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Dark energy and dark matter as due to zero point energy

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Abstract. An attempt is made to explain dark energy and dark matter of the expanding universe in terms of the zero point vacuum energy. This analysis is mainly limited to later stages of an observable nearly flat universe. It is based on a revised formulation of the spectral distribution of the zero point energy, for an ensemble in a defined statistical equilibrium having finite total energy density. The steady and dynamic states are studied for a spherical cloud of zero point energy photons. The 'antigravitational' force due to its pressure gradient then represents dark energy, and its gravitational force due to the energy density represents dark matter. Four fundamental results come out of the theory. First, the lack of emitted radiation becomes reconcilable with the concepts of dark energy and dark matter. Second, the crucial coincidence problem of equal orders of magnitude of mass density and vacuum energy density cannot be explained by the cosmological constant, but is resolved by the present variable concepts, which originate from the same photon gas balance. Third, the present approach becomes reconcilable with cosmical dimensions and with the radius of the observable universe. Fourth, the deduced acceleration of the expansion agrees with the observed one. In addition, mass polarity of a generalized gravitation law for matter and antimatter is proposed as a source of dark flow.

1. Introduction

The universe has earlier been a speculative subject of discussion, but during the latest decades it has become more of an area of strict scientific analysis, in several respects due to Hubble's important discovery of the cosmic expansion and the Big Bang hypothesis by Gamov. Astronomical observations and related theoretical work have thus resulted in a number of new points of view, as described in reviews such as those by Linde (1994), Hogan et al. (1999), Luminet et al. (1999), Turner (2003), Perlmutter (2003), Riess and Turner (2004), Crease (2007), Linder and Perlmutter (2007), and Schmidt (2005).

Shortly after the discovery of the expanding universe, it was commonly expected that the attraction force due to the cosmical mass content would slow down the expansion, and even turn it to a cosmic contraction. As a great surprise, it was then found from ingenious methods of observation of the type Ia supernovae that the expansion of the universe is at present in fact accelerating (Hogan et al. 1999; Perlmutter 2003; Riess and Turner 2004; Schmidt 2005; Linder and Perlmutter 2007). This period of acceleration may not have prevailed during the entire expansion, but appears to have started about five billion years ago. In order to account for the present acceleration, about 75% of the mass–energy content is then considered to be made of some weird gravitationally repulsive substance called 'dark energy' (Linder and Perlmutter 2007), i.e. a 'cosmological antigravity,' which can drive the universe apart. The remaining 25% has attractive gravitational interaction, but 21% of this is not even normal matter but rather some additional unknown substance called 'dark matter' (Perlmutter 2003).

The vacuum is not merely an empty space. There is a non-zero level of its ground state, the zero point energy, which derives from the quantum mechanical energy states of the harmonic oscillator, as described by Schiff (1949). An example of the related vacuum fluctuations was given by Casimir (1948) who predicted that two metal plates will attract each other when being sufficiently close together. This prediction was first confirmed experimentally by Lamoreaux (1997) for the force between a spherical and a plane metal surface, thereby also giving a proof of the existence of the zero point energy.

In a number of investigations, the author (Lehnert 2009a, b, c, 2010) has recently proposed the zero point energy and a corresponding 'gas' of zero point energy photons to be a basis for the origin of dark energy and dark matter. In this approach, a spherical cloud of such a gas has a pressure gradient and a related force representing the effect of dark energy, and a gravitating mass due to Einstein's mass–energy relation representing dark matter. This paper mainly concerns dark energy

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and dark matter on the largest cosmical scales, leaving open discussions on smaller scales such as those of galaxies.

A systematic review is given here on this latter approach, starting from astronomical observations in Sec. 2, followed by a discussion on the zero point energy concept in Sec. 3 and of the force balance of a corresponding photon gas in Sec. 4, and finally ending with some fundamental results of the present theory in Secs. 5 and 6.

2. Astronomical observations

At present, it is often stated that observations indicate the universe just to be about flat on a scale of $R_0 = 10^{26}$ m, which is the radius of its observable parts (Linde 1994), but the radius of the universe could be larger. With Hubble's discovery and the Big Bang model, a finite and growing radius can also form the basis of an expansion until the present time. Conventional wisdom says the universe is infinite, but it could be finite, merely giving the illusion of infinity (Luminet et al. 1999). On the other hand, a closed finite universe of curved space is certainly attractive from the conceptual point of view, but does not become reconcilable with an observed nearly flat geometry. The idea of a finite and flat universe then runs into its own obstacle of the apparent need for a 'cut-off' at an edge having a radius larger than R_0. An alternative concept would consist of an almost flat universe at dimensions up to R_0, combined with closed geometry on a scale being considerably larger than R_0.

These questions have not been settled, and they will not be further discussed here where the analysis is restricted to the observable parts of a nearly flat universe, on the scale of the radius R_0.

The method of surveying space-time with supernovae has become accurate enough even to measure the rate of change in the cosmic expansion. The acceleration of the radius R_0 can be determined from the measured redshift, the relative intensity of light, and the relative distance. Here we consider very distant supernovae, near the radius of visibility where the redshift $z = \Delta\lambda/\lambda \cong 1$, $\Delta\lambda$ is the shift in wavelength, and λ stands for the wavelength of the light emitted by the supernova at its position. For a given intensity of light, the diagrams of the observations yield a redshift $z_a \cong 0.8$ of an accelerated expansion instead of $z_o \cong 1$ for a constant one (Hogan et al. 1999; Perlmutter 2003). This shows that the high-redshift supernovae are fainter than would be expected for a constant expansion (Perlmutter 2003). The deviation of the redshift due to this acceleration thus corresponds to an additional increment $\Delta v = (z_o - z_a)\, c$ in velocity where c is the velocity of light. With a linear scale of $R_0/2$ relative to the universe of today, the corresponding time of passage becomes $\Delta t \cong R_0/2c$. This yields an acceleration of about $\Delta v/\Delta t \cong 4 \times 10^{-10}$ m s^{-2} for the radius R_0.

A candidate to explain the effect of dark energy and the associated acceleration is the vacuum energy, which has commonly been considered to be mathematically equivalent to the cosmological constant introduced by Einstein in 1917. However, it appears to be a remarkable and implausible coincidence that the mass density, just at the present epoch, is within a factor of two of the vacuum energy density. This would need some kind of accelerating dark energy that, unlike the cosmological constant, does not become constant as stressed by Perlmutter (2003). In addition to this crucial coincidence problem, there are considerable difficulties with the conventional interpretation of the frequency distribution of the zero point vacuum energy. Thus, the standard model of particle physics has no place for a vacuum density of the modest magnitude required by astronomical data, because simple estimates predict a vacuum energy being 10^{120} times greater. We shall later in this paper return to this important point.

The interpretation of the astronomical observations in terms of the present approach is based on and limited to the following points:

- The observable parts of the universe have a nearly flat geometry. In a first approximation, this supports a simple Euclidian treatment of the *average* behavior of an expanding universe on *scales* of the order of the radius R_0, i.e. without the introduction of the curved space effects of General Relativity.
- This does not exclude General Relativity to play an essential role at scales larger than R_0 as well as on local much smaller scales such as those of gravitational lensing and dark matter structures within galaxies.

3. The concept of zero point energy

Due to the Casimir force and its experimental confirmation, there is little doubt about the physical existence of the zero point energy. However, the so far obtained results on this force only reveal the existence of the low-frequency part of the zero point energy spectrum and do not give any information on the high-frequency part. This section begins with a reconsideration of the conventional theory on the zero point energy and related shortcomings, and ends with a revised version of its frequency spectrum.

3.1. Conventional theory

It is known from quantum mechanics that the energy of a linear harmonic oscillator with the frequency ν only assumes the values given by Terletskii (1971), Milonni (1994), and Loudon (2000), that is

$$E_k = h\nu\left(k + \frac{1}{2}\right), \qquad k = 0, 1, 2\ldots . \tag{1}$$

Utilizing the partition function and the Gibbs–Helmholtz equation, the mean energy of the ensemble

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of oscillators of all k-values, also including $k = 0$, then becomes

$$\bar{E} = \frac{1}{2}hv + \frac{hv}{\exp(hv/\bar{k}T) - 1} \tag{2}$$

according to the conventional deductions (Terletskii 1971). Here, $\bar{k}T$ is the mean energy of a classical oscillator in thermal equilibrium at the temperature T, and $\bar{k}$ stands for the Boltzmann constant.

The number of virtual field oscillators per unit volume with frequencies in the range $(v, v + dv)$ further becomes

$$dn(v) = (8\pi/c^3)v^2 dv. \tag{3}$$

On the average, the oscillators would then have the energy density

$$du(v) = \bar{E}dn(v) = (8\pi\bar{E}/c^3)v^2 dv \tag{4}$$

in the same range. The total energy density then becomes

$$u = u_0 + u_p, \tag{5}$$

where

$$u_0 = \int_0^\infty (4\pi h/c^3)v^3 dv \tag{6}$$

and

$$u_p = \int_0^\infty (8\pi h/c^3)\frac{v^3}{\exp(hv/\bar{k}T) - 1}dv. \tag{7}$$

Here, u_P is the finite and well-known contribution from Planck's radiation law. The contribution u_0 related to the zero point energy and obtained from conventional theory is on the other hand infinite. To obtain a finite total zero point energy, the doubtful proposal has sometimes been made to truncate the integral (6) at cutoff frequencies corresponding either to the Planck length or to an arbitrary high energy of 100 GeV, say. As compared to the magnitude of astrophysical data, this still leads to an excessive vacuum energy density being about 10^{120} or 10^{55} times greater than that which is expected.

Several investigators such as Riess and Turner (2004) as well as Heitler (1954) have thrown doubt upon the conventional theory on the vacuum energy and its related frequency distribution. Here, the following points indicate that some fundamental parts of the same theory are lacking:

- In the conventional analysis, the probability that an oscillator is excited to its kth state is given by the Boltzmann factor. In this factor, however, the zero point energy cancels and disappears when expression (1) is substituted into the deductions (Loudon 2000).
- The energy values, which an oscillator can assume at given frequencies, are determined by expression (1), whereas expression (2) represents the mean energy values, which an oscillator adopts in thermal equilibrium. Here, $\bar{E}$ differs from E_k for $k \geqslant 1$ of the Planck radiation part, which adapts itself to a distribution being in thermodynamic equilibrium at a finite temperature T. For the zero point energy part $k = 0$, however, conventional theory yields $\bar{E}_0 = E_0 = \frac{1}{2}hv$. This would correspond to the same probability of unity for oscillators of all frequencies v, and it evidently leads to the unphysical result of an infinite energy density u_0.
- The shortcoming of an infinite u_0 also becomes obvious when making a comparison with Planck's law, as based on a finite mean energy $\bar{k}T$ for all frequencies v. Here, $\bar{k}T$ is a given and independent parameter standing for a finite total energy of the ensemble. In the case of the zero point energy, a corresponding independent parameter is missing. In other words, this implies that the conventional deductions on the zero point energy part are *underdetermined* and not self-consistent, thus not being clearly defined for a closed system of equations. There is then no specified total energy of the corresponding ensemble of oscillators, which can form the basis of a complete statistical analysis.

3.2. Revised theory

To avoid complications and interference with the conventional deductions of Planck's radiation law, we now limit ourselves to the case $k = 0$ at zero temperature $T = 0$. There then remains an ensemble of zero point energy photons, each having the energy E_0 of (1). The number of possible states of oscillations is as before given by (3). To satisfy the condition of a fully determined statistical system, we now have to base the analysis on the concept of a given and finite total energy of the oscillators per unit volume, i.e. on an average energy,

$$\bar{E}_0 = \frac{1}{2}h\bar{v} \tag{8}$$

per zero point energy photon. Here $\bar{v}$ is the corresponding average frequency. Following Kennard (1938), the probability in statistical equilibrium of any state of energy $E_0(v)$ per member of the ensemble then becomes proportional to the Boltzmann factor

$$P_B = \exp(-E_0/\bar{E}_0) = \exp(-v/\bar{v}). \tag{9}$$

The revised form of the energy density (4) then becomes

$$du(v) = (4\pi h/c^3)v^3 \exp(-v/\bar{v})dv. \tag{10}$$

This results in a finite total and local energy density

$$u = 24\pi h\bar{v}^4/c^3. \tag{11}$$

Here, the average frequency $\bar{v}$ is so far an undetermined quantity of the state $k = 0$, in analogy with the arbitrary mean energy $\bar{k}T$ of the states $k \geqslant 1$.

3.3. The Casimir force

The Casimir force (Casimir 1948) between two conducting plates at the separation distance a is now reconsidered. Only the high-frequency modes having a wavelength smaller than or equal to $2a$ will squeeze in between the plates for frequencies larger than or equal to $\hat{v} = c/2a$. With the isotropic pressure $p = u/3$, the

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corresponding Casimir driving pressure difference in the case of a finite mean energy $\bar{E}_0$ becomes:

$$\hat{p} = \frac{1}{3}\int_0^\infty du - \frac{1}{3}\int_{\hat{\nu}}^\infty du = \left(4\pi h/3c^3\right)\int_0^{\hat{\nu}} \nu^3 \exp(-\nu/\bar{\nu})d\nu \tag{12}$$

according to (10). After introduction of the dimensionless frequency ratio $x = \nu/\bar{\nu}$ and the ratio $\hat{x} = \hat{\nu}/\bar{\nu}$ between the low-frequency limit $\hat{\nu}$ and the average frequency $\bar{\nu}$, the driving pressure reduces through partial integration to

$$\hat{p} = p_0 \Pi(\hat{x}), \qquad p_0 = 8\pi h\bar{\nu}^4/c^3, \tag{13}$$

where

$$\Pi(\hat{x}) = 1 - \left(1 + \hat{x} + \frac{1}{2}\hat{x}^2 + \frac{1}{6}\hat{x}^3\right)\exp(-\hat{x}) \tag{14}$$

for which $\Pi(0) = 0$ and $\Pi(\infty) = 1$.

For $\hat{x} \ll 1$, i.e. when the low-frequency limit is much smaller than the average frequency, the normalized pressure (14) can be written as

$$\Pi(\hat{x}) \cong \Pi^*(\hat{x})\left(1 - \frac{4}{5}\hat{x} + \frac{1}{3}\hat{x}^2 - \ldots\right), \tag{15}$$

where

$$\Pi^*(\hat{x}) = \hat{x}^4/24. \tag{16}$$

Then,

$$\hat{p}^* = p_0\Pi^*(\hat{x}) = \pi h\bar{\nu}^4\hat{x}^4/3c^3 = \pi hc/48a^4 \tag{17}$$

is the Casimir pressure limit at small $\hat{x}$. With $\bar{\lambda} = c/2\bar{\nu}$ close to this limit, we can also write

$$\hat{p} = \left(\frac{\pi hc}{48a^4}\right)\left[1 - \frac{4}{5}\frac{\bar{\lambda}}{a} + \frac{1}{3}\left(\frac{\bar{\lambda}}{a}\right)^2 - \ldots\right]. \tag{18}$$

The Casimir pressure difference thus becomes proportional to $1/a^4$ for small $\hat{x}$, where $\Pi \cong \Pi^*$ is in agreement with the analysis by Casimir (1948). The value of $\hat{p}^*$ according to (17) is exactly 10 times larger than the value deduced by Casimir, this difference so far not being explained. In the deductions by Casimir, the two terms in the second member of (12) were treated separately, and in terms of an introduced auxiliary function in combination with an expansion due to the Euler–Maclaurin formula. The range of small $\hat{x}$ applies to the original deductions by Casimir, and this range is also that first being within reach in so far made experiments.

For increasing $\hat{x}$, up to large values, the normalized pressure $\Pi(\hat{x})$ of (14) deviates more and more from the original value $\Pi^*(\hat{x})$ of an infinite $\bar{\nu}$, thereby ending up at the saturation value $\Pi(\hat{x} \to \infty) = 1$ corresponding to $\hat{p} \to p_0$ in (8). Refined experiments on this saturation tendency in the range $\hat{x} \geqslant 1$ of very small plate distances are desirable, in particular to estimate the order of magnitude of the mean frequency $\bar{\nu}$ and its related mean energy of the zero point energy photons, and the local energy density of (11).

However, such experiments will not turn out to become easy, and their interpretation is also complicated by the finite conductivity σ and related mean skin depth $\delta_S = \left(4\pi/\mu_0\sigma\bar{\nu}\right)^{1/2}$ of the plates in the required range of frequencies. This results in an effective plate distance $a_{\text{eff}} = a + 2\delta_S$. Measurements in the case of zero plate distance, $a = 0$, could on the other hand provide a way of circumambulating the extreme requirements on technical precision in the range of small and variable a. In this limit, $\bar{\nu} = c/4\delta_S$, and the Casimir pressure $\hat{p}$ of (13) then becomes a function of the plate conductivity σ only. If the predictions of the present approach come out to be correct, measurements of $\hat{p}$ as a function of σ for a set of different plate metals, such as Ag, Cu, Au, Al, Mg, Mo, W, Zn, and Ni, should become reconcilable with the same value of the mean frequency $\bar{\nu}$ of the zero point energy spectrum. If no other forces will interfere with such an experiment, it should thus lead to a determination of the maximum available Casimir force, probably with less strong requirements on plate flatness.

It should be observed that the combination of (13), (14), and (17) and the expression for the skin depth δ_S as a function of the average frequency $\bar{\nu}$ yields a Casimir pressure $\hat{p}$, which for zero plate distance a increases monotonically and strongly with $\bar{\nu}$. Here, an infinite value of $\bar{\nu}$ corresponds to the infinite energy density u_0 of the conventional theory given by (6). This leads to an infinite Casimir pressure $\hat{p}$ for conducting plates at zero distance a, in contradiction with experimental evidence, and could be taken as an argument for a finite and non-zero frequency $\bar{\nu}$.

The interpretation of the Casimir force due to (12)–(18) is sometimes encumbered with a crude misconception, in stating that a positive pressure does not cause expansion but rather gravitates, and that the Casimir force should be considered as attractive. In reality, the latter force is due to a deficit of pressure in the spacing of the plates, by which the expanding larger pressure on their external sides tends to push the plates together. The experimentally verified Casimir force is thus a clear demonstration of a real physical pressure being exerted by the zero point energy photons.

4. A gas cloud of zero point energy photons

With the local zero point energy density u defined by (11), it is now possible to proceed to the study of a corresponding gas cloud of photons, in the simple case of spherical symmetry. As based on recent observations, an expanding state of a nearly flat universe is considered, with a simple Euclidian model in a first approximation. We also neglect the small presence of normal matter, restricting the analysis to a pure zero point energy photon gas.

With the radial coordinate r of a spherical frame, the density u becomes a spatial function of r only. A radially

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outward directed pressure force arises, as given by

$$f_p = -\frac{dp}{dr} = -\frac{1}{3}\frac{du}{dr}. \tag{19}$$

This has the character of a radially outward directed 'antigravity' force, and plays here the role of dark energy. With an average total mass $\bar{E}_0/c^2$ of each photon, the integrated mass of the photon gas within the radius r becomes

$$M(r) = \int_0^r 4\pi r^2 \left(u/c^2\right) dr = (4\pi/c^2)\int_0^r r^2 u dr. \tag{20}$$

This leads to a radially inward directed gravitational force

$$f_g = -GMu/c^2 r^2, \tag{21}$$

where $G = 6.673 \times 10^{-11}$ m^3 kg^{-1} s^{-2} is the Newtonian constant of gravitation. The force (21) plays here the role of that due to dark matter.

4.1. A steady equilibrium state

A steady equilibrium state is defined by $f_p + f_g = 0$, which results in

$$-\frac{1}{3}\frac{du}{dr} = \frac{4\pi G}{c^4 r^2} u \int_0^r r^2 u dr. \tag{22}$$

This equation is normalized by introducing $\rho = r/r_c$, where r_c is a characteristic radius, and $u = u_c U(\rho)$ with u_c as a characteristic photon energy density. Multiplying (22) by r^2/u and taking the derivative with respect to r, this results in

$$\frac{d^2U}{d\rho^2} + \frac{2}{\rho}\frac{dU}{d\rho} - \frac{1}{U}\left(\frac{dU}{d\rho}\right)^2 + 2C_0 U^2 = 0, \tag{23}$$

which relation includes the dimensionless parameter

$$C_0 = 6\pi G u_c r_c^2/c^4 = 3\pi n_c r_c^2 L_p^2/\bar{\lambda}, \qquad L_p = (Gh/c^3)^{1/2}. \tag{24}$$

Here, $n_c = u_c/\bar{E}_0$ is a characteristic photon density, $\bar{\lambda} = c/\bar{\nu}$, and L_p is the Planck length. A particular solution is given by $C_0 = 1$ and $U = 1/\rho^2$ for which

$$u(r) = u_c (r_c/r)^2. \tag{25}$$

The equilibrium condition $C_0 = 1$ corresponds to a characteristic radius

$$r_c = (c^4/6\pi G u_c)^{1/2}. \tag{26}$$

The integrated mass at the distance r further becomes:

$$M(r) = 2c^2 r/3G \tag{27}$$

from combination of relations (20), (25), and (26).

These obtained results are discussed as follows:

- The Planck length $L_p = 4.05 \times 10^{-35}$ m is the smallest basic length appearing in physics. To satisfy the equilibrium condition $C_0 = 1$, (24) indicates that the radius r_c has to be of cosmic dimensions for moderate values of u_c and n_c.
- At the origin $r = 0$, the total mass (27) vanishes. The divergence of the energy density at $r = 0$ can be taken as a remnant of the earliest stage of the Big Bang. Even if each of the forces (19) and (21) also diverges at the origin, the total force vanishes there in equilibrium.
- The mass (27) increases linearly with r to the value $M(R_0) = 2c^2 R_0/3G$ at the radius of the observable universe. This form is analogous to the solution by Einstein (1950) for a steady quasi-Euclidian universe with normal matter, but here it concerns the equivalent mass of the zero point energy photon gas.
- In principle, it is possible also to extend the deductions of (19)–(27) to include a small fraction $f_n = \rho_n/\rho_d$ of normal matter. This becomes straightforward when the density of normal matter is constant per solid angle. Expression (24) is then modified by the inclusion of an additional factor $1 + f_n$ in its numerator.
- The parameter C_0 represents the ratio $|f_g/f_p|$ between the gravitation and pressure forces. Small deviations from equilibrium can in a first approximation be represented by values of C_0 differing slightly from unity. Thus, $C_0 < 1$ corresponds to pressure-dominated accelerated expansion, and $C_0 > 1$ to gravitation-dominated enhanced compression.

4.2. Small deviations from equilibrium

During the later stages of the expansion, the equilibrium solution presented here could provide a starting point also for a simple approach to the related dynamics in the case of small deviations from equilibrium. As a working hypothesis, we further adopt the often accepted view that an equilibrium between the expansion and contraction forces corresponds to a constant expansion rate, whereas a force unbalance leads to an accelerated or retarded expansion.

For this purpose, a volume element of thickness dr near the radius $r = R_0$ is considered. With the local force densities (19) and (21), and neglecting the small influence from normal matter in a first approximation, the total pressure and gravitation forces on the layer become:

$$(dF_p, dF_g) = 4\pi R_0^2 (f_p, f_g)\, dr. \tag{28}$$

From (19), (25), and (26), we then have

$$f_p \cong 2u_c r_c^2/3R_0^3 \cong c^4/9\pi G R_0^3 \tag{29}$$

in the equilibrium state, where the total force on the element is $dF = dF_p + dF_g = 0$.

Small deviations from equilibrium are now represented by a resulting total force

$$dF^* = (\delta - 1)\, dF_p, \tag{30}$$

where $\delta = 1$ corresponds to equilibrium and $\delta \neq 1$ to accelerated or retarded expansion. The mass of the volume element is:

$$dM \cong (2c^2/3G)\, dr \tag{31}$$

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as obtained from (27). The acceleration at the radius $r = R_0$ is then roughly given by

$$dF^*/dM \cong (\delta - 1)\, dF_p/dM = 2(\delta - 1)\, c^2/3R_0. \quad (32)$$

4.3. A comparison between normal and dark matter

It has to be taken into account that the physical features of dark matter could differ in many respects from those of normal matter. In the Big Bang process, the proportions of normal and dark matter may further have been different at an early stage as compared to those at the present one. Thereby, the expanding universe will incorporate ever-increasing volumes of vacuum space, and it may not have to be considered as a closed system in respect to dark energy and dark matter. This can complicate possible efforts to treat the dynamics of the expansion in terms of conventional laws of energy and matter conservation.

5. Applications to the present expanding state

The present theory can at least be applied in a concrete way to two essential features of the observed expanding universe. This concerns the cosmical scale, and the accelerated expansion. These two examples also form tests in support of the theory.

5.1. Cosmical scale at present stage of the universe

With the average density $\rho_n = 10^{-26}$ kg m^{-3} of normal matter in the present universe according to Linde (1994), and with mass contents of 21% and 4% of dark and normal matter, there is an average density $\rho_d \cong 5.3 \times 10^{-26}$ kg m^{-3} of dark matter. Using the present results of an equilibrium in a first approximation, the total amount of dark matter within a radius $r = R_0$ is then given by

$$M(R_0) = \frac{4}{3}\pi R_0^3 \rho_d = 2c^2 R_0/3G \quad (33)$$

in combination with (27). This leads to a corresponding radius

$$R_0 \cong c\,(2\pi G\rho_d)^{-1/2}. \quad (34)$$

With the given value of ρ_d, this yields $R_0 \cong 0.64 \times 10^{26}$m, due to the balance (23) in a quasi-steady state. This value, which is deduced from the present theory, thus rather well agrees with the value $R_0 = 10^{26}$ m estimated by the astronomers. It clearly shows the theory to be reconcilable with cosmical dimensions.

In this connection, it should also be noticed that combination of (11), (25), and (26) yields:

$$u(r) = u(R_0)(R_0/r)^2, \qquad u(R_0) = c^4/6\pi G R_0^2, \quad (35)$$

and

$$\bar{\nu}(r) = \bar{\nu}(R_0)(R_0/r)^{1/2}, \quad \bar{\nu}(R_0) = \left(c^7/144\pi^2 hGR_0^2\right)^{1/4}, \quad (36)$$

where $u(R_0) = 6.4 \times 10^{-10}$J m^{-3} and $\bar{\nu}(R_0) = 0.69 \times 10^{12}$ s^{-1} for $R_0 = 10^{26}$ m.

An open question concerns the position of the Earth within the expanded universe. When being observed from the Earth, the surrounding parts of the universe appear on the average to be rather uniformly distributed over the sky. This could here be taken as an indication that our position is deep inside the cloud of the universe, far from its outer parts at $r = R_0$. When taking $r = r_c$ at the position of the Earth, we would then have to assume that $r_c = R_0$. The values of $u(r_c)$ and $\bar{\nu}(r_c)$ of (35) and (36) could then become considerably larger than those at the radius R_0.

It is also noticed that the condition $C_0 = 1$ of (24) for a steady balance between the pressure gradient and the gravity can be satisfied for very small radii r_c when there are very large energy densities u_c, such as at an early stage of the Big Bang.

5.2. The accelerated expansion at the present stage

At the present stage of the accelerated expansion, the radius of the outermost observable parts of the universe is found to expand at an acceleration of about 4×10^{-10} m s^{-2} as given from astronomical data in Sec. 2. This refers to a situation where 75% of the mass–energy content is due to the expansive force of dark energy. It corresponds to the value $\delta = 5/3$ in (30). With this value and $R_0 = 10^{26}$m, the acceleration due to (32) becomes $dF^*/dM \cong 4 \times 10^{-10}$ m s^{-2}. This value thus agrees well with that from astronomical observations, and can be taken as a further support of the theory.

Combination of (32) and (34) also provides an approximate expression for the acceleration, as given by

$$dF^*/dM \cong \frac{2}{3}(\delta - 1)\, c(2\pi G\rho_n/f_n)^{-1/2} \quad (37)$$

in terms of the normal mass density ρ_n, the ratio $f_n = \rho_n/\rho_d$, and the force unbalance factor $\delta - 1$ only.

5.3. A possible effect of mass polarity

In this connection, we finally turn to a feature that could be related to the expanding universe, but not directly to the zero point energy. Thus, in relativistic mechanics, a particle at rest with the mass m_0 and the energy E obeys the relation

$$E^2 = m_0^2 c^4. \quad (38)$$

Here, we investigate the possible interpretation of this relation with E as a positive quantity and the mass

$$m_0 = \pm E/c^2 \quad (39)$$

taking positive as well as negative values. This result is applied to a generalized law of gravitation by the author (Lehnert 2011), as shortly being summarized in this paper. The strength of a steady gravitational field is:

$$\mathbf{g} = -\nabla\phi \quad (40)$$

as being due to a potential ϕ. Its source is a mass density ρ related to g by

$$-\mathrm{div}\mathbf{g} = 4\pi G\rho = \nabla^2\phi, \quad (41)$$

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where G is the constant of gravitation. The associated force density becomes

$$\mathbf{f} = \rho \mathbf{g}. \tag{42}$$

A spherically symmetric configuration is now considered with the potential

$$\phi(r) = \phi_0 (r/r_0)^{-\gamma}, \qquad \gamma > 0, \tag{43}$$

and where the mass density ρ is zero within an inner hollow region $0 \leqslant r \leqslant r_i$ and $\rho \neq 0$ in the outer region $r > r_i$. For $\gamma \geqslant 2$, the total integrated mass inside the radius r then becomes

$$M(r) = \frac{1}{G} \left(\phi_0 r_0^{\gamma}\right) \gamma \left(r_i^{-\gamma+1} - r^{-\gamma+1}\right). \tag{44}$$

The strength $\mathrm{g} = (g, 0, 0)$ is further given by

$$g(r) = -\left(\phi_0 r_0^{\gamma}\right) \frac{\gamma}{r^2} \left(r_i^{-\gamma+1} - r^{-\gamma+1}\right). \tag{45}$$

A revised renormalization procedure is now applied, aiming at a point mass concept. For this purpose, we 'shrink' the parameters $\phi_0 r_0^{\gamma}$ and $r_i^{\gamma-1}$ in such a way that

$$\phi_0 r_0^{\gamma} = c_{\phi r} \cdot \varepsilon, \qquad r_i^{\gamma-1} = c_i \cdot \varepsilon, \qquad 0 < \varepsilon \ll 1, \tag{46}$$

where ε is a smallness parameter and $c_{\phi r}$ and c_i are constants. We further rewrite expression (44) as

$$M(r) = M_0 \left[1 - (r_i/r)^{\gamma-1}\right], \qquad M_0 = \gamma c_{\phi r}/G c_i. \tag{47}$$

In the limit $\varepsilon \to 0$ and $r_i \to 0$, there is then a point mass M_0 at the origin $r = 0$, which generates a field strength

$$g(r) = -GM_0/r^2. \tag{48}$$

With another point mass M_1 at the distance r, a mutual force

$$F_{01} = M_1 g(r) = -GM_0 M_1/r^2 \tag{49}$$

then arises in the radial direction, where $F_{01} < 0$ represents attraction between M_0 and M_1.

A generalization of the gravitation law (49) now implies that the mass of (39) and the constants ϕ_0, $c_{\phi r}$, M_0, and M_1 all are allowed to have either sign. Denoting this with subscripts $(_+)$ and $(_-)$, the forces become:

$$M_{1+} g_+ = -GM_{0+} M_{1+}/r^2 = M_{1-} g_- = -GM_{0-} M_{1-}/r^2, \tag{50}$$

$$M_{1+} g_- = -GM_{0-} M_{1+}/r^2 = M_{1-} g_+ = GM_{0-} M_{1+}/r^2. \tag{51}$$

These gravitation laws are fully symmetric, in which masses of equal polarity attract each other, and masses of opposite polarity repel each other.

With an adopted positive energy E, the gravitational mass $\pm E/c^2$ can thus have either sign. As a further consequence, the inertial mass then has to take the positive value E/c^2 to be consistent with observations of the electron and positron orbits during pair formation. This is also plausible from the physical point of view, where a body of the energy content E gains total energy when being accelerated.

The obtained result can be used in a speculation about the formation of matter and antimatter at an early stage of the Big Bang. If there are equal amounts of matter and antimatter from the beginning, and if matter and antimatter have opposite mass polarities, then there is a repulsive gravitation force, which tends to separate antimatter from matter. This force can dominate the momentum balance in an electrically quasi-neutral 'ambiplasma.' Thereby matter and antimatter can be 'pushed' into two separate universes, and we just happen to live in one of these. The repulsive force in this process is further expected to generate macroscopic motions, also to appear at the present state of the expanding universe. Such motions may provide a possible and alternative explanation of the observed 'dark flow' discussed by Kashlinsky et al. (2011) among others.

6. Fundamental new results of present approach

The fundamental new results, which have come out of the present approach, can shortly be summarized as follows:

- Astronomical observations require dark energy and dark matter not to be associated with emitted radiation. This requirement is fulfilled by the present theory, which is based on the zero point energy concept.
- There is a remarkable coincidence that the mass density just at the present epoch nearly matches the vacuum energy density. This crucial problem is not resolved by a fixed cosmological constant. It can, however, be clarified in terms of the present approach in which dark energy and dark matter are variable concepts originating from the same photon gas model.
- The present approach is reconcilable with the largest observable cosmical dimension. This reveals itself in the appearance of the Planck length in the theory, and in a deduced order of magnitude of the radius of the observable universe, which agrees with astronomical data.
- The deduced acceleration of the expansion agrees with present astronomical observations. The flexibility of the present theory in respect to the coincidence problem also makes possible other rates of accelerated or even retarded expansion during various phases of development of the universe.

7. Summary and conclusions

In this paper, an attempt is made to explain the concepts of dark energy and dark matter in terms of the zero point energy of the vacuum state. The analysis is mainly

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limited to the later stages of the observable parts of a nearly flat universe, on its largest geometrical scale.

The theory is based on a revised formulation of the spectral distribution of the zero point energy, thereby abandoning the underdetermined conventional form, which leads to the unphysical result of an infinite total energy density. This revised formulation includes an ensemble of oscillators having a final total energy density, thereby constituting a closed system for a defined statistical analysis. As based on this formulation, the equilibrium and dynamics are studied of a spherical cloud of zero point energy photons. The radially outward directed 'antigravitational' force due to the pressure gradient of the cloud then represents dark energy, and the radially inward directed gravitational force due to the energy density and equivalent mass of the cloud represents dark matter.

There are four new features and results of the present approach, which support its relevance. First, the zero point energy becomes reconcilable with the lack of emitted radiation, as expected for dark energy and dark matter. Second, the crucial coincidence problem of equal orders of magnitude of mass density and vacuum energy density is resolved by the present photon gas model in which dark energy and dark matter become variable concepts of the same origin. Third, the present approach becomes reconcilable with the largest cosmical dimensions, and it leads to a deduced value of the radius of the observable universe being of the same order as that estimated from astronomical data. Fourth, the deduced acceleration of the present expansion also agrees with astronomical data.

It has finally to be observed that the solution (25) of the steady-state balance equation (23) also applies to a freely expanding flat universe in which the radius r_c increases linearly with time t, and the energy density u_c decreases as t^{-2}. Then $u_c r_c^2$ remains constant per solid angle at any position r.

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