior unsecured bonds and an average realized recovery of 31.34% for the subordinate bonds. In comparison to Altman and Kishore (1996), our results suggest that there is a *positive* recovery risk premium of 8%–14% for senior contracts and 7%–11% for subordinate contracts. The positive risk premium for the recovery rate that we find supports the view that bond investors are risk averse to how much they can recover after a firm defaults.

Table 5 shows that the correlations between the 1-year expected recovery and the 1-year default probability are negative, suggesting that on average, the recovery rate at default decreases when the likelihood of default rises. This finding

[14] Time-series averages for each firm are reported in Table IA1 of the Supplementary Material.

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TABLE 5

Expected Recovery and Default Probability Grouped by Ratings and Industries

Table 5 reports the time-series averages of expected recovery and default probability implied by the credit default swap (CDS) valuation model. The results are reported for the overall sample as well as for subsamples grouped by overall credit rating and industry sector. The average expected recovery for senior and subordinate CDS contracts is reported at 1-, 5-, and 10-year horizons. *Default Prob.* reports the average default probability for a 1-year horizon. The two columns under *Correlation* report the averages of the time-series correlations between the 1-year expected recovery and the 1-year default probability for senior and subordinate contracts. The last column reports the number of firms in each category. We obtain monthly Standard & Poor's (S&P) long-term entity credit ratings from Compustat. Industry classifications are obtained from Kenneth French's Web site. *FIR* refers to firms in the finance, insurance, and real estate sectors. *Mining/Constr.* includes firms in the mining and construction industries.

	Default Prob. (%)	Expected Recovery (%)						Correlation (%)		
		Senior			Subordinate			Senior	Sub.	
Category	1 Yr	1 Yr	5 Yr	10 Yr	1 Yr	5 Yr	10 Yr	1 Yr	1 Yr	No. of Firms
Overall	3.90	39.54	34.02	33.25	24.65	21.16	20.64	−20.85	−18.64	46
AAA/AA	1.17	61.85	49.94	45.84	33.77	30.42	28.76	−27.83	−21.31	5
A	2.03	36.08	32.01	30.34	20.35	16.12	15.01	−14.61	−14.46	12
BBB	4.22	39.25	34.31	35.32	27.42	24.38	24.74	−19.71	−13.73	9
BB	5.04	36.04	29.91	29.30	24.68	20.94	20.63	−22.45	−23.03	16
B	7.61	36.73	35.92	37.37	19.74	18.34	18.17	−27.06	−21.27	4
Mining/Constr.	2.91	36.79	29.83	30.06	19.05	19.99	21.74	−25.62	−23.79	4
Manufacturing	5.41	41.34	39.40	42.48	26.09	26.72	28.91	−22.19	−19.43	8
Communications	6.67	36.07	23.27	19.01	31.14	19.45	15.12	−9.78	−8.00	3
Retail trade	3.27	40.63	30.85	27.82	31.86	22.22	18.42	−0.42	−4.25	6
FIR	2.18	42.34	36.63	35.34	22.92	19.24	18.21	−27.80	−23.64	17
Services	5.97	33.66	31.59	30.59	21.83	20.10	20.69	−21.85	−19.41	8

is consistent with the literature documenting the decline in recovery rates during the economic downturn (i.e., when default is more likely).

We find that the average 1-year default probability is larger for firms with worse credit ratings, reflecting their higher likelihood of default. However, we do not find that firms' ratings monotonically explain the cross-sectional differences in recovery rates. This finding is consistent with Altman and Kishore (1996), who show that the bond ratings have no impact on the recovery rates once the seniority of the bonds is taken into consideration. Credit ratings, conversely, are solely assigned based on the firm's default probability, which explains why we observe a clear monotonic relationship between the risk-neutral default probabilities and the credit ratings.

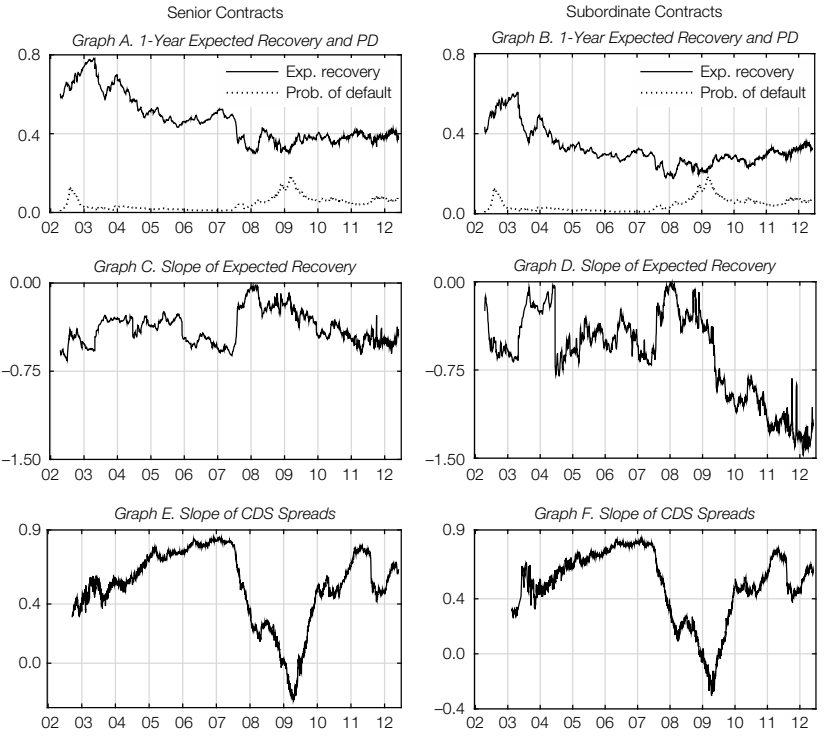
2. Time-Varying Expected Recovery

We next examine the time-varying dynamic of expected recovery rates. Figure 2 reports the time-series properties of the expected recovery dynamic for the 46 firms in our sample. Graphs A and B plot the daily time series of the 1-year expected recovery rate averaged across firms for senior and subordinate CDS contracts, respectively. For these two graphs, we also plot daily time series of 1-year default probability averaged across firms. We require that data on at least five firms are available to plot the daily cross-sectional averages. The time-series dynamics of expected recovery at other horizons are qualitatively similar and are not plotted here due to space constraint. As expected, Graphs A and B in Figure 2 show that the expected recovery rate for senior contracts is always higher than that for subordinate contracts. The average expected recovery rates for both senior and subordinate contracts are at their lowest level near the end of 2008, corresponding

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FIGURE 2
Credit-Risk Dynamics: Senior versus Subordinate

Figure 2 reports results obtained from the firm-by-firm estimation of the credit default swap (CDS) valuation model. The sample consists of the 46 single-name securities described in Table 1. The plotted time series are daily cross-firm average values. We require that data on at least five firms are available to plot the daily averages. The left-column graphs plot results for senior CDS spreads, and the right-column graphs plot results for subordinate CDS spreads. Graphs A and B plot the time series of the average 1-year expected recovery and the 1-year default probability for senior and subordinate CDS contracts. Graphs C and D plot the time series of the average slope of expected recovery. The slope of expected recovery is defined in equation (16). Graphs E and F plot the time series of the average slope of the market-observed CDS spreads. The slope of the market-observed CDS spreads is defined in equation (17). The number of firms used to calculate the time-series averages varies over time depending on data availability.



with the midst of the credit-risk crisis.[15] We note that the levels of expected risk-neutral recovery rates that we observe before and after the credit crisis are in line with the realized recovery rates reported by Moody's. In Moody's reports, the realized recovery rate for senior unsecured bonds before the crisis is approximately 55%. It then falls to 37.5% during the crisis and increases back to approximately 49% in 2010.[16]

Figure 2 shows that the expected recovery rates of senior and subordinate contracts are highly correlated. Nevertheless, there is a subtle difference between

[15]There is a sharp drop in the cross-firm expected recovery in 2003. This result is driven by the fall in expected recovery of Fannie Mae and Freddie Mac when the accounting practices of these two firms were under intense scrutiny. We discuss the recovery-rate dynamics of Fannie Mae in Section B of the Supplementary Material.

[16]For Moody's report of the recovery rates prior to the crisis, see Emery et al. (2008). For Moody's report during and after the credit crisis, see Ou, Chiu, and Metz (2011).

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their dynamics. Looking at the Graph B, we find that the expected recovery rates of subordinate contracts steadily decline between 2004 and 2008, then slowly increase back to pre-crisis levels between 2010 and 2012. However, for the senior contracts (Graph A), the expected recovery rate decreases most rapidly in the beginning of 2007 when the U.S. subprime mortgage market crisis started to unfold. The significant decline in the recovery rates during the financial crisis is not dominated by a single firm. In an unreported analysis, we find a similar sharp drop in the expected recovery rates of senior contracts across firms in different rating groups and industries. Further, Graph A of Figure 2 shows that the expected recovery rate of senior contracts in 2009–2012 does not return to pre-crisis levels as in the case for subordinate contracts.

Graphs C and D of Figure 2 plot the cross-sectional averages of the slope of expected recovery. For each day t, we calculate the average value of the expected recovery's slope by averaging the *relative* slope of expected recovery across firms. The relative slope of expected recovery is defined as

$$\frac{R^{\text{EXP}}(t, 10 \text{ years}) - R^{\text{EXP}}(t, 1 \text{ years})}{R^{\text{EXP}}(t, 10 \text{ years})}, \tag{16}$$

where $R^{\text{EXP}}(t, 1 \text{ years})$ and $R^{\text{EXP}}(t, 10 \text{ years})$ are expected recovery rates (see equation (15)) with horizons of 1 and 10 years, respectively. We use the relative slope of expected recovery instead of the raw slope so that the results are comparable across firms.

Graphs C and D of Figure 2 show that the slope of expected recovery's term structure is time varying and mostly negative throughout our sample period. Thus, on average, the term structure of the recovery rate is downward sloping. A possible explanation is that the market expects that a smaller fraction of CDSs' notional value can be recovered as the default horizon increases. Alternatively, the downward-sloping term structure of expected recovery can be linked through the recovery risk premium. Longer-maturity CDS contracts could be exposed to more uncertainty about the firm's LGD. As a result, investors are risk averse about this uncertainty and hence command a recovery risk premium for investing in longer-term debt contracts.

We compare the time-varying dynamics of the expected recovery term structure to that of CDS spreads. This exercise helps examine whether changes in the term structure of the recovery rate can be explained by changes in the term structure of CDS spreads. Graphs E and F of Figure 2 plot the daily average slopes of the market-observed CDS spreads across firms for senior and subordinate CDS contracts, respectively. The daily average slope of market spreads is calculated as the mean of the *relative* slope of CDS spreads across firms. Similar to the average slope of expected recovery, we define the relative slope of CDS spreads as

$$\frac{\text{CDS}(t, 10 \text{ years}) - \text{CDS}(t, 1 \text{ years})}{\text{CDS}(t, 10 \text{ years})}, \tag{17}$$

where $\text{CDS}(t, 1 \text{ years})$ and $\text{CDS}(t, 10 \text{ years})$ are market-observed CDS spread levels with 1 and 10 years to maturity, respectively. To save space, we only report the average slope of senior CDS spreads. Looking at Graphs C and E, we do not find

a clear relationship between the slope of expected recovery and the slope of CDS spreads on an aggregate level. The average slope of CDS spreads is positive and fairly stable up until the start of the subprime crisis. Afterward, there is a sharp drop in the average slope, resulting in a negatively sloped CDS term structure in the beginning of 2009.

An important observation from Graphs C and D of Figure 2 is that during the 2008 crisis, the slope of expected recovery is close to 0. The relatively flat term structure of expected recovery during the crisis implies that the market prices the short- and long-term CDSs in such a way that they have equally low recovery rates. The low levels of short-term expected recovery support the prevailing view that fire-sale and liquidity effects may depress the firm's ability to recover its fair value if it were to default in the midst of economic distress (e.g., Schuermann (2004), Altman (2011)).

3. Financial versus Nonfinancial Firms

Firms in the financial industry played an important role in the 2008 crisis. We examine their expected recovery dynamic further in this subsection. Out of the 46 firms in our sample, 17 of them are classified as financial firms. In Table 1, we denote financial firms with an asterisk next to the company's name. We define financial firms as those belonging to the finance, insurance, and real estate (FIR) sectors following the classification on Kenneth French's Web site (http://mba.tuck.dartmouth.edu/pages/faculty/ken.french). Most of the financial firms we study have a sample average rating of "A" or higher.

Graphs A and B of Figure 3 plot the time series of 1-year expected recovery for senior contracts alongside the time series of 1-year default probability for financial firms and nonfinancial firms, respectively. The values shown in each panel represent daily cross-sectional averages. We find that the expected recovery of firms in the financial sector falls sharply in mid-2007, which marks the beginning of the subprime crisis. We obtain similar conclusions when looking at the recovery dynamics for subordinate contracts.

Next, we look at the average slope of expected recovery for financial and nonfinancial firms. Graphs C and D of Figure 3 plot the results. Graph C shows that the slope of expected recovery for financial firms started increasing rapidly at the beginning of 2007 and stayed mostly positive from mid-2007 through 2008. This finding suggests that the expected recovery's term structure for financial firms was upward sloping during the financial crisis. Importantly, the inversion of the expected recovery's term structure is only evident for financial firms. For nonfinancial firms (Graph D), the average slope of their expected recovery did not invert but was relatively flat from mid-2007 through 2009.

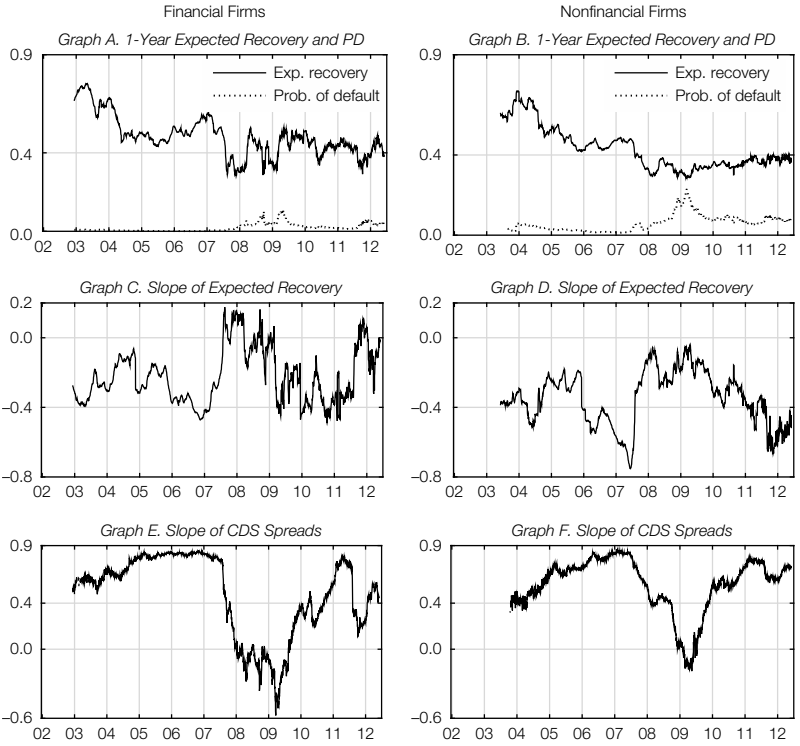
Graphs E and F of Figure 3 plot the average slope of CDS spreads for financial and nonfinancial firms, respectively. The average slope of the CDS spreads for financial firms is fairly stable up until the beginning of the subprime crisis. It then changes sharply from positively sloped to negatively sloped in 2007. In fact, the term structure of CDS spreads for financial firms remains negatively sloped for most of 2008 through 2009. During this period, it was more expensive to insure a financial firm through a 1-year senior CDS contract than through a 10-year senior CDS contract. Graph F shows that the slope of nonfinancial firms' CDS

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FIGURE 3

Credit-Risk Dynamics of Senior Contracts: Financial versus Nonfinancial Firms

In Figure 3, we plot the time series of the average credit-risk dynamics of *senior* credit default swap (CDS) contracts for financial and nonfinancial firms (see Table 1). The left-column graphs plot the results averaged across 17 financial firms, whereas the right-column graphs plot the results averaged across the other 29 nonfinancial firms. Financial firms are those categorized under the finance, insurance, and real estate sectors following the classification on Kenneth French's Web site. Graphs A and B plot the average 1-year expected recovery rate and 1-year probability of default (PD). Graphs C and D plot the average relative slope of expected recovery. Graphs E and F plot the time series of the average relative slope of market-observed CDS spreads. The relative slope of expected recovery and the relative slope of market-observed CDS spreads are calculated according to equations (16) and (17), respectively. We require that data on at least five firms are available to plot the daily averages.



spreads starts decreasing in 2007. However, it does not fall as quickly compared with that of financial firms and becomes inverted for only a few months in 2009. For both financial and nonfinancial firms, we find that the term structure of CDS spreads reverts back to being positively sloped after the subprime crisis.

Overall, Figure 3 shows that investors expected the recovery rate to be lower during the subprime crisis, which in turn raised the spread of the short-term CDS contracts relative to those at the longer horizons. Our results suggest that the fear of illiquidity and of the fire-sale effect is significantly priced in CDS spreads between 2007 and 2008, and this effect is significantly stronger for firms in the financial industry.

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D. The Impact of the Financial Crisis

In this section, we examine the impact of the 2008 subprime crisis. Graphs A and B of Figure 4 plot the average term structure of expected recovery for senior and subordinate contracts in three periods: Feb. 2007 (pre-crisis), Feb. 2008 (during the crisis), and Feb. 2011 (post-crisis). The results shown are averages of firms for which we have data available in each period. For comparison, we plot the average CDS spreads (Graph C) and the average default probabilities (Graph D) implied by the model. Graphs A and B show that the average expected recovery rate in Feb. 2007 (solid line) decreases as the horizon increases. The expected recovery rate, however, exhibits a U-shaped pattern during the 2008 subprime crisis (dotted line). The observed U-shaped pattern in the expected recovery provides an important economic insight on the market's expectation of the recovery level in 2008. That is, during the crisis, investors expect firms will recover more of their debt's value if they were to default after the crisis. These findings apply to both senior and subordinate contracts. We find that the U-shape pattern observed during the crisis disappears or becomes less evident in the 2011 post-crisis period (dashed line). Thus, we find that, on average, the term structure of expected recovery in 2011 returns to its pre-crisis shape.

Graph C of Figure 4 shows that the CDS term structure flattens during the financial crisis. The change in the shape of the CDS term structure somewhat mirrors that of the expected recovery. Before the subprime crisis, the slope of credit spreads is clearly upward sloping. However, in Feb. 2008, the CDS spreads' term structure has a hump shape; the credit spreads increase from 3 to 5 years, then slightly decrease from 5 to 10 years.

Next we look at the average term structure of cumulative default probabilities shown in Graph D of Figure 4. For each firm, the cumulative default probability at time h is calculated as $1-\mathrm{E}_t^Q[1_{\tau>t+h}]$, where $\mathrm{E}_t^Q[1_{\tau>t+h}]$ is the survival probability given in equation (4). The results shown in Graph D represent the average values across firms. We find that the term structure of default probability is slightly convex before the financial crisis, whereas during the crisis, it becomes concave. This result suggests that the relative increase in default probabilities during the financial crisis period is higher at the short horizon. Therefore, during the financial crisis, we observe changes in the term structure of default probabilities, as well as for the expected recovery.

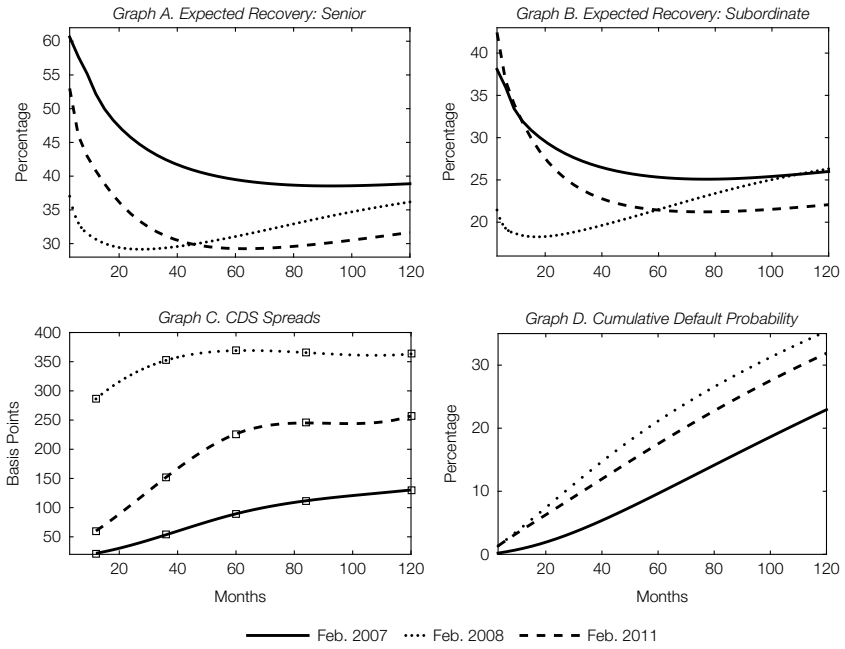
As shown earlier in Figure 3, the subprime crisis affected the recovery dynamic of financial firms more severely than others. We next examine the recovery term structure of individual financial firms before (Feb. 2007) and during (Feb. 2008) the subprime crisis. Figure 5 plots the expected recovery term structure for senior contracts. Each graph corresponds to an individual firm denoted by its ticker in the title.[17] The general pattern in Figure 5 is that the recovery term structure is, on average, upward sloping in Feb. 2008, even at extremely short horizons. This general pattern starkly differs from the downward-sloping shape of the

[17] Although we have 17 financial firms, we plot the results for 16 firms to save space and conserve the 4×4 structure of the panels. We exclude Capital One Bank in Figure 5, although its results are qualitatively similar to those of the other financial firms.

FIGURE 4

Term Structure of Expected Recovery: Before, During, and After the 2008 Crisis

For each graph in Figure 4, we plot cross-sectional average results for three different periods: Feb. 2007 (pre-crisis), Feb. 2008 (during crisis), and Feb. 2011 (post-crisis). Graphs A and B plot the average term structure of expected recovery across firms for the senior and subordinate credit default swap (CDS) contracts, respectively. Graph C plots the average senior CDS spreads observed in the market (in basis points). Graph D plots the average cumulative default probability implied by the model at various horizons. The cumulative default probability at time h is calculated as $1-E_t^Q[1_{\tau>t+h}]$, where $E_t^Q[1_{\tau>t+h}]$ is the survival probability given in equation (4). The x-axis in each graph indicates the horizon in number of months. We plot results for the pre-crisis period using a solid line and results for the post-crisis period using a dashed line. Results for the during-crisis period are plotted using a dotted line. The sample of firms for each period differs due to CDS data availability, as summarized in Table 1.



recovery term structure that we find in Feb. 2007. These findings also apply to subordinate contracts (unreported here to save space; available from the authors).

E. Changes in the Level of Expected LGD

This section discusses the impact of the 2008 crisis on the level of expected LGD. For each firm-day observation in Feb. 2008, we calculate the logarithmic changes (i.e., percentage changes) in LGDs and CDS spreads relative to their 1-year prior values. The monthly averaged values in Feb. 2008 are then calculated for each firm. Table 6 reports the cross-sectional averages over different rating classes and industries.[18] The level of CDS spread depends on two sources of risk: the firm's default risk and the LGD to the CDS's notational value. Thus, by looking at CDS spread changes in relation to changes in the firm's expected LGD, we can qualitatively examine to what extent the CDS spread changes during the 2008 crisis are due to changes in the LGD.

[18] D. R. Horton Inc. and Solectron Corp. do not have CDS data in Feb. 2008 and are excluded from Table 6. As a result, 44 firms are reported in this table.

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FIGURE 5

Expected Recovery for Individual Financial Firms: Before and During the 2008 Crisis

The sample in Figure 5 consists of the 16 financial firms described in Table 1. We plot the average term structure of expected recovery for the senior contracts of each financial firm in Feb. 2007 (pre-crisis) and in Feb. 2008 (during crisis). The title of each graph represents the ticker of the corresponding financial firm. Table 1 reports the company name corresponding to each ticker. We plot the results for Feb. 2007 using a solid line against the left *y*-axis, and we plot the results for Feb. 2008 using a dotted line against the right *y*-axis. The expected recovery levels are in percentage terms, and the *x*-axis in each graph indicates the number of months.

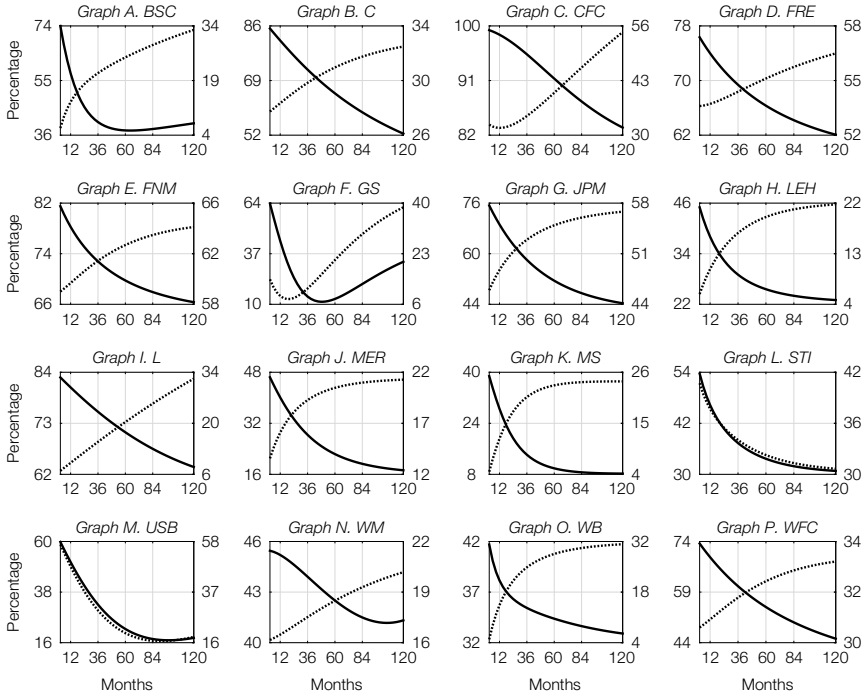


Table 6 shows that the expected LGD increases significantly at short-term horizons (i.e., from 1 to 5 years). Looking at the overall sample, the change in expected LGD for senior contracts is approximately 50% at the 1-year horizon, whereas it is negligibly small at the 10-year horizon. The largest increase in the expected LGD is for the finance, insurance, and real estate sectors (FIR). The breakdown based on credit ratings shows that the increase in LGD is larger for higher-rated firms. This finding is not surprising because most of the financial firms in our sample have AAA/AA ratings. We find similar results when looking at the change in LGD term structure for subordinate firms.

Table 6 reports the average logarithmic change in CDS spreads from the pre-crisis to during-the-crisis periods. We find that market-observed CDS spreads increase significantly across maturities, seniority, rating classes, and industries, reflecting the higher cost of insuring all types of firms during the crisis. Similar to the change in term structure of LGD, the increase in CDS spreads is larger at shorter horizons for both seniorities. For the overall sample, 1-year senior CDS spreads increase by 260.9%, which is large relative to the 121% increase for

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TABLE 6
Relative Change in LGD during the 2008 Crisis

In Table 6, we report the average logarithmic change in expected loss given default (LGD) and market credit default swap (CDS) spreads in Feb. 2008 relative to their 1-year prior values. For each day in Feb. 2008, we calculate the logarithmic change in the expected LGD and the market CDS spread relative to their 1-year prior values. The monthly averages in Feb. 2008 are then calculated for each firm. This table reports the average results for the overall sample as well as for subsamples grouped by overall credit rating and industry sector. We report the change in expected LGD and market CDS spreads (in %) for senior and subordinate CDS contracts at 1-, 5-, and 10-year horizons. The last column reports the number of firms in each category. We obtain monthly Standard & Poor's (S&P) long-term entity credit ratings from Compustat. Industry classifications are obtained from Kenneth French's Web site. *FIR* refers to firms in the finance, insurance, and real estate sectors. *Mining/Constr.* includes firms in the mining and construction industries.

	Change in Expected LGD (%)						Change in Market CDS Spreads (%)						
	Senior			Subordinate			Senior			Subordinate			
Category	1 Yr	5 Yr	10 Yr	1 Yr	5 Yr	10 Yr	1 Yr	5 Yr	10 Yr	1 Yr	5 Yr	10 Yr	No. of Firms
Overall	49.6	16.1	−1.8	29.6	7.2	−6.7	260.9	161.1	121.3	247.9	146.9	112.5	44
AAA/AA	74.1	34.3	15.9	51.2	21.4	8.9	293.6	231.5	181.9	312.3	221.0	164.5	5
A	49.4	14.6	−5.4	28.4	0.9	−14.0	309.9	200.2	145.6	280.4	180.8	135.3	12
BBB	79.1	26.7	4.8	43.1	11.7	−3.1	300.4	195.0	137.2	283.9	174.6	130.2	9
BB	28.5	7.3	−10.0	21.1	6.4	−10.4	201.8	99.3	81.6	197.1	94.7	77.2	14
B	27.2	4.8	0.8	5.4	0.9	0.3	175.8	95.8	66.0	154.0	72.7	53.9	4
Mining/Constr.	55.3	5.1	−0.7	42.5	4.3	−0.9	241.8	104.7	45.1	220.3	107.1	62.5	3
Manufacturing	24.5	−11.6	−21.8	2.7	−14.0	−23.2	184.4	124.0	92.0	160.2	103.0	79.6	7
Communications	21.8	7.0	1.6	13.1	3.9	0.8	119.9	83.0	63.0	143.7	85.4	62.2	3
Retail trade	48.3	25.0	15.8	39.6	12.6	9.0	242.9	166.5	115.1	256.3	153.4	116.1	6
FIR	70.7	28.9	6.3	41.9	17.8	3.0	328.2	224.0	169.8	314.8	213.0	157.4	17
Services	36.1	14.0	−16.3	20.9	1.4	−29.8	257.7	106.2	95.8	222.2	77.9	76.3	8

10-year senior CDS. The magnitude of change in CDS spreads is much larger relative to the change in expected LGD at all horizons. This finding suggests that the increase in CDS spreads during the crisis is largely due to the increase in default probabilities. Nevertheless, we observe a substantially large increase in LGD at short-term horizons, particularly for highly rated firms. This suggests that a sizable increase in short-term CDS spreads can be attributed to the change in LGD. We do not observe a significant change in LGD for subordinate contracts among B-rated firms, which is the lowest rating category in our sample.

Overall, Table 6 shows that the subprime crisis shock increases the LGD substantially at the short-term horizons, which then raises the level of CDS spreads. However, the changes in LGD at the longer horizons are small and even negative for some firms, suggesting a significant change in the term structure of expected recovery rates during the crisis.

VI. Robustness Check

A. Accounting for Bid–Ask Spreads

We provide evidence showing that our estimation results are robust to the inclusion of daily CDS bid–ask spreads in the estimation. We reestimate the model by incorporating daily CDS bid and ask quotes from Bloomberg. We require that the bid–ask quotes obtained from Bloomberg overlap with quotes from Markit for at least 4 consecutive years on both senior and subordinate contracts. This leaves us with sufficient data for six financial firms in our sample: Fannie Mae (FNMA), Freddie Mac (FRE), JP Morgan (JPM), Washington Mutual (WM), Wells Fargo & Co. (WFC), and Wachovia Corp (WB). The correlation between the bid–ask spreads of senior and subordinate CDS contracts for these six firms

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are highly correlated. Their correlation coefficients are greater than 0.85, suggesting that bid–ask spreads of senior and subordinate CDS relative spreads always move in tandem.

Following Pan and Singleton (2008), we account for CDS bid–ask spreads by assuming that τ-maturity senior contracts on day t are priced with normally distributed errors with conditional standard deviations of $\sigma_u^{\text{sen}}(\tau) \times |\text{BID}_t^{\text{sen}}(\tau) - \text{ASK}_t^{\text{sen}}(\tau)|$. We assume that $\sigma_u^{\text{sen}}(\tau)$ is a constant, and $\text{BID}_t^{\text{sen}}(\tau)$ and $\text{ASK}_t^{\text{sen}}(\tau)$ refer to the bid and ask spreads of the τ-maturity senior contracts on day t. Similarly, we assume that τ-maturity subordinate contracts are priced with normally distributed errors with conditional standard deviations $\sigma_u^{\text{sub}}(\tau) \times |\text{BID}_t^{\text{sub}}(\tau) - \text{ASK}_t^{\text{sub}}(\tau)|$, where the variables are defined analogously.

This error assumption affects our firm-by-firm estimation in two ways. First, it affects the measurement-error equation (13), which is used to filter the latent state variables. Second, it affects the conditional covariance matrix of CDS pricing errors, $\overline{\Omega}_t$, which enters the log likelihood in equation (14).

We conduct the robustness exercise on the six financial firms previously described. Figure 6 illustrates our findings. We plot the recovery dynamic and the average recovery term structure obtained from two estimation methods: without bid–ask spread and with bid–ask spread. The left-hand-side graphs plot the time series of the 1-year expected recovery rate. The right-hand-side graphs plot the time-series average term structure of expected recovery. Results for each firm are plotted on each row, with the firm's ticker shown on the far-left y-axis. We note that the time periods in Figure 6 differ between firms due to the difference in their CDS data availability. We find that the estimates of the recovery dynamics from the two estimation methods do not substantially differ. Both the time-series dynamic of the expected recovery rate and the term structure of expected recovery are only slightly affected when including daily CDS bids and asks in the estimation. We conclude that our model estimation results are robust to changes in the daily liquidity, as measured by bid and ask, of senior and subordinate CDS spreads.[19]

B. Price Discovery during the Financial Crisis

Norden and Weber (2012) examine the equilibrium relation of senior and subordinate CDS spreads written on 20 large European banks. They find that during the financial crisis, price discovery in the CDS market occurs in senior CDSs and that the long-run equilibrium relationship between senior and subordinate CDS spreads breaks down. We show that the price-discovery mechanism in the CDS market of U.S. entities in our sample was not significantly affected during the crisis, providing evidence that our conclusions are not driven by structural changes in the relative liquidity between senior and subordinate spreads. Due to the space consideration, we explain the methodology that we use and discuss the findings in Section C of the Supplementary Material.

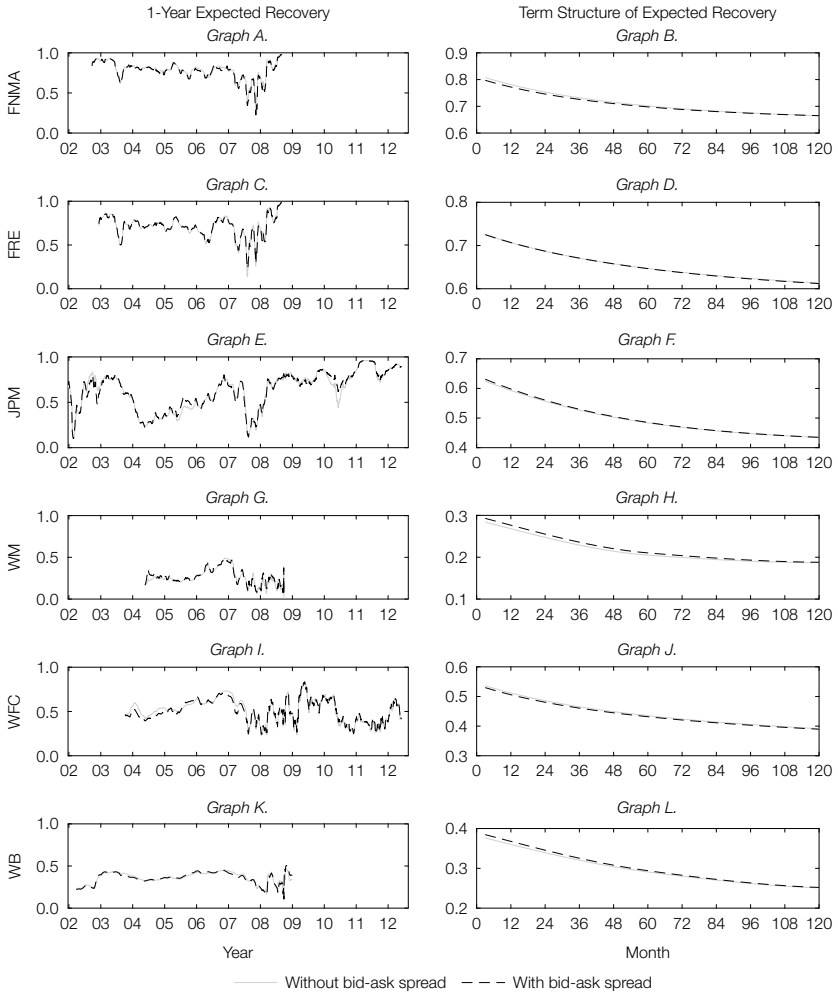
[19] We reach the same conclusion when comparing their parameter estimates. For example, we compare the parameter estimates of Fannie Mae obtained using the two estimation methods: with versus without bid–ask spread. The results are presented in Table IA2 of the Supplementary Material. We find that the estimates do not substantially differ.

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FIGURE 6
Accounting for Daily Bid–Ask Spreads

In Figure 6 we estimate the credit default swap (CDS) valuation model using the time-series data of senior and subordinate CDS contracts written on Fannie Mae (FNMA), Freddie Mac (FRE), JP Morgan, (JPM), Washington Mutual (WM), Wells Fargo & Co. (WFC), and Wachovia Corp (WB). For each firm, we use two estimation methods. The first estimation method, labeled *Without bid–ask spread*, does not account for daily CDS bid and ask spreads. In the second estimation method, labeled *With bid–ask spread*, we include daily bid and ask CDS quotes obtained from Bloomberg in the estimation; see the text for more details. We report results for each firm per each row using two graphs. The firm's ticker symbol is shown on the *y*-axis of the left-column graph. The left-column graphs plot the time series of the 1-year expected recovery. The right-column graphs plot the time-series average of expected recovery at various horizons.



C. Subperiod Analysis

We undertake a subperiod analysis to verify that our estimation results obtained using the full sample period (i.e., from Jan. 2001 to May 2012) are fairly stable. For each of the 46 firms in our sample, we reestimate the model on two subperiods. The first is from Jan. 1, 2001, to June 30, 2007, corresponding to the

pre-crisis period. The second subperiod is from July 1, 2007, to May 31, 2012, which includes the subprime crisis period and the post-crisis period; we refer to it as the *crisis/post-crisis* period. This analysis yields $2 \times 46 = 92$ new sets of parameter estimates. We compare the model parameters and the recovery-rate dynamics obtained from these two subperiods against those estimated using the full sample period. We present and discuss these results in Section D of the Supplementary Material. We summarize our findings here.

Table IA4 of the Supplementary Material examines the stability of our model estimates by looking at their parameter deviations. We find that the loading coefficients are very stable across different sample periods. The latent-factor dynamics, on the other hand, show more deviations, but their magnitudes are within a reasonable range. Figure IA2 of the Supplementary Material visually examines the recovery-rate dynamic and the default-intensity dynamic estimated from different sample periods. We find little deviation in the time series of the 1-year expected recovery and the 1-year default probability between subperiod results and those obtained using the full sample. This finding indicates that our estimates of the recovery-rate level and the default-intensity level are stable and robust to a subperiod test.

We also examine the out-of-sample pricing implications of our model. We test the model's CDS pricing performance in the crisis/post-crisis period using parameters that were estimated from the pre-crisis period. We discuss these results in Section E of the Supplementary Material.

D. Alternative Estimation Method

So far, results regarding the dynamics of default probability and recovery rate that we report are based on parameters that are estimated firm-by-firm. We estimate the model across 17 financial firms in our sample all at once (i.e., a joint estimation) to verify that the main conclusion does not change. We note that this method affects the second-stage estimation only, where the model is fitted to senior and subordinate CDS spreads. We choose firms in the financial industry because they were most severely affected by the 2008 subprime crisis, where we find that their term structure of expected recovery changes shape. We do not estimate the model on all 46 firms in our sample jointly because prior studies find that the default probability and LGD dynamics are industry specific (e.g., Chava and Jarrow (2004), Acharya et al. (2007)). This finding is supported by our results in Table 5, which show that the default probability and the expected recovery rate significantly differ across industries.

The joint-estimation method is numerically demanding because the model is simultaneously fitted to a large panel of CDS spreads from 17 different firms. The method yields one set of parameter estimates and time series of latent factors. This approach differs from the firm-by-firm estimation method, which yields 17 sets of parameter estimates and time series of latent factors (i.e., one set for each firm). Table IA6 of the Supplementary Material presents results comparing parameter estimates obtained from the joint estimation against those estimated firm-by-firm. Parameter estimates and their standard errors are reported for the joint estimation. For the firm-by-firm estimation, we report the distribution of each model parameter. We visually compare the recovery-rate and

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default-intensity levels obtained from the two estimation methods in Figure IA3 of the Supplementary Material.

Overall, we find that the joint estimation method yields parameter estimates that are representative of the median firm in the financial industry. This finding verifies that the two estimation approaches yield comparable parameter estimates. Looking at the plots of time-series estimates from the two estimation approaches (see Figure IA3 of the Supplementary Material), we reach a similar conclusion. That is, the level of expected recovery varies significantly through time and worsens during the subprime crisis period. Also, the slope of expected recovery for financial firms rises above (or is close to) 0 during and after the subprime crisis period, indicating the inversion in the term structure of expected recovery. We further discuss our findings from the joint-estimation method in Section F of the Supplementary Material.

VII. Identification

A. Simulation Studies

We first show the identification of the recovery-rate dynamic using an extensive simulation study. We simulate data for CDS spreads using the *baseline* (i.e., "true") parameters reported in Table 7. The baseline parameters are chosen such that they match the average pre-crisis spreads and expected recovery rates of the most representative industry (financial) in our sample.[20] Panel A reports factor loadings on the intensity dynamic and the subordinate LGD dynamic. Panel B reports parameters driving the latent factors X_3, X_4, and X_5.

We generate 100 sample paths of daily simulated CDS spreads using the "true" base parameters in Table 7. We allow for normally distributed noise in the simulated CDS spreads. Each simulation path consists of 1,500 daily observations, which is approximately equal to 6 years. The noise term's standard deviation level is set equal to 5% of the simulated CDS spread level.

We estimate the full CDS valuation model on 100 simulated sample paths of senior and subordinate CDS spreads. This estimation exercise yields 100 sets of parameter estimates, as well as the time series of latent state variables. We report the means and standard deviations of the parameters estimated from the simulated samples under the *MC Estimates* section in Table 7. We find that the parameters estimated from the simulation samples are very close to their true values. Importantly, any differences between the means and the true parameter values can be reconciled within 2 standard deviations. This finding suggests that the parameters in the full CDS valuation model can be identified. We report the time-series dynamic of the recovery rates estimated using the simulated data in Figure IA5 of the Supplementary Material. We find that deviations between the data-generated ("true") recovery rate and the estimated recovery rate can be reconciled within 1 standard deviation of the simulated sample.[21]

[20] The average spreads are calculated by pooling data from all financial firms. The average CDS spreads before Dec. 2007 (pre-crisis period) are 22, 35, and 45 bps for 1-, 5-, and 10-year senior contracts, respectively. The expected recovery rates for senior contracts are 50%, 44%, and 41% for 1-, 5-, and 10-year horizons, respectively.

[21] We provide an additional analysis showing that the term structure of *multiple-seniority* CDS contracts helps disentangle the stochastic default-intensity dynamic from the stochastic recovery-rate

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TABLE 7

Monte Carlo Simulation Study: True versus Estimate

Table 7 reports results from the Monte Carlo (MC) simulation study. *True value* reports the parameter values used in the simulation study. We simulate 100 paths of daily credit default swap (CDS) spreads over 1,500 days using the base-case parameters. The data for CDS spreads are simulated with normally distributed noise. We assume the noise term's standard deviation is 5% of the model-implied spread level. *MC est.* reports the mean and standard deviation (Std. dev.) of parameters estimated from the 100 simulated samples. See Table A1 in Appendix C for a summary of the model parameters. Panel A reports the factor loadings of the default-intensity dynamic, the loss given default (LGD) of senior CDSs, and the LGD of junior CDSs. Panel B reports the structural parameters that drive the factor dynamics of state variables X_3, X_4, and X_5 under the physical and risk-neutral measures.

Panel A. Factor-Loading Parameters

	Default Intensity (λ_t)			Senior LGD			Subordinate LGD			
	Constant $\alpha_0 \times 100$	X_1 $\alpha_1 \times 100$	X_2 $\alpha_2 \times 100$	Constant β_0^{sen}	X_1 β_1^{sen}	X_2 β_2^{sen}	Constant β_0^{sub}	X_1 β_1^{sub}	X_2 β_2^{sub}	X_5 β_5^{sub}
True value	0.270	−0.342	−0.680	1.094	8.641	−9.676	0.730	5.761	−6.450	0.667
MC est.										
Mean	0.275	−0.342	−0.685	1.034	8.788	−9.630	0.728	5.896	−6.499	0.671
Std. dev.	0.014	0.011	0.027	0.047	0.471	0.633	0.060	0.287	0.233	0.036

Panel B. Parameters in the Factor Dynamics of X_3, X_4, and X_5

	Intensity Factor X_3				Intensity Factor X_4				Recovery Factor X_5			
	ρ_3	ρ_3^P	$\mu_3 \times 100$	$\sigma_3 \times 100$	ρ_4	ρ_4^P	$\mu_4 \times 100$	$\sigma_4 \times 100$	ρ_5	ρ_5^P	$\mu_5 \times 100$	$\sigma_5 \times 100$
True value	0.9997	0.9986	0.0001	0.0176	0.9970	0.9981	0.0002	0.0062	0.9993	0.9831	−0.0149	1.4912
MC est.												
Mean	0.9998	0.9957	0.0001	0.0183	0.9868	0.9918	0.0002	0.0060	0.9908	0.9764	−0.0152	1.3925
Std. dev.	0.0001	0.0084	0.0000	0.0011	0.0092	0.0054	0.0000	0.0005	0.0058	0.0090	0.0011	0.1865

Additionally, we evaluate the model's performance at fitting the simulated data by looking at its pricing errors against their "true" values along three dimensions: CDS spreads, expected recovery rates, and binary CDS spreads. A binary CDS spread is priced similarly to a standard CDS contract but with a 0% recovery rate when the firm defaults (i.e., the LGD is 100%). There is no recovery risk in binary CDS contracts due to the certainty of 0% recovery, and hence their spreads reflect only the default risk. We use binary CDS spreads as our benchmark for evaluating the robustness of the estimates for the default-intensity dynamic. Due to space consideration, we discuss the results in Section G of the Supplementary Material. On average, we find that the absolute percentage errors from fitting the simulated CDS data are small and fall under 5%. The expected recovery rates and binary CDS spreads are reasonably well estimated, with absolute errors below 11% relative to their true values, respectively. These findings suggest that we can estimate the recovery and intensity dynamic fairly well.

B. Intuitive Analysis

Using a static analysis, this section illustrates how the LGD levels of senior and subordinate CDS contracts are identified using their CDS term structures jointly. Our analysis is based on the base-case parameters reported in Table 7.

dynamic. This is because the use of multiple-seniority CDS contracts imposes a stricter mechanism for identifying the default intensity than when using single-seniority CDS contracts. This feature makes it possible for the model to pick up changes in CDS spreads that arise due to stochastic recovery rates. We discuss our empirical design and results for this analysis in Section I of the Supplementary Material.

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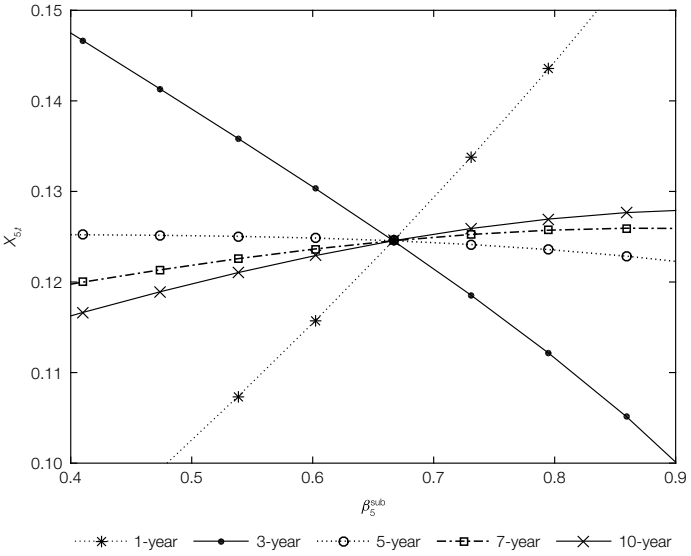
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Recall that the LGD dynamics of senior and subordinate CDSs share a common structure described by equation (6). The coefficients $\beta_i^{\text{sub}}(\beta_i^{\text{sen}})$ for $i=1,2,5$ denote the loadings of the subordinate (senior) LGD dynamic to the three state variables X_1, X_2, and X_5, respectively. However, X_5 is the recovery-specific factor that is identified using multiple-seniority CDS contracts, whereas X_1 and X_2 are identified using the default-free term structure. For ease of exposition, we set the factor loadings on X_1 and X_2 to 0. Further, recall that we normalize β_5^{sen} to 1 to facilitate the econometric identification. The relative difference between $\text{LGD}_t^{\text{sub}}$ and $\text{LGD}_t^{\text{sen}}$ in the model is therefore captured through β_5^{sub} and the level of $X_{5,t}$. For the static analysis, we examine how the level of $X_{5,t}$ and the value of its factor loading β_5^{sub} are identified from the term structures of senior and subordinate CDSs.

Figure 7 plots various combinations of $X_{5,t}$ and β_5^{sub} that keep the ratio of senior to subordinate CDS spreads with the same maturity constant.[22] Other parameters except $X_{5,t}$ and β_5^{sub} are held fixed equal to the base-case parameters

FIGURE 7
Sensitivity of CDS Term Structures to LGD Loadings

Figure 7 shows the combinations of factor loading β_5^{sub} and the level of recovery-specific factor $X_{5,t}$ in equation (6) that result in a constant ratio for the credit default swap (CDS) spread between senior and subordinate contracts. We plot the combinations of $(\beta_5^{\text{sub}}, X_{5,t})$ that keep senior-to-subordinate CDS ratios constant for 1-, 3-, 5-, 7-, and 10-year-maturity contracts. The x-axis corresponds to the value of β_5^{sub}, and the y-axis corresponds to the value of $X_{5,t}$. The coefficient β_5^{sen} is normalized to 1 in our model. For simplicity, we remove the effect of the default-free term structure on the loss given default (LGD) by setting the factor loadings on the state variables $X_{1,t}$ and $X_{2,t}$ to 0. All other parameters except β_5^{sub} and $X_{5,t}$ are held fixed equal to the base-case parameters reported in Table 7. The levels of the other state variables at time t are as follows: $X_{3,t}=0.00093$, $X_{4,t}=-0.00077$. Each $(\beta_5^{\text{sub}}, X_{5,t})$ combination determines a unique term structure of recovery rates. The model used for generating senior and subordinate CDS spreads is described in Section III.



[22] The default intensity does not impact the ratio of senior-to-subordinate CDS term structures. In Figure IA4 of the Supplementary Material, we show that default-intensity factors (i.e., X_3 and X_4) do not affect the relative CDS spreads between senior and subordinate contracts and, hence, do not impact their CDS-spread ratios.

shown in Table 7. We plot the results for five CDS maturities: 1-, 3-, 5-, 7-, and 10-year CDS spreads. Looking at CDS-spread ratios for each maturity, the relationship between $X_{5,t}$ and its factor loading β_5^{sub} is nonmonotonic, reflecting the quadratic specification of the LGD dynamic (see equation (6)). There are many possible combinations of $X_{5,t}$ and β_5^{sub} that keep the ratio of senior to subordinate CDS spreads constant. This shows that we cannot identify a unique combination of $X_{5,t}$ and β_5^{sub} using *single-maturity* senior and subordinate CDS spreads.

Figure 7, however, shows that a unique pair of $X_{5,t}$ and β_5^{sub} can be identified when using *multiple-maturity* senior and subordinate CDS spreads. The constant-CDS-ratio curves of different maturities intersect at one location, which corresponds to the base-case parameters $X_{5,t}$ and β_5^{sub}. This indicates that identification of the LGDs requires the use of multiple-maturity and multiple-seniority CDS contracts. Overall, Figure 7 illustrates, in a simple static setting, how the LGDs of senior and subordinate CDS contracts are identified using their CDS term structures.

VIII. Conclusion

This article estimates the term structure of the expected recovery rates on 46 firms individually using panel data of senior and subordinate CDSs from 2001 to 2012. We find that the recovery rates implied by CDS contracts are time varying and are negatively related to their underlying firms' default probability. In particular, we find that expected recovery rates decline substantially during the period of the 2008 financial crisis. The decline is significantly larger at the shorter horizons relative to the longer horizons. This disproportionate change results in the inversion of the recovery-rate term structure: from downward sloping before the financial crisis to upward sloping during the financial crisis. This inversion is more prominent for firms that are in distress during the crisis period (i.e., financial firms).

Our findings have important implications for how banks calculate their capital requirement for credit risk. Under the Basel III regulation, banks calculate their credit-risk capital requirement based on the expected loss rate of the defaultable securities that they hold. The expected loss rate depends on the default probability of credit-risky securities, as well as their expected recovery rate upon default. The common practice is to assume that the expected recovery rate is constant (i.e., approximately 40%), which ignores the fact that corporate debts' recovery rate is time varying and is negatively related to the underlying firm's default probability. Consequently, this could lead banks to hold (and regulators to enforce) lower capital requirements than what banks otherwise would need.

According to the new International Financial Reporting Standard (i.e., IFRS 9) scheduled for implementation in 2018, banks that are viewed as systemically important by the government are required to model the expected loan loss rate (i.e., the term structure of both LGD and default probability) when calculating their expected cash flow (see http://www.ifrs.org/current-projects/iasb-projects/financial-instruments-a-replacement-of-ias-39-financial-instruments-recognitio/Pages/financial-instruments-replacement-of-ias-39.aspx). Therefore, our research findings and the modeling framework that we introduce can provide

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valuable insights to financial risk managers who must soon comply with this new regulation change.

Appendix A. The Model in Matrix Form

Appendix A describes the model in matrix form. We use the following notations to derive solutions to default-free zero-coupon bond and CDS contracts in subsequent sections. Let X_t be a 5×1 vector containing the five latent factors that describe the model:

$$X_t = [X_{1,t} \quad X_{2,t} \quad X_{3,t} \quad X_{4,t} \quad X_{5,t}]'.$$

We can write the risk-neutral dynamic of the latent state variables in a vectorial form as

$$X_{t+1} = \mu + \rho X_t + \Sigma \varepsilon_{t+1}, \tag{A-1}$$

where μ is a 5×1 vector of factor mean, with the ith element denoted by μ_i, and ρ and Σ are diagonal 5×5 matrices containing the standard deviation and autoregressive terms of each factor, with the ith diagonal element denoted by ρ_i and σ_i, respectively. We assume the 5×1 vector of residual shocks in the factor dynamic is generated by $\varepsilon_{t+1} \sim \mathrm{N}(0, I)$, where I is the identity matrix.

The five independent latent factors in X_t can be grouped into two components. We let the factors that are identified using default-free securities be denoted by $X_t^r = [X_{1,t}\ X_{2,t}]'$. The second component is identified using multiple-seniority credit-risk securities (i.e., CDS) and is denoted by $X_t^c = [X_{3,t}\ X_{4,t}\ X_{5,t}]'$. This implies that X_t in equation (A-1) can be written as $X_t = [X_t^r\ X_t^c]'$. We apply the superscripts r and c to indicate variables that are identified from default-free term structures and CDS term structures, respectively.

Using the previously described notations, the short-rate dynamic in equation (3) can be written as

$$r_t = (\delta_0 + \delta^{r\prime} X_t^r)'\ (\delta_0 + \delta^{r\prime} X_t^r), \tag{A-2}$$

where δ_0 is a scalar, and $\delta = [\delta_1\ \delta_2]'$. The default intensity dynamic, λ_t, in equation (5) can be written compactly using vectorial notation as

$$\lambda_t = (\alpha_0 + \alpha^{r\prime} X_t^r + \alpha^{c\prime} X_t^c)'\ (\alpha_0 + \alpha^{r\prime} X_t^r + \alpha^{c\prime} X_t^c), \tag{A-3}$$

where α_0 is a scalar, $\alpha^r = [\alpha_1\ \alpha_2]'$, and $\alpha^c = [\alpha_3\ \alpha_4\ 0]'$.

Similarly, the LGD dynamic of senior and subordinate CDS contracts shown in equation (6) can be written in the following form:

$$\mathrm{LGD}_t = \exp(-(\beta_0 + \beta^{r\prime} X_t^r + \beta^{c\prime} X_t^c)'\ (\beta_0 + \beta^{r\prime} X_t^r + \beta^{c\prime} X_t^c)). \tag{A-4}$$

The loading coefficients in the senior and subordinate LGD dynamics shown equation (A-4) are

$$\begin{aligned} \beta_0 &= \beta_0^s, \\ \beta^r &= [\beta_1^s \quad \beta_2^s]', \\ \beta^c &= [0 \quad 0 \quad \beta_5^s]', \end{aligned}$$

where the subscript $s \in \{\mathrm{sen}, \mathrm{sub}\}$ in the previous equations correspond to LGD dynamics for senior and subordinate CDS contracts, respectively.

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Appendix B. Default-Free Bond Valuation

In Appendix B we use the property of a quadratic Gaussian random variable. Let

$$Q = d + a'\epsilon + \epsilon' V \epsilon, \tag{B-1}$$

where d and a are $N \times 1$ vectors, and V is an $N \times N$ matrix of coefficients. If ϵ is an $N \times 1$ vector described by a multivariate normal distribution $\epsilon \sim \mathrm{N}(0, \Gamma)$, then the Laplace transform of Q in equation (B-1) is

$$\mathrm{E}[\exp(tQ)] = \exp\left(-\frac{1}{2}\ln(\det(I - 2t\Gamma V)) + td + \frac{1}{2}ta'(\Gamma^{-1} - 2tV)^{-1}at\right), \tag{B-2}$$

where det indicates determinant. The proof of equation (B-2) is given by Mathai and Provost ((1992), p. 40).

Using the notations shown in Appendix A, the short-rate dynamic is described by

$$\begin{aligned} r_t &= (\delta_0 + \delta' X_t^r)'\,(\delta_0 + \delta' X_t^r) \\ &= \delta_0^2 + 2\delta_0 \delta' X_t^r + X_t^{r\prime} \delta\delta' X_t^r, \end{aligned} \tag{B-3}$$

where we recall that $X_t^r = [X_{1,t}\ X_{2,t}]'$. The risk-neutral dynamic of X_t^r is

$$X_{t+1}^r = \mu^r + \rho^r X_t^r + \Sigma^r \varepsilon_{t+1}^r, \tag{B-4}$$

where $\mu^r = [\mu_1\ \mu_2]'$. The 2×1 vector of residual shocks in the short-rate dynamic is denoted by $\varepsilon_{t+1}^r \sim \mathrm{N}(0, I)$, where I is the identity matrix, ρ^r and Σ^r are 2×2 diagonal matrices denoted by

$$\rho^r = \begin{bmatrix} \rho_1 & 0 \\ 0 & \rho_2 \end{bmatrix}, \quad \text{and} \quad \Sigma^r = \begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{bmatrix}.$$

Our objective is to price a default-free zero-coupon bond:

$$B(t, t+h) = \mathrm{E}_t^Q\left[\exp\left(-\sum_{j=0}^{h-1} r_{t+j}\right)\right]. \tag{B-5}$$

To solve the expectation in equation (B-5), we guess that its solution takes an exponential quadratic form:

$$B(t, t+h) = \exp(A_h + B_h' X_t^r + X_t^{r\prime} C_h X_t^r). \tag{B-6}$$

We next solve for the expressions to coefficients A_h, B_h, and C_h in equation (B-6) using recursive iterations. Let's first consider a 1-period valuation of a default-free zero-coupon bond at time $t+h-1$ maturing at time $t+h$. It can be written as

$$\begin{aligned} B(t+h-1, t+h) &= \mathrm{E}_{t+h-1}^Q[\exp(-r_{t+h-1})] = \exp(-r_{t+h-1}) \\ &= \exp(\delta_0^2 + 2\delta_0 \delta' X_{t+h-1}^r + X_{t+h-1}^{r\prime} \delta\delta' X_{t+h-1}^r), \end{aligned} \tag{B-7}$$

where we substitute expression equation (B-3) in the last line of equation (B-7). Comparing equations (B-7) and (B-6), we see that $A_1 = -\delta_0^2$, $B_1 = -2\delta_0\delta$, and $C_1 = -\delta\delta'$.

Next we consider a 2-period valuation $B(t+h-2, t+h)$:

$$\begin{aligned} B(t+h-2, t+h) &= \mathrm{E}_{t+h-2}^Q[\exp(-r_{t+h-2})\exp(-r_{t+h-1})] \\ &= \exp(-r_{t+h-2})\mathrm{E}_{t+h-2}^Q[B(t+h-1, t+h)]. \end{aligned} \tag{B-8}$$

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We can substitute in the dynamic of $-r_{t+h-2}$ following equation (B-3), and the expression for $B(t+h-1,t+h)$ from equation (B-7). This yields

$$\text{(B-9)}\quad B(t+h-2,t+h) = \exp(\delta_0^2 + 2\delta_0\delta' X^r_{t+h-2} + X^{r\prime}_{t+h-2}\delta\delta' X^r_t) \times \mathrm{E}^Q_{t+h-2}[\exp(\delta_0^2 + 2\delta_0\delta' X^r_{t+h-1} + X^{r\prime}_{t+h-1}\delta\delta' X^r_{t+h-1})],$$

where the expectation on the right-hand side of equation (B-9) is solved using the property of a quadratic Gaussian variable shown in equation (B-2). After some algebraic work and collecting terms, we compare the expression in equation (B-9) to the general solution for zero-coupon bond price, which we initially guessed in equation (B-6). For $B(t+h-2,t+h)$, the expected solution is

$$B(t+h-2,t+h) = \exp(A_2 + B_2' X^r_{t+h-2} + X^{r\prime}_{t+h-2} C_2 X^r_{t+h-2}).$$

We apply this procedure iteratively backward to solve for the bond price $B(t,t+h)$ at time t. This method shows that the coefficients A_h, B_h, and C_h in equation (B-6) can be solved recursively as follows:

$$\begin{aligned}
A_h &= -\delta_0^2 + (A_{h-1} + B_{h-1}'\mu^r + \mu^{r\prime} C_{h-1}\mu^r) \\
&\quad + \frac{1}{2}(\Sigma^{r\prime} B_{h-1} + 2\Sigma^{r\prime} C_{h-1}\mu^r)'(I - 2\Sigma^{r\prime} C_{h-1}\Sigma^r)^{-1}(\Sigma^{r\prime} B_{h-1} + 2\Sigma^{r\prime} C_{h-1}\mu^r) \\
&\quad - \frac{1}{2}\ln[\det(I - 2\Sigma^{r\prime} C_{h-1}\Sigma^r)], \\
B_h' &= -2\delta_0\delta' + (B_{h-1} + 2C_{h-1}\mu^r)'\rho^r \\
&\quad + 2(\Sigma^{r\prime} B_{h-1} + 2\Sigma^{r\prime} C_{h-1}\mu^r)'(I - 2\Sigma^{r\prime} C_{h-1}\Sigma^r)^{-1}\Sigma^r C_{h-1}\rho^{r\prime}, \\
C_h &= -\delta\delta' + \rho^{r\prime} C_{h-1}\rho^r + 2(\Sigma^{r\prime} C_{h-1}\rho^r)'(I - 2\Sigma^{r\prime} C_{h-1}\Sigma^r)^{-1}\Sigma^{r\prime} C_{h-1}\rho^r,
\end{aligned}$$

where the initial conditions are $A_1 = -\delta_0^2$, $B_1 = -2\delta_0\delta'$, and $C_1 = -\delta\delta'$.

Appendix C. Credit Default Swap Valuation

In Appendix C we solve recursively the expectations in equations (7) and (8) similar to the way that we solve for the price of a default-free zero-coupon bond. For compactness, we use the model's notation described in Appendix A. The present value of the payment by the protection buyer, $\mathrm{PB}(t,t+h)$, and the protection seller, $\mathrm{PS}(t,t+h)$, on a h periods ahead take the following forms:

$$\text{(C-1)}\quad \mathrm{PB}(t,t+h) = S\Delta \times \exp(F_h + G_h' X_t + X_t' H_h X_t),$$

$$\text{(C-2)}\quad \mathrm{PS}(t,t+h) = \exp(M_h + N_h' X_t + X_t' O_h X_t) - \exp(J_h + K_h' X_t + X_t' L_h X_t).$$

where we solve for the recursive relations of the exponential coefficients in the following section.

1. Recursive Coefficients in Equation (C-1)

The present value of the payment by the protection buyer on a payment date h periods ahead is

$$\text{(C-3)}\quad \mathrm{PB}(t,t+h) = \mathrm{E}^Q_t\left[S\Delta \times \exp\left(-\sum_{j=0}^{h-1} r_{t+j}\right) \times 1_{\tau>t+h}\right] = S\Delta \times \mathrm{E}^Q_t\left[\exp\left(-\sum_{j=0}^{h-1} r_{t+j} + \lambda_{t+j}\right)\right].$$

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Our objective is to solve the expectation on the right-hand side of equation (C-3), which has the following functional form:

$$\text{(C-4)} \quad \mathrm{E}_t^Q\left[\exp\left(-\sum_{j=0}^{h-1} r_{t+j} + \lambda_{t+j}\right)\right] = \exp(F_h + G_h' X_t + X_t' H_h X_t).$$

To solve for the recursive relations in the exponential coefficients F_h, G_h, and H_h, we first write the sum of the default intensity and the short rate in equation (C-4) as

$$\text{(C-5)} \quad \begin{aligned} r_t + \lambda_t &= (\delta_0 + \delta' X_t^r)^2 + (\alpha_0 + \alpha^{r\prime} X_t^r + \alpha^{c\prime} X_t^c)^2 \\ &= \gamma_0 + \gamma_1' X_t + X_t' \Omega X_t, \end{aligned}$$

where $\gamma_0 = \delta_0^2 + \alpha_0^2$, and

$$\gamma_1 = \begin{bmatrix} 2(\delta_0\delta' + \alpha_0\alpha^{r\prime}) \\ 2\alpha_0\alpha^{c\prime} \end{bmatrix}; \quad \Omega = \begin{bmatrix} \delta\delta' + \alpha^r\alpha^{r\prime} & \alpha^r\alpha^{c\prime} \\ \alpha^c\alpha^{r\prime} & \alpha^c\alpha^{c\prime} \end{bmatrix}.$$

We refer to Appendix A for δ, α^r, and α^c and recall that the dynamic of X_t is

$$X_{t+1} = \mu + \rho X_t + \Sigma\varepsilon_{t+1}.$$

The recursive coefficients in equation (C-4) can be derived in a similar manner as those in Appendix B. We provide their results as follows:

$$\begin{aligned} F_h &= -\gamma_0 + (F_{h-1} + G_{h-1}'\mu + \mu' H_{h-1}\mu) \\ &\quad + \frac{1}{2}(\Sigma' G_{h-1} + 2\Sigma' H_{h-1}\mu)'(I - 2\Sigma' H_{h-1}\Sigma)^{-1}(\Sigma' G_{h-1} + 2\Sigma' H_{h-1}\mu) \\ &\quad - \frac{1}{2}\ln[\det(I - 2\Sigma' H_{h-1}\Sigma)], \\ G_h' &= -\gamma_1' + (G_{h-1} + 2H_{h-1}\mu)'\rho + 2(\Sigma' G_{h-1} \\ &\quad + 2\Sigma' H_{h-1}\mu)'(I - 2\Sigma' H_{h-1}\Sigma)^{-1}\Sigma H_{h-1}\rho', \\ H_h &= -\Omega + \rho' H_{h-1}\rho + 2(\Sigma' H_{h-1}\rho)'(I - 2\Sigma' H_{h-1}\Sigma)^{-1}\Sigma' H_{h-1}\rho, \end{aligned}$$

where F_h, G_h, and H_h are computed using the recursions defined in the following section with the initial conditions $F_1 = -\gamma_0$, $G_1 = -\gamma_1$, and $H_1 = -\Omega$.

2. Recursive Coefficients in Equation (C-2)

We write equation (8) by splitting the expectation into two components, as follows:

$$\begin{aligned} \text{(C-6)}\ \mathrm{PS}(t, t+h) &= \mathrm{E}_t^Q\left[\mathrm{LGD}_{t+h-1} \times \exp\left(-\sum_{j=0}^{h-1} r_{t+j}\right) \times \exp\left(-\sum_{j=0}^{h-2} \lambda_{t+j}\right)\right] \\ &\quad - \mathrm{E}_t^Q\left[\mathrm{LGD}_{t+h-1} \times \exp\left(-\sum_{j=0}^{h-1} r_{t+j}\right) \times \exp\left(-\sum_{j=0}^{h-1} \lambda_{t+j}\right)\right] \\ &= Y^1(t, t+h) + Y^2(t, t+h). \end{aligned}$$

We derive each of the two terms in the expectation equation (C-6) separately. We start with the first term in the expectation denoted by $Y^1(t, t+h)$, which has the following solution form:

$$\text{(C-7)} \quad Y^1(t, t+h) = \exp(M_h + N_h' X_t + X_t' O_h X_t).$$

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We can solve for recursive relations for the coefficients M_h, N_h, and O_h following the same approach used in Appendix B. We provide the results as follows:

$$\begin{aligned}
M_h &= -\gamma_0 + (M_{h-1} + N_{h-1}'\mu + \mu' O_{h-1}\mu) \\
&\quad + \frac{1}{2}(\Sigma' N_{h-1} + 2\Sigma' O_{h-1}\mu)'(I - 2\Sigma' O_{h-1}\Sigma)^{-1}(\Sigma' N_{h-1} + 2\Sigma' O_{h-1}\mu) \\
&\quad - \frac{1}{2}\ln[\det(I - 2\Sigma' O_{h-1}\Sigma)], \\
N_h' &= -\gamma_1' + (N_{h-1} + 2O_{h-1}\mu)'\rho \\
&\quad + 2(\Sigma' N_{h-1} + 2\Sigma' O_{h-1}\mu)'(I - 2\Sigma' O_{h-1}\Sigma)^{-1}\Sigma O_{h-1}\rho', \\
O_h &= -\Omega + \rho' O_{h-1}\rho + 2(\Sigma' O_{h-1}\rho)'(I - 2\Sigma' O_{h-1}\Sigma)^{-1}\Sigma' O_{h-1}\rho.
\end{aligned}$$

The initial conditions for the previous recursions are $M_1 = -(\delta_0^2 + \beta_0^2)$, and

$$N_1 = -\begin{bmatrix} 2(\delta_0\delta' + \beta_0\beta^{r\prime}) \\ 2\beta_0\beta^{c\prime} \end{bmatrix}; \quad O_1 = -\begin{bmatrix} \delta\delta' + \beta^r\beta^{r\prime} & \beta^r\beta^{c\prime} \\ \beta^c\beta^{r\prime} & \beta^c\beta^{c\prime} \end{bmatrix}.$$

where β^r and β^c are defined in Appendix A.

The solution to the second term, $Y^2(t, t+h)$, in the expectation of equation (C-6) can be solved in the same manner. The solution takes the following form:

$$Y^2(t, t+h) = \exp(J_h + K_h' X_t + X_t' L_h X_t). \tag{C-8}$$

The exponential coefficients J_h, K_h, and L_h in $Y^2(t, t+h)$ can be solved recursively backward. Their solutions are as follows:

$$\begin{aligned}
J_h &= -\gamma_0 + (J_{h-1} + K_{h-1}'\mu + \mu' L_{h-1}\mu) \\
&\quad + \frac{1}{2}(\Sigma' K_{h-1} + 2\Sigma' L_{h-1}\mu)'(I - 2\Sigma' L_{h-1}\Sigma)^{-1}(\Sigma' K_{h-1} + 2\Sigma' L_{h-1}\mu) \\
&\quad - \frac{1}{2}\ln[\det(I - 2\Sigma' L_{h-1}\Sigma)], \\
K_h' &= -\gamma_1' + (K_{h-1} + 2L_{h-1}\mu)'\rho \\
&\quad + 2(\Sigma' K_{h-1} + 2\Sigma_{h-1}' L_{h-1}\mu)'(I - 2\Sigma' L_{h-1}\Sigma)^{-1}\Sigma L_{h-1}\rho', \\
L_h &= -\Omega + \rho' L_{h-1}\rho + 2(\Sigma' L_{h-1}\rho)'(I - 2\Sigma' L_{h-1}\Sigma)^{-1}\Sigma' L_{h-1}\rho,
\end{aligned}$$

with the initial conditions

$$\begin{aligned}
J_1 &= -(\delta_0^2 + \beta_0^2 + \alpha_0^2), \\
K_1 &= -\begin{bmatrix} 2(\delta_0\delta' + \beta_0\beta^{r\prime} + \alpha_0\alpha^{r\prime}) \\ 2(\beta_0\beta^{c\prime} + \alpha_0\alpha^{c\prime}) \end{bmatrix},
\end{aligned}$$

and

$$L_1 = -\begin{bmatrix} \delta\delta' + \beta^r\beta^{r\prime} + \alpha^r\alpha^{r\prime} & \beta^r\beta^{c\prime} + \alpha^r\alpha^{c\prime} \\ \beta^c\beta^{r\prime} + \alpha^c\alpha^{r\prime} & \beta^c\beta^{c\prime} + \alpha^c\alpha^{c\prime} \end{bmatrix}.$$

Table A1 provides a summary of the model parameters and parameterization.

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TABLE A1

Summary of Model Parameters and Parametrization

The model consists of five latent state variables, namely, X_i for $i=1,\ldots,5$. The first two factors $[X_1, X_2]$ are identified from the term structure of default-free London Interbank Offered Rate (LIBOR) swap rates. This is the first-stage estimation. Panel A summarizes their parameters, where the short-rate dynamic is given in equation (3). The remaining three factors $[X_3, X_4, X_5]$ are identified from the term structure of credit default swap (CDS) spreads written on senior and subordinate bonds. Panel B reports their parameters, which are estimated firm-by-firm. The parameters from Panel A and filtered latent variables X_1 and X_2 are applied to fit CDS contracts in the second-stage estimation. The dynamic of default intensity (λ_t) is described in equation (5). The loss-given-default dynamic of senior (LGD^{sen}) and subordinate CDS contracts (LGD^{sub}) is described by equation (6). Each latent state factor follows a Gaussian AR(1) process given by $X_{i,t+1}=\mu_i+\rho_i X_{i,t}+\sigma_i\epsilon_{i,t}$, for $i=1,\ldots,5$, where $\epsilon_{i,t}$ is independently and identically distributed as standard normal. We apply superscript P to denote the physical-measure (P-measure) parameters. All other parameters are estimated under the risk-neutral measure. Panel B also shows the normalization of the parameters in the model by reducing the number of parameters to be estimated. The loading coefficients α_3, α_4, and β_5^{sen} are set to 1, and $\mu_i^P=0$ for $i=3,4,5$.

Panel A. Parameters Estimated from Default-Free Bonds

		Default-Free Factors	
Parameters	Constant	X_1	X_2
Factor loadings			
Risk-free rate	δ_0	δ_1	δ_2
Factor dynamics			
Drift		μ_1	μ_2
Drift (P-measure)		μ_1^P	μ_2^P
Autoreg term		ρ_1	ρ_2
Autoreg term (P-measure)		ρ_1^P	ρ_2^P
Volatility		σ_1	σ_2

Panel B. Parameters Estimated from Senior and Subordinate CDS Contracts

		Default-Free Factors		Default-Intensity Factors		Recovery Factor
Parameters	Constant	X_1	X_2	X_3	X_4	X_5
Factor loadings						
Default intensity	α_0	α_1	α_2	$\alpha_3=1$	$\alpha_4=1$	
LGD for senior CDS	β_0^{sen}	β_1^{sen}	β_2^{sen}			$\beta_5^{sen}=1$
LGD for subordinate CDS	β_0^{sub}	β_1^{sub}	β_2^{sub}			β_5^{sub}
Factor dynamics						
Drift				μ_3	μ_4	μ_5
Drift (P-measure)				$\mu_3^P=0$	$\mu_4^P=0$	$\mu_5^P=0$
Autoreg term				ρ_3	ρ_4	ρ_5
Autoreg term (P-measure)				ρ_3^P	ρ_4^P	ρ_5^P
Volatility				σ_3	σ_4	σ_5

Supplementary Material

Supplementary material for this article is available at https://doi.org/10.1017/S0022109018000558.

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