J. Plasma Physics (1997), *vol.* 57, *part* 3, *pp.* 491–500. Printed in the United Kingdom
© 1997 Cambridge University Press

Solitary waves and corresponding double layers in relativistic plasmas with multitemperature electrons

G. C. DAS[1] and K. M. SEN[2]

[1] Plasma Physics Division, Institute of Advanced Study in Science and Technology, Khanapara, Guwahati 7 81 022, Assam, India
[2] Department of Mathematics, Modern College, Imphal 795 001, Manipur, India

(Received 12 July 1995 and in revised form 21 February 1996)

The evolution of solitary waves is studied using the KP equations derived by employing the reductive perturbation technique. It is revealed that the relativistic effect plays a role in the formation of double layers in multitemperature electron plasmas. Because of the singularity in soliton propagation introduced by the zeros in the nonlinear coefficients of the KP wave equations. Owing to the appearance of zeros in the nonlinearity, there might be a continuous modification of the KP equation. The relevant properties of compressive and rarefactive solitons as well as shock-like soliton are considered. It is possible that the layers of various solitary waves might be observed in plasmas.

1. Introduction

We have seen in recent years many investigations of solitons and double layers in two-temperature electron plasmas. Sometimes the multitemperature electrons must be considered because of the trapping of some electrons within a potential well formed in a plasma, in which the particles suffer a loss of energy. They have also been observed in the auroral plasma as well as laser-induced plasmas in the laboratory. The experiment of Nishida and Nagasawa (1986) has, in particular, confirmed the existence of rarefactive solitary waves in a multitemperature electron plasma. Although one might expect the evolution of solitary waves as an isolated phenomenon. We extend our studies to include the surroundings of such a plasma region. Among pioneering work, it is worth mentioning the observations of Jones *et al.* (1975), Goswami and Buti (1976), Buti (1980) and Nishihara and Tajiri (1981), Das *et al.* (1986) studied the formation of ion-acoustic waves as well as soliton behaviour in such plasmas. Witt and Lotko (1983), Goswami and Bujarbarua (1985), Chanteur and Raadu (1987), Nejoh (1987*a*, *b*), Raadu (1989) and Das and Sen (1991, 1994*a*, *b*) extended their investigations to study the formation of double layers in both magnetized and unmagnetized plasmas.

Most previous work considered a simplified model in which relativistic effects were ignored. However, it has been shown that this leads one to overlook important aspects of ion-acoustic waves. Recently, we have witnessed a newfound interest in studying the evolution of solitons and double layers in relativistic plasmas. The velocity of plasma particles, as compared with that of

https://doi.org/10.1017/S0022377897005382 Published online by Cambridge University Press

light, is negligibly small in the laboratory, and, as well, ion-acoustic waves exhibit non-relativistic character. But plasma particles in the solar wind propagate with high speed during solar flares, and their velocity must be taken as weakly relativistic. This affects the characteristic features of ion-acoustic solitary waves and correspondingly the double layers, and may lead to new phenomena.

Das and Paul (1985) first studied the propagation of small-amplitude ion-acoustic solitons through the KdV equation in a relativistic plasma with isothermal electrons, and showed that, because of the weakly relativistic effect, the solitons may break up into many more solitons with the formation of precursors. Das *et al.* (1988) later showed that no such peculiar behaviour is exhibited in the absence of the relativistic effect. The effect of ion temperature on the formation of solitons in relativistic plasmas has been studied by Nejoh (1987). He showed that the amplitude of the solitary waves increased with increasing relativistic effect for lower ion temperature, and precursors could be formed with breaking up the solitons into many more solitons. However, no such phenomenon is observed for higher ion temperatures. More recently, Nejoh (1992) showed further that relativistic double layers, along with other various solitary waves, could be formed in plasmas. He showed that, although the nonlinear coefficient in the KdV equation is always positive in the case of higher order nonlinearity, the coefficient in the modified KdV (mKdV) equation could be negative but independent of ion temperature, and finally the relativistic effect may play a role.

Since multitemperature electrons are known to cause a similar effect on the nonlinear coefficients in KdV equations for unidirectional solitons, we shall investigate here the evolution of other types of solitary waves in space plasmas. Before the work of Das and Sen (1991), emphasis was placed on the formation of solitary waves in simple plasmas, on the higher-potential side of the double-layer region. The present investigation is mainly addressed to the formation of solitary waves on the lower-potential side of the double layer.

2. Basic equations and derivation of the KP equation

We consider a collisionless, relativistic plasma consisting of cold ions and two-temperature electrons arising owing to the trapping of the electrons in the potential well formed in the plasma. The non-identical electron densities and their temperatures are $n_{eh,w}$ and $t_{eh,w}$ respectively. To derive the relativistic effect in isolation, neglecting electron inertia, we write the composite electron density as

$$n_e(\phi) = \frac{1}{s!}\sum_s \alpha_s \phi^{(s)},$$

where

$$\alpha_s = \frac{n_h + n_w \sigma^s}{(n_h + n_w \sigma)^s}, \qquad \sigma = \frac{t_h}{t_w}. \tag{1}$$

The effective temperature T_e of the composite electrons is given by

$$T_e = \frac{t_h t_w}{n_h t_w + n_w t_h}, \qquad \text{with} \quad n_h + n_w = 1. \tag{2}$$

https://doi.org/10.1017/S0022377897005382 Published online by Cambridge University Press

Furthermore, the ions are assumed to be cold, and the plasma dynamics is governed by the following non-dimensional equations:

continuity equation

$$\frac{\partial \bar{n}_i}{\partial \bar{t}} + \bar{\nabla} \cdot (\bar{\mathbf{n}}_i \cdot \bar{\mathbf{u}}) = 0, \tag{3}$$

equation of motion

$$\frac{\partial \bar{\mathbf{u}}^\gamma}{\partial \bar{t}} + (\bar{\mathbf{u}} \cdot \bar{\nabla})\, \bar{\mathbf{u}}^\gamma + \bar{\nabla} \bar{\phi} = 0. \tag{4}$$

The above system of equations is closed using Poisson's equation in the form

$$\bar{\nabla}^2 \bar{\phi} = \bar{n}_e - \bar{n}_i, \tag{5}$$

where $\bar{u}$ is the velocity of the ions, normalized by the ion-acoustic speed C_s, and the ion number density is $\bar{n}_i$, normalized by the unperturbed ion density n_0, the density at the equilibrium state. All other symbols have their usual meaning as given elsewhere (Das and Sen, 1994*a*, *b*).

To study the formation of finite-amplitude ion-acoustic waves in relativistic plasmas and to compare the results with observations in space as well as in the laboratory, we employ the reductive perturbation technique. The plasma parameters $\psi \equiv (n_i, u, \phi)$ are expanded in a power series as (omitting overbars herein after)

$$\left.\begin{aligned} n_i &= 1 + \varepsilon n_i^{(1)} + \varepsilon^2 n_i^{(2)} + \dots, \\ u_{x,y} &= \varepsilon^{3/2} u_{x,y}^{(1)} + \varepsilon^2 u_{x,y}^{(2)} + \dots, \\ u_z &= u_0 + \varepsilon u_z^{(1)} + \varepsilon^2 u_z^{(2)} + \dots, \\ \phi &= \varepsilon \phi^{(1)} + \varepsilon^2 \phi^{(2)} + \dots, \end{aligned}\right\} \tag{6}$$

where the ion drift u_0 is assumed to be restricted to the z direction in the equilibrium state. In matching with the expansions given by (6), the following stretched space–time coordinates are used:

$$\xi = \varepsilon x, \qquad \eta = \varepsilon y, \qquad \zeta = \varepsilon^{1/2}(z - \lambda t), \qquad \tau = \varepsilon^{3/2} t, \tag{7}$$

where λ is the unknown phase velocity of the ion-acoustic wave, to be determined later in a self-consistent manner. Using (6) and (7) in the basic equations (1)–(5) and then equating the various powers of ε, at the lowest order in ε determines the phase velocity λ as

$$\lambda = u_0 \pm \left[\frac{1}{1 + \frac{3}{2}(u_0/c)^2}\right]^{1/2}. \tag{8}$$

It may be deduced from (8) that the phase velocity λ is real if $u_0/c < \sqrt{\frac{2}{3}} \approx 0.8$, and it decreases as the relativistic effect u_0/c increases, which of course, will be shown later with the exhibition of solitary waves in plasmas. The next higher-order in ε yields the relation

$$\frac{\partial}{\partial \zeta}\left(\frac{\partial u_x^{(1)}}{\partial \xi} + \frac{\partial u_y^{(1)}}{\partial \eta}\right) = \frac{1}{\lambda - u_0}\left(\frac{\partial^2 \phi^{(1)}}{\partial \xi^2} + \frac{\partial^2 \phi^{(1)}}{\partial \eta^2}\right). \tag{9}$$

https://doi.org/10.1017/S0022377897005382 Published online by Cambridge University Press

Similarly using (9) in the system of equations derived at $O(\varepsilon^2)$, followed by a straightforward elimination of $n^{(2)}$, $u_z^{(2)}$ and $\phi^{(2)}$, we obtain the nonlinear partial differential equation in the form of the desired KP equation as

$$\frac{\partial}{\partial\zeta}\left(\frac{\partial\phi^{(1)}}{\partial\zeta}+A\phi^{(1)}\frac{\partial\phi^{(1)}}{\partial\zeta}+B\frac{\partial^3\phi^{(1)}}{\partial\zeta^3}\right)+C\left(\frac{\partial^2\phi^{(1)}}{\partial\xi^2}+\frac{\partial^2\phi^{(1)}}{\partial\eta^2}\right)=0, \tag{10}$$

where

$$A=\tfrac{1}{2}(\lambda-u_0)\frac{3-3(u_0/c)\,\lambda/c+7.5(u_0/c)^2}{1+\frac{3}{2}(u_0/c)^2},$$

$$B=\tfrac{1}{2}(\lambda-u_0), \qquad C=\tfrac{1}{2}(\lambda-u_0)^{-1}.$$

It should be noted that the dispersion coefficient B decreases as the relativistic effect u_0/c increases, while the transverse coefficient C increases with increasing relativistic effect. The nonlinear coefficient A, however, can be positive, negative or even zero; thus there may be a critical phase velocity. The behaviour of the dispersion coefficient is controlled by the relativistic effect, while the nonlinear coefficient is determined by the relativistic effect as well as by the multitemperature electron effect.

To find the stationary solution of the KP equation (10), we follow the usual procedure (Das and Sen, 1991, 1994*a*, *b*), introducing $\chi=l\xi+m\eta+n\zeta-U\tau$, where (l, m, n) are the direction cosines of the propagation direction with respect to a frame moving with velocity U. Taking the boundary condition

$$\phi^{(1)},\ \frac{d\phi^{(1)}}{d\chi},\ \frac{d^2\phi^{(1)}}{d\chi^2},\ \frac{d^3\phi^{(1)}}{d\chi^3}\to 0 \quad \text{as } |\chi|\to\infty, \tag{11}$$

the solution, after integrating (10), has the following form:

$$\phi^{(1)}=\phi_1\,\mathrm{sech}\,(\chi/\delta_1), \tag{12}$$

where

$$\phi_1=\frac{3(Cn^2+Un-C)}{An^2}, \qquad \delta_1=2\left(\frac{Bn^4}{Cn^2+Un-C}\right)^{1/2}$$

are respectively the amplitude and width of the solitary wave. The presence of the direction-cosine parameter in the expressions for the amplitude and width indicates that the solitary-wave properties are affected not only by the plasma parameters but also by the direction cosine. Again the soliton solution of (10) yields compressive or rarefactive solitary waves according to whether the nonlinear coefficient A is positive or negative. But both can exist only if $\lambda/c>\frac{2}{3}(3-\alpha_2)(5-\alpha_2)$; that is, if the parameter α_2, which depends functionally on the density and temperatures of the hot and warm electrons, does not lie in the interval $(3, 5)$. When the nonlinear coefficient A vanishes at a critical phase velocity, the amplitude of the solitary wave becomes infinite, and the solitary wave breaks up into many more solitons, resulting in the formation of precursors as predicted by Das and Paul (1985) and Nejoh (1987). In the case of $A=0$, the soliton solution of the KP equation fails to describe the soliton propagation in our plasma model. In this regard, following on from the earlier procedure for the simple cases of a plasma with negative ions, we modify the procedure to derive a modified KP equation.

https://doi.org/10.1017/S0022377897005382 Published online by Cambridge University Press

3. Derivation of the modified KP (mKP) equation

In order to study the salient features of solitons that might appear at the zeros of the nonlinear coefficient A in a relativistic plasma with multitemperature electrons, we introduce a new set of stretched space–time coordinates

$$\xi = \varepsilon^2 x, \quad \eta = \varepsilon^2 y, \quad \zeta = \varepsilon(z - \lambda t), \quad \tau = \varepsilon^3 t, \tag{13}$$

together with the appropriate modified expansions of the plasma parameters

$$\left.\begin{aligned} n_i &= 1 + \varepsilon n_i^{(1)} + \varepsilon^2 n_i^{(2)} + \dots, \\ u_{x,y} &= \varepsilon^2 u_{x,y}^{(1)} + \varepsilon^3 u_{x,y}^{(2)} + \dots, \\ u_z &= u_0 + \varepsilon u_z^{(1)} + \varepsilon^2 u_z^{(2)} + \dots, \\ \phi &= \varepsilon\phi^{(1)} + \varepsilon^2\phi^{(2)} + \dots \quad . \end{aligned}\right\} \tag{14}$$

Following a similar procedure as before, we use the relations (13) and (14) in the basic equations (1)–(5). Then, as usual, equating powers of ε determines the phase velocity λ as in expression (8). The next-higher order yields the following relations:

$$u_z^{(2)} = (\lambda - u_0)\,[\phi^{(2)} + \tfrac{1}{2}(\alpha_2 - 2)\,\phi^{(1)\,2}], \tag{15}$$

$$n^{(2)} = \phi^{(2)} + \tfrac{1}{2}\alpha_2\,\phi^{(1)\,2}. \tag{16}$$

Next, at $O(\varepsilon^3)$, with the elimination of $n^{(3)}$, $u_z^{(3)}$ and $\phi^{(3)}$, we derive a modified KP (mKP) equation in the form

$$\frac{\partial}{\partial\zeta}\left(\frac{\partial\phi^{(1)}}{\partial\zeta} + D\phi^{(1)\,2}\frac{\partial\phi^{(1)}}{\partial\zeta} + B\frac{\partial^3\phi^{(1)}}{\partial\zeta^3}\right) + C\left[\frac{\partial^2\phi^{(1)}}{\partial\xi^2} + \frac{\partial^2\phi^{(1)}}{\partial\eta^2}\right] = 0, \tag{17}$$

where

$$D = \tfrac{3}{2}(\lambda - u_0)\left[\tfrac{1}{2}(\alpha_2^2 - 2\alpha_2 + 2) - \tfrac{1}{6}\alpha_3 + \frac{1}{2c^2}\frac{(\lambda - u_0)\,(3u_0 - \lambda)}{1 + \tfrac{3}{2}(u_0/c)^2}\right],$$

and the dispersion coefficients B and C have the same values as defined earlier in (10). The higher-order nonlinear coefficient D can clearly be positive, zero or negative. As in the case of the nonlinear coefficient A in the KP equation (10), the behaviour of the higher-order nonlinear coefficient D in the mKP equation (17) is determined by the relativistic effect as well as by the presence of multitemperature electrons in the plasma.

In the region where D is positive, the mKP equation (17) admits a stationary solution

$$\phi^{(1)} = \pm\phi_2\,\mathrm{sech}\,(\chi/\delta_2), \tag{18}$$

where

$$\phi_2 = \left[\frac{6(Cn^2 + Un - C)}{Dn^2}\right]^{1/2}, \qquad \delta_2 = \left(\frac{Bn^4}{Cn^2 + Un - C}\right)^{1/2}$$

are respectively the amplitude and width of the solitary wave. The $\pm$ signs in the solution (18) show that both compressive and rarefactive solitary waves might be formed simultaneously, provided that $D > 0$ at the critical phase velocity for $A = 0$ in the KP equation. Otherwise, in the region where D is negative, the complexity of the nonlinearity leads to a shock-like solitary wave and causes a switch over from a solitary wave to a double layer. Following Das

https://doi.org/10.1017/S0022377897005382 Published online by Cambridge University Press

and Sen (1991, 1994*a*, *b*), the double-layer solution of the mKP equation (17) is given by

$$\phi^{(1)} = \pm\psi_0 \tanh\left[\left(\frac{-D}{6Bn^2}\right)^{1/2}\psi_0\chi\right], \tag{19}$$

where ψ_0 denotes the amplitude of the double layer. The thickness d of the double layer is given by

$$d = \frac{1}{\psi_0}\left(\frac{6Bn^2}{-D}\right)^{1/2}, \tag{20}$$

and its velocity U is expressed as

$$U = -\frac{1}{3n}[3C-(3C-D\psi_0^2)\,n^2]. \tag{21}$$

The solution (19), because of the $\pm$ signs, shows the existence of both compressive and rarefactive double layers, formed only when D is negative, and their properties can be seen to be determined by relativistic as well as multitemperature electron effects, and by the direction cosine of oblique propagation. The evolution of ion-acoustic solitary waves and double layers in a weakly relativistic plasma with two Maxwellian electron components has described by Yadav and Sharma (1989) in the same plasma configuration. They highlighted the fact that, although the unidirectional KdV soliton is affected by the relativistic ion drift velocity, its effect on the double-layer amplitude is negligible. It should again be noted here that the mKP equation fails to describe fully the soliton properties in the plasma owing to the appearance of zeros in the nonlinearity. To gain further insight as to how a transition takes place from an mKP$^{(+)}$ solitary wave to an mKP$^{(-)}$ double layer in a relativistic multicomponent plasma with two-temperature electrons, it is necessary to consider higher-order nonlinearity to examine the case where $D = 0$. We do this in the following section.

4. Derivation of a further modified KP (FmKP) equation and its solution

So far, we have considered the problem of deriving various KP equations through the use of a simple reductive perturbation technique, and have examined the behaviour of ion-acoustic solitary waves for the entire plasma region. However, because of the nonlinearity, which could be zero for certain plasma parameters, we have taken $A = 0$ and, using a modified perturbation scheme with higher-order nonlinearity, have derived an mKP equation and thus obtained a solitary-wave solution in the sech form. Although the singularity in soliton propagation found earlier is avoided, in the range of higher density of warm electrons the nonlinearity D could again be equal to zero, leading to similar singular behaviour in the propagation of compressive and rarefactive solitons. In this region, we do expect shock-like double layers, and so do not consider the soliton propagation in the entire plasma region. For further investigation, with the possibility of zeros in both D and A, we adopt new stretched coordinates

$$\xi = \varepsilon^3 x, \quad \eta = \varepsilon^3 y, \quad \zeta = \varepsilon^{3/2}(z-\lambda t), \quad \tau = \varepsilon^{9/2}t, \tag{22}$$

https://doi.org/10.1017/S0022377897005382 Published online by Cambridge University Press

along with the following expansions of the plasma parameters:

$$\left.\begin{aligned} n_i &= 1+\varepsilon n_i^{(1)}+\varepsilon^2 n_i^{(2)}+\dots, \\ u_{x,y} &= \varepsilon^{5/2}u_{x,y}^{(1)}+\varepsilon^{7/2}u_{x,y}^{(2)}+\dots, \\ u_z &= u_0+\varepsilon u_z^{(1)}+\varepsilon^2 u_z^{(2)}+\dots, \\ \phi &= \varepsilon\phi^{(1)}+\varepsilon^2 f^{(2)}+\dots \quad . \end{aligned}\right\} \tag{23}$$

Using (22) and (23) in the basic equations governing the plasma dynamics, we get similar relations at $O(\varepsilon)$ and $O(\varepsilon^2)$, revealing results similar to those found earlier. But at $O(\varepsilon^3)$ and by using the critical conditions $A=0$ and $D=0$, two new relations are obtained:

$$u_z^{(3)} = (\lambda-u_0)\left\{\phi^{(3)}+(\alpha_2-2)\,\phi^{(1)}\phi^{(2)} + \left[\tfrac{1}{2}(\alpha_2-2)^2+\frac{1}{2c^2}\frac{(\lambda-u_0)(3u_0-\lambda)}{1+\frac{3}{2}(u_0/c)^2}\right]\phi^{(1)\,3}\right\}, \tag{24}$$

$$n^{(3)} = \phi^{(3)}+\alpha_2\,\phi^{(1)}\phi^{(2)}+\tfrac{1}{6}\alpha_3\,\phi^{(1)\,3}. \tag{25}$$

By straightforward manipulations, Poisson's equation is found to be automatically satisfied in the system of equations derived at $O(\varepsilon^3)$. Both the critical conditions $A=0$ and $D=0$ are used in the derivation of the desired wave equation.

At the next-higher order in ε, one can eliminate $\partial u_z^{(4)}/\partial\zeta$ to obtain

$$\begin{aligned} \frac{\partial n^{(4)}}{\partial\zeta}-\frac{\partial\phi^{(4)}}{\partial\zeta} &= \frac{2}{\lambda-u_0}\frac{\partial\phi^{(1)}}{\partial\zeta}+\alpha_2\frac{\partial}{\partial\zeta}(\phi^{(1)}\phi^{(2)})+\tfrac{1}{2}\alpha_2\frac{\partial}{\partial\zeta}(\phi^{(1)\,2}) \\ &\quad+\left[3(\alpha_2-1)+\tfrac{3}{2}(\alpha_2-2)^2+\frac{\frac{3}{2}(\alpha_2-2)(3u_0-\lambda)}{c^2[1+\frac{3}{2}(u_0/c)^2]}\right]\frac{\partial}{\partial\zeta}(\phi^{(1)\,2}\phi^{(2)}) \\ &\quad+\left[\tfrac{1}{4}\alpha_2(\alpha_2-2)+\tfrac{1}{2}(\alpha_2-2)^2+\tfrac{5}{8}(\alpha_2-2)^3+\frac{5}{4}\frac{(\alpha_2-2)(\lambda-u_0)(3u_0-\lambda)}{c^2[1+\frac{3}{2}(u_0/c)^2]}\right. \\ &\qquad\left.+\frac{\alpha_3}{6}+\frac{3}{8}\frac{(\lambda-u_0)^4}{c^2}+\frac{(3u_0-\lambda)(\lambda-u_0)}{2c^2[1+\frac{3}{2}(u_0/c)^2]}\right]\frac{\partial}{\partial\zeta}(\phi^{(1)})^4 \\ &\quad+\frac{1}{\lambda-u_0}\left(\frac{\partial u_x^{(1)}}{\partial\xi}+\frac{\partial u_y^{(1)}}{\partial\eta}\right). \end{aligned} \tag{26}$$

Correspondingly, Poisson's equation at $O(\varepsilon^4)$ yields

$$\frac{\partial^2\phi^{(1)}}{\partial\zeta^2} = \phi^{(4)}+\tfrac{1}{2}\alpha_2\,\phi^{(2)\,2}+\alpha_2\,\phi^{(1)}\phi^{(3)}+\tfrac{1}{2}\alpha_3\,\phi^{(1)\,2}\phi^{(2)}+\tfrac{1}{24}\alpha_4\,\phi^{(1)\,4}-n^{(4)}. \tag{27}$$

Differentiating (27) twice with respect to ζ and then using the critical condition at $D=0$ and $A=0$, the desired FmKP equation is

$$\frac{\partial}{\partial\zeta}\left(\frac{\partial\phi^{(1)}}{\partial\zeta}+F\phi^{(1)3}\frac{\partial\phi^{(1)}}{\partial\zeta}+B\frac{\partial^3\phi^{(1)}}{\partial\zeta^3}\right)+C\left(\frac{\partial^2\phi^{(1)}}{\partial\xi^2}+\frac{\partial^2\phi^{(1)}}{\partial\eta^2}\right)=0, \tag{28}$$

https://doi.org/10.1017/S0022377897005382 Published online by Cambridge University Press

where

$$F = 2(\lambda - u_0)\left\{[\tfrac{1}{8}(\alpha_2 - 2)(5\alpha_2^2 - 14\alpha_2 + 12) + \tfrac{1}{6}\alpha_3 - \tfrac{1}{24}\alpha_4] + \frac{(1/c)(\lambda - u_0)(3(10\alpha_2 - 17)u_0/c - (10\alpha_2 - 19)\lambda/c)}{8[1 + \frac{3}{2}(u_0/c)^2]}\right\}.$$

It should be noted that the nonlinear coefficient F is again quadratic in the relativistic phase velocity λ/c. But the presence of two-temperature electrons in the plasma makes it difficult to determine analytically the regions where F may be positive or negative. In the region where F is positive, the solitary-wave solution of the FmKP equation (28) is obtained in the form

$$\phi^{(1)} = \phi_3[\mathrm{sech}\,(\chi/\delta_3)]^{2/3}, \tag{29}$$

where

$$\phi_3 = \left(\frac{10(Cn^2 + Un - C)}{Fn^2}\right)^{1/3},$$

$$\delta_3 = \frac{2}{3}\left(\frac{Bn^4}{Cn^2 + Un - C}\right)^{1/2}.$$

The solution, with the consideration of higher-order nonlinearity, describes soliton propagation for the case of $F > 0$. In the case of higher warm electron density, when $F < 0$, the soliton solution, obtained following the same procedure as described in Das and Sen (1991, 1994*a*, *b*), has a complex amplitude, which as in the case of the mKP equation, reveals shock-like solitary-wave propagation.

As a degenerate case, in the absence of the relativistic effect, the corresponding evolution equation is

$$\frac{\partial}{\partial\zeta}\left(\frac{\partial\phi^{(1)}}{\partial\zeta} + F'\phi^{(1)3}\frac{\partial\phi^{(1)}}{\partial\zeta} + B'\frac{\partial^3\phi^{(1)}}{\partial\zeta^3}\right) + C'\left(\frac{\partial^2\phi^{(1)}}{\partial\xi^2} + \frac{\partial^2\phi^{(1)}}{\partial\eta^2}\right) = 0, \tag{30}$$

where

$$F' = \tfrac{1}{12}(105 - \alpha_4), \qquad B' = C' = \tfrac{1}{2}.$$

The expression for the higher-order nonlinear coefficient F' shows clearly that it may be positive, zero or negative according to whether $\alpha_4 < 105$, $\alpha_4 = 105$ or $\alpha_4 > 105$, and thus we expect various kinds of solitary waves along with the possible formation of shock-like solitary waves in the dynamical system. Here our aim is to highlight the observation of the formation of various solitons with the change in nonlinearity, which in fact might give soliton propagation in the entire plasma, where we must have a high-order nonlinear soliton. In the case of weak nonlinearity, we have also indicated the observation of shock-like solitary waves along with compressive and rarefactive solitary waves. Here, though, we shall not repeat the derivation of the double-layer solution, but, following the procedure described in Das (1994*a*, *b*), by simple manipulations one could derive the solution for $F' < 0$. However, the present study does not have the main aim of highlighting the double layers; rather it seeks to show the existence of solitary waves in the entire plasma region.

https://doi.org/10.1017/S0022377897005382 Published online by Cambridge University Press

Although we have different types of nonlinear wave equation representing formation of different waves, the amplitude–width relations of various ion-acoustic solitary waves expressed by the sech^2, Sech and $\mathrm{Sech}^{2/3}$ profiles in plasmas with two-temperature electrons are conserved, and are expressed as follows:

$$\text{amplitude} \times \text{width}^2 = \text{constant},$$

$$\text{amplitude}^2 \times \text{width}^2 = \text{constant},$$

$$\text{amplitude}^3 \times \text{width}^2 = \text{constant}.$$

The solitary waves appearing in the plasma will be determined by the order of nonlinearity, and thus there will be the nonlinear waves of different amplitudes in the plasma. It might appear that there is a close symmetrical link between the formation of these solitary waves and the stretching coordinates employed to derive the evolution equations. The soliton wave equation might be extended to an n-times modification with higher-order nonlinearity to proceed toward the higher-concentration region of warm electrons wherein a soliton profile of type $\mathrm{sech}^{2/n}$ is expected. In this case the relation $(\text{amplitude})^n \times (\text{width})^2 = \text{constant}$ must also hold in order to satisfy energy conservation and retain the form of the soliton profile. Again, with the consideration of higher-order nonlinearity in the dynamical system, and the propagation of high-amplitude nonlinear waves in the plasma, it is always possible to observe shock-like solitary waves. These results could be the basis for experimental work. In the case of the KP equation, the solitons have either a compressive or rarefactive nature. It has been observed here, with the modification of the perturbation technique in non-relativistic plasmas, that the range of critical densities shifts towards the higher-density region of warm electrons. In the present case, because of the n-times modifications, the critical density might be in the higher range of warm electron density. Thus the procedure described here might give a full account of solitary waves in the lower range and the formation of distinct layers of waves in the plasma.

5. Discussion and conclusions

We have studied the possible evolution of various forms of ion-acoustic waves in a relativistic multicomponent plasma with cold ions and two-temperature electrons through the derivation of KP, mKP and FmKP equations. It has been shown that, under suitable conditions, both compressive and rarefactive double layers are expected. While in earlier studies the investigations were focused on the characteristic features of ion-acoustic solitary waves formed on the higher-potential side of the double-layer regions, in the present study the investigation has been extended to cover the regions on both sides of the double layers. The study appears to indicate that in some relativistic multicomponent plasmas there may evolve layer after layer of different solitary waves with different amplitudes. Here we have shown that the controlling factor is the order of nonlinearity with different-amplitude solitons. Because of this, in contrast to earlier studies, we have found compressive and rarefactive as well as shock-like solitary waves. The evolution of shock-like solitons depends on the nature of the nonlinearity in the KP equations. It has been shown clearly that the high-order nonlinearity arises for the higher range of the concentrations of

https://doi.org/10.1017/S0022377897005382 Published online by Cambridge University Press

warm electrons in a plasma for which compressive and rarefactive solitary waves are expected. The shock-like soliton waves are observed in the region of higher concentrations of warm electrons. These solitary waves are determined by the physical parameters of the plasma, and their formation may also depend on the stretched coordinates.

References

Buti, B. 1980 *Phys. Lett.* **76A**, 251.
Chanteur, G. and Raadu, M. 1987 *Phys. Fuids* **30**, 2708.
Das, G. C., Karmakar, B. and Paul, S. N. 1988 *IEEE Trans. Plasma Sci.* **16**, 22.
Das, G. C., Karmakar, B. and Singh, Kh. I. 1990 *IEEE Trans. Plasma Sci.* **18**, 86.
Das, G. C. and Paul, S. N. 1985 *Phys. Fluids* **28**, 823.
Das, G. C. and Sen, K. M. 1991 *Contrib. Plasma Phys.* **31**(6), 13.
Das, G. C. and Sen, K. M. 1994*a* *Planet. Space Sci.* **42**, 41.
Das, G. C. and Sen, K. M. 1994*b* *Earth, Moon and Planets* **64**, 47.
Das, G. C., Paul, S. N. and Karmakar, B. 1986 *Phys. Fluids* **29**, 2192.
Goswami, B. N. and Bujarbarua, S. 1985 *Phys. Lett.* **108A**, 149.
Goswami, B. N. and Buti, B. 1976 *Phys. Lett.* **57A**, 149.
Jones, W. D., Lee, A., Gleeman, S. M. and Doucet, H. 1975 *Phys. Rev. Lett.* **35**, 1349.
Nejoh, Y. 1987*a* *J. Plasma Phys.* **37**, 487.
Nejoh, Y. 1987*b* *Phys. Lett.* **123A**, 245, 1987.
Nejoh, Y. 1992 *Phys. Fluids* **B4**, 2830.
Nishida, Y. and Nagasawa, T. 1986 *Phys. Fluids* **29**, 345.
Nishihara, Y. and Tajiri, M. 1981 *J. Phys. Soc. Japan* **50**, 4047.
Raadu, M. A. 1989 *Phys. Rep.* **178**, 27.
Yadav, L. L. and Sharma, S. R. 1989 *Proceedings of International Conference on Plasma Physics, India*, p. 781.
Witt, E. and Lotko, W. 1983 *Phys. Fluids* **26**, 2176.

https://doi.org/10.1017/S0022377897005382 Published online by Cambridge University Press