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RADIOCARBON DISTANCE BETWEEN CALENDAR DATES

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ABSTRACT. The calibration procedure, and especially the nonlinear shape of the calibration curve, makes analyzing a possible dating result a far from straightforward process. This is especially so if the goal is to distinguish between two relatively close events. Proposed herein is a calculator, or alternatively a graph, which enables reading of the difference between two radiocarbon ages corresponding to their expected calendar ages. The result may surprise the less experienced ^{14}C users. Such a calculation also indicates the time periods with high or low potential for application of the wiggle-matching method.

INTRODUCTION

In archaeological investigations, the question can arise of whether two rather close events of known (expected) age can be distinguished by the radiocarbon method. The problem would be only a trivial statistical one if there were no interference from the wiggles of the calibration curve. Thus, before application of standard statistical inference, the problem needs to be linearized against the calibration curve. This task is easy when starting with the calendar age since the calibration curve is unequivocal for the calculation of the ^{14}C age (Figure 1). On the other hand, as is typical in ^{14}C determination, when one possesses the ^{14}C age and is interested in the calendar age, the result is a multimodal (almost alpine-shaped) probability distribution, which frequently appears in archaeological as well as geological articles. It is a source of trouble for simple inferences about the time axis, which is a calendric, not ^{14}C one. The inadequately recognized, relatively subtle result is that of a bias of the dating method, which is a local feature, unequally distributed along the calendar time (Walanus 2009).

A good example of such a problem would be the controversy about the authenticity of the grave of Copernicus (AD 1473–1543), in Frombork, Poland (Gingerich 2009). From Figure 2, it is clear that the date AD 1543 is not very good if it is to be distinguishable from the slightly older, and especially younger dates, by means of ^{14}C. There is a white bay of low differences in ^{14}C years around AD 1543.

Any situation where one possesses two or more samples, with a known distance between them in calendar years and known approximate calendar age, is where Figure 2 may be useful. Clearly, this is a question of how efficient wiggle-matching (Galimberti et al. 2004) would be at different calendar ages. In the figure, white bays indicate that wiggle-matching would be of little use, while dark gray approaching the time axis, like that at 800 BC, for example, would suggest that wiggle-match dating would achieve high precision.

Similar but opposite reasoning applies to single samples as well. The ^{14}C timespan defined by the $\pm\sigma$ around the date will be transformed through the calibration procedure into a calendar timespan (with, say, a 68% confidence interval), which will be relatively short or long: short for the calendar ages out of the dark bays, and long for those surrounded by a white area in Figure 2.

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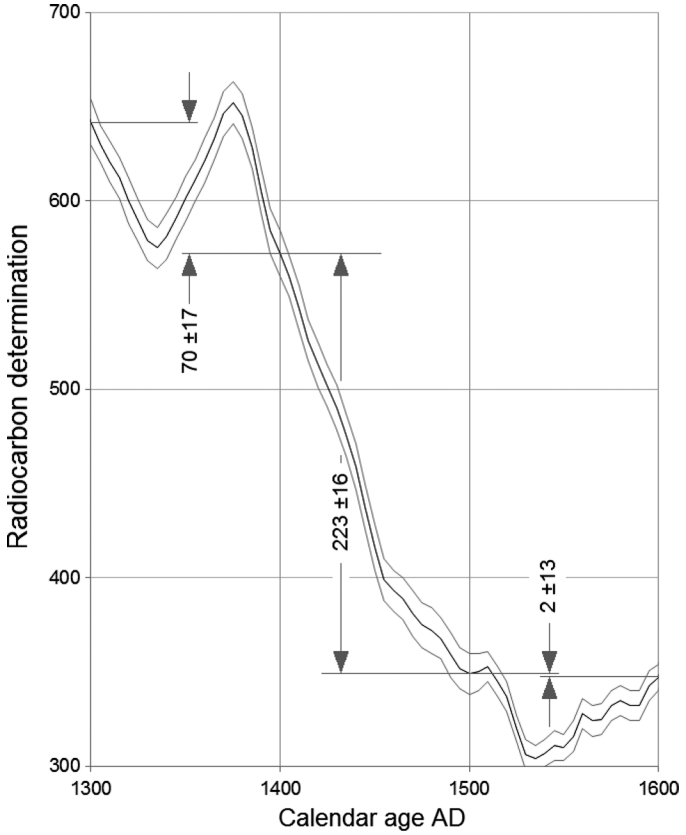


Figure 1 Fragment of the calibration curve (IntCal13, Reimer et al. 2013), with indicated distances in the ^{14}C age for pairs of calendar ages: AD 1300–1400, 1400–1500, and 1500–1600. Errors (±) are calculated according to the precision of the calibration curve, which is also shown as closely bound to the curve itself.

All of that can, of course, be read off almost directly from the calibration curve, but only almost. In Figure 2, the quantity of interest, i.e. how many ^{14}C years there are between two calendar dates, can immediately be discerned.

The map presented in the figure has been calculated in a relatively easy way, because calculation of the ^{14}C age for a given calendar age is straightforward. The measurement error of the calibration curve has not been taken into account in producing the map in the figure, because it is impossible to show errors in an isoline-type map. This drawback may be overcome by using the kind of calculator shown in Figure 3.

CALCULATION

In the calculation of the difference in ^{14}C years between two precisely given calendar dates, only the imprecision of the calibration curve has to be taken into account. This is the case using the calculator presented in Figure 3, where the error of the final result is calculated by the standard rule of the summing of two errors, namely, those of the calibration curve at the two respective points. (The rule is: the square root of the sum of the squares of the uncertainties.)

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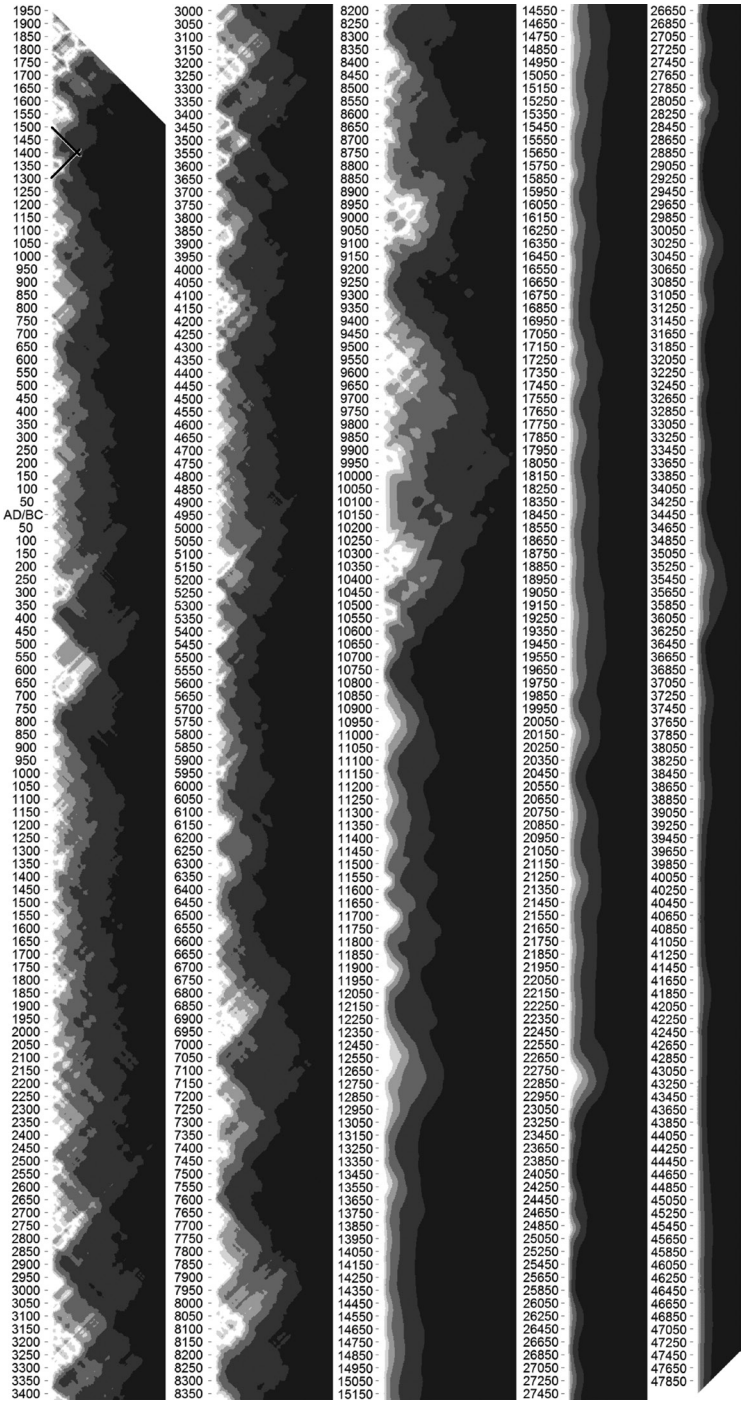


Figure 2 Map of the ^{14}C distance between two calendar ages (AD/BC). The darkest gray indicates distance >400 ^{14}C yr, white is for <25 yr, and the boundaries between are 50, 100, and 200 yr. The point corresponding to the pair of calendar dates is that where the respective diagonal lines cross (see e.g. at AD 1300–1500, where the result is about 300 ^{14}C yr). The timescale is not linear, and corresponds to the data source, i.e. IntCal13 (Reimer et al. 2013). The high-resolution plot may be found as a supplemental data file accompanying the online version of this article.

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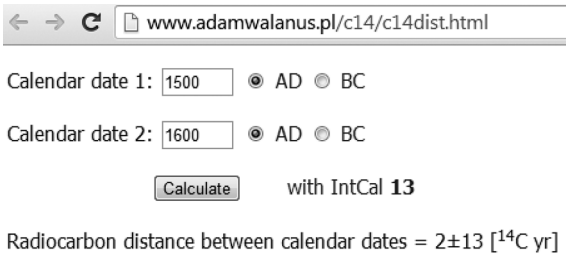


Figure 3 Calculator for the distance in ^{14}C age between two calendar dates.

Another question, already referred to, is that of the errors of the potential ^{14}C measurements of samples of a given expected calendar age. The uncertainty quoted for the ^{14}C date depends simply on the precision of the measurement and the estimate of any other associated uncertainties. Assuming two sigmas, σ_1, σ_2, of two measurements (where one would be zero, for example, for a historically dated event), the rule for the probability that, after two ^{14}C datings, the two calendar events would be differentiated as follows:

$$p = \Phi^{-1}((c - 1.64*(\sigma_1^2 + \sigma_2^2)^{1/2})/(\sigma_1^2 + \sigma_2^2 + \sigma_c^2)^{1/2})$$

where Φ^{-1} is the inverse normal cumulative distribution function, c is the distance in ^{14}C year age between two calendar dates, and σ_c is its error, both these derived from the calculator (Figure 3).

In the formulae above, it is assumed that the rule for the decision, if two ^{14}C determinations ($d_1 \pm \sigma_1$, $d_2 \pm \sigma_2$) are different, is that defined by the 5% significance level (see, however, Johnson 2013), i.e.:

$$(d_1 - d_2)/(\sigma_1^2 + \sigma_2^2)^{1/2} > 1.64$$

assuming sample 1 was *a priori* expected to be older than sample 2 (one-sided statistical test).

The value of c used above may be estimated from Figure 2, as well as in the way typically used for isoline maps. The σ_c value may, in such a case, be estimated as an error of the estimation of c itself, since that estimation is probably much less precise than the calibration curve is (even if its error is multiplied by $2^{1/2}$, on account of there being two readings of c).

CONCLUSION

The map presented in Figure 2 is certainly more complex than the plot of the calibration curve alone (Figure 1). However, the interpretation of the error band around the curve would be misleading for less mathematically oriented ^{14}C date users. The isoline map proposed is straightforward in its reading: it simply offers a single number as the answer to the question of how far apart two calendar points are, in the sense of the ^{14}C method. Even simpler is the proposed calculator. The resolution power of the ^{14}C method varies greatly along the time axis; Figure 2 is a quantitative visualization of this essential feature.

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